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Information Acquisition and Credibility in Cheap Talk

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Abstract

This paper explores the interaction between uncertain bias and endogeneous information acquisition in strategic communication. I consider an expert who is privately informed about his bias as well as about whether he is informed, in addition can also engage in costly information acquisition. In this setup, information acquisition simultaneously serves the purposes of getting informed and increasing credibility before communicating through cheap talk to a decision maker. I define the *signaling* and the *intrinsic value* of information and find the conditions under which a separating equilibrium can arise, which is the most informative as well as the welfare maximizing equilibrium. I solve for equilibria as a function of cost of information acquisition and show that communication is most precise with an initially uninformed expert at an intermediary cost value. The overall welfare is non-monotone in cost, and it increases when cost increases to enable separation. When covert information acquisition is considered, there is a tradeoff between less wasteful investment versus less communication precision compared to the overt case.

Keywords: information acquisition; cheap talk; communication; signaling; credibility.

JEL Codes: D82, D83.

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1 Introduction

Decision makers often rely on the advice of experts. However, it is difficult to assess the credibility of experts as well as their knowledge. The literature on strategic information transmission pioneered by the seminal paper Crawford and Sobel (1982) has made two most common assumptions: that the bias of the expert is known and that the expert has access to information for free. These assumptions have been relaxed separately in different papers. Uncertainty about bias has been explored in the literature on the reputation of experts in repeated cheap talk initiated by Sobel (1985) and Morris (2001). Costly information acquisition has been first introduced by Austen-Smith (1994) and recently Sobel (2013) described this area as an open question. In order to study the interaction between bias and information acquisition, in this paper both of these assumptions are relaxed: the expert may or may not be aligned, in addition he may or may not be ex-ante informed. Finally, costly acquisition of information is observable by the decision maker, which means it is a signal. These ingredients make the problem rich in that there is costly signaling and cheap talk simultaneously.

To understand the relevance of this question, consider a government consulting a policy advisor about the affects of pollution on the occurrence of lung disease. Assume the advisor can be of three types: one type who gets kickbacks from a polluting industry and hence is biased. The other type of advisor has no conflict of interest, but already has done some work and hence is informed on this issue. Finally, there is the third type of advisor who needs to incur cost in order to get informed. One way to get informed is to hire a researcher to carry out some studies, which is costly but will make later advice more credible. Then, this means even a biased adviser could mimic this behavior only to increase his credibility while making a suggestion.[1]. Similar issues arise when a CEO asks a team manager about whether to launch a new product. The manager may have a bias towards launching the product due to his career concerns. In addition, the manager may already know the potential of this product due to his experience or else he may ask his team to carry out a market research. Motivated by these settings, the question this paper asks is how does bias interact with endogenous acquisition of information which may take place before cheap talk communication.

In the setup considered, the sender's information acquisition influences the decision maker's belief about his type which determines the credibility of his communication. Hence, there are 2 different motives for information acquisition for different types of experts: the first one is the *intrinsic value of information* and the second one is its *signaling value*. The expert cares about his credibility only insofar as it affects the decision maker's (DM) action: reputation is instrumental and not intrinsic, as in Sobel (1985) and Morris (2001) who consider repeated cheap

[1] There are many instances when experts incur costs in order to produce fake information in order to increase their credibility. For instance, Andrew Wakefield, a former gastroenterologist and medical researcher who was found guilty of misconduct in his research paper in order to support a claim that MMR vaccine was linked to autism and bowel disease. It was later discovered that he had been paid by lawyers who were trying to prove that the vaccine was unsafe. Deer (2006)https://www.thetimes.co.uk/article/mmr-doctor-given-legal-aid-thousands-00ftl80msbs

talk. By endogenizing the information acquisition process, in this paper this effect is captured without dynamics, as there is an initial stage in which the expert chooses whether to invest in information acquisition or not before communication takes place. The investment stage can be thought of as a reputation building or signaling stage. Wasteful investment in information arises in order to pool with aligned types. However, investment is not always wasteful as it leads to learning for the uninformed unbiased type of expert.

An effect similar to *bad reputation*, as defined by Ely and Valimaki (2003), arises: even an unbiased informed expert ends up wastefully investing in information when the cost is low enough. This disturbs the decision making process and undermines the value of information for the uninformed type of expert, as well as resulting in lower overall welfare. When the *signaling value* of information is higher than its *intrinsic value*, there exists no equilibrium in which information acquisition is efficient. Surprisingly, the decision maker's payoff may be higher when matched with an expert who is uninformed, as this type can communicate more efficiently than the one that is ex-ante informed.

A simple model is considered with binary state of the world, $\omega \in \{0, 1\}$ and perfectly revealing signals. The decision maker's action space is $y \in [0, 1]$. With probability β the expert is biased, and with probability α he is already informed about the state of the world. The decision maker and the aligned type of expert share the same incentives and want to match the state of the world, while the biased type wants highest action. As the biased expert has state independent preferences, information has no intrinsic value for him. Indeed we could assume that this type is never informed. This assumption enables to disentangle the intrinsic value of information from its signaling value. It turns out that the incentives of the biased expert and the unbiased expert endowed with signal 1 are equivalent: both want highest action to be taken.

The expert can incur cost and acquire a perfectly revealing signal, where investment in information is observable by the DM but not its outcome. As there is a probability that the expert is already informed, that an expert does not invest in information does not reveal that he is uninformed, neither that he is biased.

First, assuming information acquisition is a binary decision with a fixed cost, I solve for the different types of equilibria that arise for different costs of information acquisition. As there is signaling, equilibrium refinements, mainly the Intuitive Criterion, is useful to rule out unreasonable equilibria and obtain uniqueness. When the cost is high enough, in the unique equilibrium no investment takes place. Below the no investment region of cost, there is the separating equilibrium which is the most informative equilibrium and the best equilibrium for the DM. In this equilibrium the only type that invests is the uninformed unbiased expert, hence there is no *wasteful* investment. This equilibrium exists for some cost values under certain parameter restrictions and fails to exist whenever the "signaling value" of information is higher than its "intrinsic value". It is less likely to arise when the probability of the expert being biased is very high as this implies that the biased type and informed (1) type have higher incentives to invest.

The signaling value of investment is higher the lower the proportion of initially informed unbiased expert. Indeed, the proportion of informed unbiased types determine the payoff for the biased types when not investing: the higher the proportion of these types, communication is more credible hence the DM's action will be higher. The DM's payoff is highest when just sufficiently many types are informed that the equilibrium is separating. However, once in the separating region, the DM's as well as the overall welfare is decreasing in the proportion of informed unbiased experts, as these types cannot separate themselves from the biased type whenever the state is 1. This is the intuition for the result that the decision maker is better off having an expert who is initially uninformed: the uninformed type can separate himself from the biased type and communicate perfectly in the separating equilibrium while the informed type is always pooled with the biased type whenever the state is 1 and in any equilibrium.

When the cost of information acquisition is low enough, all three types except the informed (0) type pool by investing in information. Only the unbiased uninformed type invests for the value of information. However, due to the "contamination" by the biased types, the communication of the unbiased type is less credible in this equilibrium.

In the region in between the pooling and separating equilibrium, there exists no pure strategy equilibria. Instead, there are mixed strategy equilibria with the feature that the unbiased uninformed type always strictly prefers investing, while the biased and the unbiased informed (1) types are indifferent. As the cost of information acquisition goes down, proportionately more biased types than informed unbiased types should be investing in order to keep the indifference condition of these types themselves.

After characterizing the equilibria for every cost region, I go on to make welfare comparisons. Unsurprisingly, the DM's welfare is highest in the separating equilibrium which is the most informative one. Surprisingly, the ex-ante total welfare over all expert types plus the DM is higher when cost increases so that the equilibrium moves from pooling to the separating equilibrium. This means less wasteful investment more than compensates for a higher information acquisition cost when the cost moves from the pooling region to the separating one.

Then, I extend to covert information acquisition in which investment is no longer observable. The only investment that can arise in this setup is the efficient one and separation never arises in this case as the biased types can pool without incurring any cost with the unbiased type. The overall welfare is higher in the covert case whenever the separating equilibrium arises due to more precise communication. Otherwise, when pooling in investment arises, the covert case leads to higher welfare due to less wasteful investment.

As an extension, I generalize the information acquisition choice from binary to a continuum of effort levels, $e \in [0, 1]$, where e is the probability of getting information and $c(e)$ is an increasing and convex cost function. I show that the result that a separating equilibrium fails to exists whenever the signaling value of information is more than the intrinsic value remains robust. In addition, in the least cost separating equilibrium there may be over investment by the unbiased type in order to separate himself from the biased type. Indeed, for some parameter values, the decision maker is better off having uncertainty about the type of the expert compared to having an unbiased expert with probability one, precisely when the over investment in information by the unbiased type more than compensates for the possibility that the expert is biased and communication is uninformative. There are also pooling equilibria at positive investment levels which survive the Intuitive Criterion. However, whenever a LCS equilibrium does exist, I argue that it will be the unique outcome through the use of a babbling threat by the decision maker.[2]

2 Literature

This paper relates closely to the literature on reputational concerns in cheap talk introduced by Sobel (1985) and Morris (2001) who consider repeated cheap talk with uncertainty about sender type. The concern about reputation in this paper is similar to Morris (2001) in that it is "instrumental" and not "intrinsic": it matters insofar as the decision maker's action can be influenced. Morris captures this in a two period setup in which the DM updates the prior about the agent's type as a function of his message and the outcome realized, whereas in my setting the decision is taken only once but the information acquisition serves as a credibility building stage. In his setting, the advice of the unbiased type of advisor is distorted in order to avoid being perceived as a biased type and to enhance his credibility in period 2. In my setup, the messages are never distorted as decision making takes place only once, but distortion takes place in form of wasteful investment in information acquisition. In Morris, as reputational concerns increase, the good advisor more incentive to be politically correct that when these concerns are sufficiently high, there is only a babbling equilibrium. In my setup, when the signaling incentive is high enough, there is always wasteful investment and no possibility for an unbiased type to separate himself from a biased one in both states of the world.

Meng (2015) considers a similar setup to Morris (2001) with two periods and endogenizes the precision of expert's information, where investment is not a signal as it is not observable, but it allows for better decision making. Reputation building enhances the incentives to invest in information for both types in period 1, and aligned experts acquire more information. The expert's objective is similar to Morris that he cares about his future communication being credible.

There is a huge literature that considers bad reputation, where distortions caused by reputational concerns lead to loss of valuable information. These usually arise when there are dynamics, or when there is an intrinsic value of reputation for the experts. Outside of the communication literature, Ely and Valimaki (2003) consider a long run player who takes a payoff relevant action facing short run players and highlight the distortional consequences today of the incentives to avoid bad reputations in the future. In terms of intrinsic value of reputation, Ottaviani and

[2] For a similar argument, see Argenziano et al. (2016) where unless the expert acquires a sufficient amount of information, the DM treats his message as a babbling one.

Sorensen and Ottaviani and Sorensen (2006a) show that reputation doesn't always give the right incentives and may lead to herding. In their setup, experts have a utility from being perceived as good and bias their recommendation in order to appear more informed. Suurmond et al. (2004) consider the effect of reputation in a delegation setup with information acquisition and an unbiased agent. In this setup, reputational concerns may be good as they incentivize the agent to acquire information when the agent doesn't know his ability. When the agent has private information about his ability, the expert may take inefficient actions in order to mimic an efficient type. The welfare effect of reputational concerns is ambiguous.

In the disclosure literature, Boujarde and Jullien (2011) consider an expert of known bias plus an intrinsic reputational concern about ability. The probability that the expert is informed depends on his ability hence disclosure enhances the expert's reputation. They study the interaction between the reputational concerns and incentives to disclose information.

The distinctive feature of my setting compared to the literature is that there is neither dynamics involved which is present in the literature on reputation, nor an intrinsic value of reputation, yet a similar effect arises due to the endogenous information acquisition and signaling motive.

This paper also relates to the literature on costly information acquisition before cheap talk. The first example is Austen-Smith (1994) who considers the transmission of costly information where information acquisition leads to perfect information and is not observable. As the expert can prove being informed but can feign ignorance, the low types can now pool with uninformed types to achieve a higher outcome, which improves communication for higher types. In recent years there has been some work on strategic communication with endogenous information acquisition, such as Pei (2015) and Argenziano et al. (2016). A common finding is that the expert truthfully transmits all the information he acquires, in other words the expert doesn't acquire information that he will not transmit. Argenziano et al. (2016) show that the sender over-invests in information compared to what the decision maker would have incurred himself, due to the use of a babbling threat by the decision maker. In my setup in the extension considered, there is an over-investment result when there is uncertainty about the expert's bias compared to having an unbiased expert for sure. The difference is that it is the expert who over invests compared to what he would have done if his type were known. Esö and Szalay (2010) consider a game in which the expert has no bias and endogenous information acquisition, and show that restricting the message set can induce the sender to acquire information more often. Deimen and Szalay (2016) also consider endogenous information when a biased expert can choose on which issues to gather information and show that communication dominates delegation. Frug (2017) consider dynamic information acquisition before cheap talk. Dur and Swank (2005) also study endogenous information acquisition and show that an unbiased advisor puts higher effort into acquiring information as the value of information is higher for them. Hence, when a decision maker is biased, it may be better for him to hire an advisor who is less biased than himself. There is no uncertainty about the bias hence no signaling motive for the expert in these papers.

In a disclosure setup, Che and Kartik (2009) show that the DM benefits having an advisor with some difference of opinion as this gives more incentives to invest in information, due to the incentive to manipulate the decision and also in order to avoid prejudice. The tradeoff is between the advisor acquiring more information versus hiding more strategically. Kartik et al. (2017) study disclosure with two experts having opposite biases, and show that information acquisition decisions of the experts are strategic substitutes.

There is also some literature in which the bias of the expert is unknown. Morgan and Stocken (2003) consider strategic communication as in Crawford and Sobel (1982) introducing uncertainty about the expert's type. They show that truthful communication cannot arise even for the unbiased type whenever the state of the world is high enough. In a disclosure setup, Wolinsky (2003) considers a setting in which the expert has uncertain bias and shows that the expert conceals more information when his bias is unknown. There are no information acquisition or signaling incentives in these settings.

In Austen-Smith (2000) cheap talk is not the only way to communicate but senders may incur some loss in utility in form of *burning money*. The wasteful investment in my setup is reminiscent of burning money, in that for some types there is no informational value but only a signaling one. However, contrary to burning money, some expert types do value information per se.

3 Model

There is a decision maker (DM) who wants to take an action, $y \in [0, 1]$. The state of the world is binary, $\omega \in \{0, 1\}$ and there is a commonly known prior probability $Pr(\omega = 1) = p_0$. The DM does not know the state of the world but has access to an expert advisor. The expert has a two dimensional private type. First, he is either unbiased with probability $(1 - \beta)$ and shares the same objective function as the DM, which is $u(\omega, y) = -(\omega - y)^2$ or he is biased and always wants the highest possible action with utility function, with utility function $u^b(\omega, y) = -(1 - y)^2$[3]. Second, with probability α the expert is perfectly informed about the state of the world ex-ante, otherwise he has the same information as the DM.[4] The decision maker does not observe whether the agent is biased or not and neither whether the expert is informed or not, but knows β and α. In addition, the agent can invest in information by incurring a cost c leading to a perfectly revealing signal. The investment in information acquisition is observable while its outcome is not. Indeed, any type of expert could invest in information acquisition, including a biased or an informed one, which is crucial in the setup. Finally, communication happens through cheap talk after the investment decision. Below are the stages of the game:

[3]This assumption about the expert type is also present in Morris (2001) and Morgan and Stocken (2003)
[4]Indeed, it could also be thought that getting informed is costless for some experts.

- the expert decides whether to acquire a signal by incurring c and become perfectly informed, binary decision $x \in \{0,1\}$.
- the expert sends a message $m \in M$ to the DM.
- the DM takes an action, $y \in [0,1]$.

The set of possible messages is $M = \{\varnothing, 0, 1\}$, where $\varnothing$ means the sender doesn't have information. The decision maker interprets the expert's message as a function of her belief about the expert's type, which depends on the different type of experts' equilibrium strategies of information acquisition.

Let us now summarize the types of experts at the beginning of the game. As the biased type of expert always wants the highest action irrespective of the state, we only have 1 type of biased expert and whether he is informed or not has no relevance.[5] It could be assumed that the biased type of agent is never informed. We can then summarize the types of experts at the beginning of the game by separating them into four:

1. biased agent
2. unbiased agent who is uninformed
3. unbiased agent who is informed with signal (1)
4. unbiased agent who is informed with signal (0)

Equilibrium Concept: I look for Perfect Bayesian Equilibrium. As this is a game of signaling followed by cheap talk, there is multiplicity of equilibria for some values of cost and the Intuitive Criterion (due to Cho and Kreps (1987) and Banks and Sobel (1987)) is used to get uniqueness in some cases. After observing x and m, the DM's updated belief is $\mu(x,m)$ about the state being 1, as a function of the strategies of each expert type in equilibrium. The optimal decision y^* for the DM when the belief is $\mu(x,m) = p$ is given by:

$$\max_y -p(1-y)^2 - (1-p)y^2 \tag{1}$$

This simplifies to $-y^2 - p_0 + 2p_0 y$ which is maximized for $y = p$. Hence, $y^* = \mu(x,m)$ for the DM. Indeed, the updating of the DM's belief takes place in two stages. The first updating happens upon observing the decision to invest, while the second updating happens upon receiving the message m.

For the biased and informed (1) agent, information has no intrinsic value as these types strictly prefer sending $m = 1$. The only situation in which they may find it valuable to invest in

[5]This would be different if the biased type's payoff was not state independent, such as in Morgan and Stocken (2003).

information acquisition is when $\mu(1,1) - \mu(0,1)$ is large enough.

Without any communication, the DM would choose $y^* = p_0$ which provides her payoff $-(p_0 - {p_0}^2)$, as well as for the unbiased agent. Then, the value of perfect information for the DM and the unbiased agent is given by $(p_0 - {p_0}^2)$. This is the gain in payoff for the unbiased type from getting informed, conditional on his communication being interpreted as truthful. This value of information is maximized when $p_0 = 0.5$, when uncertainty is highest.

4 Equilibrium Analysis

Observation 1. *In any equilibrium, the unbiased informed (0) type sends $m = 0$ without additional investment, upon which the DM chooses $y^* = 0$.*

The informed (0) type's payoff is maximized when $y = 0$ and there is no other type that would benefit from sending this message. There is also no benefit to this type of sending the same message as any other type. Hence, the DM infers that this message comes from an unbiased informed type, $\mu(0,0) = 0$ and she will choose $y = 0$. This also implies that there exists no completely babbling equilibrium in this setup. Then, from now on we will focus on the equilibrium behaviors of the 3 other types.

Observation 2. *The biased type and the unbiased informed (1) type share the same incentives as their payoff functions are given by: $-(1-y)^2$. These types have the same information acquisition strategies in strict Nash Equilibria and they always send the message $m = 1$ regardless of their investment strategy.*

As the payoff functions of these types are identical, whenever they are playing a strictly dominant strategy, they follow the same investment strategy. In case they are playing a mixed strategy, then their investment strategies can differ, which will play a role in finding the mixed strategy equilibria. This will be important, as even though both types always send $m = 1$, they have different meaning for the decision maker: the unbiased informed (1) type is communicating truthfully while the biased type's message is "babbling" as he always announces the state 1.

Observation 3. *The unbiased type of expert always sends a truthful message.*

The aligned type never has incentives to lie about the state. Whenever uninformed, he strictly prefers sending $m = \varnothing$ which is always a credible message leading to $y^* = p_0$ and whenever informed, he sends $m = \omega$.

There are 3 different equilibria in pure strategies and a continuum of semi-pooling equilibria in mixed strategies. Which equilibrium arises depends on the cost of information. After ruling out the unbiased informed (0) type, we define "pooling" and "separating" in this setup in terms of the investment decision of the two groups of expert types defined as:

1. biased type and unbiased informed (1) type,
2. unbiased uninformed type.

We focus on parameter values such that separation does exist for some cost values. Then, following is the list of all possible equilibria that arise:

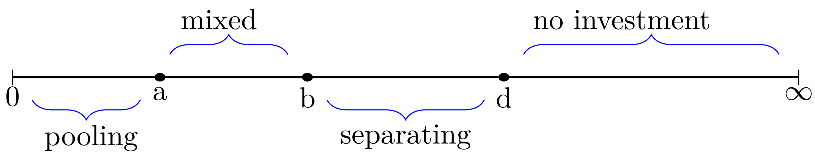
1. **Separating equilibrium:** Only the unbiased uninformed type invests upon which the DM takes communication at face value. The biased and informed (1) unbiased expert do not invest and the DM chooses $y = \hat{p} > p_0$.
2. **Pooling equilibrium:** Investment by all two groups of types. The unbiased type sends $m = \omega$ while the biased type sends $m = 1$. Upon $m = 1$, the DM chooses $y = \tilde{p} > \hat{p}$.
3. **Semi-pooling equilibria:** Biased type mixes and unbiased uninformed type invests. There is multiplicity of equilibria depending on the unbiased informed (1) type's strategy. As cost decreases, the biased types invests with higher probability.
4. **No investment equilibrium:** No type invests. Upon $m = 1$ the DM chooses $y = \hat{p}$.

The equilibrium maximizing the decision maker's and overall surplus is the separating one which is also the most informative communication equilibrium. This equilibrium only exists under certain parameter conditions: when the **signaling (reputational) value of information** is less than the **intrinsic value of information**. The signaling value of information is the gain in payoff for the group 1 types from being perceived as unbiased, in other words if their message is taken at face value, compared to the case when they do not invest and the decision maker knows that the message is coming from group 1. The intrinsic value of information is the gain in payoff for the uninformed unbiased (group 2) type from getting informed and being perceived as an unbiased type, i.e. when his message is taken at face value, compared to not having information and sending $m = \emptyset$. When this condition is violated, there is no separating equilibrium, in other words there is no equilibrium in which investment in information is efficient. In that case, the unbiased uninformed type can never be the only type that invests in information hence his communication is never perfect.

The value of information for the unbiased uninformed type is highest when p_0 is close to 0.5 while the signaling value of information decreases in p_0. For any $p_0 \geq 0.5$, there exists a region of cost values for which the separating equilibrium arises. When it does exist, the separating equilibrium arises for intermediate values of the investment cost, as for low enough cost there is pooling and for high enough cost values even the unbiased uninformed type doesn't find it profitable to invest. For separation to exist, we also need α to be high enough: sufficiently many of the unbiased types should be initially informed so that the biased types find it attractive to not invest and pool with these types.

Separating Equilibrium

Figure 1: Equilibria as a function of cost



$$a = (1-\hat{p})^2 - (1-\tilde{p})^2$$
$$b = (1-\hat{p})^2$$
$$d = p_0 - {p_0}^2$$

In this equilibrium the only type that acquires information is the uninformed unbiased type. Hence, information acquisition is efficient. Upon communication of $m = 1$ without investment, the DM's belief $\mu(0, 1)$ is given by:

$$\hat{p} = \frac{\alpha(1-\beta)p_0 + \beta p_0}{\alpha(1-\beta)p_0 + \beta}$$

When the DM receives $m = 1$ without investment and knows that it either comes from the biased agent (with probability β) or the unbiased informed (1) agent (with probability $\alpha(1-\beta)$), she chooses $y = \hat{p}$. The payoffs to both the biased and unbiased informed expert types are $-(1-\hat{p})^2$. If instead they deviate to invest, they induce the DM to choose $y = 1$ by sending $m = 1$ and obtain a payoff of 0. Hence, the condition for this deviation to not be profitable for these types is:

$$c \geq (1-\hat{p})^2 = \left[\frac{\beta(1-p_0)}{\alpha(1-\beta)p_0 + \beta}\right]^2 \tag{2}$$

For the unbiased uninformed expert who invests, payoff in case of deviating to not invest is $-(p_0 - {p_0}^2) < 0$ as in that case she would send $m = \varnothing$ and the DM would choose $y = p_0$. On the other hand, upon investing he obtains a payoff of 0 as the DM will treat his message as truthful in either state. Then, this expert prefers to invest if and only if:

$$p_0 - {p_0}^2 \geq c$$

This means, together with equation (2), that the cost values for which the separating equilibrium exists is:

$$(1-\hat{p})^2 \leq c \leq p_0 - {p_0}^2 \tag{3}$$

Then, for the separating equilibrium to exist at least for some cost values, we should have:

$$(1-\hat{p})^2 \leq p_0 - {p_0}^2$$

If the above condition is violated, then there exists no cost values for which the separating equilibrium arises. We find that $p_0 - {p_0}^2 \geq (1-\hat{p})^2$ if and only if:

$$p_0[\alpha(1-\beta)p_0 + \beta]^2 - \beta^2(1-p_0) > 0$$

which can be satisfied for p_0 and α large enough and specifically, it is always satisfied when $p_0 \geq 0.5$. As we have: $\frac{\partial(1-\hat{p})}{\partial\beta} > 0$ and $\frac{\partial(1-\hat{p})}{\partial\alpha} < 0$, this equilibrium is more likely to exist for low β and high α. When β is high enough, the incentive to pool is high. When α and $\hat{p}$ is low enough, the outside option of the biased types of pooling only with the unbiased informed types is lower, hence they will be more tempted to invest. For the uninformed type to be willing to invest, the condition $c \leq p_0 - {p_0}^2$ is easier to satisfy the closer p_0 is to 0.5, in other words when the intrinsic value of information is sufficiently high. Hence, p_0 should be high enough to increase the non investment value for the biased types, but not too much, for this condition to be satisfied.

In this region, there cannot be any equilibrium for any beliefs in which the group 1 types invest with some probability, as they do not benefit from investing even for the most favorable out of equilibrium belief, which is in the separating equilibrium. The only other possibility is the equilibrium in which no type invests.

No Investment equilibrium:

It is intuitive that when c is high enough, no type wants to invest. This is the case when:

$$c \geq p_0 - {p_0}^2$$

The right hand side is the upper boundary of the separating equilibrium, in which the only type that invests is the unbiased uninformed type. By making use of the intuitive criterion, we know that in the region $(1-\hat{p})^2 \leq c \leq p_0 - {p_0}^2$, the biased type doesn't find it profitable to invest, even when his message is credible. Then, in the region $c \geq p_0 - {p_0}^2$, the out of equilibrium belief upon investment should put probability 0 to the expert being a biased type, as this type can never gain from deviating to invest even if he were then believed to be an unbiased type. Then, the message upon a deviation to invest will be taken at face value. The gain in utility from doing so for the uninformed unbiased type defines the boundary of the no investment equilibrium. Next proposition summarizes the results until now.

Proposition 1.
- *In the region where $(1-\hat{p})^2 \leq c \leq p_0 - {p_0}^2$, the separating equilibrium, in which the only investment is made by the uninformed unbiased type, is the unique equilibrium that survives the intuitive criterion.*
- *In the region where $c > p_0 - {p_0}^2$, the unique equilibrium is the one in which there is no investment.*

Now, consider the equilibria in the region $c < (1-\hat{p})^2$. We know that there is no separating equilibrium as the biased types will have an incentive to deviate and invest if the DM expects the investment to come from an unbiased type. However, it cannot be the case that just below this threshold, the biased and informed (1) types invest with probability 1, as then $\mu(1,1) < 1$, and the biased and informed (1) types would not find it profitable to invest anymore at this cost. Then, in this region there can only be mixed strategy equilibria with semi-pooling in investment. First, we will study the pooling equilibrium which arises for lower cost values, before studying the mixed strategy equilibria in the cost region in between.

Pooling equilibrium:

We will now solve for the pooling equilibrium in which all types in group 1 and 2 invest. The out of equilibrium belief of the DM upon no investment and $m = 1$ is $\hat{p}$. By using the Intuitive Criterion, the out of equilibrium belief $\mu(0,1)$ puts probability 0 on the unbiased uninformed expert who always prefers to send $m = \varnothing$ if he were to deviate to no investment. On the other hand, the group 1 types always prefer sending $m = 1$ over any other message. Then, $\mu(0,1) = \hat{p}$ as this deviation is equally likely to come from a biased or informed (1) type.

The DM's updated belief $\mu(1,1)$ is:

$$\tilde{p} = \frac{p_0(1-\beta) + \beta p_0}{p_0(1-\beta) + \beta} = \frac{p_0}{p_0(1-\beta) + \beta}$$

First, consider the investment choice of the group 1 types. If the biased or unbiased agent were to deviate to not invest and send $m = 1$, then the DM would choose $y = \hat{p}$. Then, the following should hold for pooling equilibrium to arise:

$$-(1-\tilde{p})^2 - c \leq -(1-\hat{p})^2$$

Second, for the uninformed unbiased type, the payoff from not investing is $-(p_0 - {p_0}^2)$ as before, in which case he sends $m = \emptyset$. Then, consider the payoff of this type from investing. If the signal turns out to be 0, then the DM takes the message at face value and chooses $y = 0$ whereas if the signal is 1, this type will be pooled with the group 1 types and the decision maker will choose $\tilde{p}$. Then this type prefers investing to not if and only if:

$$-p_0(1-\tilde{p})^2 - c \geq -(p_0 - {p_0}^2)$$

These together result in:

$$c \leq min\{(1-\hat{p})^2 - (1-\tilde{p})^2, p_0 - {p_0}^2 - p_0(1-\tilde{p})^2\}$$

When $(1-\hat{p})^2 < (p_0 - {p_0}^2)$, which is the case we consider, we have $min\{(1-\hat{p})^2 - (1-\tilde{p})^2, p_0 - {p_0}^2 - p_0(1-\tilde{p})^2\} = (1-\hat{p})^2 - (1-\tilde{p})^2$. To see this, realize that the value of getting informed is higher for unbiased uninformed types than for the group 1 types: $(1-\hat{p})^2 - (1-\tilde{p})^2 < p_0 - {p_0}^2 - p_0(1-\tilde{p})^2$. This is because unbiased uninformed types get their bliss point in case the state of the world is 0, while communication is distorted when the message is 1. However, from the point of view of the group 1 types, communication is always distorted as their bliss point is 1 and $\mu(1,1) = \tilde{p} < 1$. Then, the condition for the pooling equilibrium is given by the condition for the biased and informed (1) types to be willing to invest:

$$c \leq (1-\hat{p})^2 - (1-\tilde{p})^2$$

As we have $\frac{\partial(1-\hat{p})}{\partial\beta} - \frac{\partial(1-\tilde{p})}{\partial\beta} > 0$, when β increases the incentive of the biased type to invest increases and pooling equilibrium becomes more likely.

Now consider the expression $c \leq p_0 - {p_0}^2 - p_0(1-\tilde{p})^2$ which is the condition for the unbiased uninformed types to invest in the pooling equilibrium. As $p_0 - {p_0}^2 > p_0 - {p_0}^2 - p_0(1-\tilde{p})^2$, where the left hand side is the cutoff for investment in the separating equilibrium, the condition for unbiased uninformed types to invest is easier to satisfy in the separating equilibrium. The difference between these two is due to the biased types' "crowding out" the unbiased uninformed types: investment of the biased types make information acquisition by the unbiased types less profitable, hence cost has to be lower in order to satisfy their participation.

Mixed strategy (semi-pooling) equilibria

When $c \in [(1-\hat{p})^2 - (1-\tilde{p})^2, (1-\hat{p})^2]$, there exists no pure strategy equilibrium. In this region, there exist a plethora of mixed strategy equilibria, all with the property $\hat{p} < \mu(1,1) < 1$ and other conditions that will be defined. The biased and informed (1) types will no longer play the same strategy in this region and their strategies will be such that the indifference condition for these group 1 types is satisfied for every value of c.

In this type of equilibrium, the biased sender is mixing between investing and not with σ the probability of investment. Given that, the unbiased informed (1) type is also indifferent between investing or not and invests with probability γ. Both these types always send $m = 1$. The mixing strategies are such that p', which is the updated belief of the decision maker after investment and message $m = 1$, equals the value which makes these group 1 types indifferent between investing and not.

More specifically, as c goes down, $\frac{\sigma}{\gamma}$ should be increasing in order for $\mu(1,1)$ to decrease and to keep the indifference condition satisfied, hence a larger portion of biased types should be investing in order for $\mu(1,1)$ to decrease. We have that when $\gamma = \sigma = 1$, the indifference condition is satisfied at $c = (1-\hat{p})^2 - (1-\tilde{p})^2$ which is the cutoff below which the pooling equilibrium arises. For the indifference condition to be satisfied at higher c, we need $\mu(1,1) > \hat{p}$. Then, we know that when these types are indifferent, the unbiased uninformed type strictly prefers to invest. We also know that $\sigma < 1$, as when $\sigma = 1$, $\mu(1,1) \leq \hat{p}$ for any $\gamma \leq 1$.

Given σ and γ, $\mu(1,1)$ is:

$$p' = \frac{(1-\beta)(1-\alpha)p_0 + \beta\sigma p_0 + (1-\beta)\gamma p_0}{(1-\beta)(1-\alpha)p_0 + \beta\sigma + (1-\beta)\gamma p_0} > \hat{p}$$

The belief upon no investment and message $m = 1$, $\mu(0,1)$:

$$p^* = \frac{(1-beta)\alpha(1-\gamma)p_0 + \beta(1-\sigma)p_0}{(1-\beta)(1-\gamma)\alpha p_0 + \beta(1-\sigma)}$$

For the indifference condition of the biased types (and informed (1) unbiased) to be satisfied, it should be that $c = (1-p^*)^2 - (1-p')^2$, which leads to:

$$c = \left[\frac{\beta(1-\sigma)(1-p_0)}{(1-\beta)\alpha(1-\gamma)p_0 + \beta(1-\sigma)}\right]^2 - \left[\frac{\beta(1-p_0)\sigma}{(1-\beta)(1-\alpha)p_0 + \beta\sigma + (1-\beta)\gamma p_0}\right]^2 \tag{4}$$

The left hand side of equation 4 is decreasing and the right hand side is increasing in σ for a given γ. Hence, for every c and γ, there is a σ for which this equality holds.

Finally, we will check that the unbiased uninformed type strictly prefers to continue investing. To see this, consider the condition for this type to invest:

$$-p_0(1-p')^2 - c \geq -(p_0 - {p_0}^2)$$

leading to:

$$c \leq (p_0 - {p_0}^2) - p_0(1-p')^2$$

when the indifference condition of the biased type is satisfied, $c(e) = (1-p^*)^2 - (1-p')^2$. Then the incentive compatibility of the unbiased agent also is satisfied, as $p^* > \hat{p} \rightarrow (1-p^*) < 1-\hat{p} \rightarrow (1-p^*)^2 < (1-\hat{p})^2 \rightarrow (1-p^*)^2 < (1-\hat{p})^2 < p_0 - {p_0}^2$ plus $(1-p')^2 > p_0(1-p')^2$, hence:

$$c = (1-p^*)^2 - (1-p')^2 < (p_0 - {p_0}^2) - p_0(1-p')$$

5 Welfare analysis

Now we will make welfare comparison among the types of equilibria defined. Let us summarize some results first:

- The DM is better off if facing an uninformed expert whenever there is investment. If the expert is unbiased and informed, he can never distinguish himself from the biased type when the state is 1. On the other hand, it is possible for the uninformed unbiased type to perfectly communicate when the separating equilibrium arises, which is the welfare maximizing equilibrium for the DM.

- DM's payoff (and welfare) is non-monotone in α: it should be high enough for separation as this serves as an outside option for the biased types and decreases incentives to invest. However above the level that leads to separation, the DM's payoff and total welfare decrease in α as there are more and more unbiased informed types pooling with the biased type.
- Total welfare is non-monotone in the cost of information acquisition: it is increasing as cost increases when moving from pooling to separating equilibria, however once in the separating equilibrium, total welfare decreases in cost.

5.1 The DM's payoff

It is easy to see that the DM's payoff is minimized in the equilibrium in which no type invests, which is the least informative equilibrium. Then, the comparison is between the separating and pooling equilibria.

Separating equilibrium:

$$[(1-\beta)\alpha p_0 + \beta][-(\hat{p}-\hat{p}^2)] = -\hat{p}\beta(1-p_0) \tag{5}$$

This is decreasing in α and β. As α increases, more of the informed unbiased types will be pooled with biased types, and as β increases, the DM's welfare decreases as there are more biased types.

Pooling equilibrium:

$$[(1-\beta)p_0 + \beta][-(\tilde{p}-\tilde{p}^2)] = -p_0(1-\tilde{p}) \tag{6}$$

This is decreasing in β. If $p_0 > 0.5$, this is increasing in p_0. When $p_0 < 0.5$, and β high enough it may be decreasing in p_0. It is independent of α, as in the end all types do get informed. The condition for the separating equilibrium payoff to dominate the pooling equilibrium payoff is:

$$\alpha(1-p_0) + p_0 \leq 1$$

which is always satisfied. Hence, the DM's payoff is unambiguously higher in the separating equilibrium, in which information is more precise.

The DM benefits from having some initially informed experts who pool with the biased type, as otherwise the separating equilibrium cannot arise. Inside the separating region, the DM's payoff is decreasing in α: she is better off when more types are uninformed as the informed types endowed with signal 1 are not able to separate themselves from the biased types. However, if α is very low, there are more incentives to "pool" and the separating region shrinks. Hence, the DM's payoff is non-monotone in α: it has to be high enough for biased types not to invest, but once in the separating region the DM's payoff decreases in α. Also, the DM's payoff is unambiguously decreasing in β.

No investment equilibrium:

The DM's utility in the equilibrium in which no one invests:

$$\frac{-(1-p_0)p_0(\beta+(1-\alpha)\alpha(1-\beta)^2 p_0)}{\beta+\alpha(1-\beta)p_0} \tag{7}$$

which is found by simplifying $-[\beta+(1-\beta)\alpha p_0](\hat{p}-\hat{p}^2)-(1-\beta)(1-\alpha)(p_0-{p_0}^2)$. When we compare the no investment equilibrium with the pooling in investment equilibrium, we find that pooling in investment always leads to higher payoff for the DM than no investment. Hence, no investment equilibrium provides the minimum payoff to the DM.

5.2 The expert's payoff

Separating equilibrium:

The payoff of the biased expert is $-(1-\hat{p})^2$. This is increasing in α and p_0 and decreasing in β.

The ex-ante payoff of the unbiased expert is $-\alpha p_0(1-\hat{p})^2-\alpha(1-p_0)0-(1-\alpha)c = -\alpha p_0(1-\hat{p})^2-(1-\alpha)c$. This is increasing in α, decreasing in p_0 and decreasing in β.

Pooling equilibrium:

The payoff of the biased expert is $-(1-\tilde{p})^2-c$. This is increasing in p_0 and decreasing in β.

The ex-ante payoff of the unbiased expert is $-p_0(1-\tilde{p})^2-(1-\alpha(1-p_0))c$. This is decreasing in p_0 as this type can perfectly reveal information when $\omega=0$, and it is also decreasing in β due to the contamination of the biased types. It is increasing in α as less cost will have to be incurred.

It is trivial that the biased type of agent is unambiguously better off in pooling equilibrium than in separating equilibrium, as otherwise, he would not invest in the pooling region.

On the other hand, the unbiased agent's payoff is higher in separating than in pooling equilibrium, when we insert the minimum cost at which there is separation and maximum cost at which there is pooling.

5.3 Total Welfare Comparison

Proposition 2. *The total welfare increases when moving from the pooling in investment equilibrium to the separating equilibrium at minimum cost, where it takes the highest value. Hence, welfare is maximized at the minimum cost level at which there is no wasteful investment and information transmission is the most precise. Welfare takes the minimum value in the no investment equilibrium for high enough cost values.*

First, I compare the no investment equilibrium surplus to other equilibria. As no investment equilibrium surplus is unambiguously lower than the separating equilibrium, we compare it to the pooling in investment equilibrium and we find that the payoff in no investment equilibrium is also lower than the pooling in investment equilibrium. This is intuitive: first, the DM's payoff is unambiguously higher in the pooling in investment equilibrium compared to the no investment equilibrium, as it provides more information. The welfare of the biased type also increases in the pooling in investment equilibrium as their outside option of not investing and getting $-(1-\hat{p})^2$ is still present, as it was in the no investment equilibrium. Hence, if this type does find it profitable to invest, then they must be getting a higher payoff. The same is true for the unbiased informed (1) type who would get $-(1-\hat{p})^2$ if deviating to not invest. Finally, for the unbiased uninformed type, it is true as well: if this type didn't invest they would get the payoff $-(p_0-{p_0}^2)$ which is still available if they were to deviate in the pooling equilibrium to send $m=\emptyset$.

6 Covert information acquisition

Now, we consider the case if information acquisition process were covert, in other words that the decision maker did not observe the investment made by the expert. In this case, as there is no signaling value of investment, the biased and informed 1 types will never wastefully invest, as they can now pool with the unbiased uninformed expert without having to incur any information acquisition cost. The only type that may invest is the unbiased uninformed type. There are 2 types of pure strategy equilibria as a function of the cost, as summarized below:

1. There is investment only by the uninformed unbiased type. Upon $m=0$, the DM takes action 0 and upon $m=1$, the DM takes action $\tilde{p}$. This equilibrium looks like the pooling equilibrium except that the biased and informed (1) types do not actually invest. The payoffs of the biased and informed (1) types are higher in this equilibrium compared to the pooling equilibrium discussed, as they achieve the same outcome without having to pay the investment cost.

   The condition for the unbiased uninformed type to invest is:

$$c \leq p_0 - {p_0}^2 - p_0(1-\tilde{p})^2$$

This means the unbiased type acquires information for smaller range of cost values in the covert information acquisition than in the overt case. This cutoff is above the pooling cutoff cost but below the separating equilibrium cutoff cost in the overt case studied.

2. No investment takes place. Upon $m = 1$, the DM chooses $\hat{p}$ inferring that this message is sent either by the biased or the unbiased informed (1) expert. The uninformed expert sends $m = \emptyset$ and the DM chooses $y = p_0$. This equilibrium is equivalent to the no investment equilibrium in the overt information acquisition case. This equilibrium arises for the following cost values:

$$c > p_0 - {p_0}^2 - p_0(1 - \tilde{p})^2$$

The types that gain from information acquisition being covert rather than overt are the biased and unbiased informed (1) type and only in case the cost is low enough that the uninformed unbiased type acquires information. Even though the cost does not affect these types directly as they never acquire information, the fact that the unbiased uninformed type does makes their message more credible as they can pretend to have acquired information. In this equilibrium, the payoffs of the uninformed unbiased expert and decision maker are the same as in the pooling equilibrium in the overt case while the payoff of the biased and informed (1) expert are higher. When cost is higher, we move to the no investment equilibrium in which payoffs are identical as the overt case with no investment.

The unbiased type invests in information less often and is worse off, as he can never perfectly separate himself from the biased type. This result shows that, even though the signaling value of information undermines its intrinsic value, under certain parameters overt information acquisition does strictly better than the covert one. This is true for the parameters under which the separating equilibrium does exist in the overt information acquisition.

However, when the cost of information acquisition is low enough that in the overt case there will be pooling in investment, then covert information acquisition does better than overt in terms of the overall welfare as there is no wasteful investment, although the precision of communication is equivalent. Then the next corollary follows.

Corollary 1. *The total welfare is higher in the overt case whenever the cost is such that separating equilibrium arises, while it is higher in the covert case whenever pooling equilibrium arises in the overt case.*

The tradeoff is between more informative communication versus wasteful investment in information. Whenever the separating equilibrium does exist, overt information acquisition is better as communication is most precise and investment is efficient. However, whenever the separating equilibrium doesn't exist in overt case, then covert case is better as there will never be wasteful investment.

7 Discussion: General Information Acquisition Technology

In this part, a more general information acquisition technology is considered to find some robustness properties. In addition, I show that the presence of biased types can lead to over-investment by the unbiased types. Now the cost exerted in information acquisition is $e \in [0, 1]$, where e is the probability with which the expert obtains perfect information, $c(e)$ is increasing and convex in e, $c'(e) > 0$, $c''(e) > 0$, $c(0) = 0$ and $c'(e) \to \infty$ as $e \to 1$.[6] The decision maker now observes the amount of effort exerted by the expert before communication takes place but does not know whether information did arrive, nor the realization.

Lemma 1. *Call the equilibrium effort level chosen by the unbiased uninformed expert as $\hat{e}$. The equilibrium effort level chosen by the biased and informed (1) types takes one of two values, $e \in \{0, \hat{e}\}$, in pure strategies.*

Proof. Assume effort did take any other value. Then, given the effort level of the unbiased uninformed type is $\hat{e}$, upon observing another effort level, the DM anticipates the expert to be a biased or an unbiased uninformed type. But, this would as well be the case when $e = 0$. Then, given that the biased and unbiased informed (1) types do not value information, they would choose either of the two effort levels: $\{0, \hat{e}\}$. Indeed, for some out of equilibrium beliefs, (such as $\mu(0, 1) = p_0$) which puts probability 1 to biased expert upon no investment) it is possible to have a positive but still separating effort level for these types, but this type of equilibria is worse in terms of payoff for the experts, and not more informative for the DM either. So, we will ignore the possibility of this kind of out of equilibrium beliefs and the lemma follows. □

Lemma 2. *The unbiased uninformed type always exerts a positive effort level: $\hat{e} > 0$.*

Proof. Assume that the unbiased uninformed type chose effort level 0. Then it should be that the other types do not exert any effort either, by lemma 1. We have $c'(0) = 0$. Whenever $m = 0$, the DM takes it at face value: the possibility that this message comes from a biased type is 0. Hence, upon hearing $m = 0$ after a positive effort level, the DM will infer that it is an unbiased type and take action $y = 0$. Then, only due to the probability of receiving signal 0, the expert will increase his payoff by $e(1 - p_0)(p_0 - {p_0}^2)$. Then, the minimum effort he would exert when all other types choose $e = 0$ is given by the condition $c'(e) = (1 - p_0)(p_0 - {p_0}^2)$. □

The distinctive feature of this setup is that there exist pooling equilibria at positive effort level that cannot be ruled out by the Intuitive Criterion (or other criteria). As it is not necessarily the unbiased uninformed type who benefits more from deviating to invest more when believed to be an unbiased type, the intuitive criterion fails to rule out pooling equilibria.

[6] This is a standard assumption in the literature. For example, Che and Kartik (2009) use a similar information acquisition process (followed by strategic disclosure), where $c(p)$ is convex and p is the probability that information is acquired (investigation is successful), even though the information that is acquired is still noisy but has mean equal to the state.

7.1 Separating Equilibrium

First, the separating equilibrium is one in which only the unbiased uninformed agent incurs a positive effort level, $\hat{e}$. The payoff of the unbiased uninformed type in an equilibrium in which he is the only type to exert effort is:

$$-(1-e)(p_0 - {p_0}^2) - c(e)$$

whereas without investment his payoff is: $-(p_0 - {p_0}^2)$. The first best level of investment for the unbiased type is then:

$$(p_0 - {p_0}^2) = c'(e^*) \tag{8}$$

The biased type and informed (1) type get a payoff of $-(1-\hat{p})^2$ when they do not invest and send $m = 1$. If they deviate to invest and pool with the unbiased type, they will send $m = 1$ which the DM will take at face value and their payoff will be 0. Then, the condition for these types not to deviate to invest is:

$$c(e) \geq (1-\hat{p})^2$$

If $c(e^*) > ((1-\hat{p})^2$, then at the first best level of investment of the unbiased type, there are no incentives to mimic. Then, there is a unique separating equilibrium in which the uninformed unbiased type chooses its first best investment level.

If $c(e^*) \leq (1-\hat{p})^2$, then at the first best investment level of the unbiased type, the biased type is willing to deviate and mimic. If this is the case, the least cost separating equilibrium level of effort is given by: $c(e') = (1-\hat{p})^2$. Then, we can say that whenever $c(e^*) < (1-\hat{p})^2$, there is over-investment by the unbiased type in any separating equilibrium, $e > e^*$. The separating effort level should also satisfy the participation constraint of the unbiased type, who could instead deviate to not invest and send $m = \emptyset$. The unbiased type is willing to incur any level of e which satisfies:

$$-(1-e)(p_0 - {p_0}^2) - c(e) \geq -(p_0 - {p_0}^2)$$

where the right hand side is the payoff from not investing. The above simplifies to:

$$e(p_0 - {p_0}^2) \geq c(e)$$

If there is incentive to deviate for the biased type at the best level of investment of the unbiased uninformed type, then $c(e') = (1-\hat{p})^2$ is the least cost separating effort level and as long as:

$$e'(p_0 - {p_0}^2) - (1-\hat{p})^2 \geq 0 \tag{9}$$

the LCS equilibrium will arise. Indeed, whenever the LCS equilibrium satisfies equation (9), it will be the unique equilibrium. This is possible as the DM could use a babbling threat: whenever $e < e'$, the DM treats the message as babbling and chooses $y = p_0$. In that case, the LCS equilibrium at e' will be the unique equilibrium. Then, the following proposition follows:

Proposition 3. *Given e^* the first best effort level for the unbiased uninformed type, following describes the equilibria that arise:*

- *When $c(e^*) > ((1-\hat{p})^2$, the separating equilibrium arises in which the first best effort level is chosen by the unbiased uninformed type, which is the level of effort he would choose if $\beta = 0$.*
- *When $c(e^*) \leq ((1-\hat{p})^2$, the least cost separating equilibrium effort level, $e' > e^*$, is given by $c(e') = ((1-\hat{p})^2$. This will be the unique equilibrium if $-e'(p_0 - {p_0}^2) - (1-\hat{p})^2 \geq 0$.*
- *There exists no separating equilibrium if:*

$$(p_0 - {p_0}^2) \leq (1-\hat{p})^2$$

The last part says that if even when the unbiased type exerts the maximum effort level 1, the biased type still finds it profitable to mimic him by deviating to invest, there exists no separating equilibrium. This is likely to arise if β is high, p_0 is low, or α is low. When this condition is not satisfied, there exists some $e \in [0,1]$ for which separation is possible but even then it is not guaranteed: at the LCS effort level, it may be that the payoff is lower than the payoff without investment. In that case, a separating equilibrium will not exist either.

If the separating equilibrium exists and it is the LCS one, then the DM is sometimes better off due to the presence of the biased type compared to having an unbiased expert with probability 1, due to the higher investment by the unbiased uninformed type in order to separate himself from the biased type. This is the case if the gain to the DM from the over investment of the unbiased expert outweighs the negative effect of having a biased expert with some probability who sends uninformative advice.

Let us compare this to some results from the literature before showing it in a numerical example. Argenziola et al. (2016) show that the expert overinvests compared to what the DM would have done by using an argument of babbling threat by the decision maker. There is only 1 type of expert in their setup whereas the over investment result in my setup arises because the expert wants to avoid being mimicked by the biased type. In addition, there is over investment compared to what the unbiased expert would have done without the presence of a biased expert. This result is also reminiscent of Che and Kartik (2009) who show that the DM prefers to have an advisor with different prior than herself compared to an aligned one, but in a setting in which there is no uncertainty about bias and communication happens through disclosure. The similarity is that their result comes from the higher effort exerted given that no disclosure is perceived as withholding information.

A numerical example

We will now demonstrate numerically the result that the DM's welfare may increase when introducing some uncertainty about the expert's type. Assume the cost of effort is $c(e) = e^2$, $\beta = 0.1$,$\alpha = 0.4$, $p_0 = 0.5$. For these parameters, we have $e^* = \frac{1}{8}$ as the first best investment level and $(1 - \hat{p}) = \frac{5}{28}$. Then, the DM's payoff when the expert is known to be an unbiased type (the advisor chooses $e^* = \frac{1}{8}$:

$$-(1-\alpha)\frac{7}{8}(p_0 - {p_0}^2) = -0.6 \times \frac{7}{8} \times 0.25 = -0.13125$$

When there is a probability of the advisor being biased ($\beta = 0.1$), then the LCS equilibrium has $e = \frac{5}{28}$. Then, the DM's payoff is:

$$-(0.1 + 0.9 \times 0.4 \times 0.5)\frac{5}{28}^{2} - \frac{23}{28} \times 0.9 \times 0.6 \times 0.25 = -0.11982142856$$

As we see, the DM's payoff is higher in the least cost separating equilibrium when there is a possibility that the agent is biased compared to when the advisor is unbiased with certainty. This is a hold-up problem in the sense that the advisor invests in information and the DM does not pay for it. As the expert only takes into account his own benefit but the decision is valued both by him and the decision maker, he under invests in the case when there is no uncertainty about his type. However, when uncertainty about the expert type is introduced, then it can be the case that the adviser over-invests in information. Even though there is possibility that the expert may be a biased one giving wrong advice, the over investment by the expert may overcome this effect and as a result the DM may be better off.

7.2 Pooling Equilibrium

As we established that a separating equilibrium is not always guaranteed, we now study pooling equilibria, in which all types except informed (0) unbiased invest. Upon investment and message $m = 1$, the DM's updated belief is:

$$\tilde{p} = \frac{p_0\beta + (1-\beta)p_0\alpha + (1-\beta)(1-\alpha)ep_0}{\beta + (1-\beta)p_0\alpha + (1-\beta)(1-\alpha)ep_0}$$

which simplifies to:

$$\frac{p_0\beta + (1-\beta)(p_0\alpha + (1-\alpha)ep_0)}{\beta + (1-\beta)(p_0\alpha + (1-\alpha)ep_0)}$$

and $1 - \tilde{p}$ becomes:

$$\frac{\beta(1-p_0)}{\beta + (1-\beta)(p_0\alpha + (1-\alpha)ep_0)}$$

If the unbiased type ends up not getting a signal, he sends $m = \emptyset$, whereas the biased type always sends $m = 1$. The belief $\mu(0,1) = \hat{p}$, as in the separating equilibrium:

$$\hat{p} = \frac{\alpha(1-\beta)p_0 + \beta p_0}{\alpha(1-\beta)p_0 + \beta}$$

The gain to the biased agent from investing the pooling effort level e compared to no investment is: $(1-\hat{p})^2 - (1-\tilde{p})^2$, where e affects $\tilde{p}$ and $c(e)$. As $\hat{p}$ is decreasing in e, $(1-\tilde{p})^2$ is increasing and convex in e. Then, the question remaining is to find the pooling equilibrium level of investment. The condition for the unbiased type to exert a pooling equilibrium level of effort, e, is:

$$-ep_0(1-\tilde{p})^2 - (1-e)(p_0 - {p_0}^2) - c(e) \geq -(p_0 - {p_0}^2)$$

which leads to:

$$-ep_0(1-\tilde{p})^2 + ep_0(1-p_0) \geq c(e)$$

The net value of investment for the unbiased type is convex in e. The biased type is willing to invest if and only if:

$$(1-\hat{p})^2 - c(e) \geq (1-\tilde{p})^2$$

The hand right side is decreasing in e, as $\tilde{p}$ is increasing in e. The ex-ante payoff of the unbiased type in the pooling equilibrium with effort level e is:

$$-p_0(\alpha + (1-\alpha)e)(1-\tilde{p})^2 - c(e)(1-\alpha(1-p_0))$$

Payoff of the biased type in the pooling equilibrium with effort level e:

$$-(1-\tilde{p})^2 - c(e)$$

Finally, there is a continuum of pooling equilibria which survive the intuitive criterion.

The first best investment level for the unbiased uninformed type in the pooling equilibrium is:

$$c'(e^*) = (p_0 - {p_0}^2) - p_0(1-\tilde{p})^2 + 2ep_0(1-\tilde{p})\frac{\partial \tilde{p}}{\partial e} = p_0[1 - p_0 - (1-\tilde{p})^2 + 2e(1-\tilde{p})\frac{\partial \tilde{p}}{\partial e}]$$

7.3 Equilibrium Selection

Now, we argue that in case there exists a separating equilibrium, it will be the unique equilibrium. First, if separation is possible at e^*, then this is the unique equilibrium and we are done. Otherwise, the LCS equilibrium necessarily has over investment. This will be the unique equilibrium if the payoff of the unbiased uninformed type at the LCS effort level is as good as his no investment payoff to sending $m = \emptyset$. To rule out pooling equilibria in that case, the DM uses babbling as a threat: upon observing a lower investment level than the LCS level of effort, the DM treats the message as coming from a biased agent, hence a "babbling" message.

If there exists no separating equilibrium, this is because the minimum effort level the unbiased type should exert in order to achieve separation leads to a payoff less than $-(p_0 - {p_0}^2)$ for this type or because the biased type is willing to mimic the unbiased type for any possible effort level. Then, the only type of equilibria are pooling ones, and there are multiple pooling effort levels can arise as an equilibrium.

8 Conclusion

This paper studied how concern about credibility interacts with costly information acquisition. In this setup, the ex-ante informed expert who is unbiased is unable separate himself from a biased expert whenever the state of the world is high. Due to this, the decision maker can be better off when matched with an uninformed expert who will later on get informed and communicate more efficiently. I find that an unbiased expert, as well as a biased one, can wastefully invest in information acquisition. This leads to inefficiency in decision making and lower overall welfare. Higher information acquisition cost increases the overall ex-ante welfare when the equilibrium moves from pooling to separating region.

When we extend the information acquisition technology from binary to a continuum of effort levels, it is found that the decision maker can be better off having uncertainty about the expert's bias compared to having a perfectly aligned expert with probability one. This happens when over investment by the unbiased expert more than offsets the loss in communication due to facing biased expert with some probability.

The paper studied a novel question building on several strands of the communication and signaling literature. The simplicity of the model allows for numerous extensions left for future research. An interesting feature of the model is that even though the ability to acquire information is not correlated with the type of the expert, the value of getting informed is correlated to his type. This means it is possible to use information acquisition as a screening device by taxing the experts for getting informed. While in the pooling region, the expert who is uninformed and unbiased has the highest incentives to acquire information. Hence, this type will be willing to pay more than the biased and informed (1) agents implying wasteful investment by the biased and informed experts can be avoided. Another extension could be to consider commitment by the decision maker on action as a function of investment and communication.

9 Appendix

Proof of proposition (1):

I find the separating equilibrium to be the unique equilibrium in this region by using the intuitive criterion. First, there cannot be any equilibrium in this region in which unbiased informed (1) and biased types invest with positive probability, as even when $\mu(1,1) = 1$, in this region, they do not find it profitable to invest as the cost is too high. Then, the only equilibrium that could arise is the no investment equilibrium in which even the uninformed unbiased expert doesn't invest. For some out of equilibrium beliefs, this equilibrium can arise, as discussed below.

Assume that the DM believes any type except the informed (0) unbiased one is equally likely to invest, then his belief and optimal choice will be $\tilde{p}$ which is:

$$\tilde{p} = \frac{p_0(1-\beta) + \beta p_0}{p_0(1-\beta) + \beta} = \frac{p_0}{p_0(1-\beta) + \beta} \tag{10}$$

Let us show the biased type doesn't have the incentive to incur the cost c. Now, upon the message $m = 1$, the DM chooses $y^* = \hat{p}$ as there is only biased and unbiased informed(1) types who choose $m = 1$. For the biased and unbiased informed(1), the payoff from investing should be less than that from not investing:

$$(1-\tilde{p})^2 + c \geq (1-\hat{p})^2 \tag{11}$$

As without investment and $m = 1$, the DM's belief is $\hat{p}$ as in case 1. This is equivalent to:

$$\left[\frac{\beta(1-p_0)}{p_0(1-\beta) + \beta}\right]^2 + c \geq \left[\frac{\beta(1-p_0)}{\alpha(1-\beta)p_0 + \beta}\right]^2 \tag{12}$$

For the uninformed agent, the payoff from not investing is $-(p_0 - {p_0}^2)$ and from investing it will be:

$$-(1-p_0)0 - p_0(1-\tilde{p})^2 - c \tag{13}$$

Then the condition that should be satisfied is:

$$c \geq p_0 - {p_0}^2 - p_0(1-\tilde{p})^2 \tag{14}$$

Finally, the equilibrium in which no type wants to invest, for the specified out of equilibrium beliefs, is:

$$c \geq \max\{p_0 - {p_0}^2 - p_0(1-\tilde{p})^2, (1-\hat{p})^2 - (1-\tilde{p})^2\} \tag{15}$$

Now consider that when there is a separating equilibrium, the condition $p_0 - {p_0}^2 > (1-\hat{p})^2$ is satisfied. Then, it is the case that $p_0 - {p_0}^2 - p_0(1-\tilde{p})^2 > (1-\hat{p})^2 - (1-\tilde{p})^2$. This means, the condition above becomes $c \geq p_0 - {p_0}^2 - p_0(1-\tilde{p})^2$, which is less than $p_0 - {p_0}^2$. Then, there is also a no investment equilibrium in this region. However, by using the intuitive criterion, we are able to rule out this type of equilibrium.

In the region $c \in [p_0 - {p_0}^2 - p_0(1-\tilde{p})^2, p_0 - {p_0}^2]$, the separating equilibrium is the unique equilibrium that satisfies the Intuitive Criterion. Whenever $c \geq (1-\hat{p})^2$, even for the highest belief $\mu(1,1) = 1$, the biased and uninformed (1) agent do not benefit from deviating to invest. Hence, in this region, the Intuitive Criterion suggests that the out of equilibrium belief upon investment should allocate probability 0 to the expert being a biased type (or unbiased uninformed), and it should assign probability 1 to the expert being unbiased and uninformed. Then, in the region $c \in [p_0 - {p_0}^2 - p_0(1-\tilde{p})^2, p_0 - {p_0}^2]$, the separating equilibrium is the unique equilibrium that satisfies the Intuitive Criterion.

Proof of proposition (2)**:**

Proof. The total welfare in the **pooling** equilibrium is given by:

$$-p_0(1-\tilde{p}) - \beta[(1-\tilde{p})^2 + c] - (1-\beta)[p_0(1-\tilde{p})^2 + (1-\alpha(1-p_0))c] \tag{16}$$

which simplifies to:

$$-(1-\tilde{p})^2[\beta + p_0(1-\beta)] - p_0(1-\tilde{p}) - c[1-\alpha(1-p_0)] = -\beta(1-p_0) - c[1-\alpha(1-p_0)] \tag{17}$$

Total welfare in the **separating** equilibrium is:

$$-\hat{p}\beta(1-p_0) - \beta(1-\hat{p})^2 - (1-\beta)[\alpha p_0(1-\hat{p})^2 + (1-\alpha)c] \tag{18}$$

which simplifies to:

$$(19) \qquad -\beta(1-p_0) - c(1-\alpha)(1-\beta)$$

The difference in the total welfare in equation (17 − 19) is:

$$(20) \qquad -c_p[1-\alpha(1-p_0)] + c_s(1-\alpha-\beta(1-\alpha))$$

The welfare in terms of information cancels out and what remain are the terms related to cost incurred in information acquisition. In pooling equilibrium there is more investment in information at a lower price, while in the separating equilibrium there is less investment but at a higher price. When we plug in the maximum cost at which the pooling equilibrium exists and minimum cost at which the separating equilibrium exists, it is seen that welfare in separating equilibrium is higher than in the pooling one although cost of information acquisition is higher.

At $c_p = (1-\hat{p})^2 - (1-\tilde{p})^2$ and $c_s = (1-\hat{p})^2$, equation (20) becomes:

$$(21) \qquad -(1-\hat{p})\beta(1-p_0) + (1-\tilde{p})^2[1-\alpha(1-p_0)] < 0$$

Then, although the cost of information rises, welfare increases due to no wasteful investment in information. In order to demonstrate this result, we considered the boundary cost values. Then, we have shown that the proof holds.

As expected, when we keep increasing the cost in the separating equilibrium region, the result may be reversed. When we plug in the maximum cost at which there is separation, $c = (p_0 - {p_0}^2)$, we get:

$$(22) \qquad (-1+p_0)(\beta + (1+\alpha(-1+p_0))p_0)$$

Then, when we plug this in the difference in the total welfare equation (20) becomes:

$$(23) \qquad (-1+p_0)p_0(-1+3\beta+\alpha(1-2\beta+p_0))$$

This whole expression is negative if and only if:

$$3\beta + \alpha(1 - 2\beta + p_0) > 1 \tag{24}$$

This is satisfied for certain parameters, mainly more likely to hold for low p_0.

Total welfare in the **no investment** equilibrium is:

$$-\beta(1-\hat{p})^2 - (1-\beta)[\alpha p_0(1-\hat{p})^2 + 2(1-\alpha)(p_0 - {p_0}^2) - (\hat{p} - \hat{p}^2)(\beta + (1-\beta)\alpha p_0)] \tag{25}$$

When we compare this to pooling in investment, it is seen that pooling welfare is higher. □

Proof of proposition (3)

Proof. The first two items were shown earlier.

The last condition says that even if the uninformed unbiased expert exerts effort $e = 1$ in which case his gain in payoff will be $(p_0 - {p_0}^2)$, this is still lower than the gain of the biased type and informed (1) type from investing and pooling with him which is $(1-\hat{p})^2$. The outcome of investment is not relevant for these types and they will always claim to have received signal 1. Given that this is a deviation from an equilibrium in which the only investment is made by unbiased types, when they send $m = 1$ the DM will choose $y = 1$. Hence, their payoff from investment is 0, whereas without investment their payoff was $-(1-\hat{p})^2$, which means their gain is $(1-\hat{p})^2$.

On the other hand, the gain in profit from investment for the unbiased uninformed expert is $e(p_0 - {p_0}^2)$. To see this, realize that with probability e, a signal will arrive and if it does, then the expert's payoff will be 0 as the DM will take the message at face value. If no message arrives, then the expert will send $m = \emptyset$ and get a payoff of $-(p_0 - {p_0}^2)$. Hence, the gain in payoff for the uninformed type is $e(p_0 - {p_0}^2)$.

Then, the gain in payoff from deviating to pool is higher for the biased type than the gain in information for the unbiased uninformed type even when $e = 1$ if $(p_0 - {p_0}^2) \leq (1-\hat{p})^2$. Then, for any level of e, the uninformed unbiased type could not separate himself from the biased type of expert. Hence, no separating equilibrium exists under this condition.

□

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