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Perils of quantitative easing[1]

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Abstract

Quantitative easing compromises the control of the central bank over the stochastic path of inflation.

Keywords: Quantitative easing; credit easing; inflation.

JEL Codes: D50; E31; E52

Quantitative easing (QE) focuses on the expansion of the balance sheet of the central bank and the related creation of reserves, but it does not restrict the composition of the balance sheet. In contrast, credit easing (CE) targets a specific allocation of assets, much like conventional monetary policy that restricts open market operations to Treasury bills. We show that this distinction has significant implications for the extent to which monetary policy can target the stochastic path of inflation.

We consider a stochastic cash-in-advance economy with flexible prices and a perfect, in particular complete, asset market, and we restrict attention to trades in securities of one-period maturity; trades in long-lived assets could duplicate such trades, as in Kreps (1982), and allow for a role for the maturity structure of debt, as in Cochrane (2001) or Angeletos (2002). Well-founded criticisms of the fiscal theory of the price level in Buiter (2002) and Drèze and Polemarchakis (2000) notwithstanding, we make the assumption of non-Ricardian seigniorage policy for the central bank; this, to remain in an environment that, under conventional monetary policy, yields a determinate price level. Our conclusion, that, surprisingly, has gone unnoticed, is that monetary policy, that sets a path of short-term, nominal interest rates, determines the path of expected or average inflation, but not the distribution of possible paths of inflation. The stochastic path of inflation is determined by the adjustment of the portfolio of the monetary authority over time in response to market forces and expectations. Under pure QE, without adequate targets, indeterminacy is pervasive.

Under conventional policy, a monetary authority conducts open market operations or repo transactions that conform to an ex ante determined overall portfolio composition; in particular, one that has an exclusive focus on Treasuries of short maturity. The fiscal theory of the price level in Woodford (1994) takes it for granted that a monetary authority trades exclusively in short-term, nominally risk-free bonds. This is also the case in Dubey and Geanakoplos (2003), who argue that "outside money" suffices to eliminate the indeterminacy that prevails in economies with nominally denominated assets; an important claim, because, as noted by Cass (1984, 1985) and analysed in depth in Balasko and Cass (1989) and Geanakoplos and Mas-Colell (1989), when the asset market is incomplete, nominal indeterminacy has real effects. Here, we highlight the importance of the composition of the portfolio of the monetary authority for the determinacy of the path of prices, even, the determinacy of the price level.

The possible multiplicity of stochastic inflation paths at equilibrium was clear in Bloise, Drèze, and Polemarchakis (2005) and Nakajima and Polemarchakis (2005); but there, the specification was Ricardian, equilibria were indeterminate, and the point was to demonstrate that the indeterminacy can be parametrized by the price level and a nominal martingale measure. Magill and Quinzii (2014b) developed the argument that inflationary expectations can serve as an alternative parametrization, which is more interesting. Drèze and Polemarchakis (2000) pointed out the need for "comprehensive monetary policy" that sets the stochastic path of the term structure of interest rates (or, equivalently, all state-contingent short-term rates) in order to determine the path of inflation. This theme was later developed in Adao, Correia, and Teles (2014), and Magill and Quinzii (2014a). Importantly, in this argument, the way out of indeterminacy involved targets or restrictions on the returns of assets. Our point here is that the the composition of the balance sheet of the monetary authority matters as an instrument of immediate policy relevance. Under unconventional quantitative policies, the asset side of the balance sheet of the central bank is not ex ante specified; market forces and expectations determine the value of the assets in the balance sheet as uncertainty unfolds. Variations in the value of the balance sheet then determine the stochastic path in which money is injected or withdrawn and, thus, the path of inflation. On the other hand, under credit policies, it is the explicit target for the composition of the balance sheet that allows the monetary authority to target the stochastic path of inflation: the target for the composition of the portfolio guarantees the necessary restrictions to obtain determinacy. The distinction we make between QE and CE is analogous to that of Curdia and Woodford (2011), who they define QE as lending to the private sector restricted only by a target quantity of reserves, while CE refers to the purchases of government bonds.

There is a vast and important literature on indeterminacy of monetary equilibria: Sargent and Wallace (1975) pointed out the indeterminacy of the initial price level under interest rate policy; Lucas and Stokey (1987) obtained uniqueness of a recursive equilibrium with money supply policy, but they restricted attention to inflation processes without memory; Woodford (1994) analyzed the dynamic paths of equilibria associated with the indeterminacy of the initial price level under money supply policy. In this paper, we give the exact characterization of recursive equilibria under quantitative easing with interest rate policy.

Our argument does not derive from the infinity of the horizon or the stability of a steady state; Benhabib and Farmer (1999) is a useful survey of this literature. In particular, it applies to a finite horizon, which explains that it applies to recursive equilibria. Although, as long as fiscal policy is Ricardian, the coefficient in the Taylor rule does not change the degree of indeterminacy, it affects the number of locally bounded equilibria as in Woodford (1999) and Benhabib, Schmitt-Grohe, and Uribe (2001); Benhabib, Schmitt-Grohe, and Uribe (2002) examined the interaction of non-Ricardian fiscal policy with the

Taylor rule that yields a unique equilibrium. Carlstrom and Fuerst (1998) discussed the indeterminacy of sticky-price equilibria when the nominal interest rate is zero. here, as we show, feedback rules, that set interest rates or the composition of the balance sheet as a function of future variables, are not sufficient to obtain a determinate inflation path. "Simple" inflation processes may only be compatible with conventional monetary policy.

The indeterminacy in our benchmark model is nominal; while the central bank loses the control of inflation, the indeterminacy does not affect the attainable equilibrium allocations. If the central bank were to switch to a money supply, rather than interest rate, policy, the indeterminacy would be real: it would affect real allocations. More importantly, the indeterminacy is real if prices are sticky or the asset market incomplete, as in Bai and Schwarz (2006); in Curdia and Woodford (2011), "quantitative easing in the strict sense" is ineffective, while "targeted asset purchases" matter and may be welfare-improving, if asset markets are sufficiently disrupted.

1 Unconventional policy in practice

There is general agreement on the objective of unconventional policies as a mechanism to support credit and liquidity. However the implementation of policies vary, as Bernanke (2009) explains:

> "*The Federal Reserve's approach to supporting credit markets is conceptually distinct from quantitative easing (QE), the policy approach used by the Bank of Japan from 2001 to 2006. Our approach–which could be described as 'credit easing' (CE)–resembles quantitative easing in one respect: It involves an expansion of the central bank's balance sheet. However, in a pure QE regime, the focus of policy is the quantity of bank reserves, which are liabilities of the central bank; the composition of loans and securities on the asset side of the central bank's balance sheet is incidental.*"

While no central bank has pursued pure QE or pure CE, some central banks have pursued policies much closer to CE and others policies much closer to QE. The key characteristic that makes the Fed's policies[1] closer to CE is that it sought to expand it's balance sheet while committing to a

[1] The Federal Reserve reduced the target federal funds rate to effectively zero and also implemented a number of other programs and policies which led to significant changes to the Federal Reserve's balance sheet. For a discussion of Fed policies see, for example, Bernanke (2009), Goodfriend (2011), Reis (2009) and Fawley and Christopher (2013).

specific structure and composition in doing so. As part of this, the Federal Reserve Bank of New York published how (in terms of portfolio weights) the total Large Scale Asset Purchases (LSAP) program purchases would be distributed across maturity sectors. Moreover, each two-week-long round of asset purchases would begin every-other Wednesday, when the System Open Market Account (SOMA) Desk would announce the maturity sectors in which it would be buying over the subsequent two weeks. In our model the indeterminacy is resolved by ex-ante restrictions on the composition of assets held on the central bank balance sheet.

The Fed's approach contrasts with the easing policies of other major central banks. During the first round of QE undertaken between 2001 and 2006, the Bank of Japan (BoJ) set new operational targets for monetary policy in terms of the central bank reserves held by financial intermediaries (called Current Account Balances). To achieve these targets, it made outright purchases of a long-term Japanese government bonds, stocks held by commercial banks (from October 2002 to September 2003) and asset backed securities (ABS) (from July 2003 to March 2006), but the specific portfolio of these assets was not the target of the central bank. Recent unconventional policies by the BoJ have targeted lending to banks rather than the outright purchase of assets from secondary markets; Fawley and Christopher (2013) argue that the BoJ was mainly concerned with generating reserves and provided limited restrictions on the range of assets. One may argue that the assets purchased by the BoJ were motivated by expectations of prices and premia. We show that even such a policy is insufficient to rule out indeterminacy. Ugai (2007) and Maeda et al. (2005) discuss the details of the BoJ experience with QE in more detail.

Recent unconventional policies by the ECB have also targeted lending to banks rather than the outright purchase of assets from secondary markets. Nonetheless, Fawley and Christopher (2013) argues that these programs may also be "considered pure QE in the sense that they targeted reserves and typically accepted a wide range of assets as collateral". The QE scheme of the Bank of England (BoE) is similar to the early QE scheme of the BoJ. The Monetary Policy Committee set an overall target for the amount of assets purchased through the Asset Purchase Facility (APF), the menu of assets was not the target. However, the BoE set some restrictions on the asset portfolio largely to medium- and long-term gilts makes the APF somewhat closer to the Fed, and CE, than the BoJ.[2] Given this heterogeneity of central

[2]Established in January 2009, the APF was first used as a tool of monetary policy in March 2009. Initially the APF would buy high-grade corporate bonds and government gilts with maturity 5-25 years, but more recently the APF only bought conventional gilts, though over a slightly extended maturity range starting at three years. In any given

bank implementation of unconventional policies, the theoretical distinction we highlight, albeit extreme cases of pure QE and pure CE, is of particular relevance.

2 The analytical argument

Monetary policy involves *quantitative easing* if open market operations extend to unrestricted portfolios of government bonds of different maturities or bonds issued by the private sector. It involves *credit easing* if open market operations extend beyond treasuries, but still target a specific composition for the balance sheet of the monetary-fiscal authority; as a limit case, monetary policy is *conventional* when open market operations are restricted to short term, nominally risk-free assets (Treasury bills).

Fiscal policy is *Ricardian* if it is restricted to satisfy an intertemporal budget constraint or transversality condition; equivalently, if public debt vanishes for all possible, equilibrium or non-equilibrium, values of prices and interest rates. It is *non-Ricardian*, if it is not restricted to satisfy an intertemporal budget constraint; in particular, outside money or initial liabilities of the public towards the private sector are not taxed back.

Quantitative easing generates indeterminacy indexed by a *nominal pricing measure* over states of the world. This measure determines the distribution of rates of inflation, up to a moment that is determined by the risk-free rate and non-arbitrage. Ricardian policy leaves the initial price level indeterminate as well. Determinacy and, by extension, monetary and financial stability, obtain under credit easing or monetary policy that is conventional. The indeterminacy is nominal only as long as prices are flexible, monetary policy sets nominal rates of interest, and the asset market is (effectively) complete; otherwise, there are, generically, real effects. It is worth pointing as our analysis considers a process of continual re-balancing of the monetary-fiscal authority balance sheet, our argument applies equally to the unwinding of quantitative easing as well as to the initiation of it. What is essential is the type of policy that determines the stochastic evolution of the balance sheet.

In Section 2.1 we characterize unconventional monetary policy under pure quantitative easing where the composition of the assets traded by the central bank is unrestricted. We show the indeterminacy inherent in a stochastic economy and link it to the mix of interest and non-interest bearing assets

purchase operation, a broad range of assets, such as gilts with maturities 10-25 years, were up for purchase; a reverse auction determined the allocation and prices, remained determined by market conditions.

traded by the monetary-fiscal authority. In Section 2.2 we show that this is not a consequence of non-stationary equilibria or of exogenous interest rate paths, and we make explicit the role of the composition of the portfolio of the monetary-fiscal authority portfolio in the determination of stochastic inflation rates. Interest-rate feedback rules, in the presence of pure quantitative easing is insufficient to obtain determinacy We then show that pure credit easing policies (which set portfolio weights exogenously) obtains determinacy while policies that allow for feedback rules determining the composition of assets is insufficient to rule out indeterminacy. Finally, restricting attention to "simple" inflation processes may only be compatible with conventional monetary policy.

2.1 A stochastic dynamic economy

Time, t, is discrete, and it extends into the infinite future: t = 1, ... Events, s^t, at each date are finitely many. An immediate successor of a date-event is $s^{t+1}|s^t$, and, inductively, a successor is $s^{t+k}|s^t$. Conditional on s^t, probabilities of successors are $f(s^{t+1}|s^t)$ and, inductively, $f(s^{t+k}|s^t) = f(s^{t+k}|s^{t+k-1})f(s^{t+k-1}|s^t)$.

At a date-event, a perishable input, labor, $l(s^t)$, is employed to produce a perishable output, consumption, $y(s^t)$, according to a linear technology:

$$y(s^t) = a(s^t)l(s^t), \quad a(s^t) > 0.$$

A representative individual is endowed with 1 unit of leisure at every date-event. He supplies labor and demands the consumption good, and he derives utility according to the cardinal utility index $u(c(s^t), 1 - l(s^t))$ that satisfies standard monotonicity, curvature and boundary conditions. The preferences of the individual over consumption-employment paths commencing at s^t are described by the separable, von Neumann-Morgenstern intertemporal utility function

$$u(c(s^t), 1 - l(s^t)) + \mathrm{E}_{s^t} \sum_{k>0} \beta^k u(c(s^{t+k}|s^t), 1 - l(s^{t+k}|s^t)),$$

where $0 < \beta < 1$. Balances, $m(s^t)$, provide liquidity services. Elementary securities, $\theta(s^{t+1}|s^t)$, serve to transfer wealth to and from immediate successor date-events. The price level is $p(s^t)$, and the wage rate is $w(s^t) = a(s_t)p(s^t)$, as profit maximization requires. The nominal, risk-free interest rate is $r(s^t)$.

At each date-event, the asset market opens after the uncertainty, s^t, has realized, and, as a consequence, purchases and sales in the markets for labor and the consumption good are subject to standard cash-in-advance constraints; the effective cash-in-advance constraint is[3]

$$a(s^t)p(s^t)l(s^t) \leq m(s^t).$$

Prices of elementary securities are

$$q(s^{t+1}|s^t) = \frac{\nu(s^{t+1}|s^t)}{1 + r(s^t)},$$

with $\nu(\cdot|s^t)$ a "nominal pricing measure" or transition probabilities, which guarantees the non-arbitrage relation

$$\sum_{s^{t+1}|s^t} q(s^{t+1}|s^t) = \frac{1}{1 + r(s^t)}.$$

Inductively,

$$\nu(s^{t+k}|s^t) = \nu(s^{t+k}|s^{t+k-1})\nu(s^{t+k-1}|s^t), \quad k > 1,$$

and the implicit price of revenue at successor date-events is

$$q(s^{t+k}|s^t) = \frac{\nu(s^{t+k}|s^{t+k-1})}{1 + r(s^{t+k-1}|s^t)} q(s^{t+k-1}|s^t), \quad k > 1.$$

The individual has initial wealth $\tau(s^1) = \omega$. Initial wealth constitutes a claim against the monetary-fiscal authority; alternatively, it can be interpreted as outside money. It is exogenous in a non-Ricardian specification. In a Ricardian specification, it is set endogenously so as to satisfy the transversality condition imposed on monetary-fiscal policy.

The flow budget constraint is

$$p(s^t)z(s^t) + m(s^t) + \sum_{s^{t+1}|s^t} q(s^{t+1}|s^t)\theta(s^{t+1}|s^t) \leq \tau(s^t),$$

where $z(s^t) = c(s^t) - a(s^t)l(s^t)$ is the effective excess demand for consumption.

Wealth at successor date-events is

$$\tau(s^{t+1}|s^t) = \theta(s^{t+1}|s^t) + m(s^t),$$

and, after elimination of the trade in assets, the flow budget constraint reduces to

$$p(s^t)z(s^t) + \frac{r(s^t)}{1 + r(s^t)} a(s^t)p(s^t)l(s^t) + \sum_{s_{t+1}} q(s_{t+1}|s^t)\tau(s_{t+1}|s^t) \leq \tau(s^t).$$

[3] Nakajima and Polemarchakis (2005) provide an explicit derivation.

Debt limit constraints are

$$-\tau(s^t) \leq \sum_{k>0} \sum_{s^{t+k}|s^t} q(s^{t+k}|s^t) \frac{1}{1+r(s^t)} a(s^{t+k}) p(s^{t+k}).$$

Alternatively, $\tilde{m}(s^t) = (1/p(s^t))m(s^t)$ are real balances, $\tilde{\tau}(s^t) = (1/p(s^t))$ $\tau(s^t)$ is real wealth, $\pi(s^{t+1}|s^t) = (p(s^{t+1})/(p(s^t)) - 1$ is the rate of inflation, and

$$\tilde{q}(s^{t+1}|s^t) = q(s^{t+1}|s^t)(1+\pi(s^{t+1}|s^t)) = \frac{\nu(s^{t+1}|s^t)(1+\pi(s^{t+1}|s^t))}{1+r(s^t)}$$

are prices of indexed elementary securities.

Real wealth at successor date-events is

$$\tilde{\tau}(s^{t+1}|s^t) = \left(\frac{\theta(s^{t+1}|s^t) + m(s^t)}{p(s^t)} \right) \frac{1}{1+\pi(s^{t+1}|s^t)},$$

and the flow budget constraint reduces to

$$z(s^t) + \frac{r(s^t)}{1+r(s^t)} a(s^t) l(s^t) + \sum_{s_{t+1}} \tilde{q}(s_{t+1}|s^t) \tilde{\tau}(s_{t+1}|s^t) \leq \tilde{\tau}(s^t).$$

First order conditions for an optimum are

$$\tfrac{\partial u(c(s^t),1-l(s^t))}{\partial c(s^t)} = \tfrac{\partial u(c(s^t),1-l(s^t))}{\partial l(s^t)} \left(\tfrac{a(s^t)}{1+r(s^t)} \right)^{-1},$$

$$\beta f(s^{t+1}|s^t) \tfrac{\partial u(c(s^{t+1}),1-l(s^{t+1}))}{\partial c(s^{t+1})} \tilde{q}(s^{t+1}|s^t)^{-1} = \tfrac{\partial u(c(s^t),1-l(s^t))}{\partial c(s^t)},$$

and the transversality condition is

$$\lim_{k\to\infty} \sum_{s^{t+k}|s^t} \tilde{q}(s^{t+k}|s^t) \tilde{\tau}(s^{t+k}|s^t) = 0.$$

The monetary-fiscal authority sets rates of interest and accommodates the demand for balances. It supplies balances, $M(s^t)$, and trades in elementary securities subject to a flow budget constraint that, after elimination of the trade in assets, reduces to

$$T(s^t) \leq \frac{r(s^t)}{1+r(s^t)} M(s^t) + \sum_{s^{t+1}|s^t} q(s^{t+1}|s^t) T(s^{t+1}|s^t),$$

where $T(s^t)$ and, similarly, $T(s^{t+1}|s^t)$ are obligations towards the private sector; initial obligations are $\Omega = T(s^1)$. Ricardian policy imposes on the monetary-fiscal authority the transversality condition

$$\lim_{k\to\infty} \sum_{s^{t+k}|s^t} q(s^{t+k}|s^t)T(s^{t+k}|s^t) = 0$$

or, equivalently, as prices vary, it sets the initial claims of the private sector as

$$\Omega = \frac{r(s^1)}{1+r(s^1)}M(s^1) + \sum_{t>0}\sum_{s^t|s^1} \frac{r(s^t|s^1)}{1+r(s^t|s^1)}q(s^t|s^1)M(s^t|s^1).$$

For equilibrium, it is necessary and sufficient that the excess demand for output vanishes:

$$z(s^t) = c(s^t) - a(s^t)l(s^t) = 0.$$

From the first order conditions for an optimum, this determines the path of employment and consumption:

$$\frac{\partial u(c(s^t), 1-l(s^t))}{\partial c(s^t)} = \frac{\partial u(c(s^t), 1-l(s^t))}{\partial l(s^t)} \left(\frac{a(s^t)}{1+r(s^t)}\right)^{-1},$$

and, in turn, the prices of indexed elementary securities:

$$\beta f(s^{t+1}|s^t) \frac{\partial u(c(s^{t+1}), 1-l(s^{t+1}))}{\partial c(s^{t+1})} \tilde{q}(s^{t+1}|s^t)^{-1} = \frac{\partial u(c(s^t), 1-l(s^t))}{\partial c(s^t)}.$$

The initial price level serves to guarantee that, at equilibrium, the transversality condition of the monetary-fiscal authority holds. If monetary-fiscal policy is Ricardian, the price level remains indeterminate. If it is non-Ricardian, in that initial claims are given, then the equilibrium path of nominal asset prices determines the present-discounted value of unindexed transfers and so the initial price level.

More importantly, without further restrictions, as is the case under QE, the decomposition of equilibrium asset prices into an inflation process, $\pi(\cdot|s^t)$, and a nominal pricing measure, $\nu(\cdot|s^t)$, remains indeterminate: if the nominal pricing measure, $\nu(\cdot|s^t)$, is specified arbitrarily, the inflation process, $\pi(\cdot|s^t)$, adjusts to implement the equilibrium; that is, to satisfy

$$\tilde{q}(s^{t+1}|s^t) = \frac{\nu(s^{t+1}|s^t)(1+\pi(s^{t+1}|s^t))}{1+r(s^t))}.$$

The determinacy in Woodford (1994) highlights the importance of the present value of the monetary-fiscal authority budget constraint in the determination of the price level. We examine here, and what is often overlooked, is that the stochastic evolution of government wealth is essential for the determination of the stochastic path of prices. That Woodford (1994) restricts attention to conventional policy, in which case the portfolio of the monetary-fiscal authority is composed solely of Treasury bills, obscures this second point. Our results are not dependent on the infinite horizon of the economy, or the open-endedness of the policies that we describe. In previous versions of this paper we showed that our results remain valid in a finite horizon economy.

It may be confusing that we abstract from fiscal transfers after the initial period; we only do so because their implications are straightforward and do not affect the argument. It is worth pointing out, however, that the dichotomy between the nominal pricing measure and the initial price level that obtains when transfers are indexed, no longer holds when transfers are not indexed: the nominal pricing measure, indeterminate under quantitative easing, affects the aggregate volume of claims against the monetary-fiscal authority and, as a consequence, the initial price level as well.

Under QE, the nature of the interest-elasticity of money demand does not determine the stationary equilibrium path, though it determines the stability of the path. Following Sims (1994), the introduction of a portfolio of securities (rather than the single risk-free bond that was considered) under a policy of QE leaves the difference equations, that otherwise determine the unique path of money, to depend on the state-contingent return on the portfolio of the monetary-fiscal authority and the portfolio-to-money supply ratio. A given (stationary) distribution of portfolio returns, that satisfies the no-arbitrage condition given by the fixed short-term nominal interest rate, then corresponds to a stationary distribution of portfolios, even for the fiscal policy rules that were considered there. Put simply, adequate consideration of the interest-elasticity of money demand guarantees a stationary distribution, but not a unique one. In the following section we examine the nature of stationary equilibria, abstracting from consideration of the stability of stationary equilibria and, therefore, also from the interest-elasticity of money demand.

2.2 A stationary economy

The argument extends to stationary economies and stationary equilibria or steady states.

The resolution of uncertainty follows a stationary stochastic process. Elementary states of the world are s, finitely many, and transition probabilities are $f(s'|s)$.

Rates of interest, $r(s)$, determine the path of consumption, $c(s)$, and employment, $l(s)$, at equilibrium, which, in turn, determine the prices of indexed elementary securities:

$$\beta f(s'|s)\frac{\partial u(c(s'),1-l(s'))}{\partial c(s')}\tilde{q}(s'|s)^{-1}=\frac{\partial u(c(s),1-l(s))}{\partial c(s)}$$

or

$$\tilde{Q}=\beta Du(s)^{-1}FDu(s').$$

Here,

$$Du(s)=diag\left(\ldots,\frac{\partial u(c(s),1-l(s))}{\partial c(s)},\ldots\right)$$

is the diagonal matrix of marginal utilities of consumption, and

$$F=(f(s'|s)) \quad \text{and} \quad \tilde{Q}=(\tilde{q}(s'|s))$$

are, respectively, the matrices of transition probabilities and of prices of indexed elementary securities.

With

$$\tilde{y}=(\ldots\frac{r(s)}{1+r(s)}a(s)l(s)\ldots)'$$

the vector of net, real expenditures on balances at equilibrium, real claims against the fiscal-monetary authority at the steady state,

$$\tilde{\tau}=(\ldots\tilde{\tau}(s),\ldots)',$$

are determined by the equation

$$\tilde{y}+\tilde{Q}\tilde{\tau}=\tilde{\tau} \quad \text{or} \quad \tilde{\tau}=(I-\tilde{Q})^{-1}\tilde{y};$$

since $0<\beta<1$,

$$(I-\tilde{Q})^{-1}=\sum_{k=0}^{\infty}\tilde{Q}^k=\sum_{k=0}^{\infty}\beta^kDu(s)^{-1}F^kDu(s'),$$

and, since F is a Markov transition matrix, while $\tilde{y}\gg 0$, the real claims against the monetary-fiscal authority at the steady state are strictly positive:

$$\tilde{\tau}\gg 0.$$

Non-Ricardian monetary-fiscal policy determines the initial price level by setting exogenously the level of initial nominal claims; otherwise, the price level remains indeterminate.

More importantly, the decomposition of equilibrium asset prices into an inflation process, $\pi(\cdot|s)$, and a nominal pricing measure, $\nu(\cdot|s)$, remain indeterminate:

$$\tilde{Q} = R^{-1}N \otimes \Pi,$$

where $\otimes$ denotes the Hadamard product. Here,

$$R = diag(\ldots, (1 + r(s)), \ldots)$$

is the diagonal matrix of interest factors, and

$$N = (\nu(s'|s)) \quad \text{and} \quad \Pi = ((1 + \pi(s'|s)))$$

are, respectively, the Markov transition matrix of "nominal pricing transition probabilities" and the matrix of inflation factors.

Alternative specifications of the stochastic process of inflation serve to characterize the set of equilibria and to highlight the role of the balance sheet policy of the monetary-fiscal authority.

The role of the balance sheet policy is the focus of the analysis here; it was not dealt with in Drèze and Polemarchakis (2000) or Bloise, Drèze, and Polemarchakis (2005).

QE: In the absence of restrictions on the balance sheet of the monetary fiscal authority, which is the case under QE, the set of steady state equilibria is indexed by the nominal pricing transition probabilities, $\nu(\cdot|s)$, that can be set arbitrarily, while the inflation factors, $\pi(\cdot|s)$, adjust to implement the equilibrium; alternatively, the inflation factors are set arbitrarily, up to a scale effect, and the nominal pricing transition probabilities adjust to implement the equilibrium.

The argument is as follows: with

$$(1 + \pi(s'|s)) = h(s)\gamma(s'|s),$$

an arbitrary (for the moment) decomposition of the inflation process into a term (the scale effect) that depends only on the current state and a term of (relative) inflation factors and is Markovian, the equilibrium condition takes the form

$$\tilde{Q} = R^{-1}N \otimes H\Gamma;$$

here, H be the diagonal matrix of the $h(s)$ and Γ the matrix of the $\gamma(s'|\ s)$.

Given Γ, there are H, N that guarantee equilibrium; the argument is straightforward:

$$\tilde{Q} = R^{-1}N \otimes \Pi \Rightarrow \tilde{Q} = R^{-1}N \otimes H\Gamma \Rightarrow \tilde{Q} \oslash \Gamma = (R^{-1}H)N,$$

the last step, since H is a diagonal matrix.[4]

Since N is Markovian if and only if it is non-negative and $N\mathbf{1}_S = \mathbf{1}_S$,

$$(\tilde{Q} \oslash \Gamma)\mathbf{1}_S = (R^{-1}H)\mathbf{1}_S,$$

which allows us to solve for $h(s)$.

With H, Γ in hand, we can solve for N, that shall indeed, be Markovian.

If N is given, there are H, Γ (or, equivalently, Π) that guarantee equilibrium.

Taylor rules: We now show that the indeterminacy obtained is not ruled out by interest-feedback rules. Any process can be written uniquely as

$$(1 + \pi(s'|s)) = h(s)\gamma(s'|s), \quad \gamma(s'|s) = \frac{\delta(s'|s)}{f(s'|s)}, \quad \sum_{s'} \delta(s'|s) = 1,$$

in which case,

$$h(s) = \mathrm{E}_{s'}(1 + \pi(s'|s));$$

With $r(s)$ not set exogenously, but as a function of $h(s)$, this is a Taylor (1993) rule, and indeterminacy persists. In other words, policy that specifies the path of nominal interest rates as a function of expected inflation, does not pin down the stochastic path of inflation.[5]

Evidently, with $r(s)$ not set exogenously, but as a function of $h(s)$, equilibrium requires solution of the equation

$$\frac{h(s)}{1 + r(h(s))} = \sum_{s' \in S} \frac{f(s'|s)}{\gamma(s'|s)} \beta \frac{\frac{\partial u(c(h(s')), 1 - l(h(s')))}{\partial c(h(s'))}}{\frac{\partial u(c(h(s)), 1 - l(h(s)))}{\partial c(s)}},$$

where the allocation, as a function of $h(s)$, is solved from the individual optimality conditions. If a solution to this system of equations exists and is unique, for example if the function/rule is linear, then the solution still depends on the (arbitrarily chosen) Γ.

CE: Alternatively,

$$(1 + \pi(s'|s)) = h(s)\gamma(s'|s), \quad \gamma(s'|s) = \frac{\delta(s'|s)}{\tilde{\tau}(s')}, \quad \sum_{s'} \delta(s'|s) = 1,$$

[4] $\oslash$ denotes element-by-element (Hadamard) division.

[5] That the Taylor rule does not depend on realized rates of inflation is appropriate for (stochastic) steady-state equilibria.

in which case, $\delta(s'|s)$ are portfolio weights that determine the composition of assets in the balance sheet of the monetary-fiscal authority.

Monetary-fiscal policy conducted as CE sets the composition of the balance sheet; that is, it sets explicit positive portfolio weights, $\delta(s'|s) > 0$; claims against the monetary-fiscal authority in real terms, $\tilde{\tau}(s)$, are determined, at the steady-state, by fundamentals, and, as a consequence, under CE, the matrix Γ is determined.

Since

$$N\mathbf{1}_S = \mathbf{1}_S \quad \Leftrightarrow \quad H\mathbf{1}_S = (R\tilde{Q} \oslash \Gamma)\mathbf{1}_S,$$

the Markov transition matrix, N, is well defined ($h \gg 0$) and determinate; it follows that the equilibrium is determinate as well.

Under conventional monetary-fiscal policy, the portfolio of the monetary-fiscal authority consists of Treasury bills, nominally risk-free bonds of short maturity. Here, this corresponds to one-period nominally risk-free bonds: $\delta(s'|s) = 1/S$.

In Eggertsson and Woodford (2003), determinacy obtains for arbitrary, but, importantly, fixed portfolio weights in the balance sheet of the monetary-fiscal authority; similarly, in Curdia and Woodford (2011) the composition of the portfolio depends only on date t variables or, in our terminology, state s variables. On the other hand, if the portfolio weights are chosen by policy to depend on endogenous nominal variables at s', such as the stochastic rate of inflation, then indeterminacy obtains. To be explicit, consider, for example, that the portfolio weights depended on expectations of the future nominal value of wealth: $\delta(s'|s) = [\tilde{\tau}(s')(1+\pi(s'|s))]/[\sum_{s'} \tilde{\tau}(s')(1+\pi(s'|s))] \Rightarrow h(s) = \sum_{s'} \tilde{\tau}(s')(1+\pi(s'|s))$. In a model where there are long-dated securities, setting portfolio weights as a function of expected nominal asset prices would have the same outcome. This contrasts with Magill and Quinzii (2014b) and Adao, Correia, and Teles (2014), where explicit targets for asset prices, independent of equilibrium, pin down portfolio weights.

Simple inflation processes: It is instructive to consider whether restricting the inflation process to depend endogenously only on either the current or future state is compatible with an equilibrium policy choice. Suppose that the inflation process, which is endogenous, is restricted to take the form

$$(1 + \pi(s'|s)) = h(s)b(s'),$$

where $b(s) > 0$ is positive function of the fundamentals of the economy determined at the steady state and, as a consequence,

$$N \otimes \Pi = HNB.$$

Here,

$$b = (\dots, b(s), \dots), \quad \text{and} \quad h = (\dots, h(s), \dots),$$

and B and H are the associated diagonal matrices.

Then,

$$\tilde{Q} = R^{-1} N \otimes \Pi \quad \Leftrightarrow \quad R\tilde{Q}B^{-1} = HN,$$

which determines the inflation process as well as nominal pricing probabilities, since

$$N\mathbf{1}_S = \mathbf{1}_S \quad \Leftrightarrow \quad N = \left(diag(R\tilde{Q}B^{-1}\mathbf{1}_S)\right)^{-1} R\tilde{Q}B^{-1},$$

a Markov transition matrix, as required.

This is indeed the case under conventional monetary policy.

Real wealth at successor date-events is

$$\tilde{\tau}(s') = \left(\frac{\theta(s'|s) + m(s)}{p(s)}\right) \frac{1}{1 + \pi(s'|s)},$$

and conventional monetary policy requires that

$$\theta(s'|s) = \theta(s)$$

or

$$(1 + \pi(s'|s)) = \underbrace{\left(\frac{\theta(s) + m(s)}{p(s)}\right)}_{h(s)} \underbrace{\frac{1}{\tilde{\tau}(s')}}_{b(s')}.$$

Suppose, instead, that inflation is restricted to depend endogenously only on the future state,

$$(1 + \pi(s'|s)) = h(s')b(s),$$

and, as a consequence,

$$N \otimes \Pi = BNH.$$

In this case,

$$\tilde{Q} = R^{-1} N \otimes \Pi \quad \Leftrightarrow \quad B^{-1}R\tilde{Q} = NH,$$

and

$$N\mathbf{1}_S = \mathbf{1}_S \quad \Leftrightarrow \quad N = B^{-1}R\tilde{Q}\left(diag((B^{-1}R\tilde{Q})^{-1}\mathbf{1}_S)\right)$$

that need not be positive. In other words equilibrium inflation may be restricted, as in Lucas and Stokey (1987) and Curdia and Woodford (2011), to depend endogenously only on the current state but not only on the future one. However such a restriction precludes analysis of the effects of unconventional monetary policy on changes in the composition of the balance sheet of the monetary-fiscal authority and their subsequent determination of the stochastic path of inflation.

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