PROPERTY RIGHTS AND LONG-RUN CAPITAL

Julio Dávila

Center for Operations Research and Econometrics
Université c. de Louvain
Belgium

Abstract. The fact that some private capital eventually slides into the public domain, in which it keeps being productive without having to be remunerated —*e.g.* R+D investments as patents expire, or institutional framework and public infrastructures funded by taxed household savings and income— inefficiently distorts downwards capital accumulation. This may have dire consequences on households' level of consumption: in a neoclassical infinitely-live agents setup with *constant returns to scale* on aggregate capital and labor —as opposed to the increasing returns generated by aggregate capital in Romer (1986)— the planner's steady state consumption is almost 39% higher than the market one, for standard empirically supported values for the share of capital income, the consumption of physical capital, and discounting. This is established also for overlapping generations economies, where the missed aggregate consumption is a more telling because of less astonishing but still whopping 6,7%. I provide next a tax and subsidy balanced policy able to decentralise the planner's steady state without resorting to the (impracticable) extension of property rights otherwise needed to address the problem. It consists of (i) subsidising the rental rate of capital by an amount equal to the depreciation/obsolescence rate of the capital sliding into the public domain, and (ii) taxing households' borrowing against future dividends.

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1. Introduction

Each generation passes on subsequent generations the results of its achievements, both physical (properties, infrastructures, facilities, ...) and intangible (technology, know-how, institutions, organisations, culture, ...),[1] the creation of all of them having required the use of resources and being productive, and hence being capital in a broad sense. Some of this capital is passed on through the trade or bequest of individual property rights —this is notably the case for real state and production facilities, but also for intellectual property rights to the extent they have not expired yet. But also some of it just slides eventually into the public domain —this is certainly the case for expired intellectual property rights,[2] and for institutional organisational schemes too (*e.g.* judiciary, urban planning,...) for which resources were required; but it is also true for any physical infrastructures publicly built out of taxed private capital or savings, *etc.*[3] This paper explores thus the consequences for capital accumulation and consumption of at least some capital eventually escaping individual property rights.

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[1]Some of them actually fall into both categories like, for instance, cities, metropolitan areas or regions, with their combined nature of public infrastructures and organisations.

[2]Property rights over the technologies resulting from R+D investments are temporarily protected by law to allow the investor to get a return from the investment and, hence, supposedly to incentivise growth-enhancing R+D activities. There is a heated debate about whether the current patent system implementing those property rights is actually the most adequate for spurring innovation. For instance, Boldrin and Levine (2013) point that the evidence shows no correlation between the number of patents and productivity, and highlight that the rent-seeking nature of *patenting aims rather at preventing further innovation* from competitors, which typically builds on previous innovations. On the other hand, Gould and Gruben (1996) find that intellectual property protection is an important determinant of growth, although this seems to hinge on the openness of the country to international competition, without which it can be detrimental to growth. Also, in a Romer-style endogenous growth model Saint-Paul (2003) argues the crowding out effects of free blueprints on proprietary innovation and, hence, its negative impact on growth and welfare. At any rate, besides the issue of what drives innovation and what incentives are at play, there is the fact that technology is the result of investment, and is hence capital, but one whose property rights are protected only temporarily and thus eventually falls into the public domain.

[3]Strictly speaking, some commonly held physical capital is actually subject to property rights of state institutions —the *res publicae* and *res universitatis* of municipalities in Roman law, as opposed to things not subject to property rights at all by their nature (*res communes*) or by lack of appropriation (*res nullius*). Notwithstanding, for all purposes, I will consider it to be freely available in the public domain —at least for residents or citizens, depending on the case at hand— since the relevant feature characterising it is the fact of not being subject to *individual* property rights.

I show in this paper that the progressive drift of proprietary investments into the public domain distorts capital accumulation away from the optimal level that would be chosen by a utilitarian planner unconstrained by property rights. More specifically, the model shows analytically the essentials of why the gradual drift of capital into the public domain prevents the markets to deliver, under laissez-faire, the optimal level of capital accumulation.

The mechanism behind the result is simple enough. Some of the savings that are lent to firms as capital by households are used to create ways to increase productivity that will eventually become public domain[4] —*e.g.* R+D private investments, given the time limits to patents and copyrights, or taxed savings and income funding the intangible organisational and legal framework in which the economy operates, as well as public infrastructures, but these are far from being the only examples.[5] As a result of this, firms effectively operate using not only the capital they borrow, but also the capital coming from prior private investments that has fallen into the public domain. Since only capital on which property rights can be enforced is remunerated,[6] savers do not take into account the impact that their loans to firms have on the future capital in the public domain, and hence on the future productivity of factors and on output through this channel.

The idea that firms may operate with more capital (intangible or not) than the one they have to remunerate might remind of the mechanism at the heart of the contribution made in Romer (1986). Notwithstanding, the mechanism introduced in this paper is distinct from the one underpinning Romer (1986). Indeed, increasing returns to scale are —among others elements— a *key* ingredient to the results in Romer (1986), while the misallocation of resources that depresses capital accumu-

[4]Some savings are even deliberately privately invested in capital intended, from the start, to be in the public domain, as it is the case for open source and shareware software. This paper is nonetheless not about such instances of capital deliberately accumulated to be freely available, but rather on the consequences of proprietary capital eventually sliding into the public domain.

[5]A related problem is that of firms' investment in the human capital of their employees. Such investments can be substantial, but they stop being "proprietary" for the firm if the employees quit for another job (since embodied in them). In a sense, such human capital investments by firms, while being proprietary to the employees, have the potentiality of becoming a *de facto* (excludable) public good provided by each firm to the industry.

[6]This is not to say that the productivity of public domain capital is not appropriated by anyone. It is indeed: it accrues firms' profits and eventually feeds into firms' owners wealth as distributed dividends. In effect, with proprietary capital eventually sliding into the public domain even constant returns to scale firms make profits. Nonetheless, even if these profits are distributed to households, the latter will — since they do not manage the firms themselves— still fail to internalise the effect in their saving decision.

lation in this paper takes place even with a neoclassical, constant returns to scale technology. In a nutshell, Romer (1986) is driven by a technological assumption that I don't need here, while the results of this paper are driven by vanishing property rights —and the impossibility to restore them, as it will be seen below. Moreover —although admittedly not as much a definitive conceptual difference as the previous one— the positive externality that investments have on everybody else's productivity is contemporaneous at every period in Romer (1986), while in the case of capital falling into the public domain the externality exerted by the latter takes place across time and is therefore intertemporal. But there is still another crucial difference with the mechanism in Romer (1986) explained next.

By sliding into the public domain, the productivity of the capital doing so is not remunerated to its original investors, but is rather fed into the profits of the firms operating with it for free.[7] Even in an aggregate model, where the representative household is both the lender to the firm —so that it receives the returns to privately held capital— and the owner of the firm —so that it receives the distributed profits too— and therefore receives the entirety of the productivity of the capital used by the firm (whether privately or publicly held) the channel through which this productivity is received matters for the saving decision of the household. Namely, the productivity of capital in the public domain does not incentivise savings, while that of capital privately held does. This differentiated impact would be even more obvious with heterogeneous agents of which some are lenders and others owners, or all are both but to different extents. Thus, in another crucial difference with Romer (1986), since technology is in the latter linearly homogeneous *"with respect to the factors that receive compensation"* firms do not make profits in Romer (1986) and therefore this differentiated impact of the remuneration of capital —whether as return to loans or distributed dividends— on households' saving decision cannot be captured by the framework in Romer (1986).

As a consequence, private investments differ from those that a planner able to take into account the effect of public domain capital would choose. Specifically, in the case of infinitely-lived agents I explicitly show below that the market accumulates too little capital —leading to miss an almost 39% higher (!) steady state consumption that, for standard empirically supported values for the parameters, the planner would deliver. Interestingly enough, in the overlapping generations case this is in

[7]As it will be shown below, while with free entry positive profits allow firms to enter the market driving each firms' profits down to zero, aggregate profits —which amount to the unremunerated productivity of the capital in the public domain— will remain constant at a positive level and will be distributed as dividends to the owners of the firms, the households.

general only seen indirectly through the subsidy to capital returns required by the policy decentralising the planner's allocation. Nonetheless, with additional assumptions on the production and utility functions the gap can be explicitly computed in the overlapping generations setup too, pointing to the market missing a possibly more realistic but still significant 6.7% higher steady state aggregate consumption delivered by the planner.

In order to address the problem, I provide a policy allowing to steer decentralised choices towards the planner's allocation of choice. Since at the heart of the problem there lays an expiration of property rights,[8] it might seem that a simple all-encompassing extension of property rights would be enough. Nonetheless, since this is clearly impracticable —some of this capital cannot be appropriated (institutions, organisations,...) or is not advisable to be so (because, for instance, of the perverse incentives on innovation of extending indefinitely intellectual property rights pointed by the literature, see Boldrin and Levine (2013))[9]— it is important to devise an implementable policy that avoids running into generating additional inefficiencies. For that purpose, the policy put forward in this paper requires instead (i) to subsidise the rental rate of capital by an amount equal to the depreciation/obsolescence factor of the capital sliding into the public domain, (ii) to tax households' borrowing against future dividends. While the first element of this policy —i.e. the subsidisation of capital returns— may be expected (although probably not the exact rate at which this needs to be done), its second element —i.e. taxing households' reliance on credit against future dividends distributed by firms— only makes full sense once one understands the differentiated impact on the incentives to save of, on the one hand, the return to privately held capital and, on the other hand, the dividends received from the productivity of the unremunerated capital in the public domain.

In what follows, Section 2 presents the model with two variants for its demographics —infinitely-lived agents and overlapping generations respectively. Section 3, addressing the issue first in the infinitely-lived agents economy, establishes that the market necessarily under-accumulates capital due to part of it falling into the public domain. It provides also an assessment of the size of the inefficiency for standard, empirically supported parameter values of the model. Section 4 addresses then the

[8] Of which there is none in Romer (1986), which is driven by increasing returns.

[9] Many considerations about the public interest for patents to expire, to be entirely phased out, or even to invest in freely available technology —for instance, developing open source software as means to invest in network building— are not addressed in this paper (for the case against patents see Boldrin and Levine (2013); for an overview of the economics of open source see Lerner, Josh and Tirole (2005)). While extremely interesting, they address a different point from the one being made here.

question for an overlapping generations setup —along with a new assessment of the size of the inefficiency— which provides additional insight on the way the externality operates and allows to provide a policy decentralising the planner's choice as a market outcome. This is done through a subsidy on capital returns, and a tax on households' borrowing against future dividends. Section 5 concludes.

2. The model

Consider an economy in which part of savings or capital becomes, after some period of time, non-proprietary and falls into the public domain —*e.g.* the part of investments devoted to R+D that leads to proprietary technology for a limited amount of time only, or the one that is taxed and used to fund the institutional and legal framework or some public infrastructure. Specifically, let N_t be the (possibly constant) population at t, and k_t be the amount of savings lent to firms by the representative household at t, so that the total investment at t used in production at $t+1$ is $N_t k_t$. Without loss of generality, assume that a fraction α of investments becomes public domain after one period, the complement fraction $1-\alpha$ remains proprietary and depreciates each period by a factor $\delta \in (0,1)$. Public domain capital, on the other hand, depreciates or obsolesces each period by a factor $\phi \in (0,1)$, so that only a fraction ϕ of it remains productive after each period,[10] and is not reversible into consumption good.

Total capital available at any period t is therefore

$$K_t = N_{t-1}k_{t-1} + \sum_{i=2}^{+\infty}[(1-\alpha)\delta^{i-1} + \alpha\phi^{i-1}]N_{t-i}k_{t-i} \tag{1}$$

Notwithstanding, firms need to remunerate only

(1) the investment $N_{t-1}k_{t-1}$from savings made at $t-1$
(2) and proprietary older capital $(1-\alpha)\delta^{i-1}N_{t-i}k_{t-i}$, for $i = 2, 3, \dots$ resulting from previous investments

that is to say

$$N_{t-1}k_{t-1} + (1-\alpha)\sum_{i=2}^{+\infty}\delta^{i-1}N_{t-i}k_{t-i} \tag{2}$$

[10]Note that the depreciation factor of proprietary capital δ and public domain capital ϕ need not be the same.

but, crucially, not capital resulting from prior investments that has already slid into the public domain, i.e.

$$\alpha \sum_{i=2}^{+\infty} \phi^{i-1} N_{t-i} k_{t-i}. \tag{3}$$

For the sake of simplifying expressions and without loss of generality (see footnote 12 below), I will consider below —with no consequence for the main point of the paper— the extreme case in which $\alpha = 1$, so that capital available at t

$$K_t = N_{t-1} k_{t-1} + \sum_{i=2}^{+\infty} \phi^{i-1} N_{t-i} k_{t-i} \tag{4}$$

consists of newly saved capital and all the previously saved capital that has become public domain at previous periods —*i.e.* the sum in the second term in the right-hand side above.

The production function $F(K_t, N_t)$ is neoclassical, *i.e.* returns to scale are constant in labor and available capital —both proprietary and in the public domain— as opposed to the overall increasing returns to scale that are the keystone of the setup considered in Romer (1986).

Finally, as for the demographics, I will consider next both an infinitely-lived agents and an overlapping generations setup. Specifically, a normalised unit of labor is supplied inelastically by each household

(1) when young, if the economy consists of 2-period-lived overlapping generations, and the cohort sizes change each period by a constant factor n, so that, for all $i = 1, 2, \ldots$

$$N_t = n^i N_{t-i} \tag{5}$$

(2) each period, if agents are infinitely-lived, with a constant population (*i.e.* $n = 1$) normalised to 1, so that $N_t = 1$, for all t.

3. Market under-accumulation: the infinitely-lived agents economy case

3.1 The planner's allocation for the infinitely-lived agents economy.

A planner is not constrained by property rights. It just aims at maximising the increasing and concave utility u —discounted by a factor β— that a representative household derives from a sequence of consumptions c_t while satisfying, at each period t, the feasibility constraint. Specifically, the planner chooses the sequence of nonnegative c_t and k_t solving

$$\begin{aligned} &\max_{c_t, k_t} \sum_{t=1}^{+\infty} \beta^{t-1} u(c_t) \\ &c_t + k_t \leq F\Big(\sum_{i=1}^{+\infty} \phi^{i-1} k_{t-i}, 1\Big) \end{aligned} \tag{6}$$

where $\beta \in (0,1)$ and, in each constraint, *i.e.* for all $t = 1, 2, \ldots$, trivially $k_{t-i} = 0$ for $i > t$, given some initial endowment $k_0 > 0$. Note that the feasibility constraint conveys the assumption of irreversibility of capital.

The planner's choice must, therefore, necessarily satisfy, for each $t = 1, 2, \ldots$, and some positive multipliers $\lambda_t, \lambda_{t+1}, \ldots$, the condition

$$\begin{pmatrix} \beta^{t-1} u'(c_t) \\ 0 \end{pmatrix} = \lambda_t \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \lambda_{t+1} \begin{pmatrix} 0 \\ -F_K^{t+1} \end{pmatrix} + \lambda_{t+2} \begin{pmatrix} 0 \\ -F_K^{t+2} \cdot \phi \end{pmatrix} + \ldots \tag{7}$$

where F_K^{t+j} stands for the marginal productivity of capital at $t+j$, *i.e.*

$$F_K^{t+j} = F_K\Big(\sum_{i=1}^{+\infty} \phi^{i-1} k_{t+j-i}, 1\Big) \tag{8}$$

and from which the next characterisation easily follows.

Proposition 1. *In the infinitely-lived agents economy in Section 2, a planner's allocation* $\{c_t, k_t\}_{t\in\mathbb{N}}$ *is characterised by*

$$1 = \sum_{j=1}^{+\infty} \beta^j \frac{u'(c_{t+j})}{u'(c_t)} F_K\Big(\sum_{i=1}^{+\infty} \phi^{i-1} k_{t+j-i}, 1\Big) \phi^{j-1} \tag{9}$$

and the feasibility constraint, for all $t = 1, 2, \ldots$, *given some initial endowment* $k_0 > 0$ *—and trivially* $k_{t-i} = 0$ *for* $i > t$.

We will now compare this necessary characterisation of the planner's choice with that of the market allocation next.

3.2 The market allocation for the infinitely-lived agents economy.

A household behaving competitively aims instead at maximising its utility under its budget constraint, choosing its sequences of consumptions c_t and savings k_t that are lent to firms as capital solving

$$\begin{aligned} &\max_{c_t,k_t} \sum_{t=1}^{+\infty} \beta^{t-1} u(c_t) \\ &c_t + k_t \leq w_t + r_t k_{t-1} + \pi_t \end{aligned} \tag{10}$$

given the sequence of aggregate profits π_t it receives as firm owner,[11] and the sequences of factor prices w_t and r_t, determined at equilibrium by their marginal productivities, *i.e.*

$$\begin{aligned} w_t &= F_L\Big(\sum_{i=1}^{+\infty} \phi^{i-1} k_{t-i}, 1\Big) \\ r_t &= F_K\Big(\sum_{i=1}^{+\infty} \phi^{i-1} k_{t-i}, 1\Big) \end{aligned} \tag{11}$$

where, for all $t = 1, 2, \ldots$, trivially $k_{t-i} = 0$ for $i > t$, given some initial endowment $k_0 > 0$.

The household's choice therefore necessarily satisfies at equilibrium, for all t and some positive multipliers λ_t, λ_{t+1},

$$\begin{pmatrix} \beta^{t-1} u'(c_t) \\ 0 \end{pmatrix} = \lambda_t \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \lambda_{t+1} \begin{pmatrix} 0 \\ -F_K^{t+1} \end{pmatrix} \tag{12}$$

where F_K^{t+1} stands, as before, for the marginal productivity of capital at $t+1$, and from where the next characterisation easily follows.

[11] The productivity of the capital in the public domain $\pi_t = F_K\big(\sum_{i=1}^{+\infty} \phi^{i-1} k_{t-i}, 1\big) \cdot \sum_{i=2}^{+\infty} \phi^{i-1} k_{t-i}$ is not remunerated to any owner of production factors and hence feeds aggregate profits, which are distributed as dividends to firm owners. Note that even though free entry in the industry might drive *per firm* profits down to zero if an unbounded number of firms enter the market —and hence the same aggregate profits above are reaped by an increasing number of firms— from the linear homogeneity of F these same *aggregate* profits remain constant at a positive level regardless the number of firms. It should be noticed that, accordingly, capital in the public domain is implicitly here nonproprietary but excludable (*e.g.* commons), which applies to any capital whose use may suffer from congestion (*e.g.* infrastructure, urban networks, judiciary, police forces,... but not technology).

Proposition 2. *In the infinitely-lived agents economy in Section 2, in which some private capital eventually falls into the public domain, a market allocation* $\{c_t, k_t\}_{t\in\mathbb{N}}$ *is characterised by*

$$1 = \beta \frac{u'(c_{t+1})}{u'(c_t)} F_K\Big(\sum_{i=1}^{+\infty} \phi^{i-1} k_{t+1-i}, 1\Big) \tag{13}$$

and the budget constraint,[12] *for all* $t = 1, 2, \ldots,$ *given some initial endowment* $k_0 > 0$ *—and trivially* $k_{t-i} = 0$ *for* $i > t$*.*

The characterisations provided in Propositions 1 and 2 allow to compare the planner's and market steady state allocations next.

3.3. The planner *vs* the market steady states in the infinitely-lived agents economy.

When it comes to comparing the steady state allocations that the market and the planner would deliver for the economy, in the framework of the infinitely-lived agents economy of Section 2, the previous characterisations point to a clear-cut result: at the steady state the market accumulates less capital than the planner would, as the next proposition establishes.

Proposition 3. *In the infinitely-lived agents economy in Section 2, in which some private capital eventually falls into the public domain, the market steady state level of capital* $\bar{k}$ *is unique and smaller than the planner's unique steady state level* k^**.*

Proof. From the characterisations (9) and (13) of Propositions 1 and 2 above it follows that each steady state —i.e. the planner's k^* and the market $\bar{k}$— is characterised respectively by

$$\begin{aligned} 1 &= \beta F_K\Big(\frac{k^*}{1-\phi}, 1\Big)\frac{1}{1-\beta\phi} \\ 1 &= \beta F_K\Big(\frac{\bar{k}}{1-\phi}, 1\Big) \end{aligned} \tag{14}$$

[12] Which is equivalent to the feasibility constraint.

From these conditions and the decreasing marginal productivity of capital it follows that both $\bar{k}$ and k^* are unique. They also imply straightforwardly that $k^* > \bar{k}$ —since $\beta, \phi \in (0,1)$.[13] $\square$

The impact, in the infinitely-lived agents economy, of private capital sliding into the public domain is therefore very clear: the market accumulates too little capital.

In order to get a grasp of by how much might the market be missing the optimal level of capital because of the gradual slide of the latter —through a number of ways— into the public domain, consider a standard value for the share of income remunerating capital for a Cobb-Douglas production function with normalised total factor productivity $F(K,L) = K^\alpha L^{1-\alpha}$, *i.e.* $\alpha = 1/3$, and a value of $\beta = .98$ corresponding approximately to a discounting by a rate of 2%, as well as a value of $\phi = .85$ corresponding to a 15% consumption of fixed capital.[14] In this case, it follows from (14) easily[15] that the ratio of the planner's steady state level of capital to the market one is

$$\frac{k^*}{\bar{k}} = \left(\frac{1}{1-\phi\beta}\right)^{\frac{1}{1-\alpha}} = \left(\frac{1}{1-.85\cdot .98}\right)^{\frac{1}{1-\frac{1}{3}}} = 14.6530 \qquad (15)$$

so that the market's steady state level of capital accumulation is way too low —the planner would choose to save/invest almost fifteen times more! More informative of the potential impact on the households' well-being, in terms of consumption, the

[13] If not all capital slides into the public domain, i.e. $\theta \in (0,1)$, then the respective steady states are characterised by

$$1 = \beta F_K\Big(\Big[\frac{1-\theta}{1-\delta} + \frac{\theta}{1-\phi}\Big]k^*, 1\Big)\Big[\frac{1-\theta}{1-\beta\delta} + \frac{\theta}{1-\beta\phi}\Big]$$
$$1 = \beta F_K\Big(\Big[\frac{1-\theta}{1-\delta} + \frac{\theta}{1-\phi}\Big]\bar{k}, 1\Big)$$

and the same conclusion follows.

[14] The consumption of fixed capital, or CFC, captures in national accounts the depreciation of aggregate capital stock. It is the difference between the gross investment (aggregate gross fixed capital formation) and net investment (net fixed capital formation) or between the Gross National Product and Net National Product. It has remained in the vicinity of 15% for major economies like the US and Germany, for instance, since the 80's (source: AMECO annual macro-economic database of the European Commission's Directorate General for Economic and Financial Affairs).

[15] Isolating the steady state level of capital in each equation and dividing the planner's by the market one.

ratio of the planner's to the market is, at the steady state, (approximately)

$$\frac{c^*}{\bar{c}} = \frac{(14.6530\bar{k})^{\frac{1}{3}} - 14.6530\bar{k}}{\bar{k}^{\frac{1}{3}} - \bar{k}} \tag{16}$$

with —from the second condition in (14)—

$$\bar{k} = (\alpha\beta)^{\frac{1}{1-\alpha}}(1-\phi) = (\frac{1}{3} \cdot .85)^{\frac{1}{1-\frac{1}{3}}}(1 - .85) = 0.0226 \tag{17}$$

so that

$$\frac{c^*}{\bar{c}} = \frac{(14.6530 \cdot 0.0226)^{\frac{1}{3}} - 14.6530 \cdot 0.0226}{0.0226^{\frac{1}{3}} - 0.0226} = 1.3866 \tag{18}$$

That is to say, for empirically reasonable values of basic parameters, the market fails to deliver the almost 39% more consumption that households would be allocated by a hypothetical planner. This simple assessment points to a huge market inefficiency, even if the exact figure should be taken with a pinch of salt: the point of the exercise is clearly not precision, but rather to see that the inefficiency is substantial, pointing to a far from negligible order of magnitude. It would be nonetheless interesting to have an estimate of the actual size of the inefficiency that followed from empirical data.

Having now a rough idea of the size of the market inefficiency, what would be necessary for the market to deliver the planner's steady state? From the conditions in (14) characterising both the market and the planner's steady state, it follows that subsidising the market return to capital by a factor τ, which will have as affect transform the market condition in (14) above into

$$\begin{aligned} 1 &= \beta F_K(\frac{k^*}{1-\phi}, 1)\frac{1}{1-\beta\phi} \\ 1 &= \beta F_K(\frac{\bar{k}}{1-\phi}, 1) \cdot \tau \end{aligned} \tag{19}$$

would make $\bar{k}$ be k^* if

$$\tau = \frac{1}{1-\beta\phi} \tag{20}$$

which for the values of the parameters considered —*i.e.* $\beta = .98$ and $\phi = .85$— requires $\tau = 5.99$, that is to say it requires to distort the representative household's choice subsidising the return to savings to be almost six times (!) bigger than the return to capital, and pay for it by means of some non-distortionary lump-sum tax in income.

The enormity of the intervention is obviously due to the minimalist character of the framework, which asks from the savings in physical capital —the only possible means of saving here— to do all the heavy-lifting. Addressing this same question, in the next section, in an overlapping generations setup instead, which allows for the introduction of other saving means, the policy allowing to offset in the market allocation the inefficiency arising from capital sliding into the public domain will take a less extreme form. Moreover, the richer model will provide more insight about why households interacting through the market miss so spectacularly so much surplus.

4. Market under-accumulation: the overlapping generations economy case

Because of the need to distinguish, in the 2-period-lived (say, young and old) representative agent overlapping generations economy in Section 2, variables relating to different generations as well as time periods, we will use superscript t to identify t's generation choice variables like, among others, the intertemporal profile of consumption c_0^t, c_1^t when young and old respectively, or the amount k^t lent to firms by generation t's representative household for production at $t+1$.

As previously, I will characterise next the steady state allocation that the planner, unconstrained by property rights, would choose, and I will compare it to the market steady state of the economy. From the systems of equations characterising the optimal steady state and the market steady state will follow the policy that is necessary to correct the depressing effect of capital sliding into the public domain on capital accumulation in an overlapping generations setup.

4.1 The planner's problem in the overlapping generations economy.

A utilitarian planner would choose an allocation of each period's output between consumption for the agents alive in the period and investment for future production that maximises a weighted sum of the utilities of all households under each period's feasibility constraint —expressed below in *per* young terms, for a population growth factor n—

$$\max_{c_0^t, c_1^t, k^t \geq 0} \sum_{t=1}^{+\infty} \eta^{t-1} \big(u(c_0^t) + \beta u(c_1^t)\big)$$
$$c_0^t + \frac{c_1^{t-1}}{n} + k^t \leq F\Big(\frac{1}{\phi} \sum_{i=1}^{+\infty} (\frac{\phi}{n})^i k^{t-i}, 1\Big), \forall t = 1, 2, \dots \tag{21}$$

—with $k^{t-i} = 0$ for $i > t$ trivially— given some initial c_1^0, k^0, a discount factor η for future generations, the households' own discounting β of old age utility, and the depreciation/obsolescence factor ϕ for capital in the public domain.

From the problem above follows the next characterisation of the planner's choice linking, on the one hand, the contribution of savings k^t at any given period t to the marginal productivity of capital at all future periods $t + j$, for all $j = 1, 2, \dots$ to, on the other hand, the marginal rates of intertemporal substitution of consumption for all agents between t and each $t + j$.

Proposition 4. *In the overlapping generations economy in Section 2, an allocation $\{c_0^t, c_1^t, k^t\}_{t \in \mathbb{N}}$ chosen by the planner satisfies*

$$1 = \frac{1}{\phi} \sum_{j=1}^{+\infty} \left[F_K\Big(\frac{1}{\phi} \sum_{i=1}^{+\infty} (\frac{\phi}{n})^i k^{t+j-i}, 1\Big) (\phi\beta)^j \prod_{h=0}^{j-1} \frac{u(c_1^{t+h})}{u(c_0^{t+h})} \right] \tag{22}$$

as well as each period feasibility constraint binding (with $k^{t-i} = 0$ for $i > t$ trivially).

Proof. The solution to the planner's problem is necessarily characterised, for some $\lambda^{t+i} > 0$ with $i = 0, 1, 2, \dots,$ by

$$\begin{aligned} \begin{pmatrix} \eta^{t-1} u'(c_0^t) \\ \eta^{t-1} \beta u'(c_1^t) \\ 0 \end{pmatrix} = \lambda^t \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} &+ \lambda^{t+1} \begin{pmatrix} 0 \\ \frac{1}{n} \\ -\frac{1}{\phi} F_K\big(\frac{1}{\phi} \sum_{i=1}^{+\infty} (\frac{\phi}{n})^i k^{t+1-i}, 1\big) \frac{\phi}{n} \end{pmatrix} \\ &+ \lambda^{t+2} \begin{pmatrix} 0 \\ 0 \\ -\frac{1}{\phi} F_K\big(\frac{1}{\phi} \sum_{i=1}^{+\infty} (\frac{\phi}{n})^i k^{t+2-i}, 1\big) (\frac{\phi}{n})^2 \end{pmatrix} \\ &+ \dots \end{aligned} \tag{23}$$

for all $t = 1, 2, \ldots$, that is to say, by the following conditions on the marginal rates of substitution

(1) within generations

$$\frac{1}{\beta}\frac{u'(c_0^t)}{u'(c_1^t)} = \frac{\lambda^t}{\lambda^{t+1}} \cdot n \tag{24}$$

(2) and across generations

$$\frac{u'(c_0^t)}{u'(c_0^{t+i})} = \eta^i \frac{\lambda^t}{\lambda^{t+i}} \tag{25}$$

and

$$\frac{1}{\beta}\frac{u'(c_0^t)}{u'(c_1^{t+i})} = \eta^i \frac{\lambda^t}{\lambda^{t+i}} n \tag{26}$$

on top of

$$1 = \frac{1}{\phi}\sum_{j=1}^{+\infty}\left[F_K\Big(\frac{1}{\phi}\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t+j-i}, 1\Big)(\frac{\phi}{n})^j \frac{\lambda^{t+j}}{\lambda^t}\right] \tag{27}$$

and

$$c_0^t + \frac{c_1^{t-1}}{n} + k^t = F\Big(\frac{1}{\phi}\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t-i}, 1\Big) \tag{28}$$

The necessary condition obtains then from repeated direct substitutions of the intra-generational intertemporal marginal rate of substitution (24) into (27). □

From the previous characterisation it can be obtained that of the unique symmetric allocation that a planner treating equally all generations would choose, that is to say the characterisation of the planner's steady state in Proposition 5 next.

Proposition 5. *In the overlapping generations economy in Section 2, the egalitarian planner's steady state is characterised by the unique profile of life-cycle consumptions and savings* c_0, c_1, k *solving*

$$\begin{aligned} \frac{1}{\beta}\frac{u'(c_0)}{u'(c_1)} &= n = F_K(\frac{k}{n-\phi}, 1) + \phi \\ c_0 + \frac{c_1}{n} + k &= F(\frac{k}{n-\phi}, 1) \end{aligned} \tag{29}$$

given n, ϕ.

Proof. We will see first that the system above characterises any steady state, and then we will see that there is a solution to the system and only one.

From Proposition 4 and its proof, the symmetric limit allocation resulting from the planner treating all generations increasingly equally as $\eta \to 1$ —so that $\frac{u'(c_0^t)}{u'(c_0^{t+i})} \to 1$ for all t and all i, and hence so that $\eta^i \frac{\lambda^t}{\lambda^{t+i}} \to 1$ and thus $\frac{\lambda^t}{\lambda^{t+i}} \to 1$— is necessarily characterised by

$$\frac{1}{\beta}\frac{u'(c_0)}{u'(c_1)} = n \tag{30}$$

—which implies $\phi\beta\frac{u'(c_1)}{u'(c_0)} < 1$ whenever $n > 1$,[16] so that the series next is convergent— and[17]

$$1 = \frac{1}{\phi}F_K\Big(\frac{k}{n-\phi},1\Big)\sum_{j=1}^{+\infty}\Big(\phi\beta\frac{u'(c_1)}{u'(c_0)}\Big)^j \tag{31}$$

i.e., replacing the series by its value and rearranging terms,

$$\frac{1}{\beta}\frac{u'(c_0)}{u'(c_1)} = F_K\Big(\frac{k}{n-\phi},1\Big) + \phi \tag{32}$$

since $\phi\beta\frac{u'(c_1)}{u'(c_0)} < 1$.[18]

A planner steady state is clearly locally unique since it is a regular zero of the left-hand side of the planner's steady state equations

$$\begin{aligned} u'(c_0) - n\beta u'(c_1) &= 0 \\ F_k(\frac{k}{n-\phi},1) + \phi - n &= 0 \\ c_0 + \frac{c_1}{n} + k - F(\frac{k}{n-\phi},1) &= 0 \end{aligned} \tag{33}$$

[16]Indeed, equivalently $\beta\frac{u'(c_1)}{u'(c_0)} = \frac{1}{n} < 1$, since $n > 1$, from which $\phi\beta\frac{u'(c_1)}{u'(c_0)} < 1$ given that $\phi < 1$ too.

[17]Since $K_t = \sum_{i=1}^{+\infty}\phi^{i-1}k^{t-i}N_{t-i}$ and $N_t = n^i N_{t-i}$, then $\frac{K_t}{N_t} = \frac{1}{\phi}\sum_{i=1}^{+\infty}\left(\frac{\phi}{n}\right)^i k^{t-i}$ which at a steady state becomes $\frac{k}{n-\phi}$.

[18]Obtaining this characterisation from the one in Proposition 4 requires an argument in the limit as $\eta \to 1$, since for an equal weight for all generations in the planner's problem, i.e. $\eta = 1$, the planner's objective is not well defined.

In effect,

$$\begin{vmatrix} u''(c_0) & -n\beta u''(c_1) & 0 \\ 0 & 0 & F_{KK}(\frac{k}{n-\phi},1)\frac{1}{n-\phi} \\ 1 & \frac{1}{n} & 1-F_K(\frac{k}{n-\phi},1)\frac{1}{n-\phi} \end{vmatrix} = \\ -F_{KK}(\frac{k}{n-\phi},1)\frac{1}{n-\phi}\left[n\beta u''(c_1)+\frac{1}{n}u''(c_0)\right] < 0. \tag{34}$$

But it is globally unique too, since if c_0, c_1, k and c_0', c_1', k' were two distinct steady states for the planner, then necessarily $k = k'$ —since the (injective) marginal productivity of capital must match $n - \phi$ for both of them— and $c_0 < c_0'$ would imply $c_1 < c_1'$, which cannot be —since the *per* young aggregate consumption each period must match the common $F(\frac{k}{n-\phi}, 1) - k$. Therefore $c_0 = c_0'$ and from the feasibility constraint $c_1 = c_1'$ too.

The existence follows from the fact that the planner's steady state equations are also the first-order conditions of

$$\max_{c_0,c_1,k\geq 0} u(c_0) + \beta u(c_1) \\ c_0 + \frac{c_1}{n} + k \leq F(\frac{k}{n-\phi}, 1) \tag{35}$$

for which the existence of a solution is guaranteed by the usual differentiably strict concavity of u —which ensures the continuity of the planner's objective and the strict convexity of the utility upper contour sets— and the strict concavity of F with respect to capital —which makes the constrained set of the planner to be compact. $\square$

In the next sections I will characterise now the market steady state.

4.2. Firms' problem.

The stock of capital available for production at t is generation $t-1$'s aggregate savings in physical capital K^{t-1} plus the stock of depreciated capital available for production at $t-1$, that is to say ϕK_{t-1}, that is now in the public domain, *i.e.*

$$\begin{aligned} K_t &= K^{t-1} + \phi K_{t-1} \\ &= k^{t-1}N_{t-1} + \phi \sum_{i=1}^{+\infty} \phi^{i-1} k^{t-1-i} N_{t-1-i} \end{aligned} \tag{36}$$

or, in *per* young terms,

$$\frac{K_t}{N_t} = \frac{k^{t-1}}{n} + \frac{1}{n}\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t-1-i} \tag{37}$$

Only the proprietary part of the stock of capital available for production at t in the first term of the right-hand side of (36) is remunerated by firms, while the fraction ϕ of capital built from all previous loans from households to firms and now in the public domain, represented by the remaining terms, is not.

Firms maximise profits at t choosing how much capital to borrow and how much labor to hire —which at equilibrium need be $K^{t-1} = k^{t-1}N_{t-1}$ (*i.e.* the aggregate of the amount k^{t-1} lent to firms by each of the N_{t-1} households at $t-1$) and N_t respectively— given the rental rate of capital r_t, the wage w_t, and the amount of private capital that has fallen into the public domain ϕK_{t-1}, that is to say

$$\max_{K^{t-1},N_t} F(K^{t-1} + \phi K_{t-1}, N_t) - r_t K^{t-1} - w_t N_t \tag{38}$$

Note that, in its objective in (38), the firm does not have to remunerate the non-proprietary capital ϕK_{t-1} in the public domain that it uses. Factor prices are hence determined, at equilibrium, by

$$\begin{aligned} r_{t+1} &= F_K(K_{t+1}, N_{t+1}) \\ w_t &= F_L(K_t, N_t) \end{aligned} \tag{39}$$

As a consequence, firms make at t the following aggregate profits

$$\pi_t = F_K(K^{t-1} + \phi K_{t-1}, N_t)\phi K_{t-1} \tag{40}$$

or, equivalently, the per old aggregate profits

$$\frac{\pi_t}{N_{t-1}} = F_K\Big(\frac{k^{t-1}}{n} + \frac{1}{n}\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t-1-i}, 1\Big) \cdot \sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t-1-i} \tag{41}$$

It is worth reminding now[19] that —because of the linear homogeneity of the production function— since *aggregate* profits are positive at every period, free entry of firms in the market drives, at any given t, the level of *each firm*'s profits to zero, but not the level of *aggregate* profits π_t, which remains positive as the product of the positive marginal productivity of capital and the positive amount of capital in the public domain. As a result, the ownership of the firms —which entitles to being distributed dividends— is traded across generations.

[19]See footnote 10 above.

4.3 Households' problem in the overlapping generations economy.

As a consequence of the firms distributing as dividends the profits obtained from the non remunerated productivity of the capital in the public domain, firm ownership has a return, and can therefore be used by households as a means of saving. The representative household born at t can therefore now transfer wealth *from its first period into the second* in three ways: (i) lending to firms to get at $t+1$ the income r_{t+1} *per* unit of capital lent, (ii) holding real monetary balances[20] and, *moreover*, (iii) taking a stock in the ownership of firms in order to be distributed an equal share d_{t+1} of the aggregate profits π_{t+1} made by firms at $t+1$ —so that $d_{t+1} = \pi_{t+1}/N_t$— as well as the resale value of the firms at $t+1$. Besides, we are going to assume that households can also transfer wealth *from its second period into the first* by (iv) borrowing from perfectly competitive financial intermediaries operating through the lives of all generations, the former lending back to new generations the funds reimbursed by the latter.

Thus, let k^t be the amount lent by the representative household born at t to firms, m^t be the household real balances, and s^t be the net saving in assets other than these two, that is to say the *net position* resulting from investing in firm ownership and borrowing from the competitive financial intermediaries against the future income from firm ownership. If $s^t > 0$, household t is therefore investing in firm ownership more than it may be borrowing from its second period income. If $s^t < 0$ instead, household t is rather borrowing more from its second period income than it is investing in firm ownership and hence, effectively, transferring wealth from the second period to the first, which capital or money does not allow.[21]

[20]Money is introduced in the model for the benchmark equilibrium to be optimal. In effect, in the Diamond (1965) setup that is at the foundation of the current one, in the absence of a bubbly asset in which to be able to save, there is no hope for the market to implement the planner's choice, independently of whether the additional effect of public domain capital studied here is included or not. The reason is that it is the presence of such an asset which allows the market allocation to replicate the planner's link between the return to capital and the population growth factor. In more precise terms, generically, no non-monetary equilibrium can decentralise the planner's allocation, neither in Diamond (1965) nor in this paper's setup.

[21]For the ease of its interpretation, a positive s^t can be thought of as the amount payed by each household born at t to the households born at $t-1$ for the firm ownership, i.e. for the right to receive its dividends and the value of its resale to its n children paying each s^{t+1}. The claim on future dividends, and hence the possibility to borrow against them, is what allows to extend the interpretation of s^t to that of a net position that can be negative as well as positive.

Household t's choice must therefore satisfy the budget constraints

$$\begin{aligned} c_0^t + k^t + s^t + m^t &\leq w_t \\ c_1^t &\leq r_{t+1}k^t + d_{t+1} + s^{t+1}n + \frac{p_t}{p_{t+1}}m^t \end{aligned} \tag{42}$$

—where d_{t+1} is the *per* owner distributed profits— given the wage w_t, the rental rate of capital r_{t+1}, the level of prices during the household's lifetime p_t, p_{t+1}, the profits made by firms when old π_{t+1}, the *per* young net position s^{t+1} of generation $t+1$,[22] and the population growth factor n. Moreover, the household's net position s^t —of savings in firm ownership minus borrowing against the future profits and resale value— must be bounded below by the present value of the net revenue from firm ownership, *i.e.*

$$r_{t+1}s^t \geq d_{t+1} + s^{t+1}n \tag{43}$$

must hold for all t. Indeed, should household t choose to hold a negative net position s^t between borrowing against firm ownership income and saving in terms of acquiring the firm ownership itself —which is a possible equilibrium outcome if generation $t+1$ does so as well and $s^{t+1}n$ more than offsets d_{t+1}— the amount borrowed cannot exceed the present value of the future net income against which the borrowing takes place.

Therefore, household t solves the problem

$$\begin{aligned} \max_{c_0^t, c_1^t, k^t, m^t \geq 0, s^t \in \mathbb{R}} \quad & u(c_0^t) + \beta u(c_1^t) \\ c_0^t + k^t + s^t + m^t &\leq w_t \\ c_1^t &\leq r_{t+1}k^t + d_{t+1} + s^{t+1}n + \frac{p_t}{p_{t+1}}m^t \\ r_{t+1}s^t &\geq d_{t+1} + s^{t+1}n \end{aligned} \tag{44}$$

the solution of which is necessarily characterised by the first-order conditions

$$\begin{pmatrix} u'(c_0^t) \\ \beta u'(c_1^t) \\ 0 \\ 0 \\ 0 \end{pmatrix} = \lambda_0^t \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \\ 1 \end{pmatrix} + \lambda_1^t \begin{pmatrix} 0 \\ 1 \\ -r_{t+1} \\ -\frac{p_t}{p_{t+1}} \\ 0 \end{pmatrix} + \mu^t \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -r_{t+1} \end{pmatrix} \tag{45}$$

[22] Note that, according to the interpretation of s^t as a net position, the amount received or reimbursed at $t+1$ by household t will equal at equilibrium the aggregate of the (positive or negative, respectively) net positions of its n children.

for some muyipliers $\lambda_0^t, \lambda_1^t, \mu^t > 0$, along with the constraints binding, or equivalently

$$\begin{aligned}\frac{1}{\beta}\frac{u'(c_0^t)}{u'(c_1^t)} = \frac{p_t}{p_{t+1}} = r_{t+1} \\ c_0^t + k^t + s^t + m^t = w_t \\ c_1^t = r_{t+1}k^t + d_{t+1} + s^{t+1}n + \frac{p_t}{p_{t+1}}m^t \\ r_{t+1}s^t = d_{t+1} + s^{t+1}n\end{aligned} \tag{46}$$

Note that, on top of the first order conditions and the binding budget constraints, the last constraint imposing a lower bound on borrowing delivers the (equilibrium) non-arbitrage condition according to which the household is indifferent, at its optimal choice, between lending to the firm or buying it, if it is to do both. It also follows from the first-order conditions above that the value of firm ownership for the household is $\mu^t = u'(c_0^t)/r_{t+1} > 0$.

The optimising behaviour of households in (46) and firms in (39) and (41) determines, when compatible, a competitive equilibrium of this economy, as stated in the next section.

4.4. Competitive equilibrium of the overlapping generations economy.

A competitive equilibrium of the overlapping generations economy is therefore characterised by the following conditions.

Proposition 6. *In the overlapping generations economy in Section 2, in which some private capital eventually falls into the public domain, a competitive equilibrium is characterised by a consumption profile c_0^t, c_1^t, a loan to firms k^t, a net position in firms ownership and borrowing s^t, a real balance m^t, and distributed profits d_{t+1}, for each agent born in each period t, as well as prices p_t, for all t, such* *that*

$$
\begin{aligned}
\frac{1}{\beta}\frac{u'(c_0^t)}{u'(c_1^t)} = \frac{p_t}{p_{t+1}} &= F_K\Big(\frac{1}{\phi}\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t+1-i}, 1\Big) \\
c_0^t + k^t + s^t + m^t &= F_L\Big(\frac{1}{\phi}\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t-i}, 1\Big) \\
\frac{c_1^t}{n} = F_K\Big(\frac{1}{\phi}\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t+1-i}, 1\Big)\frac{k^t}{n} + \frac{d_{t+1}}{n} &+ s^{t+1} + \frac{p_t}{p_{t+1}}\frac{m^t}{n} \\
F_K\Big(\frac{1}{\phi}\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t+1-i}, 1\Big)s^t &= d_{t+1} + s^{t+1}n \\
d_{t+1} = F_K\Big(\frac{k^t}{n} + \frac{1}{n}\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t-i}, 1\Big) &\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t-i} \\
\frac{p_t}{p_{t+1}}\frac{m^t}{m^{t+1}} &= n
\end{aligned}
\tag{47}
$$

Proof. The first four lines follow from the household choice in (46) with the factor prices replaced by the marginal productivities of factors according to the firms' optimal behavior in (39). The fifth line is the equilibrium *per* owner profits (41) distributed at each $t + 1$.

The sixth line is equivalent to the feasibility of the allocation of resources for this economy, and can be obtained in the usual way adding up the budget constraints of the agents alive at any given period t —of which there are n young agents *per* old one— and taking into account the homogeneity of degree 1 of the production function, *i.e.* adding up

$$
c_0^t + k^t + s^t + m^t = w_t \tag{48}
$$

and

$$
\frac{c_1^{t-1}}{n} = r_t\frac{k^{t-1}}{n} + \frac{d_t}{n} + s^t + \frac{p_{t-1}}{p_t}\frac{m^{t-1}}{n} \tag{49}
$$

which with (FP) and the feasibility constraint

$$
c_0^t + \frac{c_1^{t-1}}{n} + k^t = F\Big(\frac{1}{\phi}\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t-i}, 1\Big) \tag{50}
$$

amounts to

$$\frac{p_t}{p_{t+1}}\frac{m^t}{m^{t+1}} = n \tag{51}$$

at any given t. □

It should be noted, first, that the equilibrium conditions imply that different generations choose, necessarily, a different mix of monetary savings and net position of stocks and borrowing, for savings other than loans to firms, as the next proposition establishes.

Proposition 7. *In an overlapping generations economy in Section 2, in which some private capital eventually falls into the public domain, there is no competitive equilibrium in which the representative agent monetary savings m^t and the net position in investing in firm ownership and borrowing s^t are constant.*

Proof. In effect, should $m^t = m$ hold for some m and all t, then from the last equation in the competitive equilibrium system (47) above

$$\frac{p_t}{p_{t+1}} = n \tag{52}$$

would hold, and should $s^t = s$ hold, then the no-arbitrage condition in the fourth line of (47) would require that distributed profits be zero, since then

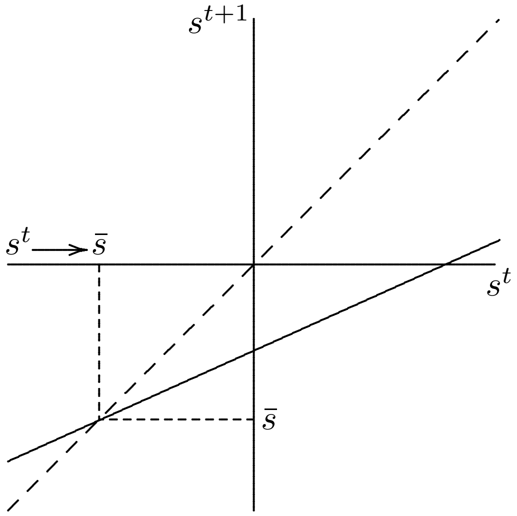
$$d_{t+1} = \left[F_K\Big(\frac{1}{\phi}\sum_{i=1}^{+\infty}\big(\frac{\phi}{n}\big)^i k^{t+1-i}, 1\Big) - n\right]s = 0 \tag{53}$$

according to the equilibrium condition $F_K\big(\frac{1}{\phi}\sum_{i=1}^{+\infty}\big(\frac{\phi}{n}\big)^i k^{t+1-i}, 1\big) = \frac{p_t}{p_{t+1}}$ and (52) above. However, profits distributed at $t+1$ *are* positive, since

$$d_{t+1} = F_K\Big(\frac{1}{\phi}\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t-i}, 1\Big)\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t-i} > 0 \tag{54}$$

from which the conclusion follows. □

Therefore, a competitive equilibrium steady state will be an allocation where only the consumption profile, the loans to firms and the *total* (but not the composition) of *savings* in instruments *other than loans to firms* will stay constant, at say s, as shown in the Proposition 8 next. It is shown there too that, in the relevant case in which the sum of real balances and savings in firm ownership net of borrowing do not explode, (i) the share of real balances m^t within s converges to zero, so that in the limit the net position s^t of *firm ownership minus borrowing asymptotically replaces money* as the bubbly asset in the economy; and (ii) $s < 0$, meaning that, in the limit, *each generation borrows when young against future income from firm ownership more than it pays for it*, the funds of the loan being provided by the repayments to the financial intermediaries made by the previous generation.



Proposition 8. *In the overlapping generations economy in Section 2, in which some private capital eventually falls into the public domain, a competitive equilibrium steady state is characterised by a constant profile of consumptions c_0, c_1 and a constant loan to firms k, for all generations, as well as a constant growth factor of real balances m^{t+1}/m^t satisfying*

$$\begin{aligned} \frac{1}{\beta}\frac{u'(c_0)}{u'(c_1)} = n\frac{m^{t+1}}{m^t} &= F_K\Big(\frac{k}{n-\phi},1\Big) \\ c_0 + k + s &= F_L\Big(\frac{k}{n-\phi},1\Big) \\ \frac{c_1}{n} &= F_K\Big(\frac{k}{n-\phi},1\Big)\frac{k}{n-\phi} + s \end{aligned} \tag{55}$$

—where $s = \frac{d}{r-n}$, with $d = r\frac{k}{n-\phi}\phi$ and $r = F_K\big(\frac{k}{n-\phi},1\big)$— which determines c_0, c_1, k, and $\frac{m^{t+1}}{m^t}$.

Moreover, for all t,

$$s^t + m^t = s \tag{56}$$

and, if $r < n$, the household net position s^t of savings in ownership of the firm minus borrowing converge to $s < 0$, while positive real balances m^t converge to zero,[23] *i.e.*

$$\begin{aligned} \lim_{t\to+\infty} s^t &= s < 0 \\ \lim_{t\to+\infty} m^t &= 0. \end{aligned} \tag{57}$$

Proof. From Propositions 6 and 7, a competitive equilibrium steady state is therefore characterised by the conditions next, where consumptions c_0^t, c_1^t, capital savings k^t, and distributed profits d_{t+1} —but not s^t or m^t (nor p_t, *a fortiori*)— stay constant at levels c_0, c_1, k, and d,

$$\begin{aligned} \frac{1}{\beta}\frac{u'(c_0)}{u'(c_1)} = \frac{p_t}{p_{t+1}} &= F_K\Big(\frac{1}{\phi}\sum_{i=1}^{+\infty}\big(\frac{\phi}{n}\big)^i k, 1\Big) \\ c_0 + k + s^t + m^t &= F_L\Big(\frac{1}{\phi}\sum_{i=1}^{+\infty}\big(\frac{\phi}{n}\big)^i k, 1\Big) \\ \frac{c_1}{n} = F_K\Big(\frac{1}{\phi}\sum_{i=1}^{+\infty}\big(\frac{\phi}{n}\big)^i k, 1\Big)\frac{k}{n} + \frac{d}{n} &+ s^{t+1} + \frac{p_t}{p_{t+1}}\frac{m^t}{n} \\ F_K\Big(\frac{1}{\phi}\sum_{i=1}^{+\infty}\big(\frac{\phi}{n}\big)^i k, 1\Big)s^t &= d + s^{t+1}n \\ d = F_K\Big(\frac{k}{n} + \frac{1}{n}\sum_{i=1}^{+\infty}\big(\frac{\phi}{n}\big)^i k, 1\Big) &\sum_{i=1}^{+\infty}\big(\frac{\phi}{n}\big)^i k \\ \frac{p_t}{p_{t+1}}\frac{m^t}{m^{t+1}} &= n \end{aligned} \tag{58}$$

Note that, nonetheless, from the second line above, the aggregate $s^t + m^t$ has necessarily to be a constant, say s, at a competitive equilibrium steady state, even though s^t and m^t are not.[24]

[23] If $r \geq n$, savings invested in both real balances and the net position of savings in firm ownership minus borrowing diverge.

[24] Should m^t be constant, then s^t would be constant too (second equation in (58)) and $\frac{p_t}{p_{t+1}} = n$ (last equation), which in turn would imply the marginal productivity of capital to be n too (first equation) and hence d to be zero (fourth equation), which it is not since it is positive (fifth equation).

Thus, after substituting the sixth equation into the first line and replacing the series by their value, a competitive equilibrium steady state is characterised by

$$\begin{aligned} \frac{1}{\beta}\frac{u'(c_0)}{u'(c_1)} &= n\frac{m^{t+1}}{m^t} = F_K\Big(\frac{k}{n-\phi},1\Big) \\ c_0 + k + s^t + m^t &= F_L\Big(\frac{k}{n-\phi},1\Big) \\ \frac{c_1}{n} = F_K\Big(\frac{k}{n-\phi},1\Big)\frac{k}{n} &+ \frac{d}{n} + s^{t+1} + \frac{p_t}{p_{t+1}}\frac{m^t}{n} \\ F_K\Big(\frac{k}{n-\phi},1\Big)s^t &= d + s^{t+1}n \\ d &= F_K\Big(\frac{k}{n-\phi},1\Big)\frac{k}{n-\phi}\phi \\ \frac{p_t}{p_{t+1}}\frac{m^t}{m^{t+1}} &= n \end{aligned} \tag{59}$$

or equivalently, after substituting the last two equations into the third line,

$$\begin{aligned} \frac{1}{\beta}\frac{u'(c_0)}{u'(c_1)} &= n\frac{m^{t+1}}{m^t} = F_K\Big(\frac{k}{n-\phi},1\Big) \\ c_0 + k + s^t + m^t &= F_L\Big(\frac{k}{n-\phi},1\Big) \\ \frac{c_1}{n} = F_K\Big(\frac{k}{n-\phi},1\Big)\frac{k}{n-\phi} &+ s^{t+1} + m^{t+1} \\ F_K\Big(\frac{k}{n-\phi},1\Big)s^t &= d + s^{t+1}n \end{aligned} \tag{60}$$

whose first three lines are those in (55), with $s^t + m^t = s$, as requested. It remains to be checked that the no-abitrage condition in the equation above implies that s is the value claimed and that the convergence of s^t to s obtains. Indeed, from the no-arbitrage condition at the steady state, which can be rewritten as

$$s^{t+1} = \frac{r}{n}s^t - \frac{d}{n} \tag{61}$$

—where $d = r\frac{\phi}{n-\phi}k$ and $r = F_K\big(\frac{k}{n-\phi},1\big)$ are, respectively, the profits distributed to each agent and the return to capital at a competitive equilibrium steady state—it follows that, whenever $r < n$, the value for s^t converges to

$$s = \frac{d}{r-n} < 0. \tag{62}$$

—and therefore m^t converges to 0— as claimed. $\square$

A few remarks are now in order. Firstly, from conditions (29) and (55) it follows that the planner's steady state cannot be decentralized through markets under laissez-faire. In particular, the market leads the agents to consume too early at the steady state, in the sense of choosing an intertemporal marginal rate of substitution smaller than the planner's, as made precise in the Proposition 9 next.

Proposition 9. *In the overlapping generations economy in Section 2, in which some private capital eventually falls into the public domain, the* (*unique*) *planner's steady state cannot be decentralized as a laissez-faire competitive markets outcome. In particular, the market makes households choose a profile of consumptions whose intertemporal marginal rate of substation is smaller than the planner's.*

Proof. Indeed, note first that, since $s^t + m^t = s$, for $m^t > 0$, it must be that $s^t < s < 0$, so that s^t converges to s from the left, decreasing in absolute value. Therefore, $m^t = s - s^t$ is decreasing, so that $m^t / m^{t+1} > 1$ which, from the equilibrium condition

$$\frac{p_t}{p_{t+1}} \frac{m^t}{m^{t+1}} = n \tag{63}$$

implies $p_t / p_{t+1} < n$.

Now, if $\bar{c}_0, \bar{c}_1$ is the competitive equilibrium steady state profile of consumption, while the profile chosen by the planner is c_0^*, c_1^*, it follows from the respective characterisations in (55) and (29) that

$$\frac{1}{\beta} \frac{u'(\bar{c}_0)}{u'(\bar{c}_1)} = \frac{p_t}{p_{t+1}} < n = \frac{1}{\beta} \frac{u'(c_0^*)}{u'(c_1^*)} \tag{64}$$

as claimed. $\square$

Interestingly enough, it is not immediate in this overlapping generations setup —as opposed to what in happened the infinitely-lived agents case— whether the market lends too few or too much capital to firms, compared to what the planner would choose. Indeed, since at the competitive equilibrium steady state $p_t / p_{t+1} < n$, it follows from (55) and (29) that

$$F_K(\frac{\bar{k}}{n - \phi}, 1) = \frac{p_t}{p_{t+1}} < n = F_K(\frac{k^*}{n - \phi}, 1) + \phi \tag{65}$$

where $\bar{k}$ is the steady state *per* young market level of capital, and k^* is the planner's. As a consequence, it holds that

$$F_K(\frac{\bar{k}}{n-\phi}, 1) < n > F_K(\frac{k^*}{n-\phi}, 1) \tag{66}$$

so that the *per* young market level of capital $\bar{k}$ could, in principle, be smaller or bigger than the planner's k^*.

Note however that, while the planner's steady state *per* young level of capital k^* needs to be such that

$$n - \phi = F_K(\frac{k^*}{n-\phi}, 1) \tag{67}$$

the competitive equilibrium steady state equations (55) pin down the market steady state *per* young level of capital to be $\bar{k}$ such that

$$\frac{1}{\beta}\frac{u'(F_L(\frac{\bar{k}}{n-\phi}, 1) - \bar{k} - s)}{u'(F_K(\frac{\bar{k}}{n-\phi}, 1)\frac{\bar{k}}{n-\phi}n + sn)} = F_K(\frac{\bar{k}}{n-\phi}, 1) \tag{68}$$

which —for the sake of assessing the wedge between the market *per* young steady state capital accumulation and the planner's— in the case of $u(c) = \ln c$ and $F(K,N) = K^\alpha N^{1-\alpha}$ take (after some algebra) respectively the form

$$\begin{aligned} &\alpha\left(\frac{k^*}{n-\alpha}\right)^{\alpha-1} - n + \phi = 0 \\ n\frac{1}{\beta}\frac{\bar{k}}{n-\alpha}&\left[\alpha\left(\frac{\bar{k}}{n-\alpha}\right)^{\alpha-1} - n + \phi\right] = \\ &\left[\alpha\left(\frac{\bar{k}}{n-\alpha}\right)^{\alpha-1} - n\right]\left[(1-\alpha)\left(\frac{\bar{k}}{n-\alpha}\right)^{\alpha} - k\right] - \alpha\left(\frac{\bar{k}}{n-\alpha}\right)^{\alpha}\phi \end{aligned} \tag{69}$$

that can be further simplified to the following equation in $x = \frac{\bar{k}}{n-\phi}$

$$\alpha(1-\alpha)x^{2\alpha-1} - n(1-\frac{\alpha}{\beta})x^\alpha - n(1-\frac{1}{\beta})(n-\phi)x = 0 \tag{70}$$

Specifically, for the profile of parameters we considered in Section 3.3, i.e. $\alpha = 1/3$, $\beta = .98$, $\phi = .85$, and a population growth factor $n = 1.007$ —empirical value for the USA in 2016[25]— the latter has a single root at (approximately)

$$\frac{\bar{k}}{n-\phi} = 0.1939 \tag{71}$$

[25]Source https://data.worldbank.org/indicator/SP.POP.GROW.

while from (67) the same value is for the planner (approximately)

$$\frac{k^*}{n-\phi} = \left(\frac{n-\phi}{\alpha}\right)^{\frac{1}{\alpha-1}} = 3.0936 \tag{72}$$

so that the planner to market steady state *per* young capital ratio is (approximately)

$$\frac{k^*}{\bar{k}} = 15.9546 \tag{73}$$

In other words, once more the market's steady state level of capital accumulation is way too low: the planner would choose to save/invest almost sixteen times more(!). This wedge is of the same order of magnitude than the on found for the infinitely-lived agents economy —in that case $\frac{k^*}{\bar{k}} = 14.6530$— which points to the robustness of the size of the inefficiency created by the slide of capital into the public domain.

While the size of this gap is still shocking, the implied gap in terms of aggregate consumption might be more realistic while being still sizeable: a more informative estimate of the potential impact on the households' well-being —namely the ratio of the planner's to the market steady state *per* young aggregate consumption— is (approximately)

$$\frac{c_0^* + \frac{c_1^*}{n}}{\bar{c}_0 + \frac{\bar{c}_1}{n}} = \frac{(15.9546\bar{k})^{\frac{1}{3}} - 15.9546\bar{k}}{\bar{k}^{\frac{1}{3}} - \bar{k}} \tag{74}$$

with

$$\bar{k} = 0.1939(1.007 - 0.85) = 0.0304 \tag{75}$$

so that

$$\frac{c_0^* + \frac{c_1^*}{n}}{\bar{c}_0 + \frac{\bar{c}_1}{n}} = \frac{(15.9546 \cdot 0.0304)^{\frac{1}{3}} - 15.9546 \cdot 0.0304}{0.0304^{\frac{1}{3}} - 0.0304} = 1.0674 \tag{76}$$

That is to say, for empirically reasonable values of basic parameters, the market fails to deliver the almost 7% more *per* young aggregate consumption that households would be allocated by a hypothetical planner. This is a substantially smaller estimate of the inefficiency due to the slide into public domain of private capital —compared to the one delivered by the infinitely-lived agents setup— but substantially big, nonetheless, to be ignored too. At any rate, in spite of the crudeness of the quantitative exercise, whose point is clearly not precision, it unequivocally points to a significant cost of not addressing the problem by means of some offsetting policy. What that policy should be in this setup is presented next.

4.5. Market implementation of the planner's steady state through a subsidy on capital returns and a tax on households' borrowing.

If households see their returns from loans to firms subsidised or taxed at a rate τ and their net position on firm ownership and borrowing against it taxed or subsidised by a factor σ, the household born at t would face instead

$$\begin{gathered}\max_{c_0^t, c_1^t, k^t, m^t, s^t} u(c_0^t) + \beta u(c_1^t) \\ c_0^t + k^t + s^t + m^t \leq w_t \\ c_1^t \leq (r_{t+1} + \tau)k^t + d_{t+1} + \sigma s^{t+1} n + \frac{p_t}{p_{t+1}} m^t \\ (r_{t+1} + \tau) s^t \geq d_{t+1} + \sigma s^{t+1} n\end{gathered} \tag{77}$$

given the wage w_t, the rental rate of capital r_{t+1}, the level of prices during his lifetime p_t, p_{t+1}, the profits received as dividends when owner d_{t+1}, the household $t+1$'s net positions s^{t+1}, and the population growth factor n.

As a consequence, the choice of a household born at t necessarily satisfies

$$\begin{pmatrix} u'(c_0^t) \\ \beta u'(c_1^t) \\ 0 \\ 0 \\ 0 \end{pmatrix} = \lambda_0^t \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \\ 1 \end{pmatrix} + \lambda_1^t \begin{pmatrix} 0 \\ 1 \\ -(r_{t+1} + \tau) \\ -\frac{p_t}{p_{t+1}} \\ 0 \end{pmatrix} + \mu^t \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -(r_{t+1} + \tau) \end{pmatrix} \tag{78}$$

for some $\lambda_0^t, \lambda_1^t, \mu^t > 0$, along with the binding constraints, or equivalently

$$\begin{gathered}\frac{1}{\beta} \frac{u'(c_0^t)}{u'(c_1^t)} = \frac{p_t}{p_{t+1}} = r_{t+1} + \tau \\ c_0^t + k^t + s^t + m^t = w_t \\ c_1^t = (r_{t+1} + \tau)k^t + d_{t+1} + \sigma s^{t+1} n + \frac{p_t}{p_{t+1}} m^t \\ (r_{t+1} + \tau) s^t = d_{t+1} + \sigma s^{t+1} n\end{gathered} \tag{79}$$

As before, firms distribute at $t+1$ to each household born at t dividends

$$d_{t+1} = F_K\left(\frac{k^t}{n} + \frac{1}{n} \sum_{i=1}^{+\infty} \left(\frac{\phi}{n}\right)^i k^{t-i}, 1\right) \sum_{i=1}^{+\infty} \left(\frac{\phi}{n}\right)^i k^{t-i} \tag{80}$$

and factor prices are

$$\begin{aligned} r_{t+1} &= F_K(K_{t+1}, N_{t+1}) \\ w_t &= F_L(K_t, N_t) \end{aligned} \tag{81}$$

The market clearing condition can again be obtained adding up the budget constraints of the agents alive at any given period t, of which there are n young agents *per* old one, *i.e.* adding up

$$c_0^t + k^t + s^t + m^t = w_t \tag{82}$$

and

$$\frac{c_1^{t-1}}{n} = (r_t + \tau)\frac{k^{t-1}}{n} + \frac{d_t}{n} + \sigma s^t + \frac{p_{t-1}}{p_t}\frac{m^{t-1}}{n} \tag{83}$$

which after taking into account the feasibility condition amounts to

$$m^t = \tau\frac{k^{t-1}}{n} + (\sigma - 1)s^t + \frac{p_{t-1}}{p_t}\frac{m^{t-1}}{n} \tag{84}$$

holding at any given t.

A competitive equilibrium is therefore characterised, under such a policy, by the following conditions.

Proposition 10. *In the overlapping generations economy in Section 2, in which some private capital eventually falls into the public domain, a competitive equilibrium under a policy that (i) subsidises/taxes the returns to capital at a rate τ and (ii) taxes/subsidises the net position in firms ownership and borrowing by a factor σ, is characterised by a consumption profile c_0^t, c_1^t, a loan to firms k^t, a net position in firms ownership and borrowing s^t, a real balance m^t, and distributed profits d_{t+1},* *for each agent born in each period t, as well as prices p_t, for all t, such that*

$$\frac{1}{\beta}\frac{u'(c_0^t)}{u'(c_1^t)} = \frac{p_t}{p_{t+1}} = F_K(\frac{1}{\phi}\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t+1-i}, 1) + \tau$$

$$c_0^t + k^t + s^t + m^t = F_L(\frac{1}{\phi}\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t-i}, 1)$$

$$\frac{c_1^t}{n} = \left[F_K(\frac{1}{\phi}\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t+1-i}, 1) + \tau\right]\frac{k^t}{n} + \frac{d_{t+1}}{n} + \sigma s^{t+1} + \frac{p_t}{p_{t+1}}\frac{m^t}{n}$$

$$\left[F_K(\frac{1}{\phi}\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t+1-i}, 1) + \tau\right]s^t = d_{t+1} + \sigma s^{t+1} n \tag{85}$$

$$d_{t+1} = F_K(\frac{k^t}{n} + \frac{1}{n}\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t-i}, 1)\sum_{i=1}^{+\infty}(\frac{\phi}{n})^i k^{t-i}$$

$$m^t = \tau\frac{k^{t-1}}{n} + (\sigma - 1)s^t + \frac{p_{t-1}}{p_t}\frac{m^{t-1}}{n}$$

It is noteworthy that the argument underpinning Proposition 7 in Section 4.4 does not hold true under this policy, so that it does not rule out anymore the possibility of a steady state equilibrium in which s^t and m^t also are constant. As a matter of fact, the equilibrium that decentralises the planner's steady state is indeed an equilibrium in which s^t and m^t do stay constant, as Proposition 11 next establishes. This policy finances a subsidy to the return to capital through a tax on household debt issued against future dividends.

Proposition 11. *In the overlapping generations economy in Section 2, in which some private capital eventually falls into the public domain, the planner's steady state is decentralised as a competitive equilibrium steady state by a period-by-period balanced policy subsidising the returns to capital by the depreciation/obsolescence factor, i.e.*

$$\tau = \phi \tag{86}$$

by means of taxing debt issued against future profits at the positive rate $\sigma - 1$, with

$$\sigma = 1 - \frac{\phi k^*}{ns} \tag{87}$$

since $s < 0$ at a stable steady state.

Proof. Should there be values for c_0, c_1, k, s, m, d, and p_t/p_{t+1} such that

$$\begin{aligned}
\frac{1}{\beta}\frac{u'(c_0)}{u'(c_1)} = \frac{p_t}{p_{t+1}} &= F_K(\frac{k}{n-\phi}, 1) + \phi \\
c_0 + k + s + m &= F_L(\frac{k}{n-\phi}, 1) \\
\frac{c_1}{n} = \left[F_K(\frac{k}{n-\phi}, 1) + \phi\right]\frac{k}{n} + \frac{d}{n} + \sigma s &+ \frac{p_t}{p_{t+1}}\frac{m}{n} \\
d + \sigma s n &= \left[F_K(\frac{k}{n-\phi}, 1) + \phi\right] s \\
d &= F_K(\frac{k}{n-\phi}, 1)\frac{k}{n-\phi}\phi \\
m &= \phi\frac{k}{n} + (\sigma - 1)s + \frac{p_{t-1}}{p_t}\frac{m}{n}
\end{aligned} \tag{88}$$

for a given σ, they would characterise a competitive equilibrium steady state under the policy of subsidising returns at a rate ϕ and taxing debt by a factor σ. In particular, the system including σ as endogenous variable and augmented by the additional equation balancing taxes and subsidies

$$\phi k + (\sigma - 1)ns = 0 \tag{89}$$

pins down a balanced policy implementing the planner's steady state. Indeed, from (89) the last equation in (88) implies $p_t/p_{t+1} = n$ so that the first line becomes

$$\frac{1}{\beta}\frac{u'(c_0)}{u'(c_1)} = n = F_K(\frac{k}{n-\phi}, 1) + \phi \tag{90}$$

which is the first line of the planner's system in (29). Moreover, the equations in the second, third, fifth, and sixth lines imply the feasibility of the allocation.

The equation (89) pins down the necessary σ to be

$$\sigma = 1 - \frac{\phi k}{ns} > 1 \tag{91}$$

for a stable steady state, and hence it is a tax on borrowing against dividends to be received in the second period —since accordingly $s < 0$— as it increases the amount that households must repay.

The existence of a solution to (88,89) —and hence of a policy decentralising the planner's steady state— follows from the existence of the latter, specifically with $k = k^*$. $\square$

It is worth noting that for the Cobb-Douglas production function assumed in Section 3.3, σ can (after some algebra) be expressed as

$$\sigma = 1 - \frac{n-\phi}{n}\left[1 - \frac{n}{\alpha}\left(\frac{k^*}{n-\phi}\right)^{1-\alpha}\right] \tag{92}$$

which, with the profile of parameters considered —specifically, $\alpha = 1/3$ and $\phi = .85$ so that $\frac{k^*}{n-\phi} = 3.0936$— and considering a population growth factor of $n = 1.007$,[26] delivers an approximate σ equal to

$$\sigma = 1 - 0.1559 \cdot (1 - 3.021 \cdot (3.0936)^{0.66}) = 1.8440$$

while $\tau = \phi = .85$. In other words, in order to decentralise the planner's steady state, the returns to capital need to be subsidised by 85% (!) while borrowing in excess of firm ownership investment needs to be taxed by 84,4% (!). These are still huge numbers, although one order of magnitude smaller than the intervention required in the infinitely-lived agents setup (of about a 600% subsidy!). With all likelihood, a more detailed model capturing more closely the features of an actual economy would continue to bring the estimate of the required policy intervention closer to reasonable numbers.

5. Conclusion

The models presented in this paper aim at pointing to a source of inefficiency that seems to have been overlooked until now. Namely, that the impossibility of maintaining property rights on the capital generated by all past investments distorts households consumption/saving decision away from the optimal one, with potentially very significant consequences for the long-run steady state level of consumption. Although the models are deliberately stripped of any details other than those essential to make the point, for that very same reason the mechanisms shown to be at play will, with all likelihood, stay in any other model with additional elements to make it more apt to match empirical evidence.

[26] US population growth factor in 2016. Source https://data.worldbank.org/indicator/SP.POP.GROW.

The simple estimates of the deviations of the market from the planner's levels of consumption and capital accumulation, are intended only to give an idea of the size of the inefficiency and the potential gains from addressing it. While the specific figures obtained for these estimates are to be taken, therefore, with a pinch of salt, they unequivocally point to these gains to be significant. The sheer size of the estimated interventions required to decentralise through the market the planner's allocation are more likely to be the result of the parsimonious nature of the models than a hard result: already when a more realistic demographics was introduced when we moved from a dynastic setup to an overlapping generations one, the subsidy to the return on savings needed was reduced to less than a sixth. In more structured and detailed models a further convergence to "sensible" numbers is to be expected.

Empirically supported, detailed models that incorporates the mechanisms uncovered here should be able to deliver quantified policy recommendations able to undo the inefficiencies pointed at in this paper, and to steer the market outcome towards the optimal one that a planner would choose.

References

1 Boldrin, M. and D.K. Levine (2013), "The Case Against Patents", *Journal of Economic Perspectives*, vol. 27, no. 1, pp. 3–22.

2 Diamond, P.A. (1965), "National debt in a neoclassical growth model", *American Economic Review*, vol. 16, no. 3, pp. 531–538.

3 Gould, D.M. and W.C. Gruben (1996), "The Role of Intellectual Property Rights in Economic Growth", *Journal of Development Economics*, vol. 48, pp. 323–350.

4 Lerner, Josh and Jean Tirole, "The Economics Of Technology Sharing: Open Source and Beyond", Journal of Economic Perspectives, 2005, v19(2,Spring), 99-120. In a Romer-style endogenous growth model Saint-Paul (2003)

5 Romer, Paul M., 1986, "Increasing Returns and Long-Run Growth", *Journal of Political Economy*, 94(5), 1002–1037

6 Saint-Paul, Gilles, 2003, "Growth Effects of Non-Proprietary Innovation", *Journal of the European Economic Association: Papers and Proceedings*, 1, 429–439