Contracting with Type-dependent Naïveté

Matteo Foschi,
(University of Leicester and RES Fellow)

META4, University of York — 18^{th} March 2016

Assumption 1. Individuals form biased expectations about their abilities.

Assumption 1. Individuals form biased expectations about their abilities.

Assumption 2. There is correlation between the bias and the abilities.

Assumption 1. Individuals form biased expectations about their abilities.

Assumption 2. There is correlation between the bias and the abilities.

Empirical Evidence:

- Skills and awareness:
  Svenson (1981); Chi et al. (1982); Dunning and Kruger (1999); Dunning et al. (2003); Banneret al. (2008).
- Overconfidence and "self-efficacy":
  Dittrich at al. (2005); Bankset al. (2007); Moore and Healy (2008); Ferraro (2010).
- Projection-bias:
  Lichtenstein et al. (1982); Loewenstein et al. (2003) (theoretical); Conlin et al. (2007).

Research Question

I study optimal contracting in a principal-agent model where:

- agents **differ in** their **productivity** and in their **beliefs** about it;

Research Question

I study optimal contracting in a principal-agent model where:

- agents **differ in** their **productivity** and in their **beliefs** about it;
- **beliefs** are **biased** — i.e. agents are naïve;

Research Question

I study optimal contracting in a principal-agent model where:

- agents **differ in** their **productivity** and in their **beliefs** about it;
- **beliefs** are **biased** — i.e. agents are naïve;
- **the bias depends on the productivity itself**.

Research Question

I study optimal contracting in a principal-agent model where:

- agents **differ in** their **productivity** and in their **beliefs** about it;
- **beliefs** are **biased** — i.e. agents are naïve;
- **the bias depends on the productivity itself**.

I connect two literatures:

- Sequential screening (Courty and Li, 2000; Reiche, 2008; Kovác and Krähmer, 2013; Deb and Said, 2015; Evans and Reiche, 2015; Grubb, 2015).
- Contracting with naïve agents (O'Donoghue and Rabin, 2001; Eliaz and Spiegler, 2006, 2008; Asheim, 2007; Gilpatric, 2008; Heidhues and Köszegi, 2010).

(Main) Result:

Key novelty $\Rightarrow$

(Main) Result:

Key novelty $\Rightarrow$ Principal's updating of her beliefs.

(Main) Result:

Key novelty $\Rightarrow$ Principal's updating of her beliefs.

It generates a new **trade-off** for the principal between:

(Main) Result:

Key novelty $\Rightarrow$ Principal's updating of her beliefs.

It generates a new **trade-off** for the principal between:

- **"taking advantage" of the most naïve agents** in the population and

(Main) Result:

Key novelty $\Rightarrow$ Principal's updating of her beliefs.

It generates a new **trade-off** for the principal between:

- **"taking advantage" of the most naïve agents** in the population and
- **designing "efficient" contracts** for the most widespread type of agent.

The Model

Two-period principal(employer)-agent(worker) model where:

Period 1: the employer seeks to hire a worker from a population.
He does so by offering contract $\boldsymbol{w(e)}$.

Period 2: if he accepted $\boldsymbol{w(e)}$, the worker carries out a task.
The effort he exerts for the task is $\boldsymbol{e \in [0,1]}$.

Effort is perfectly observable (no moral hazard).

The Model — The Employer

Given $w(e)$, hiring a worker that exerts e, the employer obtains:

$$\Pi = y(e) - w(e)$$

$y(e) \rightarrow$ production function. Increasing and concave in e.

The Model — Worker's Utility and Productivity

A worker can be:

Productive $\rightarrow$ $U_P(w(e)) = w(e) - \theta_P e$

or $\theta_P < \theta_U$

Unproductive $\rightarrow$ $U_U(w(e)) = w(e) - \theta_U e,$

There is a fraction λ of productive workers in the model.

In Period 1 both workers and the employer have priors over θ.

The Model — Timing & Efficiency

Period 1

- Employer offers $w(e)$.
- Workers accept or reject a contract.
- They both have priors on θ.

Period 2

- Workers learn θ and then pick e.

The Model — Timing & Efficiency

Period 1

- Employer offers $w(e)$.
- Workers accept or reject a contract.
- They both have priors on θ.

Period 2

- Workers learn θ and <u>then</u> pick e.

A worker with $\theta = \theta_i$ choosing $e^* : y'(e^*) = \theta_i$, exerts **efficient effort**.

Period 1 Beliefs

The employer has **unbiased beliefs** (i.e. $\Pr\{\theta = \theta_P\} = \lambda$).

Period 1 Beliefs

The employer has **unbiased beliefs** (i.e. $\Pr\{\theta = \theta_P\} = \lambda$).

Workers have **differently biased beliefs**. They are naïve.

A worker can be:

Optimistic $\rightarrow$ $\Pr\{\theta = \theta_P\} = \phi$

or where $\phi > \lambda > \delta$.

Pessimistic $\rightarrow$ $\Pr\{\theta = \theta_P\} = \delta$.

Type-Dependent Naïveté

The distribution of beliefs is conditional on the productivity of the worker.

That is, $\Pr\{\delta|\theta_P\} \neq \Pr\{\delta|\theta_U\}$:

Type-Dependent Naïveté

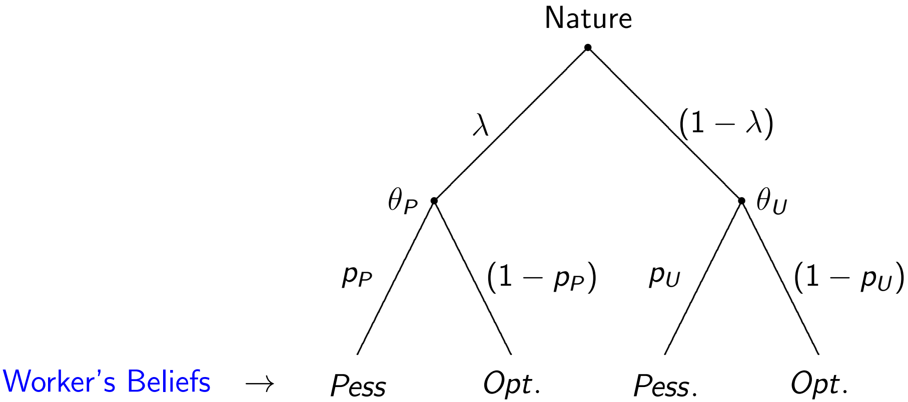
The distribution of beliefs is conditional on the productivity of the worker.

That is, $\Pr\{\delta|\theta_P\} \neq \Pr\{\delta|\theta_U\}$:

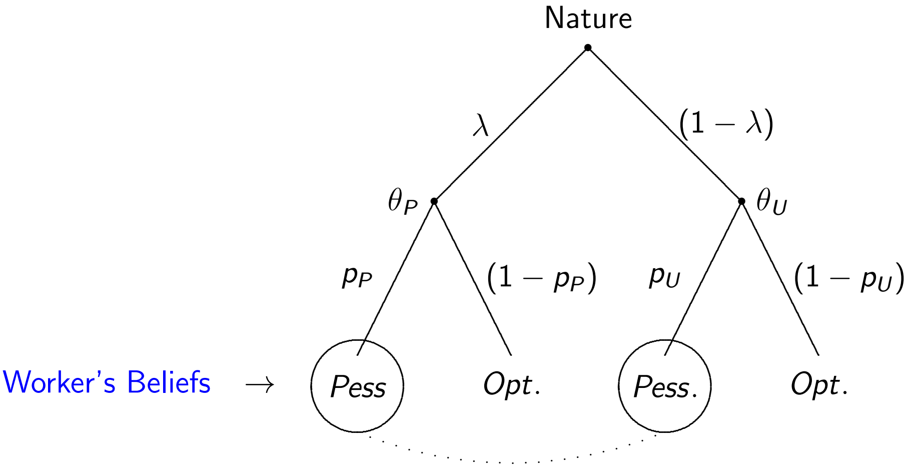
$$\Pr\{\delta|\theta_P\} = p_P$$

$$\Pr\{\delta|\theta_U\} = p_U$$

Type-Dependent Naïveté



Type-Dependent Naïveté



Screening

The employer designs contracts that:

- in **period 1**, screen among workers with **different beliefs**;
- in **period 2**, screen among workers with **different productivity**.

The Problem

The employer solves the following problem:

$$\max_{\{w_j(e)\}_{j=\delta,\phi}} E(\Pi)$$

$$\text{s.t. } (IR_\delta),\ (IR_\phi),$$
$$(IC_\delta),\ (IC_\phi),$$
$$(IC_{P,\delta}),\ (IC_{U,\delta}),$$
$$(IC_{P,\phi}),\ (IC_{U,\phi}).$$

Expanded Constraints

The Problem

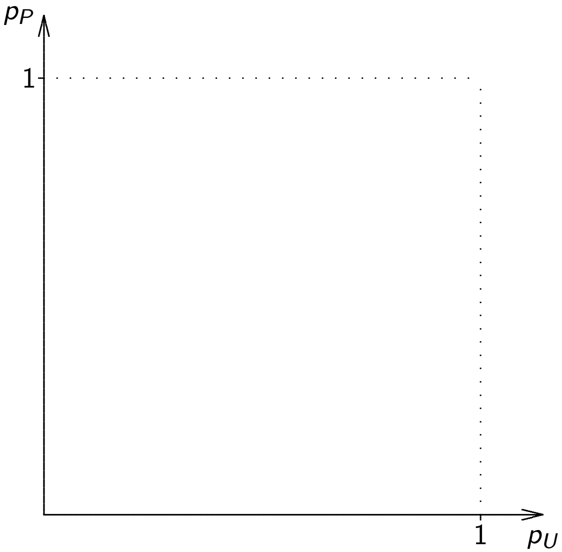
$$\max_{\{w_j(e)\}_{j=\delta,\phi}} E(\Pi)$$

$$\text{s.t. } (IR_\delta) \text{ and } (IC_\phi) \text{ (binding)}$$

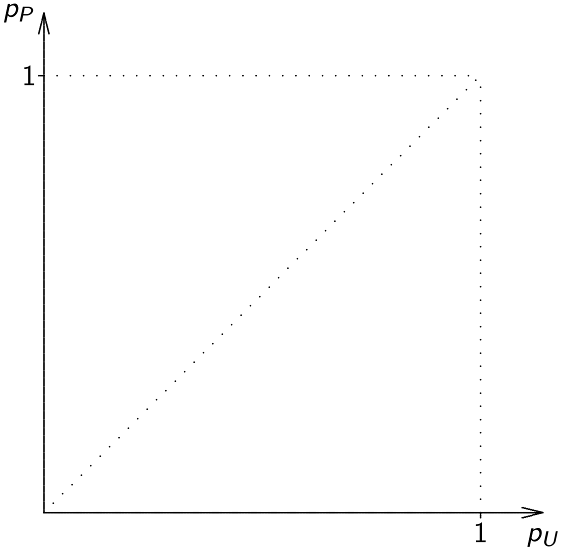
$$(IC_{P,\delta}),\ (IC_{U,\delta}),$$

$$(IC_{P,\phi}),\ (IC_{U,\phi}).$$

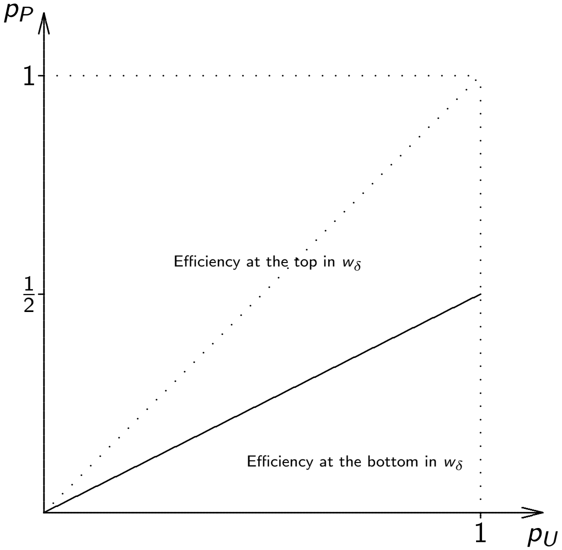
Efficiency with Imperfect Correlation ($\delta = \frac{1}{3}, \lambda = \frac{1}{2}, \phi = \frac{2}{3}$)



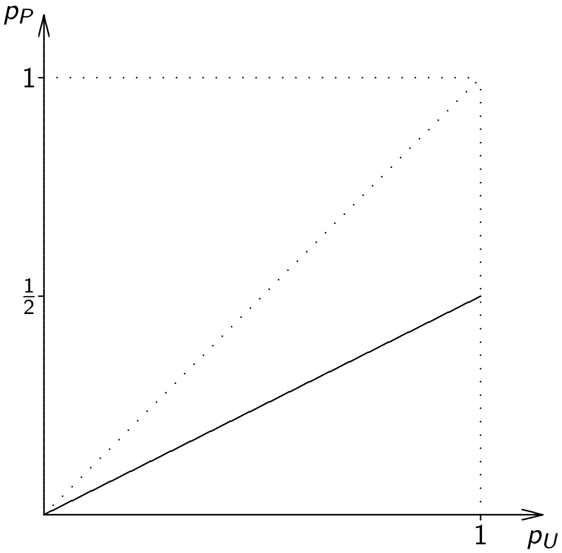
Efficiency with Imperfect Correlation ($\delta = \frac{1}{3}, \lambda = \frac{1}{2}, \phi = \frac{2}{3}$)



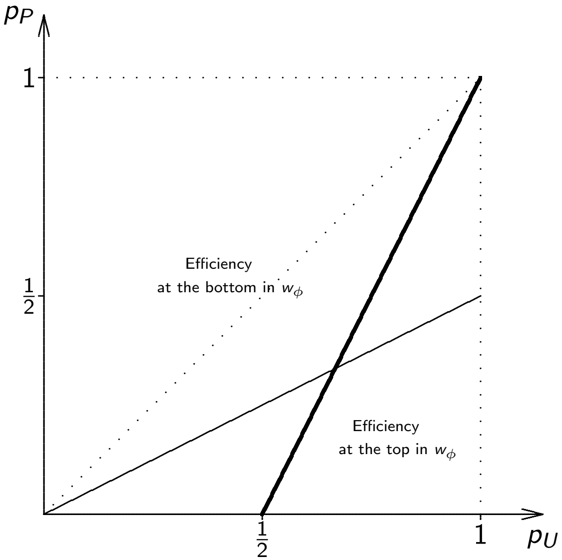
Efficiency with Imperfect Correlation ($\delta = \frac{1}{3}, \lambda = \frac{1}{2}, \phi = \frac{2}{3}$)



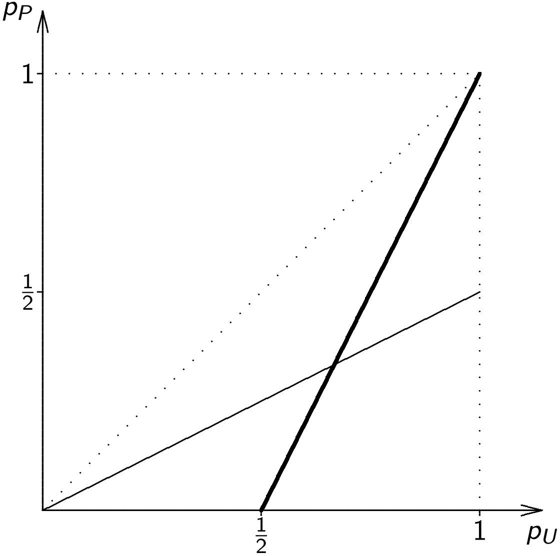
Efficiency with Imperfect Correlation ($\delta = \frac{1}{3}, \lambda = \frac{1}{2}, \phi = \frac{2}{3}$)



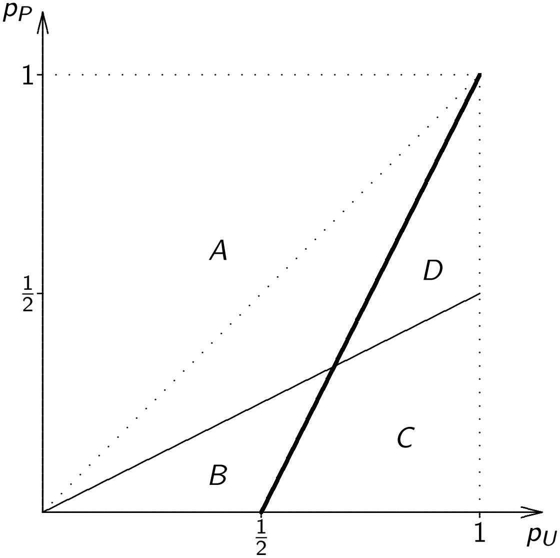
Efficiency with Imperfect Correlation ($\delta = \frac{1}{3}, \lambda = \frac{1}{2}, \phi = \frac{2}{3}$)



Efficiency with Imperfect Correlation ($\delta = \frac{1}{3}, \lambda = \frac{1}{2}, \phi = \frac{2}{3}$)



Efficiency with Imperfect Correlation ($\delta = \frac{1}{3}, \lambda = \frac{1}{2}, \phi = \frac{2}{3}$)



Area	Efficiency
A	top in w_δ bottom in w_ϕ
B	bottom in w_δ bottom in w_ϕ
C	bottom in w_δ top in w_ϕ
D	top in w_δ top in w_ϕ

Contracts:

Area A Area B

Area C Area D

Imperfectly Correlated Type Dimensions

Let $p_P, p_U \notin \{0, 1\}$

Imperfectly Correlated Type Dimensions

Let $p_P, p_U \notin \{0,1\}$

$$\max_{w_j(e)} E(\Pi)$$

$$\text{s.t. } (IR_\delta) \text{ and } (IC_\phi) \text{ (binding)}$$

$$(IC_{P,\delta}),\ (IC_{U,\delta}),$$

$$(IC_{P,\phi}),\ (IC_{U,\phi}).$$

Imperfectly Correlated Type Dimensions

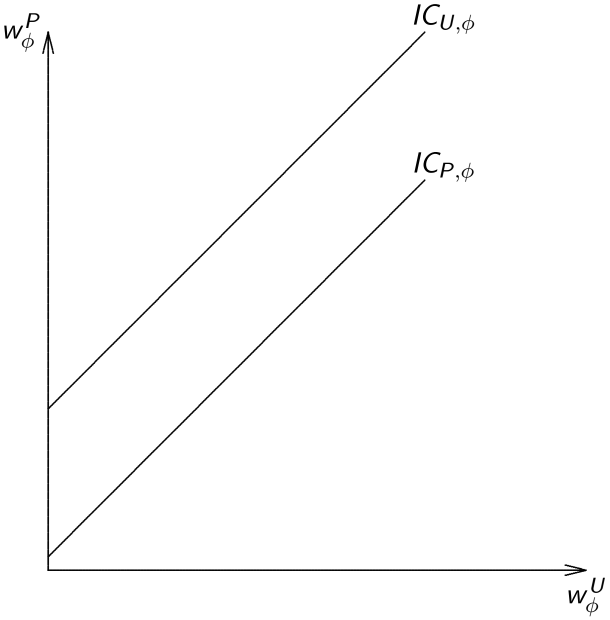
Let $p_P, p_U \notin \{0, 1\}$

$$\max_{w_j(e)} E(\Pi)$$

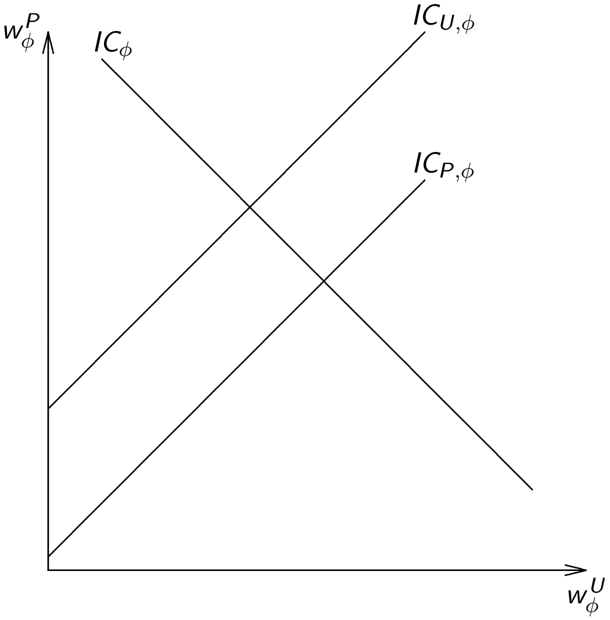
s.t. (IR_δ) and (IC_ϕ) (binding)

$$w_\delta^P - w_\delta^U \leq \theta_U(e_\delta^P - e_\delta^U) \qquad (IC_{U,\delta})$$
$$w_\delta^P - w_\delta^U \geq \theta_P(e_\delta^P - e_\delta^U) \qquad (IC_{P,\delta})$$
$$w_\phi^P - w_\phi^U \geq \theta_P(e_\phi^P - e_\phi^U) \qquad (IC_{P,\phi})$$
$$w_\phi^P - w_\phi^U \leq \theta_U(e_\phi^P - e_\phi^U). \qquad (IC_{U,\phi})$$

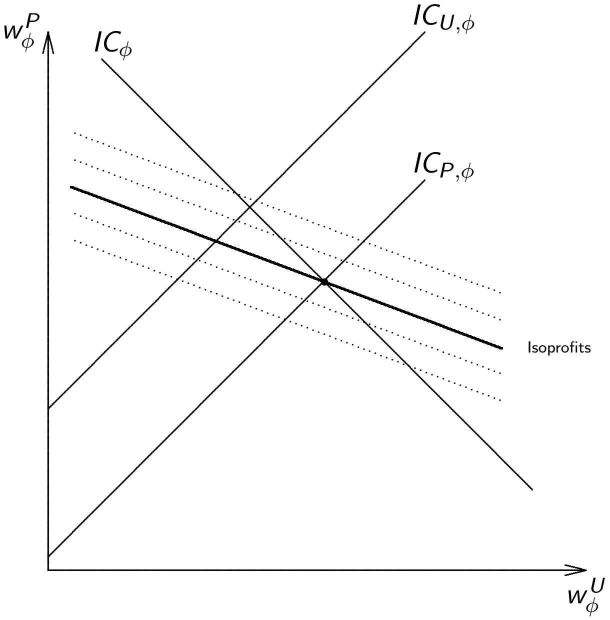
Efficiency for Optimistic Workers



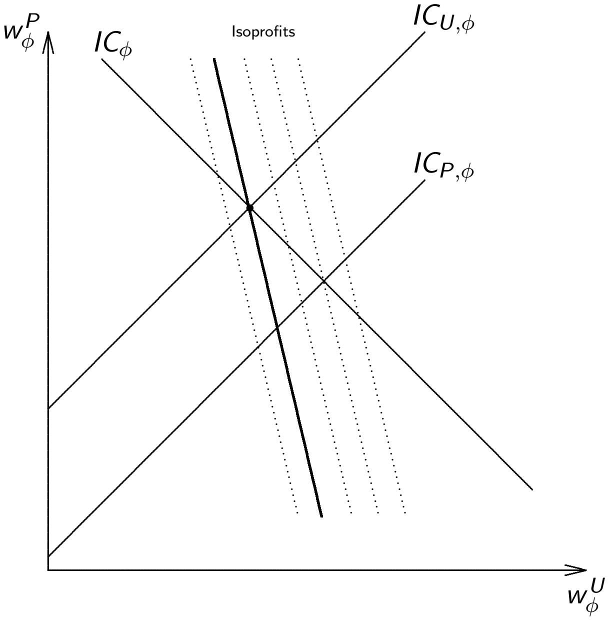
Efficiency for Optimistic Workers



Efficiency for Optimistic Workers



Efficiency for Optimistic Workers



Efficiency for Optimistic Workers

Result 1

If the employer has a strong updated belief that optimistic workers are unproductive, or unproductive optimistic workers are naïve enough, efficiency is at the bottom in the contract for optimistic workers. That is, if

$$\Pr\{\theta_P|\phi\} \leq \qquad \Pr\{\theta_U|\phi\}, \tag{1}$$

then $(IC_{U,\phi})$ *binds.*

Efficiency for Optimistic Workers

Result 2

If the employer has a strong updated belief that optimistic workers are unproductive, or unproductive optimistic workers are naïve enough, efficiency is at the bottom in the contract for optimistic workers. That is, if

$$\Pr\{\theta_P|\phi\} \leq \frac{\phi}{1-\phi}\Pr\{\theta_U|\phi\}, \tag{1}$$

then $(IC_{U,\phi})$ *binds.*

Efficiency for Optimistic Workers

Result 2

If the employer has a strong updated belief that optimistic workers are unproductive, or unproductive optimistic workers are naïve enough, efficiency is at the bottom in the contract for optimistic workers. That is, if

$$\Pr\{\theta_P|\phi\} \leq \tfrac{\phi}{1-\phi} \Pr\{\theta_U|\phi\}, \qquad (1)$$

then $(IC_{U,\phi})$ *binds.*

A similar result holds for pessimistic workers...

Efficiency for Pessimistic Workers

Result 3

If the employer has a strong updated belief that pessimistic workers are productive, or productive pessimistic workers are naïve enough, efficiency is at the top in the contract for pessimistic worker. That is, if:

$$\Pr\{\theta_U|\delta\} \leq \frac{1-\delta}{\delta} \Pr\{\theta_P|\delta\}, \quad (2)$$

then $(IC_{P,\delta})$ binds.

Bunching of Pessimistic Workers

Bunching of Pessimistic Workers

Result 4

Pessimistic workers are separated if and only if:

$$\frac{\delta}{\phi} \geq \Pr\{\phi\} \tag{3}$$

Conclusion

Naïveté's Type-Dependance has strong implications for the efficiency of optimal contracts.

Conclusion

Naïveté's Type-Dependance has strong implications for the efficiency of optimal contracts.

The principal faces a trade-off:

To design efficient contracts either for the most naïve workers in the population, or for the most widespread ones.

Conclusion

Other Results:

If **naïveté and productivity are perfectly correlated**:

- full efficiency is achieved,
- (under some conditions) productive workers obtain zero surplus,
- (under some conditions) pessimistic workers are assigned a contract that induces (imaginary) pooling.

If **naïveté and productivity are imperfectly correlated**:

- (under some conditions) pessimistic workers are bunched together,

Future Research

- relax perfect observability of effort,

Future Research

- relax perfect observability of effort,
- heterogenous distribution of beliefs across equally productive workers
- what if agents could invest in their abilities before the contracting stage?

THANK YOU!

Constraints

$$\max_{\{w_j^i\}_{j=\delta,\phi,i=P,L}} \lambda\left[p_P(y(e_\delta^P) - w_\delta^P) + (1-p_P)(y(e_\phi^P) - w_\phi^P)\right] + $$
$$+(1-\lambda)\left[p_U(y(e_\delta^U) - w_\delta^U) + (1-p_U)(y(e_\phi^U) - w_\phi^U)\right] \quad (4)$$

$$\delta(w_\delta^P - \theta_P e_\delta^P) + (1-\delta)(w_\delta^U - \theta_U e_\delta^U) \geq 0$$
$$\phi(w_\phi^P - \theta_P e_\phi^P) + (1-\phi)(w_\phi^U - \theta_U e_\phi^U) \geq 0$$
$$\delta(w_\delta^P - \theta_P e_\delta^P) + (1-\delta)(w_\delta^U - \theta_U e_\delta^U) \geq \delta(w_\phi^P - \theta_P e_\phi^P) + (1-\delta)(w_\phi^U - \theta_U e_\phi^U)$$
$$\phi(w_\phi^P - \theta_P e_\phi^P) + (1-\phi)(w_\phi^U - \theta_U e_\phi^U) \geq \phi(w_\delta^P - \theta_P e_\delta^P) + (1-\phi)(w_\delta^U - \theta_U e_\delta^U)$$
$$w_\delta^P - \theta_P e_\delta^P \geq w_\delta^U - \theta_P e_\delta^P$$
$$w_\delta^U - \theta_U e_\delta^U \geq w_\delta^P - \theta_U e_\delta^P$$
$$w_\phi^P - \theta_P e_\phi^P \geq w_\phi^U - \theta_P e_\phi^P$$
$$w_\phi^U - \theta_U e_\phi^U \geq w_\phi^P - \theta_U e_\phi^P$$

Back

Optimal Contracts

for pessimistic unproductive and optimistic productive workers

$$y'(e_\delta^U) = \theta_U,$$
$$w_\delta^U = E_\delta(\theta)e_\delta^P + \theta_U(e_\delta^U - e_\delta^P)$$

$$y'(e_\phi^P) = \theta_P,$$
$$w_\phi^P = (E_\delta(\theta) - E_\phi(\theta))e_\delta^P + \theta_P e_\phi^P$$

$$e_\phi^U = 0,$$
$$w_\phi^U = (E_\delta(\theta) - E_\phi(\theta))e_\delta^P$$

$$e_\delta^P = \begin{cases} 1 & \text{if (\textbf{??}) holds} \\ e_\delta^U & \text{if (\textbf{??}) fails,} \end{cases}$$
$$w_\delta^P = E_\delta(\theta)e_\delta^P$$

Back

Optimal Contracts in Area A

$$y'(e_\delta^U) = \frac{E_\delta(\theta) - (1 - E(p))E_\phi(\theta) - p_P\lambda\theta_P}{(1-\lambda)p_U},$$

$$w_\delta^U = E_\delta(\theta)e_\delta^U$$

$$y'(e_\phi^P) = \frac{(1 - E(p))E_\phi(\theta) - (1-\lambda)(1-p_U)\theta_U}{(1-p_P)\lambda},$$

$$w_\phi^P = (E_\delta(\theta) - E_\phi(\theta))e_\delta^P + E_\phi(\theta)e_\phi^P$$

$$y'(e_\phi^U) = \theta_U,$$

$$w_\phi^U = (E_\delta(\theta) - E_\phi(\theta))e_\delta^U + E_\phi(\theta)e_\phi^P + \theta_U(e_\phi^U - e_\phi^P)$$

$$y'(e_\delta^P) = \theta_P,$$

$$w_\delta^P = E_\delta(\theta)e_\delta^U + \theta_P(e_\delta^P - e_\delta^U)$$

Back

Optimal Contracts in Area B

$$y'(e_\delta^U) = \theta_U,$$
$$w_\delta^U = E_\delta(\theta)e_\delta^P - \theta_U(e_\delta^P - e_\delta^U)$$

$$y'(e_\phi^P) = \frac{(1 - E(p))E_\phi(\theta) - (1-\lambda)(1-p_U)\theta_U}{(1-p_P)\lambda},$$
$$w_\phi^P = (E_\delta(\theta) - E_\phi(\theta))e_\delta^P + E_\phi(\theta)e_\phi^P$$

$$y'(e_\phi^U) = \theta_U,$$
$$w_\phi^U = (E_\delta(\theta) - E_\phi(\theta))e_\delta^P + E_\phi(\theta)e_\phi^P + \theta_U(e_\phi^U - e_\phi^P)$$

$$y'(e_\delta^P) = \frac{E_\delta(\theta) - (1 - E(p))E_\phi(\theta) - p_P(1-\lambda)\theta_P}{\lambda p_U},$$
$$w_\delta^P = E_\delta(\theta)e_\delta^P$$

Back

Optimal Contracts in Area C

$$y'(e_\delta^U) = \theta_U,$$
$$w_\delta^U = E_\delta(\theta)e_\delta^P - \theta_U(e_\delta^P - e_\delta^U)$$

$$y'(e_\phi^P) = \theta_P,$$
$$w_\phi^P = (E_\delta(\theta) - E_\phi(\theta))e_\delta^P + E_\phi(\theta)e_\phi^U + \theta_P(e_\phi^P - e_\phi^U)$$

$$y'(e_\phi^U) = \frac{(1 - E(p))E_\phi(\theta) - \lambda(1 - p_U)\theta_P}{(1 - p_P)(1 - \lambda)},$$
$$w_\phi^U = (E_\delta(\theta) - E_\phi(\theta))e_\delta^P + E_\phi(\theta)e_\phi^U$$

$$y'(e_\delta^P) = \frac{E_\delta(\theta) - (1 - E(p))E_\phi(\theta) - p_P(1 - \lambda)\theta_U}{\lambda p_U},$$
$$w_\delta^P = E_\delta(\theta)e_\delta^P$$

Back

Optimal Contracts in Area D

$$y'(e_\delta^U) = \frac{E_\delta(\theta) - (1 - E(p))E_\phi(\theta) - p_U\lambda\theta_P}{(1-\lambda)p_P},$$
$$w_\delta^U = E_\delta(\theta)e_\delta^U$$

$$y'(e_\phi^P) = \theta_P,$$
$$w_\phi^P = (E_\delta(\theta) - E_\phi(\theta))e_\delta^U + E_\phi(\theta)e_\phi^U + \theta_P(e_\phi^P - e_\phi^U)$$

$$y'(e_\phi^U) = \frac{(1 - E(p))E_\phi(\theta) - \lambda(1 - p_U)\theta_P}{(1-p_P)(1-\lambda)},$$
$$w_\phi^U = (E_\delta(\theta) - E_\phi(\theta))e_\delta^U + E_\phi(\theta)e_\phi^U$$

$$y'(e_\delta^P) = \theta_P,$$
$$w_\delta^P = E_\delta(\theta)e_\delta^U + \theta_P(e_\delta^P - e_\delta^U)$$

Back