

Securitization and Aggregate Investment Efficiency*

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Abstract

This paper studies the efficiency of competitive equilibria in economies where the expansion of investment is facilitated by securitization. We show that the use of securitization is generally associated with constrained inefficient aggregate investment, thereby justifying regulatory intervention in markets for securitized assets. We examine the effectiveness of three policy instruments to address this inefficiency: ex-ante capital / leverage requirements, skin-in-the game (retention) requirements and initiatives to improve transparency in the securitization process. We find that leverage/capital restrictions and improved transparency can increase welfare in our environment, but that forcing originators to hold additional skin-in-the game can never increase welfare.

JEL codes: D52, D53, E44, G18, G23.

Keywords: Securitization, pecuniary externalities, financial frictions, macroprudential regulation, fire-sales, incomplete markets, retention requirements, skin in the game.

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1 Introduction

Securitization refers to the process by which financial intermediaries transform illiquid assets, such as corporate loans or mortgages, into marketable securities. As a result, securitization allows loans which were traditionally held to maturity by originators, to be sold (at least in part) as securitized assets to outsiders. Typically, a special purpose vehicle (SPV) is set up by a sponsoring financial intermediary, which purchases a pool of assets from the sponsor and/or other originators. These purchases are financed by sales of asset-backed securities (ABS) to institutional investors, that are backed by the cash-flows from the pool. One of the appeals of ABS is that they can be designed to suit institutional investors' preferences for relatively safe assets. The rise of this originate-to-distribute model has led to significant alterations in global capital markets and the nature of financial intermediation. In the US for example, non-agency ABS outstanding grew from approximately \$11.3 billion in 1986 to over \$1.36 trillion in 2015, and coincided with the increase of specialty non-bank lenders such as Countrywide Financial. Despite a number of flaws in the securitization process, highlighted in the fallout of the financial crisis of 2007-2009, growth in securitization is generally viewed as having increased overall credit availability and lowered the cost of credit in advanced economies.¹

Financial intermediaries value securitization because it allows them to alter the size and composition of their balance sheets when markets for the underlying assets are incomplete. Specifically, motivated by standard liquidity / risk-management considerations or binding regulatory requirements, intermediaries can use securitization to shed asset risk. This allows them to re-deploy capital towards alternative, possibly more profitable, investment opportunities. However, as is well known from the theoretical literature, competitive equilibria can exhibit excessive aggregate investment when markets are incomplete.² Thus, while securitization may raise investment through enhanced risk-sharing, it may cause over-investment, and as such the welfare implications of securitization are unclear. The main contribution of this paper is to show that aggregate investment is always constrained inefficient in economies where securitization is useful in expanding investment.

We develop a dynamic general equilibrium model of investment with three periods ($t = 0, 1, 2$). In period 0, risk-neutral borrowers with limited capital, whom we refer to as intermediaries, finance risky investment projects by creating and selling safe debt (ABS) backed

¹For surveys on securitization, see [Gorton and Metrick \(2012\)](#), and/or [Segoviano et al. \(2015\)](#). We provide evidence on the growth of ABS and demand from institutional investors in Appendix C, for both the United States and Europe. See also [Acharya and Schnabl \(2010\)](#) and [Claessens et al. \(2012\)](#) for discussions on the myriad factors behind the growth of ABS.

²See for example [Lorenzoni \(2008\)](#). Generally speaking, opening additional markets does not necessarily increase overall welfare, a result first established by [Hart \(1975\)](#).

by the investments' cash-flows. Investment returns are subject to both idiosyncratic and aggregate risk. In creating safe debt, intermediaries are limited by their capital and the riskiness of returns as they retain residual risk. Projects may succeed early (period 1) yielding complete returns, or partly succeed late (period 2) yielding partial returns, or fail late returning nothing. The overall fraction of intermediaries' projects that succeed is uncertain, and depends on the underlying state of the economy. By engaging in securitization at period 0, individual intermediaries can shed idiosyncratic risk, thereby increasing their capacity to create and issue safe debt. Importantly, and in line with empirical evidence, we assume that frictions in the securitization process limit the amount of idiosyncratic risk that intermediaries can shed. This is modeled in the form of a skin-in-the-game constraint on originators. Thus, while ex-ante homogeneous, intermediaries will differ at period 1 as returns on their individual investments may be early or late, with the degree of heterogeneity dependent upon the amount of securitization at $t = 0$.³ At period 1, we assume those with early returns (early types) always have sufficient funds to service their debt obligations and invest in new opportunities. On the other hand, if there is a recession late types require outside funding to undertake new investment opportunities and service debt. Financial frictions rule out state-contingent contracts at period 0, and new borrowing at period 1 (investor preferences rule out default). To raise funds, late types sell their late investments to early types via a spot market (fire-sale). However, they may be constrained in their ability to raise funds, in which case they will have to forgo new positive NPV investments (credit-crunch). The extent to which late borrowers are constrained will depend directly on their investment and securitization decisions at time 0, and indirectly through the prevailing asset price, which is a function of both the aggregate funds early types have (demand), and the aggregate quantity of assets for sale (supply). Crucially, as atomistic intermediaries do not anticipate the impact of their period 0 decisions on the asset price at period 1, a pecuniary externality arises that results in socially excessive investment and securitization at time 0 whenever late types are constrained at period 1. In fact, we show that the reduction in return variability via securitization is only valuable when late types are constrained, which is precisely when aggregate investment in the economy is constrained inefficient.

To better understand the role of securitization in our model, note that a reduction in the variability of asset returns via securitization mitigates the impact of financial frictions by moving funds from early to late types at period 1. This substitutes for contingent contracts or direct borrowing, which might otherwise provide such transfers but are assumed to be

³Intermediation here may be viewed as market finance, which is often also referred to as shadow banking. We abstract from regulatory arbitrage motives for securitization, see for example [Acharya et al. \(2013\)](#). We also ignore tax considerations as discussed in [Han et al. \(2015\)](#).

unavailable. As a result, intermediaries can create more safe debt for investors by increasing pledgeable income when asset returns are late. This is the standard partial equilibrium view of how more securitization can lead to higher leverage and investment.⁴ However, our framework is novel in that it highlights an aspect of securitization previously unstudied; that securitization also affects spot market prices by altering the distribution of cash in the market. Specifically, with more securitization at period 0, demand for assets at time 1 declines since the funds of early types are reduced. On the supply side, late types require less funds and have more assets to sell. Overall, we show there is a reduction in the price of assets at period 1, creating a transfer from late to early types and thereby exacerbating the financial frictions and the pecuniary externality.

Another key contribution of this paper is that it provides a framework to analyze the welfare implications of policies designed to curb excessive leverage and limit securitization in the financial sector. We show that leverage restrictions, akin to those outlined in the Basel III reforms, can be welfare improving when the competitive equilibrium is characterized by over-investment. Importantly, since securitized lending is at the heart of the (mostly unregulated) shadow banking sector, our results suggest broader regulation of the financial industry may be valuable.

It is plausible that welfare could be improved by recently enacted policies forcing originators to hold more skin-in-the-game, since securitization affects leverage indirectly. For example, the retention requirements specified in the U.S. Dodd-Frank Act and the European Capital Requirements Directive. These policies are interesting not only because they are a part of the current policy discussion, but because they also apply to both regulated and unregulated entities. It is also reasonable to assume that they require significantly less information than direct restrictions on the balance sheet, such as capital or leverage constraints. In our model, the total effect of forcing more skin-in-the-game can be decomposed into a direct effect and a price effect. There is a direct tightening of the constraints on late intermediaries, which reduces the resources available to intermediaries in a bad state of the world, causing them to reduce leverage ex-ante. On the other hand, reduced aggregate investment increases the price of assets in a fire-sale, which in turn increases the returns to intermediaries in the fire-sale and results in increased leverage. The direct effect is obvious and provides an intuitive rationale for tightening constraints as a means to reduce excessive investment. However, this is undone by the price effect, leaving the negative impact of the tighter constraint to dominate. This makes clear that the rationale for policies to increase skin-in-the-game as a means to reduce excessive leverage cannot rely solely on partial

⁴See for example [DeMarzo \(2005\)](#), [Coval et al. \(2009\)](#), [Gorton and Metrick \(2012\)](#), and [Kiff and Kisser \(2014\)](#).

equilibrium arguments.

Finally, we consider the effect of policies aimed at improving transparency in securitization markets. Such policies can be interpreted as reducing the required skin-in-the-game, which is welfare-improving in our environment in the absence of implementation costs. Thus, through the lens of this model at least, it would seem that efforts to improve transparency conflict with policies forcing originators to hold more skin-in-the game.

Related Literature

This paper presents a novel model of a fire-sale induced credit-crunch. Unlike previous models, such as [Lorenzoni \(2008\)](#) and [Stein \(2012\)](#), both demand (total cash held by early types) and supply (total assets for sale held by late types) in the fire-sale are endogenously determined in our framework. Endogenizing cash in the market is necessary to examine the ex-ante risk-sharing function of securitization. Thus, we are able to show that more securitization simultaneously increases the quantity of assets available for sale, while at the same time reducing funds available on the demand side. As a result, our model captures situations where outside liquidity is endogenously very limited, as we might expect in a severe financial crisis.⁵

Our paper is most closely related to [Gennaioli et al. \(2013\)](#), where the main role of securitization is also to pool idiosyncratic risk, allowing the financial sector to increase investment through higher leverage.⁶ Our framework is a generalization of their benchmark model with rational expectations. The key difference is that we assume the existence of frictions in the securitization process that limit risk-sharing, which together with financial frictions lead to constrained inefficient competitive equilibria in our model. [Gennaioli et al. \(2013\)](#) focus on a different inefficiency associated with securitization, namely the inability of investors to recognize aggregate risk. However, under rational expectations, the use of securitization in [Gennaioli et al. \(2013\)](#) is completely efficient, whereas in our environment this is not the case. In fact, securitization is privately valuable in our environment *only* when aggregate investment is constrained inefficient.

There is a growing literature on securitization, which for the most part focuses on security design and the contractual features arising from asymmetric information. For example, [DeMarzo and Duffie \(1999\)](#) and [DeMarzo \(2005\)](#) examine security design problems in static settings whereas [Hartman-Glaser et al. \(2012\)](#) focus on a dynamic problem. [Hanson and Sunderam \(2013\)](#) consider a security design problem incorporating endogenous information

⁵For example, in the financial crisis of 2007-2009, the lack of outside capital was partly evidenced by the US government's decision to rescue institutions through mergers rather than seeking recapitalizations.

⁶[Diamond \(1984\)](#) first showed risk-pooling by financial intermediaries can increase investment and welfare.

acquisition by investors. [Shleifer and Vishny \(2010\)](#) develop a novel model of securitization where intermediaries sell assets to maximize fee revenue from intermediation. In contrast, this paper focuses on the general equilibrium effects of securitized lending and the associated welfare implications, taking the inefficiencies in the securitization process as given. [Gale and Gottardi \(2015\)](#) also examine the impact of pecuniary externalities on ex-ante capital structure of intermediaries. Debt is preferable to equity in their model due to tax advantages, and they identify the impact of fire-sales by bankrupt firms on the ex-ante capital structure. Our paper differs in that we abstract from tax advantages of debt and the possibility of default in equilibrium, while focusing instead on the role of securitization on ex-ante capital structure.

Finally, this paper is related to the extensive literature on pecuniary externalities which arise from incomplete markets. This literature goes back to the seminal work of [Hart \(1975\)](#), [Diamond \(1980\)](#), [Stiglitz \(1982\)](#), [Geanakoplos and Polemarchakis \(1986\)](#) and [Greenwald and Stiglitz \(1986\)](#). In our application, market incompleteness precludes individuals from equalizing marginal returns to investment. This is similar to the type of friction studied in [Shleifer and Vishny \(1992\)](#), [Gromb and Vayanos \(2002\)](#), [Caballero and Krishnamurthy \(2001\)](#), [Allen and Gale \(2004\)](#), [Lorenzoni \(2008\)](#), [Farhi et al. \(2009\)](#), [Davila et al. \(2012\)](#), and [He and Kondor \(2016\)](#), among others.⁷ This paper shows how pecuniary externalities may result in inefficient investment when the securitization process is plagued by frictions. Thus, we link the literature on investment with incomplete markets and asset securitization. This allows us to study welfare and examine policies in a well-understood framework.

2 Model

We consider a three-period economy ($t = 0, 1, 2$), populated by a measure one of both investors and intermediaries.

2.1 Investor's Problem

Investors are endowed with wealth w at $t = 0$, and have preferences:

$$U = E_0 \left[c_0 + \beta \min_{\omega \in \Omega} \{c_{2,\omega}\} \right], \quad (1)$$

⁷[Krishnamurthy \(2010\)](#) and [Brunnermeier and Oehmke \(2013\)](#) survey the literature on aggregate implications of financial frictions. [Davila \(2015\)](#) provides a discussion of several key papers in this literature, including the impact of various modeling assumptions on welfare analysis. In his terminology, we model a “terms-of-trade” externality.

where c_0 is consumption at $t = 0$, $c_{2,\omega}$ is consumption at period 2 in state $\omega \in \Omega$, and $\beta \in (0, 1)$ is a discount factor. These preferences for investors are similar to those in [Stein \(2012\)](#) and [Gennaioli et al. \(2013\)](#), and capture evidence that a large class of investors that purchase ABS, such as pension or mutual funds, have a strong desire for safety.⁸ Investors do not have direct access to investment opportunities, but can save by purchasing safe debt claims, as well as risky assets from intermediaries. As we show in the next section however, they do not purchase any risky assets in equilibrium. We denote investors' purchases of safe debt (savings) by B , which pays a gross return r per unit at $t = 2$.⁹ Debt purchases are chosen to maximize (1), subject to the following budget constraints at $t = 0, 2$:

$$c_0 + B \leq w, \tag{2}$$

$$c_{2,\omega} \leq rB \quad \forall \omega \in \Omega. \tag{3}$$

2.2 Intermediary's Problem

Intermediaries are risk-neutral and endowed with k resources at $t = 0$. For simplicity, we assume that intermediaries do not discount the future, and so are indifferent between consumption at $t = 0, 1, 2$. Each intermediary has access to risky investment projects at $t = 0$, which can either succeed and return R_0 , or fail and return nothing. Projects may succeed early at $t = 1$, or late at $t = 2$, and returns are subject to both idiosyncratic and aggregate risk. The probability of success at $t = 1$ is identical and independent across intermediaries and varies with the aggregate state ω at period 1, where $\omega \in \Omega \equiv \{g, b\}$. The state g captures a “good” or growth state where intermediaries' projects are successful, whereas b captures a “bad” state where relatively few intermediaries' projects succeed. The probability that the good state occurs at $t = 1$ is q_1 , while the probability of the bad state is $1 - q_1$. We assume that all projects succeed early if the state is good at $t = 1$, whereas only a fraction $\alpha < 1$ succeed early if the state is bad. Moreover, remaining projects provide only a fraction $\theta < 1$ of their original return upon success.¹⁰ The probability that remaining projects succeed at $t = 2$ depends on the aggregate state ω at $t = 2$, which may also be either good or bad. The probability of success at $t = 2$ is equal to 1 if the good state occurs,

⁸See [Bernanke et al. \(2011\)](#), as well as [Stein \(2012\)](#) and [Gennaioli et al. \(2013\)](#) for discussions on the desire for safety by institutional investors that represent savers. A demand for safety can also be interpreted as a convenience yield if there is a demand for the use of ABS as collateral in sale and repurchase agreements, see [Gorton and Metrick \(2012\)](#).

⁹The use of short-term safe debt generates qualitatively similar results in our environment.

¹⁰If we interpret intermediary investments as mortgages for example, then a bad state not only results in more failures, but the underlying homes may also be worth less. Alternatively, θ may capture early failures within an intermediary's portfolio of mortgages. Such early failures were characteristic of sub-prime mortgages issued in the United States prior to the recent crisis.

or 0 if the bad state is realized. The probability of the good state at $t = 2$, conditional on the good state at $t = 1$ is $q_{2,g}$, while the probability is $q_{2,b}$, conditional on the bad state at $t = 1$. Thus, the expected gross returns on period 0 investments at $t = 0$ can be written as

$$E_{\pi}R_0 \equiv [q_1 + (1 - q_1)(\alpha + (1 - \alpha)q_{2,b}\theta)] R_0. \quad (4)$$

Projects that do not succeed at $t = 1$ become “impaired” in two ways, since the return upon success is reduced by $(1 - \theta)R_0$, and the probability of failure increases from $(1 - q_1)(1 - \alpha)(1 - q_{2,b})$ to $1 - q_{2,b}$.

Each intermediary also has access to new risky investment projects at $t = 1$, that either succeed or fail at $t = 2$. The gross return per unit of investment is R_1 in the case of success and zero otherwise. We assume that returns on $t = 1$ investments are perfectly correlated across intermediaries, thus they all succeed if the state is good at $t = 2$ or they all fail if the state is bad at $t = 2$. Assuming that new investments are not subject to any idiosyncratic risk is not crucial, but allows us to focus on one round of securitization. Also, this implies that returns on new investments in the bad state at $t = 1$ are perfectly correlated with returns on existing investments. This captures evidence of increased correlation across asset returns in bad times.

Investment at any period requires intermediaries to incur non-pecuniary costs $C(I)$. We assume $C(\cdot)$ is a strictly convex function such that $C', C'' > 0$ and $C(0) = C'(0) = 0$. We interpret these costs as effort required to find and maintain quality investments, and assume these costs are small enough to ensure investment at $t = 1$ is always socially worthwhile.

ASSUMPTION 1. (*Positive NPV opportunities at $t = 1$*) $q_{2,\omega}R_1 - C'(R_0(k + w)) > 1 \forall \omega$.

This assumption allows for a credit-crunch at $t = 1$, which is necessary for the existence of a constrained inefficient equilibrium in our model. We discuss the importance of this assumption following Proposition 2.

At the start of $t = 0$, each intermediary invests I_0 while holding Y_0 in cash. Investments and reserves are financed with intermediary wealth k and through funds raised by issuing claims to investors. Intermediaries issue long-term risk-less debt D at $t = 0$, promising a gross return r at $t = 2$. They may also sell cash flows associated with $S_0 \leq (1 - a)I_0$ units of their own investment, where $a \in [0, 1)$ is an exogenous “skin-in-the-game” requirement limiting asset sales. We do not model the reasons for skin-in-the-game explicitly, but this can be justified by the existence of frictions in the securitization process, and/or regulatory requirements as discussed in Section 1.¹¹ By the same token, intermediaries can purchase

¹¹Skin in the game can arise as an optimal contractual arrangement due to informational asymmetries

cash flows T_0 from other intermediaries. We interpret T_0 as the cash-flows derived from a pool of all other intermediaries' assets. Although an intermediary's own projects have the same expected payoffs per unit as the pool of other intermediaries' assets, due to diversification the latter bear no idiosyncratic risk, only aggregate risk. This is important because this diversification allows intermediaries to increase pledgeable cash-flows in the bad state when their asset returns may be late. In other words, cash-flows from the pool of assets T_0 provide better collateral than the cash-flows from the intermediary's own investment I_0 . Such collateral may be valuable if frictions limit the ability of intermediaries to raise funds.

The purchases and sales of cash-flows by intermediaries are interpreted as a standard form of securitization. Each intermediary can be viewed as creating a "special-purpose vehicle" (SPV) that purchases a pool of intermediary assets and issues ABS. In this interpretation, intermediaries' retain the most junior "tranche" (equity) in the SPV while the senior "tranche" (safe debt) is sold to investors with a commitment by the intermediary to provide the SPV with a liquidity guarantee of rD . For ease of exposition, we refer to the junior tranche as ABS and the senior tranche as debt, however these can both be interpreted as ABS. Securitization in our model thus amounts to pooling idiosyncratic risk across intermediaries.¹²

The decisions of intermediaries at $t = 1$ consist of investing in new opportunities, purchasing or selling securitized assets from other intermediaries, selling cash flows against their own $t = 0$ investments that have not yet been realized, or holding cash. In the good state, all intermediaries are identical as all $t = 0$ investments succeed and are realized early. As a result, there is no motive for trade, and each intermediary makes $I_{1,g}$ new investments and holds $Y_{1,g}$ in cash. In the bad state, early intermediaries have relatively more funds available for investment, and thus intermediaries with late returns may have access to relatively profitable investment opportunities that cannot be exploited. We denote early types by e and late types by l . Early intermediaries invest an amount $I_{1,b}^e$ at $t = 1$, while late intermediaries invest $I_{1,b}^l$. Funds may be transferred between intermediaries through the exchange of assets or via the sale of remaining cash-flows on $t = 0$ investment. Importantly, given investors' preferences, intermediaries have no other means to generate funds from outsiders.¹³ We also

between originators of securities and outsiders, as modeled in [DeMarzo and Duffie \(1999\)](#). This can arise in other interpretations, such as [Gorton and Pennacchi \(1995\)](#), where this type of structure arises to address moral hazard. [Cerasi and Rochet \(2014\)](#) also provide a model of securitization of this type, in which banks hold an equity tranche to maintain proper incentives.

¹²We do not distinguish between originating intermediaries, sponsoring intermediaries and "special-purpose vehicles" (SPV). Ignoring the difference between the latter two is not vital since these are typically artificial constructs, operating according to a set of pre-specified rules. We ignore the difference between the former two for tractability. A detailed discussion of the process can be found in [Gorton and Metrick \(2012\)](#).

¹³As $t = 1$ investments may yield nothing and investors are infinitely risk averse, they never lend additional funds at time 1. This can be relaxed without altering the qualitative nature of our results, as long as the possibility that late intermediaries may be financially constrained is retained.

assume that funds can only be raised from other intermediaries via asset sales.

ASSUMPTION 2. (*Market Incompleteness*) *Intermediaries cannot write state-contingent contracts and cannot directly borrow and lend to each other at $t = 1$.*

The missing markets we assume are a precondition for securitized lending in our model, since the existence of contingent securities at period 0 or frictionless borrowing at period 1 eliminates the value in securitizing assets. While we are agnostic about the specific market failure(s) that result in borrowing constraints, the literature has highlighted a number of possibilities.¹⁴ Ruling out *all* borrowing at $t = 1$ is done for ease of exposition and is not vital for our results. A discussion on relaxing Assumption 2 can be found in Section 4.2.1.

Denote early intermediaries' period 1 purchases of securitized assets by $T_{1,b}^e$. Late intermediaries' sales of securitized assets are $-T_{1,b}^l$, while sales of remaining cash-flows on $t = 0$ assets are $S_{1,b}^l$. Cash holdings of early and late intermediaries are denoted $Y_{1,b}^e$ and $Y_{1,b}^l$ respectively. We now formally define the intermediary's problem, which is to choose I_0 , S_0 , T_0 , D , Y_0 , $I_{1,g}$, $I_{1,b}^e$, $I_{1,b}^l$, $T_{1,b}^e$, $T_{1,b}^l$, $S_{1,b}^l$, $Y_{1,g}$, $Y_{1,b}^e$, and $Y_{1,b}^l$ to maximize expected profits at $t = 0$, denoted Π_0 :

$$\Pi_0 = q_1 \cdot \Pi_{1,g} + (1 - q_1) \cdot (\alpha \Pi_{1,b}^e + (1 - \alpha) \Pi_{1,b}^l) - C(I_0) - rD, \quad (5)$$

where

$$\begin{aligned} \Pi_{1,g} &= q_{2,g} R_1 I_{1,g} + Y_{1,g} - C(I_{1,g}), \\ \Pi_{1,b}^e &= q_{2,b} (R_1 I_{1,b}^e + \theta R_0 [(1 - \alpha) T_0 + T_{1,b}^e]) + Y_{1,b}^e - C(I_{1,b}^e), \\ \Pi_{1,b}^l &= q_{2,b} (R_1 I_{1,b}^l + \theta R_0 [I_0 - S_0 + (1 - \alpha) T_0 + T_{1,b}^l - S_{1,b}^l]) + Y_{1,b}^l - C(I_{1,b}^l). \end{aligned}$$

The first term in (5), $\Pi_{1,g}$, is expected profit at $t = 1$ in the good state. In the good state, recall that all $t = 0$ projects succeed early and the proceeds are either re-invested in new opportunities, with gross expected returns $q_{2,g} R_1 I_{1,g} - C(I_{1,g})$, or held as reserves, $Y_{1,g}$. Note that as intermediaries are all identical in this case, and each has sufficient funds to repay investors, there is no trade. The second term in (5) is expected profit at period 2 in the bad state. Expected profits are a weighted sum of early and late types' profits, $\Pi_{1,b}^e$ and $\Pi_{1,b}^l$ respectively. Profits for early types consist of expected returns on new investment, $q_{2,b} R_1 I_{1,b}^e - C(I_{1,b}^e)$, reserves carried into period 2, $Y_{1,b}^e$, and late returns on securitized assets purchased either at $t = 0$ or $t = 1$, $\theta R_0 [(1 - \alpha) T_0 + T_{1,b}^e]$. Similarly, profits for late types

¹⁴For example, limits to borrowing may be justified by the presence of asymmetric information as in [Stiglitz and Weiss \(1981\)](#), limited commitment following [Kehoe and Levine \(1993\)](#), or moral hazard as in [Gorton and Pennacchi \(1995\)](#).

consist of investment returns, $q_{2,b}R_1I_{1,b}^l - C(I_{1,b}^l)$, reserves $Y_{1,b}^l$, and late returns on assets not sold, $\theta R_0 [I_0 - S_0 + (1 - \alpha)T_0 + T_{1,b}^l - S_{1,b}^l]$. Finally, the last two terms in (5) capture the costs of investment in the initial period, $C(I_0)$, and debt repayment, rD . Intermediaries maximize (5) subject to the following set of constraints:

$$(\lambda_0) I_0 + p_0(T_0 - S_0) + Y_0 \leq k + D, \quad (6)$$

$$(\lambda_{1,g}) I_{1,g} + Y_{1,g} \leq R_0(I_0 + T_0 - S_0) + Y_0, \quad (7)$$

$$(\lambda_{1,b}^e) I_{1,b}^e + p_1T_{1,b}^e + Y_{1,b}^e \leq R_0(I_0 - S_0) + \alpha R_0T_0 + Y_0, \quad (8)$$

$$(\lambda_{1,b}^l) I_{1,b}^l + p_1(T_{1,b}^l - S_{1,b}^l) + Y_{1,b}^l \leq \alpha R_0T_0 + Y_0, \quad (9)$$

$$(\mu_{1,S}) S_0 + S_{1,b}^l \leq (1 - a)I_0, \quad (10)$$

$$(\mu_{1,T}) 0 \leq T_{1,b}^l + (1 - \alpha)T_0, \quad (11)$$

$$(\eta_{1,g}) rD \leq Y_{1,g}, \quad (\eta_{1,b}^l) rD \leq Y_{1,b}^e, \quad (\eta_{1,b}^e) rD \leq Y_{1,b}^l. \quad (12)$$

Inequality (6) is the budget constraint at $t = 0$, which requires investment costs, net purchases at price p_0 and reserves be no greater than equity and debt. Expressions (7)-(9) are the budget constraints at $t = 1$ in the good state, and for the early and late intermediaries in the bad state respectively. Early intermediaries projects have been successful, resulting in $R_0(I_0 - S_0)$ more funds than late types. They can use the returns from their individual investments, along with securitized assets, to purchase assets from late ones at a price p_1 or invest in new opportunities and reserves. Late intermediaries use returns from securitized assets, plus funds raised from asset sales, to finance new investment and reserves. Constraints (10) and (11) ensure that individual and securitized asset sales are feasible and satisfy the skin-in-the-game requirement (we ignore the analogous constraints on early types since they are never binding in equilibrium). The final set of constraints are the intermediaries' collateral constraints that ensure debt is always repaid.¹⁵ The solution to this problem is characterized in Appendix A, where the Lagrange multipliers associated with each constraint are given in brackets above.

3 Equilibrium

In this section we characterize the competitive market equilibrium. The intermediary problem is to choose investment, reserves, trade and debt levels to maximize expected prof-

¹⁵In our environment, the long term contracts between intermediaries and investors are renegotiation-proof. To see this, note that we could simply re-interpret the contracts as short-term, which are then always rolled over at $t = 1$ in equilibrium. However, the inability to commit to long term contracts may be a potentially important source of inefficiency in a more general environment.

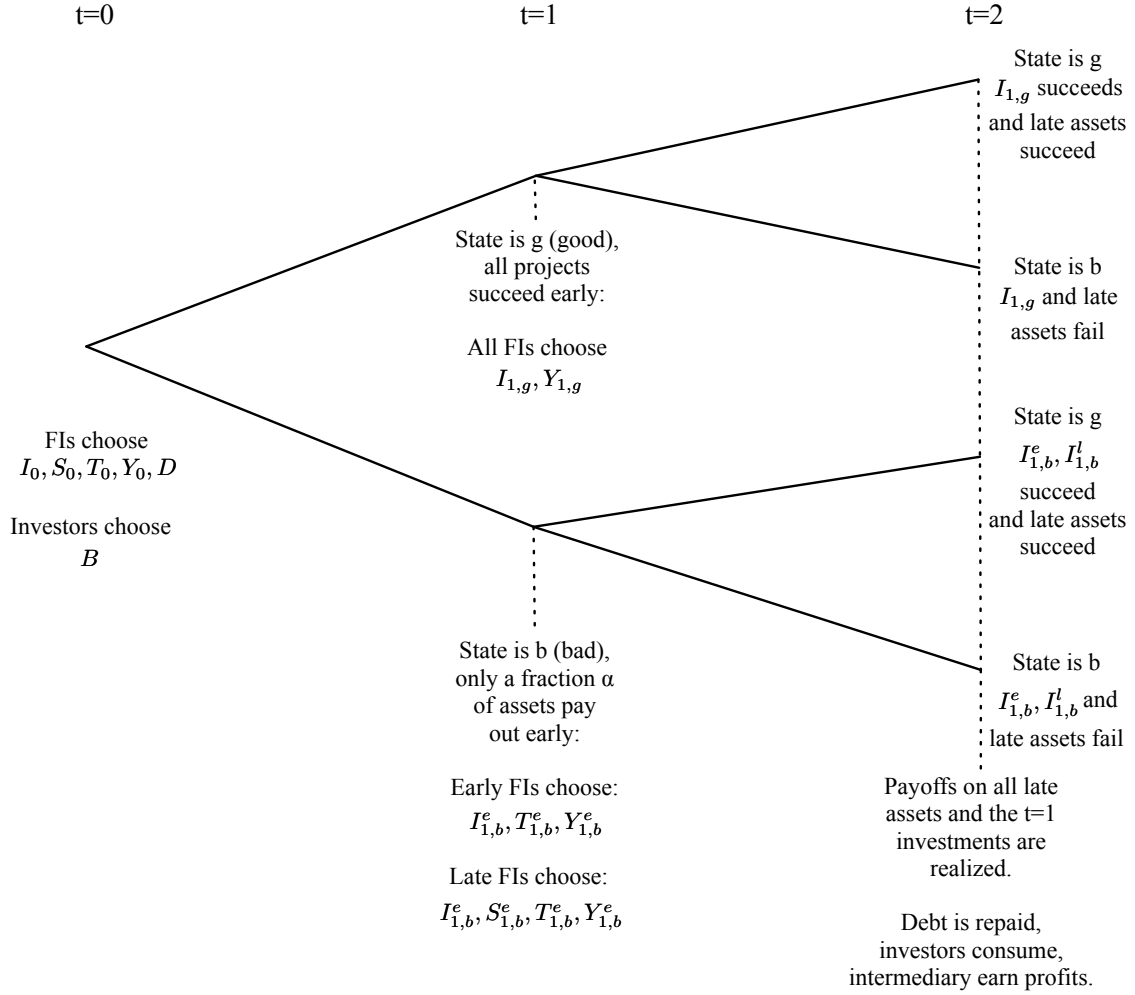


Figure 1: Timing.

its subject to budget, collateral, sales, and investors' participation constraints. The investor problem is to choose how much debt and securities issued by intermediaries to purchase (if any), and savings to maximize expected utility of consumption subject to budget constraints. The price of debt, r , and the prices of securities p_0, p_1 , are taken as given by intermediaries and investors. Our concept of equilibrium is characterized formally in the following definition.

DEFINITION 1. *A symmetric competitive equilibrium consists of prices r, p_0, p_1 , and choices of investment $I_0, I_{1,g}, I_{1,b}^e, I_{1,b}^l$, reserves $Y_0, Y_{1,g}, Y_{1,b}^e, Y_{1,b}^l$, asset sales and purchases at $t = 0, S_0, T_0$, asset purchases and sales at $t = 1, T_{1,b}^e, S_{1,b}^l - T_{1,b}^l$, debt D issued for each intermediary, and a choice of debt purchases B for each investor, such that given prices:*

1. Investors maximize expected utility (1) s.t. (2) and (3),
2. Intermediaries maximize expected profits (5) s.t. (6)-(12),
3. Markets clear:
 - (a) $B = D$ (market for debt at $t = 0$),
 - (b) $T_0 = S_0$ (market for assets at $t = 0$),
 - (c) $\alpha T_{1,b}^e = (1 - \alpha) (S_{1,b}^l - T_{1,b}^l)$ (market for assets at $t = 1$).

We now describe the optimal decisions of investors and intermediaries given prices, and then show how market clearing determines equilibrium prices.

3.1 Optimal Decisions of Investors

Investors save by purchasing claims from intermediaries. Since investors value risky assets at their lowest possible realization, they are priced out of the market for risky assets by intermediaries. More specifically, given that all risky investments may fail and return nothing, investors value these at 0 while intermediaries value them at their net present values. As a result, investors purchase only the risk-free debt issued by intermediaries if their break-even condition on funds lent, $r \geq \beta^{-1}$, is satisfied. This participation constraint places a lower bound on the equilibrium interest rate with investors willing to supply funds inelastically as long as this condition is met. We summarize investor behavior in the Lemma below:

LEMMA 1. *Investors demand only safe debt, where $B = \begin{cases} 0, & \text{if } r < \beta^{-1} \\ [0, w], & \text{if } r = \beta^{-1} \\ w, & \text{if } r > \beta^{-1} \end{cases}$.*

3.2 Optimal Decisions of Intermediaries

The following assumption allows us to focus on non-trivial equilibria in which intermediaries have incentives to borrow at $t = 0$.

ASSUMPTION 3. (*Positive Leverage*) $E_\pi R_0 - \beta^{-1} - C'(k) + q_1(R_0 - \beta^{-1})(q_{2,g}R_1 - 1 - C'(R_0k)) > (1 - q_1)(1 - \alpha)r(q_{2,b}R_1 - 1)$.

Assumption 3 ensures that the marginal benefit of debt is positive when $D = 0$. This condition holds whenever $t = 0$ investment returns are sufficiently high. We use it to establish the following preliminary result.

LEMMA 2. *In equilibrium, intermediaries do not hold cash reserves at $t = 0$ ($Y_0 = 0$), and $0 < D < w$ when investor wealth is sufficiently high.*

Proof. See Appendix B. □

It is never optimal for intermediaries to finance reserve holdings at $t = 0$ via debt. To see this, note that for every unit of debt raised at $t = 0$, intermediaries must generate $r - 1 \geq \beta^{-1} - 1 > 0$ at $t = 1$ to service this additional unit of debt. Given Assumption 3, some debt is always valuable so that leverage is not zero. Returns to borrowing eventually become negative however, due to the convexity of investment costs, and thus there is a limit to the amount of resources intermediaries can absorb. We assume throughout the paper that w is sufficiently large to guarantee that all wealth is not absorbed, i.e., $D < w + k$.¹⁶

We now consider the optimal reserve holdings, trade and investment decisions by intermediaries at period 1, taking as given prices and period 0 decisions. Note that neither returns from new investments at $t = 1$, nor late returns on $t = 0$ investments can be pledged to repay investors at $t = 2$. This is because investors value these pledges at the lowest possible return, which is zero. Thus, intermediaries must carry reserves equal to at least rD into period 2 to ensure that debt is repaid irrespective of the state at $t = 1$. Next, consider trade between intermediaries. When the state at $t = 1$ is good, intermediaries are identical as all $t = 0$ investments succeed early, and hence there is no trade. In this case, intermediaries simply set aside the required reserves and invest the remainder in new opportunities since these are always worthwhile, from Assumption 1. Hence, $I_{1,g} = R_0 I_0 - rD$. In the bad state, intermediaries differ at period 1 in that the proportion α of intermediaries projects are successful. These early types receive the full return on the fraction of $t = 0$ investments that were not securitized. The fraction $1 - \alpha$ of late types do not receive early returns on their own investments. Due to securitization, all intermediaries also receive a fraction of the early returns from other intermediaries' projects. For a given p_1 , early types can use their funds to either invest in new opportunities or purchase assets from late types. The amount of new investment, $I_{1,b}^e$, equates the marginal return to investment with the marginal return on purchasing assets. The former is simply $q_{2,b}R_1 - C'(I_{1,b}^e)$, while the latter is $q_{2,b}\theta R_0/p_1$, as $q_{2,b}\theta R_0$ is the net present value on $t = 0$ investments in the bad state. Note that for higher values of p_1 , the return on purchasing assets is lower and therefore more investment is undertaken and fewer assets are purchased by early types. Analogously, for lower values of p_1 , early types purchase more assets, and invest less. The investment and sales decisions by late types involve a similar trade-off. By selling $t = 0$ assets, late types forgo the returns, but can increase new investment and/or generate reserves required to service debt. Sales

¹⁶This does not affect the qualitative nature of the results and is done to simplify the welfare analysis.

consist of securitized assets on hand, $-T_{1,b}^l$, as well as any of their own investments which were not sold at $t = 0$, $S_{1,b}^l$. Late types may be constrained if they run out of assets to sell, in which case the multiplier on the sales constraints will bind ($\mu_{1,S}, \mu_{1,T} > 0$).

LEMMA 3. *Investment, sales and purchases of assets, and cash reserves in the bad state at $t = 1$ are as follows:*

Early types:

$$\begin{aligned} I_{1,b}^e &: q_{2,b}R_1 - C'(I_{1,b}^e) = \frac{q_{2,b}\theta R_0}{p_1}, \\ T_{1,b}^e &= \frac{R_0(I_0 - S_0) + \alpha R_0 T_0 - rD - I_{1,b}^e}{p_1}, \\ Y_{1,b}^e &= rD. \end{aligned}$$

Late types:

$$\begin{aligned} I_{1,b}^l &: q_{2,b}R_1 - C'(I_{1,b}^l) = \frac{q_{2,b}\theta R_0}{p_1} + \frac{\mu_{1,T}}{(1 - q_1)(1 - \alpha)}, \\ S_{1,b}^l - T_{1,b}^l &= \min \left[\frac{I_{1,b}^l + rD - \alpha R_0 T_0}{p_1}, (1 - \alpha)T_0 + (1 - a)I_0 - S_0 \right], \\ Y_{1,b}^l &= rD. \end{aligned}$$

Proof. See Appendix B. □

As is clear from Lemma 3, if $\mu_{1,T} > 0$, equilibrium investment levels will differ across types at $t = 1$. As a result, when late types are constrained at $t = 1$, intermediaries always find it optimal to securitize as much as possible.

LEMMA 4. $\mu_{1,T} > 0 \iff I_{1,b}^e > I_{1,b}^l > 0 \iff \mu_{1,S} > 0$. *Moreover, when constrained, intermediaries prefer to securitize as much as possible at $t = 0$, i.e., $S_0 = (1 - a)I_0$, and $S_{1,b}^l = 0$.*

Proof. See Appendix B. □

Late intermediaries may only trade assets at $t = 1$ to generate funds for investment. If late intermediaries cannot raise sufficient funds, $I_{1,b}^e > I_{1,b}^l$. Furthermore, when $\mu_{1,T} > 0$, securitized assets are worth more than individual investments, since they provide relatively more resources to late types who value them more. As result, being constrained at $t = 1$ means that intermediaries will securitize to the extent possible at $t = 0$.

Finally, we describe the optimal choice of investment at $t = 0$, which is determined by the following first order condition on I_0 :

$$E_\pi R_0 - r + q_1(R_0 - r)(q_{2,g}R_1 - 1 - C'(I_{1,g})) + (1 - q_1)\alpha(R_0 - r)(q_{2,b}R_1 - 1 - C'(I_{1,b}^e)) \\ + (1 - q_1)(1 - \alpha)(-r)(q_{2,b}R_1 - 1 - C'(I_{1,b}^l)) + \mu_{1,S}(1 - a) = C'(I_0). \quad (13)$$

The marginal return to a unit of investment at $t = 0$, given that $D > 0$ is $E_\pi R_0 - r$ plus the marginal returns from re-investing early returns at $t = 1$. When the state at $t = 1$ is good, each additional unit of I_0 (financed by one unit of D) generates $R_0 - r$ units of resources at $t = 1$ that can be reinvested for a net expected return $q_{2,g}R_1 - 1 - C'(I_{1,g})$. Similarly, when the state at $t = 1$ is bad, another unit of I_0 using borrowed funds generates $R_0 - r$ units of resources for the early types and $-r$ units for the late types. These can be reinvested at net returns of $q_{2,b}R_1 - 1 - C'(I_{1,b}^e)$ and $q_{2,b}R_1 - 1 - C'(I_{1,b}^l)$. If the sales constraint binds at $t = 0$, increasing I_0 provides an additional benefit: it raises by $(1 - a)$ units the quantity of assets late types can sell at $t = 1$, thereby mitigating the financial frictions intermediaries face at $t = 1$. Moreover, the value of relaxing the sales constraint ($\mu_{1,S}$) depends on the anticipated asset price p_1 . A higher price lowers this value while a lower price raises it. Thus, when intermediaries are constrained, securitization affects the level of ex-ante investment through both the level of a and the price p_1 . The optimal choice of I_0 simply equates the marginal benefit of investment, the left hand side of (13), with its marginal cost, $C'(I_0)$.¹⁷

3.3 Market Clearing

From the optimal choices of investors and intermediaries, we can infer that r must satisfy the following bounds $E_\pi R_0 - C'(k) \geq r \geq \beta^{-1}$. Since demand for debt is downward sloping and supply is perfectly elastic at a price of β^{-1} , we have $r = \beta^{-1}$ in equilibrium.

Consider the $t = 0$ market for securitized assets. It is shown in Lemma 4 that when constrained at $t = 1$, $S_0 = T_0 = (1 - a)I_0$. When unconstrained, intermediaries are indifferent over their choices of T_0 and S_0 . Regardless of the choices of T_0 and S_0 , any candidate equilibrium price \tilde{p}_0 must clear the market, and thus $S_0(\tilde{p}_0) - T_0(\tilde{p}_0) = 0$. Inspecting the intermediaries' problem, it is clear that \tilde{p}_0 has no effect on the budget, since all agents are identical and net purchases are zero. Thus optimal choices are determined by the first order conditions from the intermediaries' problem at a given p_1 , which are provided in Appendix

¹⁷If intermediaries incur losses at $t = 1$, these will be borne by their equity which would then place an upper bound on I_0 . In such a case, investment at $t = 1$ is zero, which is never true if the intermediary is constrained as shown in the proof of Lemma 4. The constrained case is the main focus of the paper and thus we ignore the possibility that intermediary equity is entirely wiped out at $t = 1$.

A. The $t = 0$ price that clears the market satisfies

$$p_0 = \frac{a\mu_{1,S}^* + C'(I_0^*)}{\lambda_0^*} + 1. \quad (14)$$

In an unconstrained equilibrium, $p_0 = C'(I_0^*)/\lambda_0^* + 1$ is simply the marginal cost of time 0 investment. When constrained, p_0 reflects the fact that securitized assets are relatively more valuable in this type of equilibrium, as they provide more resources to late types in the bad state of the world.

We now consider the determination of p_1 . From the optimal choices of intermediaries described in Lemma 3, we focus on prices in the range $q_{2,b}\theta R_0 \geq p_1 \geq \theta R_0/R_1$. To understand these bounds, note that if p_1 were to exceed the conditional return on assets, early types would not be willing to purchase them, since they can always invest in new projects that earn positive profit. Thus, at the equilibrium, assets will only trade at fire-sale prices (i.e. below NPV). On the other hand, if p_1 is below $\theta R_0/R_1$, early types do not make any new investments as buying up cheap assets is more profitable (and thus late types do not invest either).¹⁸ The following proposition ensures the existence of a unique constrained equilibrium, which is the main focus of subsequent analysis.

PROPOSITION 1. *Given Assumptions 1-3, a symmetric competitive equilibrium exists. The equilibrium may be constrained such that $\mu_{1,T} > 0$, in which case it is unique.*

Proof. See Appendix B. □

A sufficient condition for the existence of a constrained equilibrium is provided in the proof of Proposition 1. Intuitively, late types are constrained in equilibrium when the value of $t = 0$ assets is relatively low in the bad state (i.e. θ is small), $t = 1$ returns are high and/or when there is more heterogeneity across intermediaries at $t = 1$. Most importantly, the extent to which intermediaries can securitize assets also determines if late types will be constrained, and to what extent.

4 Welfare

In this section, we examine the efficiency of allocations at the competitive equilibrium. It is instructive to begin by characterizing the first-best allocation, as it helps to clarify the role of market incompleteness and securitization in the subsequent analysis.

¹⁸We ignore the possibility that there is zero aggregate investment at $t = 1$. In this case, the equilibrium is unconstrained (see Lemma 4), and I_0 is simply determined by the collateral constraints.

4.1 First-Best

We consider a planner that maximizes total expected surplus in the economy subject to the participation of investors. We focus on the point of the first-best frontier at which investors receive a utility of w , the level they achieve in the market outcome. This is the simplest way to compare the allocations achieved by a planner, with those that result in the competitive equilibrium. Assuming that the planner places an equal weight on all intermediaries, the planner's problem is to choose *aggregate* quantities $I_0, D, Y_0, I_{1,g}, I_{1,b}^e, I_{1,b}^l, Y_{1,g}, Y_{1,b}$ to maximize welfare Π^P where

$$\begin{aligned} \Pi^P = & q_1 [q_{2,g}R_1I_{1,g} - C(I_{1,g}) + Y_{1,g}] + (1 - q_1) [\alpha(q_{2,b}R_1I_{1,b}^e - C(I_{1,b}^e)) \\ & + (1 - \alpha)(q_{2,b}R_1I_{1,b}^l - C(I_{1,b}^l)) + Y_{1,b}] + (1 - q_1)(1 - \alpha)q_{2,b}\theta R_0I_0 - C(I_0) - rD, \end{aligned} \quad (15)$$

subject to the participation constraint of investor, $r = 1/\beta$, and the following budget and debt repayment constraints:

$$(\lambda_0) I_0 + Y_0 \leq k + D, \quad (16)$$

$$(\lambda_{1,g}) I_{1,g} + Y_{1,g} \leq R_0I_0 + Y_0, \quad (17)$$

$$(\lambda_{1,b}) \alpha I_{1,b}^e + (1 - \alpha)I_{1,b}^l + Y_{1,b} \leq \alpha R_0I_0 + Y_0, \quad (18)$$

$$(\eta_{1,g}) rD \leq Y_{1,g}, \quad (\eta_{1,b}) rD \leq Y_{1,b}. \quad (19)$$

We refer to the solution of this problem as the first-best, the salient features of which are outlined in the proof of the following result.

LEMMA 5. *In the first-best, investment is equalized across intermediaries at $t = 1$ in every state.*

Proof. See Appendix B. □

The planner maximizes the profits of all intermediaries jointly subject to a single budget constraint in each state. This implies that investment by late and early types is always equated at $t = 1$ since investment technologies across intermediaries are identical and exhibit decreasing returns to scale. This first-best outcome can also be achieved in a decentralized competitive equilibrium when intermediaries can write contingent contracts at $t = 0$ to transfer resources from early to late types at $t = 1$. This is shown in the proof of Lemma 5. It is also straightforward to show that the first-best obtains in the case where $a = 0$, in which case securitization fully completes markets.

4.2 Incomplete Markets, Securitization and Efficiency

When the aggregate state or individual type information is not ex-ante contractible and borrowing ex-post is infeasible, markets are incomplete and late intermediaries that have insufficient funds to repay debt and invest are forced to generate funds via asset sales on the spot market. Securitization changes the distribution of returns at $t = 1$, moving resources from early to late types, thereby substituting for contingent contracts and limiting the need for asset sales. This is precisely how securitization substitutes for missing markets in our environment.

To understand the nature of the inefficiency associated with securitization in the competitive equilibrium, it is necessary to establish a welfare benchmark. Consider a planner subject to the same market restrictions as intermediaries. Such a planner cannot directly re-allocate funds from early to late types at $t = 1$ in the bad state. Instead, the planner must rely on asset sales to achieve re-allocations across types. As a result, the planner cannot always equalize marginal returns to investment across types at $t = 1$ as in the first-best allocation. Thus, a second-best planning problem is nearly identical to the intermediaries', except that the planner's choices at $t = 0$ reflect aggregate quantities and thus the planner can fully account for any price effects that the choice of these aggregates have on $t = 1$ decisions. As in Section 4.1, we fix investor utility to that which obtains in the market equilibrium, which constrains the price of debt to $r = 1/\beta$.

Rather than directly compare the competitive equilibrium with the second-best allocation, we take an alternative approach to establish the inefficiency of the market equilibrium. Consider a perturbation of aggregate investment at $t = 0$, at the competitive equilibrium, such that the change is equal across intermediaries. Such a perturbation is financed by borrowing from investors and thus raises leverage of the intermediation sector, but does not change r . Using the envelope theorem, the change in welfare from such a perturbation is:

$$\frac{d\Pi_0}{dI_0} = \frac{dp_1}{dI_0} \cdot \alpha T_{1,b}^{e*} \left(\frac{\lambda_{1,b}^{l*}}{1-\alpha} - \frac{\lambda_{1,b}^{e*}}{\alpha} \right) = \frac{dp_1}{dI_0} \left[\frac{\mu_{1,T}^*}{p_1^*} (1-\alpha)(1-a)I_0^* \right]. \quad (20)$$

If $d\Pi_0/dI_0 \neq 0$, the equilibrium is constrained inefficient and a planner could engineer a Pareto improvement, even when subject to the same market incompleteness as intermediaries. Equation (20) captures a price effect which represents the difference between the individuals' first order condition on $t = 0$ investment and that from the second-best planner's problem.¹⁹ This price effect is non-zero when the marginal return on investment differs

¹⁹The perturbation generally affects both prices p_0 and p_1 , however changes in p_0 have no impact on time 0 intermediaries at the equilibrium, since each has net securitized assets purchases of zero.

across early and late types, captured here by the difference in the date 1 multipliers on the intermediaries' budget constraints in the bad state, weighted by the relevant population sizes. This is true precisely when late types are constrained in their ability to raise funds at $t = 1$, i.e., $\lambda_{1,b}^l/(1 - \alpha) - \lambda_{1,b}^e/\alpha > 0 \iff \mu_{1,T} > 0$. We summarize the above discussion in the following result.

PROPOSITION 2. *If late intermediaries are constrained in their ability to raise funds, i.e., $\mu_{1,T} > 0$, then the competitive equilibrium is constrained inefficient, in that a planner facing the same constraints as the private market can engineer a Pareto improvement. Moreover, assets are securitized if and only if aggregate investment is constrained inefficient.*

Proof. See Appendix B. □

The intuition behind the inefficiency is as follows. Generally, individual investment decisions at $t = 0$ impact the price of assets at $t = 1$ in the bad state. In the unconstrained case, price changes represent a redistribution of resources across intermediaries, which are irrelevant for welfare as they are risk-neutral. When $\mu_{1,T} > 0$ however, the price of assets in the bad state at $t = 1$ affects the ability of late types to raise funds, which in turn impacts aggregate investment at $t = 1$. Since atomistic intermediaries do not take into account the effect of their $t = 0$ investment decisions on the $t = 1$ asset price, there is a pecuniary externality that renders the competitive equilibrium inefficient. Moreover, securitization is only valuable when the competitive equilibrium is inefficient, as this is precisely when intermediaries are constrained and value the additional ex-ante insurance that securitization provides.

Assumption 1 ensures that all intermediaries have access to positive NPV investments at $t = 1$. This is crucial for the existence of constrained inefficient equilibria characterized by over-investment. To see this, consider the case in which late intermediaries exhaust their investment opportunities at $t = 1$. In this case, they are not constrained and therefore no pecuniary externality exists. Now consider the other extreme case in which early intermediaries have no investment opportunities. In this case, early types value assets sold by late types at NPV or $\theta R_0/R_1$. Therefore, even though late intermediaries may be constrained in their ability to raise funds, no pecuniary externality can exist as changing $t = 1$ resource allocations cannot raise the price any further.

The following proposition characterizes the link between frictions in the securitization process and the efficiency of the competitive equilibrium.

PROPOSITION 3. *When frictions associated with the securitization process are relatively small, the competitive market equilibrium is first-best. When frictions associated with securitization are sufficiently large, the competitive equilibrium is constrained inefficient. Formally,*

there exists an $\bar{a} \in (0, 1)$, such that for $a < \bar{a}$, the competitive market equilibrium is first-best, while for $a > \bar{a}$, late types are constrained and the competitive equilibrium is constrained inefficient.

Proof. See Appendix B. □

To understand this result, recall that the extent to which late intermediaries are constrained depends on a . When a large enough proportion of assets can be securitized, the late types will not be constrained in raising funds at $t = 1$. In this case, the marginal return to $t = 1$ investment is equalized across intermediaries and thus $t = 0$ decisions reflect the full social cost and benefit of investment (i.e., the allocation is first-best). In the extreme case in which $a = 0$, this is obvious since all assets can be securitized and thus intermediaries have the same resources at $t = 1$. On the other hand, if the frictions associated with securitization are sufficiently large, the competitive equilibrium is constrained inefficient. Importantly, in this case the competitive market equilibrium is not only inefficient relative to the first-best, but the second-best as well. Thus, micro level frictions in the securitization process lead to constrained inefficient investment at the macro level. We further characterize the nature of the inefficiency as follows.

COROLLARY 1. *When the competitive market equilibrium is constrained inefficient, time 0 investment may be either too large or too small relative to the second-best. If $\alpha R_0 - 1/\beta > 0$, there is under-investment, while $1/\beta - \alpha R_0 > a(1 - \alpha)R_0$ is sufficient for over-investment.*

Proof. See Appendix B. □

From Equation (20), it is clear that the sign of the inefficiency is determined entirely by the sign of dp_1/dI_0 . To determine the sign of the price effect, note that $\alpha R_0 - 1/\beta$ is the marginal change in aggregate cash in the market at $t = 1$ in the bad state that arises from an additional unit of I_0 . If this is positive, an increase in I_0 increases cash in the market and puts upward pressure on the asset price, thus making dp_1/dI_0 positive. The price impact from investment at $t = 0$ on cash in the market at $t = 1$ is not considered by atomistic individuals and as a result I_0 is too small from a social perspective. Analogously, if $\alpha R_0 - 1/\beta$ is sufficiently negative, more investment at $t = 0$ reduces cash in the market in the bad state of the world and this reduces the fire-sale price and results in over-investment.

4.2.1 Permitting Borrowing Between Intermediaries at $t = 1$

Assumption 2 is crucial to the inefficiency described in (20). If intermediaries could pledge all cash flows to lenders at $t = 1$, the resulting equilibrium would be efficient. However, we

can relax Assumption 2 without affecting the qualitative results of the model, as long as there is some restrictions on $t = 1$ borrowing. Consider a contingent contract in which the lender receives a gross per-unit return ρ when the borrowers' investment is successful and nothing otherwise. This is not restrictive since both parties are risk-neutral. Importantly, we assume that the borrower cannot commit to repayment at $t = 2$. Should the borrower attempt to renege, the lender could seize the fraction ϕ of the borrower's assets. Thus, borrowing B is limited to satisfy the following pledgeability constraint: $\rho B \leq \phi \cdot (\text{total assets at } t = 2)$. Note that early types are both the purchasers of securitized assets as well as lenders, and that risk-neutrality ensures $R_0/p_1 = \rho$.

As in the case with asset sales only, we can perturb aggregate investment at the equilibrium. Denote the multiplier on the pledgeability constraint by $\mu_{1,b}$.

$$\frac{d\Pi^*}{dI_0} = \frac{dp_1}{dI_0} \left[\frac{\mu_{1,T}^*}{p_1^*} (1-l)(1-a)I_0^* + \mu_{1,b}^* B^* \frac{\rho^*}{p_1^*} \right]. \quad (21)$$

Equation (21) is identical to (20), except for the additional non-negative term $\mu_{1,b}^* B^* \rho^* / p_1^*$ in the square brackets. The price effects of changes in I_0 , which also affects ρ through the arbitrage condition in this case, have an impact on welfare whenever late types are constrained at the equilibrium. Thus, while borrowing may have quantitative implications, the externality is completely analogous since the pledgeability constraint binds if and only if the sales constraint binds.

4.3 Over-investment and Securitization

In this section we focus on over-investment, the case in which the competitive equilibrium is characterized by excessively high leverage ex-ante and low asset prices ex-post. We show below that increasing the amount of securitization that is feasible will further reduce prices in a fire-sale, thus it is reasonable to think that more securitization could be undesirable relative to the second-best. To formalize this type of argument, consider the impact of a change in a on the intermediary at the competitive equilibrium.

$$\frac{d\Pi_0}{da} = -\mu_{1,S}^* I_0^* + \alpha T_{1,b}^{e*} \left(\frac{\lambda_{1,b}^{t*}}{1-\alpha} - \frac{\lambda_{1,b}^{e*}}{\alpha} \right) \frac{dp_1}{da}. \quad (22)$$

First note that this derivative is zero in the unconstrained case. This is because investment at early and late types is identical when unconstrained, and thus an increase in a simply amounts to a redistribution of cash at $t = 1$ from late to early types (all of whom are risk-neutral). When constrained, the first term in the expression above is a direct effect,

which is always negative and captures the fact that an increase in a restricts the ability of late intermediaries to generate funds at $t = 1$. The second term is an indirect effect, which captures the change in the equilibrium price due to changes in the distribution of cash-in-the-market at $t = 1$. The sign of the indirect effect is determined by dp_1/da , since the bracketed term in (22) is strictly positive in a constrained equilibrium. If the price effect is negative, then a smaller value of a , i.e., more securitization, improves welfare unambiguously. On the other hand, if the price effect is positive and dominates the direct effect, less securitization leads to higher welfare. We show in the proof of Proposition 4 below that dp_1/da is in fact positive in the over-investment case, but that the total effect of an increase in a is generally negative.

PROPOSITION 4. *In the over-investment case, $dp_1/da > 0$ and $d\Pi_0/da < 0$ at the equilibrium, i.e., welfare is increasing in the extent of securitization.*

Proof. See Appendix B. □

At a given price, an increase in a reduces investment at $t = 0$. This is because a higher value of a changes the distribution of cash in the market for a given I_0 , resulting in a greater proportion of returns for early types that value cash less in a constrained equilibrium. On the other hand, a reduction in I_0 puts upward pressure on the price of assets at $t = 1$. This price effect makes assets more valuable, encouraging higher I_0 , and opposing the direct effect described above. We are left with a somewhat counterintuitive conclusion; that more securitization leads to more investment/leverage, which is welfare improving *even when investment/leverage is excessive*.

5 Policy

The recent financial crisis has resulted in significant regulatory reforms worldwide, a number of which are directed at markets for securitized assets. The most important being the Dodd-Frank Act in the United States, and the regulatory reform initiatives across the European Union.²⁰ There are many subtleties, and a number of differences in the reforms across jurisdictions. However, we can group these into two broad categories: retention and transparency requirements.

²⁰The Dodd-Frank Wall Street Reform and Consumer Protection Act was signed July 21, 2010, but continues to adapt new rules, put in place by a number of regulatory agencies to implement its (broad) goals. In the European Union, securitization reform arises from various sections of the Basel II and III Accords, the Capital Requirements Directive, as well as a variety of others. As a part of the “Action Plan on Building a Capital Markets Union”, the European Commission published draft regulations on September 2015, which outline significant changes (if implemented) to the way in which securitization markets function in Europe.

Retention requirements, set to 5% skin-in-the-game in the US and EU reforms, translate to imposing a higher level of a in our environment.²¹ We have shown in Proposition 4 that increases in a are welfare-reducing in a constrained equilibrium, and have no welfare implications in the unconstrained case. This is true despite the fact that increased skin-in-the-game serves to reduce socially excessive leverage and raises prices in a fire-sale.

Reforms directed at improving transparency in securitization markets, which generally focus on due diligence, disclosure and the role of credit rating agencies, can also be interpreted as impacting the market level of a in our model. Presumably, a more transparent market would serve to reduce the frictions associated with securitized lending. Thus, Proposition 4 would suggest these policies would be beneficial from a welfare perspective, absent any costs of implementation. Again, we emphasize that improvements in transparency lead to reductions in a are welfare improving when leverage is excessive, even though they will generate an increase in leverage and lower prices in a fire-sale. It is worth noting at this point that reducing frictions in securitization markets, while at the same time forcing originators to hold more skin-the-game, is somewhat contradictory. Our model suggests focusing on the former and eschewing the latter.

Our previous discussion applies to policies that are directly targeted at securitization. However, a number of regulatory reforms enacted as part of the Basel III agreement are also designed to limit excessive investment and/or leverage in the financial sector. One such reform is a restriction on leverage that is independent of asset risk. In our model, as ex-ante investment may be inefficient, a direct leverage restriction (or capital requirement as the two are equivalent in our model), can improve welfare. However, this type of regulation presents a number of practical difficulties that arise from asymmetric information between intermediaries and regulators.²² Perhaps more importantly, many participants in securitization markets do not fall under the regulatory umbrella, i.e., are shadow banks. Thus, unlike the policies discussed above, tying the hands of regulated entities through leverage and/or capital controls would likely result in more resources flowing into the shadow sector. This would seem to be a major hurdle facing regulators going forward and represents an important area of further research.

Another important policy-relevant aspect of the model is that the price of securitized assets at $t = 0$ is informative about the nature of the equilibrium at $t = 1$. As is clear

²¹Article 122a of the European Capital Requirements Directive and Section 941 of the U.S. Dodd-Frank Act both require a five percent minimum retention rate by securitizers or originators, with exceptions for various types of underlying assets. Notably, “qualified” residential mortgage backed securities, which are backed by loans that meet a specific underwriting criteria.

²²For instance, identical portfolios can have significantly different levels of leverage depending on the accounting treatment of derivatives (GAAP versus IFRS).

from (14), it captures the costs associated with being constrained at $t = 1$. Thus, our model predicts that high prices for securitized assets today, that is the divergence of prices for securitized assets from fundamentals, predicts an inefficient fire-sale and a credit-crunch tomorrow. This is consistent with the behavior of prices for ABS before and after the recent crisis. As a summary of our results, we highlight the policy implications of the model in the following Proposition.

PROPOSITION 5. *When the competitive equilibrium is characterized by over-investment:*

- *Welfare decreases when skin-in-the-game requirements are tightened. As a result, welfare is increased if more transparency reduces skin-in-the-game.*
- *Direct leverage restrictions are welfare increasing.*

These results follow directly from the analysis in Section 4. The first point is simply a restatement of Proposition 4. To see the second point, note that a reduction in leverage in our framework is equivalent to a perturbation that reduces the initial investment I_0 , for all intermediaries. Equation (20) characterizes the welfare impact of a change in leverage, which is negative in the case of over-investment, as shown in the proof to Corollary 1.

6 Conclusion

We study the efficiency of competitive equilibria in an incomplete markets economy where securitization improves risk-sharing, and thus can raise overall investment. We find that by its very nature, securitization has unintended consequences that render the competitive market equilibrium inefficient. Specifically, while the additional insurance offered by securitization can increase overall investment, it may also lead to a pecuniary externality whereby the resulting expansion in investment is socially excessive. Two types of regulatory interventions can mitigate this over-investment problem: the imposition of direct leverage restrictions and/or transparency improvements in securitization markets. On the other hand, forcing issuers of securitized assets to hold more skin-in-the-game, which is currently an important part of regulatory reforms in the US and Europe, is never welfare improving.

References

- Acharya, Viral, and Philipp Schnabl, 2010, Do Global Banks Spread Global Imbalances? Asset-Backed Commercial Paper during the Financial Crisis of 2007–09, *IMF Economic Review* 58, 37–73.
- Acharya, Viral, Philipp Schnabl, and Gustavo Suarez, 2013, Securitization without risk transfer, *Journal of Financial Economics* 107, 515–536.
- Allen, Franklin, and Douglas Gale, 2004, Financial Intermediaries and Markets, *Econometrica* 72, 1023–1061.
- Bernanke, Ben, Carol Bertaut, Laurie Demarco, and Steven Kamin, 2011, International capital flows and the returns to safe assets in the United States, 2003–2007, International Finance Discussion Papers 1014, Board of Governors of the Federal Reserve System (U.S.).
- Brunnermeier, Markus, and Martin Oehmke, 2013, *Bubbles, Financial Crises, and Systemic Risk* (Elsevier, Amsterdam).
- Caballero, Ricardo, and Arvind Krishnamurthy, 2001, International and domestic collateral constraints in a model of emerging market crises, *Journal of Monetary Economics* 48, 513–548.
- Cerasi, Vittoria, and Jean-Charles Rochet, 2014, Rethinking the regulatory treatment of securitization, *Journal of Financial Stability* 10, 20–31.
- Claessens, Stijn, Zoltan Pozsar, Lev Ratnovski, and Manmohan Singh, 2012, Shadow Banking: Economics and Policy, IMF Staff Discussion Note 12/12, International Monetary Fund.
- Coval, Joshua, Jakub Jurek, and Eric Stafford, 2009, The economics of structured finance, *Journal of Economic Perspectives* 23, 3–26.
- Davila, Eduardo, 2015, Dissecting Fire Sales Externalities, Working paper, NYU Stern.
- Davila, J., J. Hong, P. Krusell, and J.V. Rios-Rull, 2012, Constrained efficiency in the neoclassical growth model with uninsurable idiosyncratic shocks, *Econometrica* 80, 2431–2467.
- DeMarzo, Peter, 2005, The pooling and tranching of securities: A model of informed intermediation, *Review of Financial Studies* 18, 1–36.

- DeMarzo, Peter, and Darrell Duffie, 1999, A Liquidity-Based Model of Security Design, *Econometrica* 67, 65–99.
- Diamond, Douglas, 1984, Intermediation and Delegated Monitoring, *Review of Economic Studies* 51, 393–414.
- Diamond, Peter, 1980, Efficiency with Uncertain Supply, *Review of Economic Studies* 47, 645–651.
- Farhi, E., M. Golosov, and A. Tsyvinski, 2009, A Theory of Liquidity and Regulation of Financial Intermediation, *Review of Economic Studies* 76, 973–992.
- Gale, Douglas, and Piero Gottardi, 2015, Capital Structure, Investment, and Fire Sales, *Review of Financial Studies* 28, 2502–2533.
- Geanakoplos, John, and Herakles Polemarchakis, 1986, Existence, regularity, and constrained suboptimality of competitive allocations when the asset market is incomplete, in W.P. Heller, R.M. Ross, and D.A. Starrett, eds., *Uncertainty, Information and Communication: Essays in Honor of Kenneth Arrow*, volume 3, 65–95 (Cambridge University Press).
- Gennaioli, Nicola, Andrei Shleifer, and Robert Vishny, 2013, A Model of Shadow Banking, *Journal of Finance* 68, 1331–1363.
- Gorton, Gary, and Andrew Metrick, 2012, Securitization, Working Paper 18611, National Bureau of Economic Research.
- Gorton, Gary, and George Pennacchi, 1995, Banks and loan sales marketing nonmarketable assets, *Journal of Monetary Economics* 35, 389–411.
- Greenwald, Bruce, and Joseph Stiglitz, 1986, Externalities in Economies with Imperfect Information and Incomplete Markets, *Quarterly Journal of Economics* 101, 229–264.
- Gromb, Denis, and Dimitri Vayanos, 2002, Equilibrium and welfare in markets with financially constrained arbitrageurs, *Journal of Financial Economics* 66, 361–407.
- Han, Joongho, Kwangwoo Park, and George Pennacchi, 2015, Corporate Taxes and Securitization, *Journal of Finance* 70, 1287–1321.
- Hanson, Samuel, and Adi Sunderam, 2013, Are There Too Many Safe Securities? Securitization and the Incentives for Information Production, *Journal of Financial Economics* 108, 565–584.

- Hart, Oliver, 1975, On the Optimality of Equilibrium when the Market Structure is Incomplete, *Journal of Economic Theory* 11, 418–443.
- Hartman-Glaser, Barney, Tomasz Piskorski, and Alexei Tchisty, 2012, Optimal securitization with moral hazard, *Journal of Financial Economics* 104, 186–202.
- He, Zhiguo, and Peter Kondor, 2016, Inefficient Investment Waves, *Econometrica* 84, 735–780.
- Kehoe, Timothy, and David Levine, 1993, Debt-Constrained Asset Markets, *Review of Economic Studies* 60, 865–888.
- Kiff, John, and Michael Kisser, 2014, Asset Securitization and Optimal Retention, *Journal of Financial Stability* 10, 32–49.
- Krishnamurthy, Arvind, 2010, Amplification mechanisms in liquidity crises, *American Economic Journal: Macroeconomics* 2, 1–30.
- Lorenzoni, Guido, 2008, Inefficient Credit Booms, *Review of Economic Studies* 75, 809–833.
- Segoviano, Miguel, Bradley Jones, Peter Lindner, and Johannes Blankheim, 2015, Securitization: The road ahead, IMF Staff Discussion Note 15/01, International Monetary Fund.
- Shleifer, Andrei, and Robert Vishny, 1992, Liquidation values and debt capacity: A market equilibrium approach, *Journal of Finance* 47, 1343–1366.
- Shleifer, Andrei, and Robert Vishny, 2010, Unstable banking, *Journal of Financial Economics* 97, 306–318.
- Stein, Jeremy, 2012, Monetary policy as financial-stability regulation, *Quarterly Journal of Economics* 127, 57–95.
- Stiglitz, Joseph, 1982, The inefficiency of the stock market equilibrium, *Review of Economic Studies* 49, 241–261.
- Stiglitz, Joseph, and Andrew Weiss, 1981, Credit rationing in markets with imperfect information, *American Economic Review* 71, 393–410.

A For Online Publication: Solution to the Intermediary Problem

The necessary conditions for the intermediary problem, given in (5)-(12), are as follows:

$$I_0 : [(1 - q_1)(1 - \alpha)q_{2,b}\theta + \lambda_{1,g} + \lambda_{1,b}^e]R_0 + \mu_{1,S}(1 - a) - \lambda_0 - C'(I_0) \leq 0 \quad (23)$$

$$T_0 : (1 - q_1)(1 - \alpha)q_{2,b}\theta R_0 + (\lambda_{1,g} + (\lambda_{1,b}^e + \lambda_{1,b}^l)\alpha)R_0 + \mu_{1,T}(1 - \alpha) - \lambda_0 p_0 \leq 0 \quad (24)$$

$$S_0 : -(1 - q_1)(1 - \alpha)q_{2,b}\theta R_0 - (\lambda_{1,g} + \lambda_{1,b}^e)R_0 - \mu_{1,S} + \lambda_0 p_0 \leq 0 \quad (25)$$

$$D : -r + \lambda_0 - r(\eta_{1,g} + \eta_{1,b}^e + \eta_{1,b}^l) \leq 0 \quad (26)$$

$$I_{1,g} : q_1(q_{2,g}R_1 - C'(I_{1,g})) - \lambda_{1,g} \leq 0 \quad (27)$$

$$I_{1,b}^e : (1 - q_1)\alpha(q_{2,b}R_1 - C'(I_{1,b}^e)) - \lambda_{1,b}^e \leq 0 \quad (28)$$

$$T_{1,b}^e : (1 - q_1)\alpha q_{2,b}\theta R_0 - \lambda_{1,b}^e p_1 \leq 0 \quad (29)$$

$$I_{1,b}^l : (1 - q_1)(1 - \alpha)(q_{2,b}R_1 - C'(I_{1,b}^l)) - \lambda_{1,b}^l \leq 0 \quad (30)$$

$$T_{1,b}^l : (1 - q_1)(1 - \alpha)q_{2,b}\theta R_0 - \lambda_{1,b}^l p_1 + \mu_{1,T} = 0 \quad (31)$$

$$S_{1,b}^l : -(1 - q_1)(1 - \alpha)q_{2,b}\theta R_0 + \lambda_{1,b}^l p_1 - \mu_{1,S} \leq 0 \quad (32)$$

$$Y_0 : -\lambda_0 + \lambda_{1,g} + \lambda_{1,b}^e + \lambda_{1,b}^l \leq 0 \quad (33)$$

$$Y_{1,g} : q_1 - \lambda_{1,g} + \eta_{1,g} = 0 \quad (34)$$

$$Y_{1,b}^e : (1 - q_1)\alpha - \lambda_{1,b}^e + \eta_{1,b}^e = 0 \quad (35)$$

$$Y_{1,b}^l : (1 - q_1)(1 - \alpha) - \lambda_{1,b}^l + \eta_{1,b}^l = 0 \quad (36)$$

First consider the intermediaries' problem at $t = 1$, taking prices r , p_0 , p_1 , and quantities I_0 , S_0 , T_0 and D as given. Since intermediaries have access to positive NPV projects at $t = 1$ (Assumption 1), it follows from (27), (28), and (30) that the budget constraints bind: $\lambda_{1,g}$, $\lambda_{1,b}^e$, $\lambda_{1,b}^l > 0$. This implies that intermediaries hold exactly the amount of cash needed to repay debt, and surplus funds are invested. Formally, combining (27)-(30) with (34)-(36) implies $\eta_{1,g}$, $\eta_{1,b}^e$, $\eta_{1,b}^l > 0$, and thus

$$Y_{1,g} = Y_{1,b}^e = Y_{1,b}^l = rD. \quad (37)$$

We now pin down investment and trade. If the state at $t = 1$ is good, then all intermediaries are identical with resources $R_0 I_0 - rD$. There is no motive for trade since investment opportunities are identical, thus all intermediaries invest $I_{1,g} = R_0 I_0 - rD$. If the state at $t = 1$ is bad, early intermediaries' purchase assets at price p_1 according to (29). Combining this with (28) and the corresponding budget constraint we obtain:

$$I_{1,b}^e : q_{2,b}R_1 - C'(I_{1,b}^e) = \frac{q_{2,b}\theta R_0}{p_1}, \quad (38)$$

$$T_{1,b}^e = \frac{R_0(I_0 - S_0) + \alpha R_0 T_0 - rD - I_{1,b}^e}{p_1}. \quad (39)$$

If late intermediaries can raise sufficient funds to repay D and to invest at $t = 1$, the sales constraint does not bind and thus $\mu_{1,T} = 0$. In this case, the marginal returns on investment

between early and late types are equalized and $\alpha\lambda_{1,b}^e = (1-\alpha)\lambda_{1,b}^l$, so that $I_{1,b} \equiv I_{1,b}^e = I_{1,b}^l$. When $\mu_{1,T} > 0$, late intermediaries are financially constrained at $t = 1$. Hence, $I_{1,b}^e > I_{1,b}^l$ as late types can not generate enough funds to equalize investments. Using (30) and (31) we can then write

$$I_{1,b}^l = \min \{ I_{1,b}^e, (1-a)(\alpha R_0 + p_1(1-\alpha))I_0 - rD \} \quad (40)$$

$$S_{1,b}^l - T_{1,b}^l = \min \left\{ \frac{I_{1,b}^l + rD - \alpha R_0 T_0}{p_1}, (1-\alpha)(1-a)I_0 \right\}. \quad (41)$$

Using (24), (25), and (28)-(31) the multipliers can be written:

$$\mu_{1,T} = p_1(1-q_1)(1-\alpha)(C'(I_{1,b}^e) - C'(I_{1,b}^l)), \quad (42)$$

$$\mu_{1,S} = \frac{\mu_{1,T}}{p_1}((1-\alpha)p_1 + \alpha R_0). \quad (43)$$

To solve for the optimal choices at $t = 0$, note that the budget constraint binds, so that $\lambda_0 > 0$. Using (33) and (26), it is clear that if debt is positive (Lemma 2), then no cash is held. We can then recover the quantity of debt via the budget constraint for given I_0 , S_0 and T_0 so that:

$$Y_0 = 0, \quad (44)$$

$$D = I_0 + p_0(T_0 - S_0) - k. \quad (45)$$

Using the above results, the optimal choices of I_0 , S_0 and T_0 are then characterized by the corresponding first-order conditions (23)-(25) when $\mu_{1,T} = 0$. When late intermediaries are constrained $S_0 = (1-a)I_0$, as shown in Lemma 4, and I_0 , T_0 are characterized by (23) and (24).

B For Online Publication: Proofs

Proof of Lemma 2

Proof. To show that intermediaries hold no cash at $t = 0$, combine (26) with (34)-(36) to obtain $\lambda_0 \leq r(\lambda_{1,g} + \lambda_{1,b}^e + \lambda_{1,b}^l)$. Substituting this into (33) gives $(1-r)(\lambda_{1,g} + \lambda_{1,b}^e + \lambda_{1,b}^l) < 0$ as $r \geq \beta^{-1} > 1$ and thus $Y_0 = 0$.

We now show that $D > 0$, which is true when the marginal return to debt exceeds the cost at $I_0 = k$, where I_0 is characterized by (13). Setting $\mu_{1,S} = I_{1,b}^l = (q_{2,b}R_1 - 1 - C'(I_{1,b}^e)) = 0$, $I_{1,g} = R_0k$, and $r = \beta^{-1}$, yields the expression in Assumption 3, which is sufficient for $D > 0$. Finally, to show that $D < w + k$ for sufficient w , rewrite the necessary condition for an optimal I_0 , evaluated at $I_0 = w + k$.

$$E_\pi R_0 - r + q_1(R_0 - r)(q_{2,g}R_1 - 1 - C'((R_0 - \beta^{-1})(w + k) + R_0k)) + (1 - q_1)\alpha(R_0 - r)(q_{2,b}R_1 - 1 - C'(I_{1,b}^e)) + (1 - q_1)(1 - \alpha)(-r)(q_{2,b}R_1 - 1 - C'(I_{1,b}^l)) + \mu_{1,S}(1 - a) \leq C'(w + k) \quad (46)$$

Setting $I_{1,b}^e = 0$, and $(1 - q_1)(1 - \alpha)(-r)(q_{2,b}R_1 - 1 - C'(I_{1,b}^l)) = 0$ gives:

$$E_\pi R_0 - r + q_1(R_0 - r)(q_{2,g}R_1 - 1 - C'((R_0 - \beta^{-1})(w + k) + R_0k)) + (1 - q_1)\alpha(R_0 - r)(q_{2,b}R_1 - 1) + \mu_{1,S}(1 - a) \leq C'(w + k), \quad (47)$$

If (47) holds, then (46) must hold as well. From the first order conditions, we have $\mu_{1,S}(1 - a) = (1 - q_1)(1 - \alpha)[q_{2,b}R_1 - C'(I_{1,b}^l)]p_1 - q_{2,b}\theta R_0(1 - a)$. Setting price at the upper bound $p_1 = q_{2,b}\theta R_0$, and $I_{1,b}^l = 0$, we bound $\mu_{1,S}(1 - a) \leq (1 - q_1)(1 - \alpha)q_{2,b}\theta R_0[q_{2,b}R_1 - 1](1 - a)$. Note that the bound on $\mu_{1,S}$ is not a function of w , while the other terms on the left hand side of (47) are strictly decreasing in w , and the right hand side is strictly increasing. Therefore, (47) must be strict for sufficiently large w . □

Proof of Lemma 3

Proof. The expressions for $I_{1,b}^e$, $T_{1,b}^e$, $I_{1,b}^l$, and $S_{1,b}^l - T_{1,b}^l$ are derived directly from the first-order conditions of the intermediaries' problem in Appendix A. Cash holding at time 1 is determined by the debt repayment constraints, which are all binding. To show this, combine (27) and (34) to obtain:

$$\eta_{1,g} = \lambda_{1,g} - q_1 \geq q_1(q_{2,g}R_1 - 1 - C'(I_{1,b}^e)) > 0,$$

(28) and (35) to obtain $\eta_{1,b}^e \geq (1 - q_1)\alpha(q_{2,b}R_1 - 1 - C'(I_{1,b}^e)) > 0$, and (30) and (36) to get $\eta_{1,b}^l \geq (1 - q_1)(1 - \alpha)(q_{1,b}R_1 - 1 - C'(I_{1,b}^l)) > 0$, where all inequalities follow from Assumption 1. □

Proof of Lemma 4

Proof. Equations (28) and (29) yield

$$C'(I_{1,b}^e) = q_{2,b} \left(R_1 - \frac{\theta R_0}{p_1} \right). \quad (48)$$

Similarly, using (30) and (31) we have

$$C'(I_{1,b}^l) = q_{2,b} \left(R_1 - \frac{\theta R_0 + \frac{\mu_{1,T}}{(1-q_1)(1-\alpha)}}{p_1} \right). \quad (49)$$

Hence, $\mu_{1,T} = 0 \Leftrightarrow I_{1,b}^e = I_{1,b}^l$. Moreover, when $\mu_{1,T} > 0$, $C'(I_{1,b}^l) < C'(I_{1,b}^e) \implies I_{1,b}^e > I_{1,b}^l$ as $C(\cdot)$ is convex. To show that investment must be positive in the constrained case, suppose $I_{1,b}^l = 0$ and $\mu_{1,S} = 0$. From (30) we have $\lambda_{1,b}^l = (1 - q_1)(1 - \alpha)q_{2,b}R_1$, and thus from (32) we have $\mu_{1,S} = (1 - q_1)(1 - \alpha)q_{2,b}(p_1 - \theta R_1)$. Therefore $p_1 > \theta R_1 > q_{2,b}\theta R_1$, which implies the price exceeds NPV and thus $I_{1,b}^l$ cannot be zero.

Adding (24), and (25) and dividing through by $\alpha(1 - \alpha)$, we obtain

$$\left[\frac{\lambda_{1,b}^l}{1 - \alpha} - \frac{\lambda_{1,b}^e}{\alpha} \right] R_0 + \frac{\mu_{1,T}}{\alpha} \leq \frac{\mu_{1,S}}{\alpha(1 - \alpha)}. \quad (50)$$

Now note that the square-bracketed term is proportional to $\Delta C \equiv C'(I_{1,b}^e) - C'(I_{1,b}^l)$ which is strictly positive when $\mu_{1,T} > 0$ as $I_{1,b}^e > I_{1,b}^l$. Hence, $\mu_{1,T} > 0 \implies \mu_{1,S} > 0$. To show the reverse, assume $\mu_{1,S} > 0$. This implies that the marginal benefit of increasing $S_0 + S_{1,b}^l$ is strictly positive. In other words, using the left-hand sides of (25) and (32) we have:

$$\begin{aligned} - (1 - q_1)(1 - \alpha)q_{2,b}\theta R_0 - (\lambda_{1,g} + \lambda_{1,b}^e)R_0 - \mu_{1,S} + \lambda_0 p_0 \\ - (1 - q_1)(1 - \alpha)q_{2,b}\theta R_0 + \lambda_{1,b}^l p_1 - \mu_{1,S} = 0. \end{aligned} \quad (51)$$

Using (31) and (24) we have:

$$\left[\frac{\lambda_{1,b}^l}{1 - \alpha} - \frac{\lambda_{1,b}^e}{\alpha} \right] R_0 + \frac{\mu_{1,T}}{\alpha} \geq \frac{\mu_{1,S}}{\alpha(1 - \alpha)}. \quad (52)$$

Note that the LHS is always non-negative and strictly positive only when $\mu_{1,T} > 0$. Since the LHS is bounded below by a positive number as $\mu_{1,S} > 0$, it must be the case that $\mu_{1,T} > 0$. Finally, to show that $S_0 = (1 - a)I_0$, and $S_{1,b}^l = 0$, we add (23) and (25) to obtain $[\lambda_0(p_0 - 1) - C'(I_0)]/a = \mu_{1,S}$, when investment and sales are non-negative. This implies that $p_0 > 1 + C'(I_0)/\lambda_0$. Hence, it is profitable to invest and then sell assets at $t = 0$. Thus, $S_0 = (1 - a)I_0$, and therefore $S_{1,b}^l = 0$ as there are no assets to sell at period 1. \square

Proof of Proposition 1

Proof. Begin with the equilibrium at $t = 1$. At a price equal to NPV ($p_1 = q_{2,b}\theta R_0$), demand is zero because early types have access to positive NPV investments, while supply is positive for the same reason and thus excess demand is negative. At $p_1 = \theta R_0/R_1$, demand

is positive since buying assets dominates investment for early types, while supply is zero, and thus excess demand is positive. We conclude that an equilibrium in the asset market at $t = 1$ exists since excess demand, characterized in (53), is a continuous function. Note that $q_{2,b}\theta R_0 > \theta R_0/R_1 = q_{2,b}\theta R_0/q_{2,b}R_1$, since $q_{2,b}R_1 > 1$ by assumption.

Given arbitrary feasible $t = 0$ choices I_0, T_0, S_0 and D , excess demand at $t = 1$ is

$$z(p_1) = \max \left\{ \frac{\alpha R_0(I_0 + T_0 - S_0) - rD - I_{1,b}}{p_1}, \alpha \frac{R_0(I_0 - S_0) + \alpha R_0 T_0 - rD - I_{1,b}^e}{p_1} - (1 - \alpha)^2 T_0 \right\}, \quad (53)$$

where optimal $t = 1$ investment is characterized by (28)-(31) and we denote $I_{1,b} = I_{1,b}^l = I_{1,b}^e$ in the unconstrained case. For prices at which sales are not constrained, we have

$$z'(p_1) = -\frac{\frac{dI_{1,b}}{dp_1}}{p_1} - \frac{\alpha R_0(I_0 + T_0 - S_0) - rD - I_{1,b}}{p_1^2} < 0, \quad (54)$$

where the inequality follows from $dI_{1,b}/dp_1 = q_{2,b}\theta R_0/C''(I_{1,b}^e)p_1^2 > 0$ and the fact that $\alpha R_0(I_0 + T_0 - S_0) - rD - I_{1,b} \geq 0$, since aggregate investment cannot exceed aggregate resources at $t = 1$. When constrained,

$$z'(p_1) = -\alpha \frac{\frac{dI_{1,b}^e}{dp_1} + T_{1,b}^e}{p_1} < 0, \quad (55)$$

where $dI_{1,b}^e/dp_1 = \frac{q_{2,b}\theta R_0}{C''(I_{1,b}^e)p_1^2} > 0$. Since excess demand is monotone in either case, the equilibrium at $t = 1$ is unique.

We now consider $t = 0$. From (24) and (25), it is clear that for sufficiently low p_0 , $S_0 = 0$ and $T_0 \geq 0$. Alternatively, for sufficiently high p_0 , $T_0 = 0$, while $S_0 \geq 0$. Since excess demand is continuous, there exists an equilibrium p_0 that clears the market at $t = 0$. From the first order conditions on the intermediary problem we have

$$p_0 = \frac{a\mu_{1,S}^* + C'(I_0^*)}{\lambda_0} + 1, \quad (56)$$

for non-negative values of T_0 and S_0 . In the unconstrained case, $p_0 = C'(I_0)/\lambda_0 + 1$, which is simply the marginal cost of investment. This price must obtain at the equilibrium since there is no collateral motive for securitized assets, which implies that supply is zero for any price lower and demand is zero for any price higher. In this case, $p_0 = C'(I_0)/\lambda_0 + 1$ and any value of $S_0 = T_0 \in [0, (1 - a)I_0]$ represents an equilibrium. If the equilibrium is constrained, the price is determined by (56) and $S_0 = T_0 = (1 - a)I_0$, by Lemma 4. To pin down the value of I_0^* , note that in equilibrium $S_0^* = T_0^*$ so that from (23) we have that I_0^* is characterized as follows:

$$E_\pi R_0 + q_1(R_0 - r)(q_{2,g}R_1 - 1 - C'(I_{1,g}) + (1 - q_1)(\alpha R_0(q_{2,b}R_1 - 1 - C'(I_{1,b}^e)) - r(q_{2,b}R_1 - 1 - C'(I_{1,b}^l)) + (1 - q_1)\alpha r(C'(I_{1,b}^e) - C'(I_{1,b}^l)) + \mu_{1,S}(1 - a) = r + C'(I_0). \quad (57)$$

The solution to this equation exists due to Assumption 3 and is unique as the second-order condition follows from $C''(\cdot) > 0$. In the unconstrained case, the optimal choice of I_0 is independent of S_0 and T_0 and is uniquely characterized by the following first-order condition:

$$E_\pi R_0 + q_1(R_0 - r)(q_{2,g}R_1 - 1 - C'(I_{1,g})) + (1 - q_1)(\alpha R_0 - r)(q_{2,b}R_1 - 1 - C'(I_{1,b})) = r + C'(I_0). \quad (58)$$

Finally, we show that parameters exist such that the equilibrium may be constrained. Posit an unconstrained equilibrium. From (28)-(31), we have

$$p_1 = \frac{q_{2,b}\theta R_0}{q_{2,b}R_1 - C'(I_{1,b})}, \quad (59)$$

where $I_{1,b} = \alpha R_0 I_0 - rD$ must hold to satisfy the aggregate resource constraint at the equilibrium. Substituting (59) into (9) gives aggregate supply

$$(1 - \alpha)(S_{1,b}^l - T_{1,b}^l) = (1 - \alpha) \cdot \frac{\alpha R_0(I_0 - T_0)(q_{2,b}R_1 - C'(I_{1,b}))}{q_{2,b}\theta R_0}. \quad (60)$$

If parameters exist such that (60) exceeds $(1 - \alpha)^2(1 - a)I_0$, then the equilibrium can not be unconstrained. Using $I_0 - T_0 \geq aI_0$, $k + w > I_0$, and simplifying yields the following sufficient condition for aggregate supply to exceed $(1 - \alpha)^2(1 - a)I_0$:

$$\frac{\alpha a}{q_{2,b}\theta(1 - a)(1 - \alpha)} \cdot [q_{2,b}R_1 - C'(\alpha R_0(k + w) - \beta^{-1}w)] > 1. \quad (61)$$

Since $q_{2,b}R_1 - C'(\alpha R_0(k + w) - \beta^{-1}w) > 1$ due to Assumption 1, we can further simplify to obtain the following sufficient condition:

$$\frac{\alpha a}{(1 - a)(1 - \alpha)} > q_{2,b}\theta. \quad (62)$$

□

Proof of Lemma 5

Proof. We first outline the salient features of the solution to the planning problem described in (15)-(19). It is straightforward to show that all budget and debt repayment constraints bind, that no cash reserves are held at $t = 0$ and the interior solution is unique since $-C''(\cdot) < 0$. Combining the first order conditions on $t = 1$ investment in the bad state gives

$$(1 - q_1)(q_{2,b}R_1 - C'(I_{1,b}^e)) = (1 - q_1)(q_{2,b}R_1 - C'(I_{1,b}^l)) \implies I_{1,b}^e = I_{1,b}^l = I_{1,b}. \quad (63)$$

Furthermore,

$$I_{1,g} = (R_0 - r)I_0 + rk, \quad (64)$$

$$I_{1,b} = (\alpha R_0 - r)I_0 + rk, \quad (65)$$

where we have used $D = I_0 - k$. Investments at $t = 0$ and $t = 1$ are thus related as follows:

$$E_\pi R_0 - r + q_1(R_0 - r)(q_{2,g}R_1 - 1 - C'(I_{1,g})) + (1 - q_1)(\alpha R_0 - r)(q_{2,b}R_1 - 1 - C'(I_{1,b})) = C'(I_0). \quad (66)$$

The planner equates the marginal cost of investment at $t = 0$ with the marginal benefit of investment across the states at $t = 1$, where investments are equalized across intermediary types in the low state.

Contingent Securities

We now introduce contingent securities traded at $t = 0$, conditional on individual type at $t = 1$ when the state is bad (there is no gains from trading securities that pay off when the state is good since all intermediaries are identical in this state). Further, there is no motive for trade at $t = 1$ and we ignore this possibility. The security pays the owner one unit, conditional on the bad state if no early returns have been realized. Denote by ζ_l and ζ_e the quantities of this security purchased or sold, and ρ_0 the corresponding price. The intermediaries' problem is:

$$\begin{aligned} \Pi_0 = & q_1[q_{2,g}R_1I_{1,g} - C(I_{1,g}) + Y_{1,g}] + (1 - q_1) (\alpha[q_{2,b}R_1I_{1,b}^e - C(I_{1,b}^e) + Y_{1,b}^e] \\ & + (1 - \alpha)[q_{2,b}R_1I_{1,b}^l - C(I_{1,b}^l) + Y_{1,b}^l]) + (1 - q_1)(1 - \alpha)q_{2,b}\theta R_0I_0 - C(I_0) - rD, \end{aligned} \quad (67)$$

subject to:

$$(\lambda_0) I_0 + \rho_0(\zeta_l - \zeta_e) + Y_0 \leq k + D, \quad (68)$$

$$(\lambda_{1,g}) I_{1,g} + Y_{1,g} \leq R_0I_0 + Y_0, \quad (69)$$

$$(\lambda_{1,b}^e) I_{1,b}^e + Y_{1,b}^e \leq R_0I_0 - (1 - \alpha)\zeta_e + Y_0, \quad (70)$$

$$(\lambda_{1,b}^l) I_{1,b}^l + Y_{1,b}^l \leq -\alpha\zeta_l + Y_0, \quad (71)$$

$$(\eta_{1,g}) rD \leq Y_{1,g}, \quad (72)$$

$$(\eta_{1,b}^e) rD \leq Y_{1,b}^e, \quad (73)$$

$$(\eta_{1,b}^l) rD \leq Y_{1,b}^l. \quad (74)$$

All budget constraints bind, $Y_0 = 0$, and all collateral constraints will bind, so that $Y_{1,g} = Y_{1,b}^e = Y_{1,b}^l = rD$. We can then rewrite the objective and obtain the following necessary optimality conditions:

$$I_0 : q_1(R_0 - r)(q_{2,g}R_1 - C'(I_{1,g})) + \lambda_{1,b}^e(R_0 - r) + \lambda_{1,b}^l(-r) = 0, \quad (75)$$

$$I_{1,g} : q_1 [q_{2,g}R_1 - C'(I_{1,g})] = \lambda_{1,g}, \quad (76)$$

$$I_{1,b}^e : (1 - q_1)\alpha [q_{2,b}R_1 - C'(I_{1,b}^e)] = \lambda_{1,b}^e, \quad (77)$$

$$I_{1,b}^l : (1 - q_1)(1 - \alpha) [q_{2,b}R_1 - C'(I_{1,b}^l)] = \lambda_{1,b}^l, \quad (78)$$

$$\zeta_l : q_1(q_{2,g}R_1 - C'(I_{1,g}))(-rp_0) + \lambda_{1,b}^e(-rp_0) + \lambda_{1,b}^l(\alpha - rp_0) = 0, \quad (79)$$

$$\zeta_e : q_1(q_{2,g}R_1 - C'(I_{1,g}))(rp_0) + \lambda_{1,b}^e(rp_0 - (1 - \alpha)) + \lambda_{1,b}^l(rp_0) = 0. \quad (80)$$

Using (79) and (80), we obtain $\lambda_{1,b}^e/\alpha = \lambda_{1,b}^l/(1 - \alpha)$, which substituted into the first order conditions for $t = 1$ investment yield $I_{1,b}^e = I_{1,b}^l = I_{1,b}$. Thus, trading securities at $t = 0$

permits agents to equate the marginal returns to investment at $t = 1$. Write the foc for I_0 after re-arranging terms as follows:

$$E_\pi R_0 - r + q_1(R_0 - r)(q_{2,g}R_1 - 1 - C'(I_{1,g})) + (1 - q_1)(\alpha R_0 - r)(q_{2,b}R_1 - 1 - C'(I_{1,b})) = C'(I_0). \quad (81)$$

This is identical to (66), hence the market allocation when Arrow securities are traded is Pareto efficient. \square

Proof of Proposition 2

Proof. Differentiating the Lagrangian for the intermediaries' problem with respect to aggregate I_0 yields

$$\frac{d\Pi_0}{dI_0} = \frac{dp_1^*}{dI_0} \left[\frac{\mu_{1,T}^*}{p_1^*} (1 - \alpha)(1 - a)I_0^* \right], \quad (82)$$

using the market clearing condition $\alpha T_{1,b}^{e*} = -(1 - \alpha)T_{1,b}^{l*}$. This is equation (20) in Section 4.2, which we provide here for convenience. Since the price effect in (82) is non-zero, the equilibrium is constrained inefficient whenever $\mu_{1,T} > 0$.

To show that intermediaries value securitization only when the competitive equilibrium is inefficient, note that when unconstrained, $\mu_{1,S} = \mu_{1,T} = 0$, and $I_{1,b}^e = I_{1,b}^l = I_{1,b}$. In this case the allocation in the competitive equilibrium coincides with the planning solution. To see this, note that in equilibrium $S_0^* = T_0^*$, as the securitization market must clear at $t = 0$. This implies that $I_0^* = D^* + k$, where I_0^* is pinned down by the first-order condition (13). When intermediaries are not constrained, this condition is identical in equilibrium to (66), the corresponding condition for the planner. Importantly, the determination of I_0^* is independent of p_1^* , S_0^* and T_0^* as $I_{1,g}^* = R_0 I_0^* - \beta^{-1} D^*$, $I_{1,b}^* = \alpha R_0 I_0^* - \beta^{-1} D^*$. Thus, securitization has no value when intermediaries are unconstrained as its use cannot improve welfare. When intermediaries are constrained, securitization has value as I_0^* is then indirectly a function of p_1 via (13) as $t = 1$ investment is a function of p_1 . \square

Proof of Proposition 3

Proof. It is straightforward to show that a constrained equilibrium is not possible for a sufficiently small value of a . In an unconstrained equilibrium, $S_0, S_{1,b}^l$ and I_0 are not functions of a . Thus, assuming $S_0 > 0$, the sales constraint $S_0 + S_{1,b}^l \leq (1 - a)I_0$ implies that there is a unique $0 < \bar{a} < 1$ such that the intermediary is constrained whenever $a > \bar{a}$. \square

Proof of Corollary 1

Proof. From (20), $\text{sign}(d\Pi_0^*/dI_0) = \text{sign}(dp_1^*/dI_0)$. Multiplying the market clearing condition at $t = 1$ by p_1^* and noting that $T_{1,b}^{l*} = -(1 - \alpha)T_0^* = -(1 - \alpha)(1 - a)I_0^*$ due to the binding sales constraint implies that $p_1 \alpha T_{1,b}^{e*} = p_1(1 - \alpha)^2(1 - a)I_0^*$. Differentiating with respect to I_0 noting that $S_0^* = T_0^*$ gives:

$$\frac{dp_1^*}{dI_0} = \frac{\alpha((a + \alpha(1-a))R_0 - 1/\beta) - (1-\alpha)^2(1-a)p_1^*}{\alpha \frac{q_{2,b}\theta R_0}{C''(I_{1,b}^{e*})p_1^{*2}} + (1-\alpha)^2(1-a)I_0^*}. \quad (83)$$

The denominator is strictly positive since $\frac{q_{2,b}\theta R_0}{C''(I_{1,b}^{e*})p_1^{*2}} > 0$ as $C''(\cdot) > 0$. Thus, the sign of dp_1^*/dI_0 is determined solely by the numerator. First, we characterize parameters to support the over-investment case. This case occurs when

$$p_1^* > \frac{\alpha((a + \alpha(1-a))R_0 - 1/\beta)}{(1-\alpha)^2(1-a)}. \quad (84)$$

Assume that (62) holds so that in equilibrium $\mu_{1,T} > 0$. Then, a sufficient condition for over-investment is that $a + \alpha(1-a) \leq \frac{1}{\beta R_0}$ which ensures the LHS of (84) is 0. As $p_1^* \geq \theta R_0/R_1 > 0$ in equilibria with positive investment at $t = 1$, the condition ensures over-investment in equilibrium.

Under-investment occurs when the inequality in (84) is reversed. From (60) we have the following upper bound on the price in a constrained equilibrium:

$$p_1^* < \bar{p}_c = \frac{\alpha a R_0}{(1-\alpha)(1-a)}. \quad (85)$$

Therefore, a sufficient condition for under-investment is

$$\frac{\alpha a R_0}{(1-\alpha)(1-a)} < \frac{\alpha((a + \alpha(1-a))R_0 - 1/\beta)}{(1-\alpha)^2(1-a)} \iff \alpha R_0 > 1/\beta, \quad (86)$$

which is the expression in Corollary 1. Note that as (62) and the conditions for over- and under-investment must hold simultaneously, the former may be viewed as restrictions on R_0 or β . □

Proof of Proposition 4

Proof. Differentiating welfare with respect to a at the optimum we have

$$\frac{d\Pi_0^*}{da} = -\mu_{1,S}I_0^* - \frac{dp_1^*}{da} [\lambda_{1,b}^{e*}T_{1,b}^{e*} + \lambda_{1,b}^{l*}(T_{1,b}^{l*} - S_{1,b}^{l*})] \quad (87)$$

$$= \frac{\mu_{1,T}^*I_0^*}{p_1^*} \left[(1-\alpha)(1-a) \frac{dp_1^*}{da} - \alpha R_0 - (1-\alpha)p_1^* \right], \quad (88)$$

where we have used (43), and $T_0^* = S_0^* = (1-a)I_0^*$. We ignore the price effects arising from changes in p_0^* , since intermediaries have net zero sales at $t = 0$, and thus changes in p_0^* do not alter the budget and in turn do not affect investment or welfare. The $t = 1$ market-clearing condition, $\alpha T_{1,b}^{e*} = -(1-\alpha)T_{1,b}^{l*}$ can be expressed as:

$$\alpha \frac{((a + \alpha(1-a))R_0 - 1/\beta) I_0^* + \beta^{-1}k - I_{1,b}^{e*}}{p_1^*} = (1-\alpha)^2(1-a)I_0^*. \quad (89)$$

Denote $M = (1 - \alpha)^2(1 - a)p_1 - \alpha((a + \alpha(1 - a))R_0 - \beta^{-1})$, where $M > 0$ is the condition for over-investment derived in the proof of Corollary 1. Differentiating both sides of (89) with respect to a and solving for dp_1^*/da we have:

$$\frac{dp_1^*}{da} = \frac{\alpha R_0 + (1 - \alpha)p_1^*}{(1 - \alpha)(1 - a)} \cdot \left[\frac{(1 - \alpha)(1 - a) \left((1 - \alpha)I_0^* - \frac{M}{\alpha R_0 + (1 - \alpha)p_1^*} \frac{\partial I_0^*}{\partial a} \right)}{M \frac{\partial I_0^*}{\partial p_1} + \frac{\alpha q_{2,b} \theta R_0}{p_1^{*2} C''(I_{1,b}^*)} + (1 - \alpha)^2(1 - a)I_0^*} \right], \quad (90)$$

where we have used $\frac{dI_{1,b}^{e*}}{da} = \frac{q_{2,b} \theta R_0}{p_1^{*2} C''(I_{1,b}^{e*})} \frac{dp_1^*}{da}$ and $\frac{dI_0^*}{da} = \frac{\partial I_0^*}{\partial a} + \frac{\partial I_0^*}{\partial p_1} \frac{dp_1^*}{da}$. Define the partial derivatives as follows:

$$\frac{\partial I_0^*}{\partial a} = \frac{(\alpha R_0 + (1 - \alpha)p_1)(1 - q_1)(1 - \alpha) (C'(I_{1,b}^{l*}) - C'(I_{1,b}^{e*}) + C''(I_{1,b}^{l*})M_l I_0^*)}{q_1(R_0 - r)^2 C''(I_{1,g}^*) + (1 - q_1)(1 - \alpha) C''(I_{1,b}^{l*})M_l^2 + C''(I_0^*)}, \quad (91)$$

$$\frac{\partial I_0^*}{\partial p_1} = \frac{-\frac{M_e(1 - q_1)\alpha q_{2,b} \theta R_0}{p_1^2} + (1 - q_1)(1 - \alpha)^2(1 - a)(q_{2,b} R_1 - C'(I_{1,b}^{l*}) - C''(I_{1,b}^{l*})M_l I_0^*)}{q_1(R_0 - r)^2 C''(I_{1,g}^*) + (1 - q_1)(1 - \alpha) C''(I_{1,b}^{l*})M_l^2 + C''(I_0^*)}, \quad (92)$$

$$= \frac{\frac{\mu_{1,T}(1 - a)(1 - \alpha)}{p_1} + \frac{M \lambda_{1,b}^*}{\alpha p_1} - C''(I_{1,b}^{l*})M_l I_0^*}{q_1(R_0 - r)^2 C''(I_{1,g}^*) + (1 - q_1)(1 - \alpha) C''(I_{1,b}^{l*})M_l^2 + C''(I_0^*)}, \quad (93)$$

where

$$M_e = aR_0 + \alpha(1 - a)R_0 - \beta^{-1}, \quad (94)$$

$$M_l = (1 - a)(\alpha R_0 + (1 - \alpha)p_1) - \beta^{-1}. \quad (95)$$

Then, $d\Pi_0^*/da > 0$ when the following is true

$$\frac{dp_1^*}{da} > \frac{\alpha R_0 + (1 - \alpha)p_1^*}{(1 - a)(1 - \alpha)} \iff \Omega \equiv \left[\frac{(1 - \alpha)(1 - a) \left((1 - \alpha)I_0^* - \frac{M}{\alpha R_0 + (1 - \alpha)p_1^*} \frac{\partial I_0^*}{\partial a} \right)}{M \frac{\partial I_0^*}{\partial p_1} + \frac{\alpha q_{2,b} \theta R_0}{p_1^{*2} C''(I_{1,b}^*)} + (1 - \alpha)^2(1 - a)I_0^*} \right] > 1. \quad (96)$$

Using $q_{2,b} R_1 - \frac{q_{2,b} \theta R_0}{p_1^*} = C'(I_{1,b}^{e*})$ from the first order conditions, rewrite Ω as

$$\Omega = \frac{\epsilon}{\epsilon + \phi}, \quad (97)$$

where

$$\epsilon = (1 - \alpha)^2(1 - a) \left[I_0^* + \frac{(1 - q_1)M}{\delta} (C'(I_{1,b}^{e*}) - C'(I_{1,b}^{l*}) - C''(I_{1,b}^{l*})M_l I_0^*) \right], \quad (98)$$

$$\phi = \frac{q_{2,b} \theta R_0}{p_1^*} \left(\frac{(1 - q_1)(1 - \alpha)^2(1 - a)M}{\delta} + \frac{\alpha}{p_1^* C''(I_{1,b}^{e*})} - \frac{MM_e(1 - q_1)\alpha}{\delta p_1^*} \right), \quad (99)$$

$$\delta = (1 - q_1)(1 - \alpha) C''(I_{1,b}^{l*})M_l^2 + C''(I_0^*) > 0. \quad (100)$$

Manipulation gives

$$\phi = \frac{(1 - q_1)q_{2,b}\theta R_0}{p_1\delta} \left(\frac{M^2}{p_1} + \frac{\alpha\delta}{p_1 C'''(I_{1,b}^{e*})} \right) > 0. \quad (101)$$

$M_l < 0$ in the over-investment case. To see this, note that M_l is strictly increasing in p_1 . Evaluating M_l at the maximum constrained price $\bar{p}_c = a\alpha R_0/(1-\alpha)(1-a)$, which is described in (85). This gives $M_l(\bar{p}_c) = \alpha R_0 - \beta^{-1}$, which must be negative in the over-investment case as shown in the proof of Corollary 1. Thus, both ϵ and ϕ are strictly positive and $\Omega < 1$, implying $d\Pi_0^*/da < 0$.

Finally, if $M > 0$ and $M_l < 0$, then $\partial I_0^*/\partial a < 0$ and $\partial I_0^*/\partial p_1 > 0$. From (90), it follows that $dp_1/da > 0$.

□

C For Online Publication: Evidence on growth of ABS and safe instruments

The panels below provide evidence on ABS and Money Market Funds in the US and Europe respectively. The top left figure in the first panel below shows the amount of ABS outstanding in the United States over the period 1986-2015. As is clear, the supply of securitized assets has increased dramatically over this period, despite a slow-down following the 2007-2009 financial crisis. The data exclude securitized assets issued by Fannie Mae, Freddie Mac and Ginnie Mae. The top left figure shows the amount of ABS issued every year over the same period. The bottom left figure shows the amount of money market instruments outstanding in the United States over the period 1996-2015. This figure shows that demand for money-like instruments also dramatically expanded over this period. The bottom right figure plots the total ABS outstanding and the total money market funds. This shows that these two series have generally moved together over time, lending credibility to our assumption that the increased demand for safe (money-like) assets has contributed to the growth in ABS. The following figures show the corresponding data for Europe.

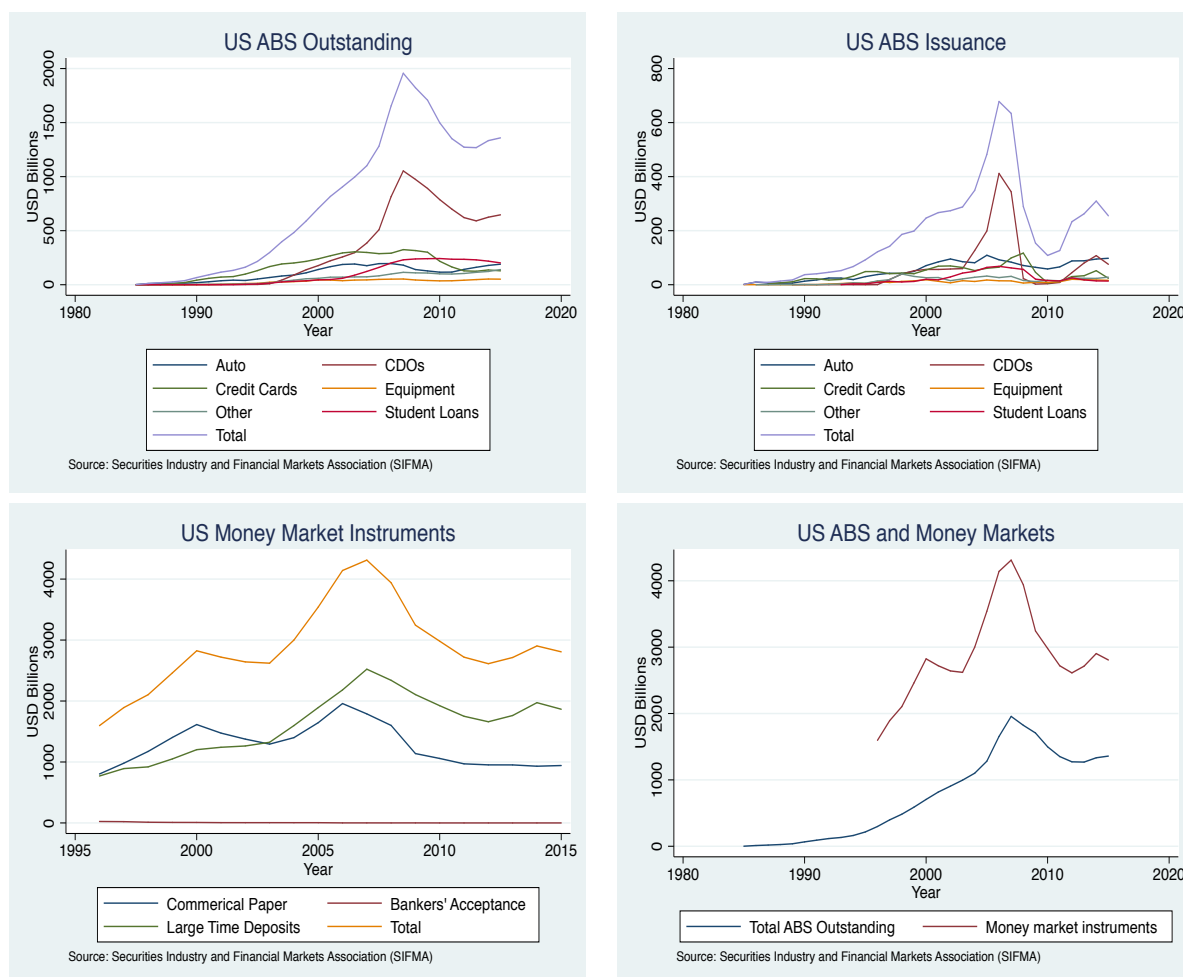


Figure 2: US: ABS and Money Markets

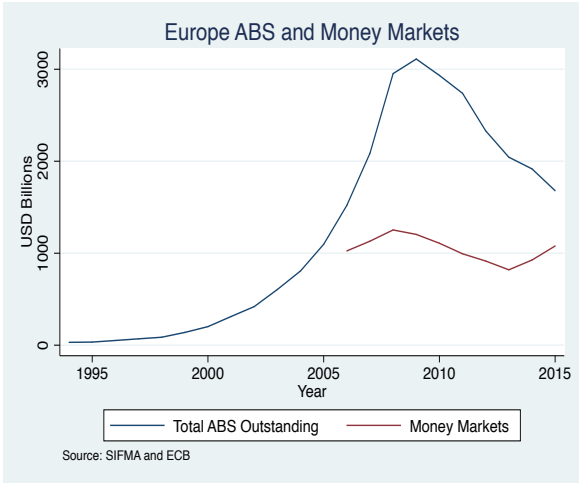
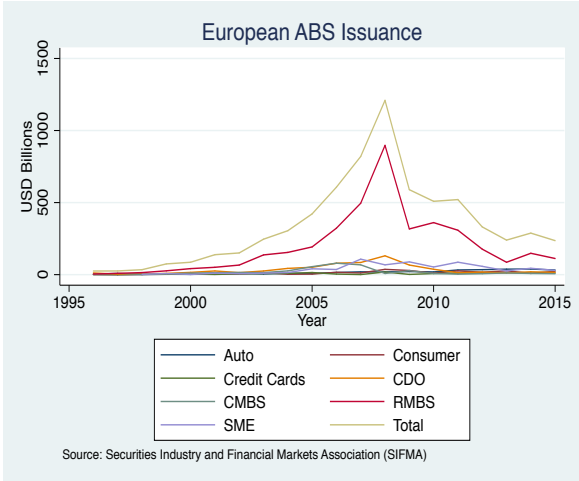
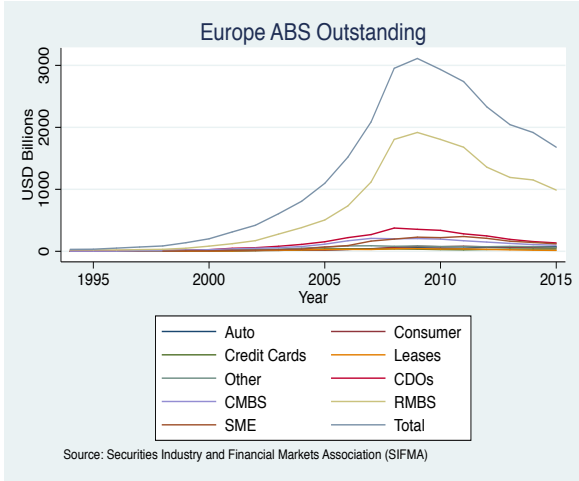


Figure 3: Europe: ABS and Money Markets