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Self-Control in the Retailing Industry: Inducing Rejection of Loyalty Schemes

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ABSTRACT. When consumers register with loyalty schemes, or open a 'customer account', offered by large retailers, they allow retailers to study their purchasing behaviour over time. Via personalised offers and discounts, retailers can then use this information to price discriminate. I study the effect on consumer welfare of this discrimination, assuming several different levels of informativeness within the schemes. When schemes are *uninformative* about consumer preferences they are certain to hurt consumer surplus. When they are fully or partially *informative*, an increase in aggregate consumer surplus can take place under some conditions. Pareto Improvements are never possible. The model studies groceries and online industries where temptation and self-control are an issue.

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1. Introduction

The industry of retailing is dense with loyalty scheme practices. From 'loyalty cards' offered by large grocery stores to 'customer accounts', required to shop with online retailers, our wallets (physical and digital) are full of passwords, fidelity cards, reward points and so on. The economics literature has focused most of its attention on loyalty schemes as a tool for competition or to increase consumer life-time value (i.e. their loyalty to the firm). When customers register with these schemes, however, they also allow the firm to observe their purchasing behaviour over time. That is, they allow the retailer to study their preferences. I, therefore, suggest a new approach to loyalty schemes. I ask: how does a retailer use the information granted by loyalty schemes? How does its use affect consumer surplus? Should loyalty schemes be regulated in

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any way? In answering these questions I make a crucial assumption. Consumers are tempted and they suffer from problems of self-control.

It has been extensively documented, both in the media and in the academic literature, how consumers make 'impulsive purchases' — unplanned decisions to buy products or services made just before a purchase — both on- and off-line.[2] In this paper temptation and self-control play two key roles. First, they increase consumer willingness to pay when facing their purchase decisions. Second, they may drive consumers away from the seller because of their fear of engaging in impulsive purchasing. This second aspect is key to my findings.

My results outline the pricing strategy of a seller who has access to loyalty scheme technologies. I show how, in this framework, loyalty schemes have two main purposes. First, when they are *informative* towards consumer preferences they become a tool for price discrimination. Second, they always allow the retailer to distinguish consumers who give in to impulse purchasing from those who are driven away from the store for fear of being too tempted.

I show that a social planner who values the welfare and surplus of consumers should ban loyalty scheme programs that are not oriented towards understanding consumer preferences. In other words, whenever loyalty schemes do not allow retailers to better tailor their prices to consumer preferences, their only effect is to raise retailers profits at the expense of consumer surplus. This contrasts with the general opinion that consumer tracking always hurts consumers.[3]

When loyalty schemes are used to study consumer preferences, third degree price discrimination becomes possible. In equilibrium, some groups of consumers may purchase a certain good at a special price, valid only for them. This good could be one they would not buy at the normal market price. When this is the case, the introduction of loyalty schemes creates positive effects on consumer welfare. Therefore, a general statement on welfare effects of 'informative' loyalty schemes cannot be made. I derive conditions for these types of schemes to always hurt aggregate consumer surplus.

In the two period model I develop, a multi-product seller offers her goods in a 'superstore' to a continuum of buyers who differ in their temptation levels and valuation of the goods. The superstore offers good x and a tempting good z. She is the sole seller of z, while x is also offered by other stores at a price fixed by the market. In period 1, the retailer sets the price of z and consumers decide whether to "enter the superstore", or one of the smaller stores. In period 2, consumers purchase either x, z or nothing at

[2]For evidence on temptation in the grocery and food industries see Thompson, Locander, and Pollio (1990), Baumeister (2002) and Houser, Reiley, Urbancic, et al. (2008). For evidence on impulse buying during online purchasing see Jeffrey and Hodge (2007) and Park, Kim, Funches, and Foxx (2012). I further motivate this assumption in Section 4 where I also discuss the media attention paid to the matter.

[3]Some examples: "Getting to know you", *The Economist*, September 11th 2014, "How retailers are watching shoppers emotions", *The Economist*, June 8th 2017 and O'Neil (2017).

all. Temptation kicks in in period 2, but consumers anticipate their self-control cost in period 1 (á la Gul and Pesendorfer, 2001).

Before the start of the game, the superstore can offer each consumer the option to register with the loyalty scheme program. These schemes are modelled as a partition of the consumer type space. If a consumer accepts the scheme, the superstore observes to which element of the partition the consumer belongs. In other words, the superstore is capable of dividing the customers who register in groups — for example: non-tempted, tempted, very tempted and so on. The more precise the information granted by the schemes the finer is the partition, i.e. the more the groups the superstore divides her consumers into. In the paper, I solve for the price of equilibrium for all possible relevant partitions and derive conditions for them to hurt consumer surplus.

The main mechanism behind the welfare results of the paper lies in the (dis)incentives of the seller to increase the price of the tempting good. Increasing the price of good z increases revenues per purchase. Some consumers give in to temptation and keep purchasing the good. Some others, however, exert self-control cost to resist this temptation. When this self-control cost is too much to bear, consumers are driven away from the store, and purchase x in one of the other ones. What if the seller had a way to increase the price only for those willing to give in to temptation? By setting a high general price and reducing the price of tempting goods for loyalty schemes subscribers alone, she avoids tempting those who choose not to subscribe. In this way, she succeeds in keeping her customers inside her store.

At first glance, this practice does not seem to hurt consumers in any way, but there's a catch. By eliminating the problem of scaring away tempted consumers, the retailer's incentive to keep the price low is decreased. This is because, if she were to increase the price by a certain amount, those consumers who are afraid of impulse purchasing, would no longer be driven away. Hence, the equilibrium price paid by (at least some) consumers increases when loyalty schemes are introduced. This is in sharp contrast with the results of the literature on 'hard evidence'.[4]

The paper is organised as follows. In Section 2, I describe the industry structure and the retailers that operate in it. In Section 3, I describe the timing of the model. In Section 4, I motivate the assumption of tempted consumers further and describe their preferences. In Section 5, I describe the full information equilibrium and the key role played by temptation in the model. In Section 6, I describe and motivate my modelling of loyalty schemes. In Section 7 and 8, I describe the optimal algorithm governing the pricing strategy of the retailer for any possible relevant partition. I also study the welfare effects of loyalty schemes. In Section 9, I discuss the relevant literature. I conclude the paper in Section 10.

[4]Mainly Bergemann, Brooks, and Morris (2015) and Pram (2017), as I discuss in Proposition 8.

2. Industry

A continuum of (heterogenous) consumers is interested in the purchase of one of two imperfectly substitute goods, x and z. These are offered in a market composed of a large set of small stores and a larger store, the "superstore" (her). Good x is sold in every shop of the market at fixed price p_x.[5] Good z is sold only in the superstore at the endogenously set price p_z. Each unit of good offered costs the stores c_x for x and c_z for z. I assume $c_x \geq c_z$. The idea behind this assumption is that since good z is sold only by one store it can be thought of as an innovative product with a lower production cost.[6]

Let an offer be a couple (a, p_a) specifying the good offered, a, and the price required for the purchase, p_a. Given the above, the superstore offers a menu of *offers* $M_m = \{\underline{0}, (x, p_x), (z, p_z)\}$ in her store. Each one of the smaller stores offers menu $M_s = \{\underline{0}, (x, p_x)\}$. Offer $\underline{0} = (0, 0)$ is called the *null offer* and it represents the assumption that a consumer is always free to make no purchase when facing a menu. If a consumer purchases good $a = x, z$ the store obtains $\pi(a, p_a) = p_a - c_a$.

3. Order of Play

There are two periods. At the beginning of time all consumers are outside the stores. In period 1 the superstore sets price p_z and each consumer decides whether to enter the superstore or one of the smaller stores. In period 2 each consumer makes his purchase from the menu offered by the store he entered in period 1.

Formally, in period 1 the superstore sets p_z, implicitly defining her menu of offers M_m. Each consumer compares menu M_m with M_s.[7] Deciding which "store to enter" therefore represents a choice among menus. The consumer enters the store that offers the menu he prefers, according to his preferences described below. In period 2, each consumer chooses an offer from the menu selected in period 1.

The superstore also has at her disposal a system of loyalty schemes. These schemes allow her to offer good z at price p_A to consumers that accept, and p_R to those who do not. If the superstore uses the loyalty scheme technology, in period 1 she sets both p_A and p_R. In this case, consumers have a set of three menus to choose from: $M_A = \{\underline{0}, (x, p_x), (z, p_A)\}$, faced if they accept the scheme and enter the superstore,

[5] The fixed price assumption can have several explanations. For example it can be a result of monopolistic competition, regulation or advertisement of the good by the original producer. Another alternative explanation is the existence of a competitive market with switching costs.

[6] On the other hand, another interpretation could be for z to represent a peculiar product, one that's not sold everywhere, such as for example no lactose milk as opposed to x being normal milk. In this case $c_z > c_x$ would sound more appropriate. I study this case in Appendix C. Assuming $c_z > c_x$ affects the results qualitatively only in terms of the set of consumers screened out of the market in equilibrium, when information is private. This, however, makes the solution computationally heavier without adding interesting insights or violating any of the policy implications of the model.

[7] Technically, each consumer also has the option to enter no store. Later I assume this is not the case, and that each consumer has at least the weak incentive to enter one of the smaller stores.

$M_R = \{\underline{0}, (x, p_x), (z, p_R)\}$, faced if they reject the scheme and enter the superstore, $M_s = \{\underline{0}, (x, p_x)\}$, faced if they enter into any of the other stores.

4. Tempted Consumers

I assume that consumers suffer from temptation and self-control problems. To see why temptation plays an important role in the kind of markets I study, one can think about the obvious connection between food and self-control. More broadly, evidence of 'impulsive purchasing' has been found in both online (Jeffrey and Hodge, 2007; Park, Kim, Funches, and Foxx, 2012) and offline markets (Thompson, Locander, and Pollio, 1990; Baumeister, 2002; Houser, Reiley, Urbancic, et al., 2008). In addition, there is a growing interest in how sellers and retailers take advantage of consumer temptation by rearranging products in the store, or offering particular discounts.[8] In 2014 the *Financial Times* reported the decision of the UK's largest grocer, Tesco, to ban sweets and chocolates from checkout tills in all stores. In 2013 the UK's Department of Health asked grocers to stop inducing "impulse purchases" in their stores.[9] Consumers in the markets I consider are tempted, and retailers take this temptation into account in their selling stategies.

To account for the temptation and self-control problem of consumers I model their preferences à la Gul and Pesendorfer (2001). This approach assumes that consumers suffer from temptation at the time of purchase, and anticipate their self-control cost before facing the purchasing decision. This translates as follows in the present model.

In period 2, when choosing from menu M of the store they entered, consumer i decides which offer to buy according to:

$$\max_{a \in M} [U(a, p_a) + V_i(a, p_a)] \tag{1}$$

Function U is called the (net) *commitment utility* while function V is called the (net) *temptation utility*. To understand the difference between these two functions, consider U as the base utility that the individual obtains from the good, free of temptation. Function V, instead, measures the impulses of the individual at the moment of purchase. When given a menu, he considers both his commitment and his temptation and takes his decision.

In period 1, instead, choosing from a set of different menus $\{M_A, M_R, M_s\}$ consumer i selects the one that maximises his *self-control preferences*. That is, he evaluates each menu according to:

$$W_i(M_j) = \max_{a \in M_j} [U(a, p_a) + V_i(a, p_a)] - \max_{a \in M_j} V_i(a, p_a) \quad j = 1, ...N. \tag{2}$$

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[8] Among others: "Temptation at checkout", J. Almy and M.G. Wootan, *Centre for Science in the Public Interest*, August 2015; "The checkout aisle that leads us not into temptation", A. Craig, *The Telegraph*, May 23rd, 2014; "The Psychology Of Shopping: How Grocery Stores Make Rational Spending Nearly Impossible", C. Weller, *Medical Daily*, November 21st, 2013.

[9] "Tesco to ban sweet temptations at tills", D. Robinson, *Financial Times online*, May 22nd, 2014.

This is the utility the consumer enjoys facing menu M_j. It is composed of the utility he obtains by making the purchase minus the temptation utility that he is foregoing because he is exerting self-control, represented by the offer that would maximise his temptation utility, $\max_{a \in M_j} V_i(a, p_a)$. To understand the intuition behind (2), notice that if offer γ maximises $U+V$ and ω maximises V, then $W(\cdot) = U(\gamma) - [V(\omega) - V(\gamma)]$ where $V(\omega) - V(\gamma)$ is known as the *self-control cost* of choosing γ over ω. If the latter is too high, the consumer does not accept the menu in the first place.

In this paper, I assume quasi-linearity of utility functions:

$$U(a, p_a) = u(a) - p_a \tag{3}$$

$$V_i(a, p_a) = \phi_i v(a) - p_a. \tag{4}$$

A good a is *tempting*, independently of consumer type, if it grants more temptation than commitment (gross) utility, $v(a) > u(a)$. Heterogeneity of preferences is captured by parameter ϕ_i, which is distributed uniformly in $[0, 1]$. It represents the level of temptation of consumer i and, in particular, it captures the idea that consumers value the tempting features of a in different ways. Hence, it defines a consumer.[10] The parameter is not multiplied by the price. First of all, this approach allows for the sign of $V_i(a, p_a)$ to be heterogenous among consumers. This generalises the approach to cases where $V_i(a, p_a) < 0$ for some a and some i, i.e. where consumer i is tempted not to buy. Second, this simplifies the analysis considerably, at no particular cost in terms of generality.[11]

From here on, I say that $(a, p_a) \succ_T (b, p_b)$ for i, i.e. offer for good a is *more tempting* than offer for good b, if and only if $V_i(a, p_a) > V_i(b, p_b)$.

In what follows I make two assumptions on the parameters of the model that restrict the problem to interesting and tractable cases.

Assumption 1. *Every consumer prefers buying x as opposed to buying nothing, that is $U(x, p_x) + V_i(x, p_x) \geq 0$, for all ϕ_i.*

Assumption 1 implies that $p_x \leq \frac{u(x)}{2}$. That is, the price of good x is lower than the half of the commitment utility obtained by good x. It also states that even the non tempted consumer ($\phi_i = 0$) finds it optimal to buy good x.

In Appendix A, I discuss this assumption in more detail. First of all, I show how it also implies that every consumer has the incentive to enter one of the small stores that offers menu M_s, as opposed to entering none (obtaining $W = 0$). Second, I discuss how dropping this assumption would lead to the same qualitative results as long as $U(x, p_x) \geq 0$. Assuming $U(x, p_x) < 0$, however, would make x an "unattractive" good.

[10]Notice that, following the existing literature on temptation models, I assume the negative part of utility the consumer gets from paying price p is equal to the actual transfer p itself. This is true also for the temptation utility.

[11]All results follow through but equilibrium conditions and derivations become more "klunky".

A good that does not yield positive commitment utility is a good that the consumer does not need in general and that only (potentially) satisfies the urges and impulses at the moment of purchase. This would not be in line with the applications discussed in this paper and it therefore falls out of its scope.

Assumption 2. *Independently of consumer types, the following are true:*

i) $v(a) \geq u(a)$ *for all* $a = x, z$.
ii) $v(z) > v(x)$
iii) $u(0) = 0$.
iv) $u(x) = u(z) \geq 0$.

Assumption 2 ranks the gross utility levels obtained by consuming good x or z. Point i) is standard in the literature and assumes that temptation act as an 'overvaluation' of the characteristics of a product. Temptation utility v values the same good at least as much as utility u. Given $\phi_i \in [0, 1]$, assuming $v(a) \geq u(a)$ for all a allows for $V_i(a, p_a)$ to be larger, equal or smaller than $U(a, p_a)$ for some ϕ_i. Point ii) defines z to be the most tempting good in the market. Point iii) is a normalisation without loss of generality. Point iv) states that goods x and z satisfy the same basic needs. In other words, if it were not for their tempting aspects the goods would have been perfect substitutes.

Following ii) and iv), in what follows I refer to the distance between $v(z)$ and $v(x)$ as the *degree of substitutability* between x and z. The closer (further) these two utility levels are, the closer (farther) substitutes are the goods.

5. Full Information

Let temptation levels be common knowledge. Since the superstore is able to first-order price discriminate among consumers the loyalty scheme tool has no use and the seller simply charges each consumer their reservation price. Given Assumption 1, for each consumer i she sets a price $p_z^*(\phi_i)$ solving:

$$\max_{p_z} [p_z - c_z] \tag{5}$$

$$\text{s.t.} \quad W_i(M_m(p_z)) \geq W_i(M_s) \tag{PC}$$

$$U(z, p_z) + V_i(z, p_z) \geq U(x, p_x) + V_i(x, p_x). \tag{IC}$$

The first constraint ensures that the consumer is willing to enter the superstore as opposed to one of the smaller stores in the market. The second constraint ensures that the consumer purchases z instead of x. The following Proposition describes the equilibrium under full information.

Proposition 1. *If temptation levels are commonly known, the optimal pricing scheme for z is non-monotonic in ϕ_i:*

$$p_z^*(\phi_i) = \begin{cases} \frac{1}{2}\phi_i\left(v(z) - v(x)\right) + p_x & \text{if } \phi_i < \phi^* \\ 2p_x - \phi_i v(x) & \text{if } \phi_i \in \left[\phi^*, \frac{p_x}{v(x)}\right] \\ p_x & \text{if } \phi_i > \frac{p_x}{v(x)} \end{cases} \tag{6}$$

$$\text{where} \quad \phi^* \equiv \frac{2p_x}{v(z)+v(x)} \tag{7}$$

represents the consumer with the highest willingness to pay. The superstore is willing to sell z to every consumer, since $\pi\left(z, p_z^*(\phi_i)\right) \geq \pi(x, p_x)$ for all ϕ_i.[12]

Figure 1 shows the optimal level of $p_z^*(\phi_i)$ for each value of ϕ_i — i.e. the ex-ante willingness to pay of each consumer in the market.[13] From the figure it is already evident the key role that temptation and self-control play in this model. Their presence results in a non-monotonicity of the consumers' willingness to pay in their own valuation of the good — which is higher when ϕ is larger. This implies that the most 'valuable' customer for the superstore is not the one with the highest valuation of the good.[14]

Studying the bold line in Figure 1, we see that low valuation consumers are not interested in the tempting aspects of z enough for the superstore to charge them a high price. For these consumers $\underline{0} \succ_T (z, p_z^*(\phi_i))$ holds. As ϕ_i increases, a consumer's valuation of the good increases, the IC becomes slack and the superstore can raise p_z to make the constraint bind again. This is true for all ϕ_i below ϕ^*. I define these consumers as *weakly tempted*. Consumers beyond ϕ^*, instead, are tempted by z enough that $(z, p_z^*(\phi_i)) \succ_T \underline{0}$. This temptation may render the purchase of z ex-ante sub-optimal. That is, their ex-ante willingness to pay for good z is decreasing in their temptation level, ϕ_i. I define these consumers as *strongly tempted*. Among these there are consumers so tempted that they find even (x, p_x) to be a tempting offer, i.e. $(x, p_x) \succ_T \underline{0}$. These are the most tempted consumers in the market, $\phi_i > \frac{p_x}{v(x)}$. Ex-ante, they exert self-control effort and ignore completely the tempting features of good z. That is, in order to give in to temptation and enter to purchase z, they cannot be charged more than p_x for it.

6. Loyalty Schemes

Suppose now that temptation levels are the private information of consumers. In this section, I study the decision of the superstore to use a loyalty scheme technology.

[12] This changes when $c_z > c_x$ since $(p_x - c_z) < (p_x - c_x)$ and it is no longer profitable to sell good z to some consumers in the market.

[13] In the figure, for simplicity I assume that $v(z) = 3v(x)$ so that the absolute value of the slope is the same before and after ϕ^*. This carries no particular meaning and leads to no particular consequences besides making the graph pleasant to the eye. I assume this in all the graphs of the paper. Of course, my results hold for $v(z) \neq 3v(x)$.

[14] For a review of the model without temptation and self-control preferences, and a study of its differences with my results, see Appendix D.

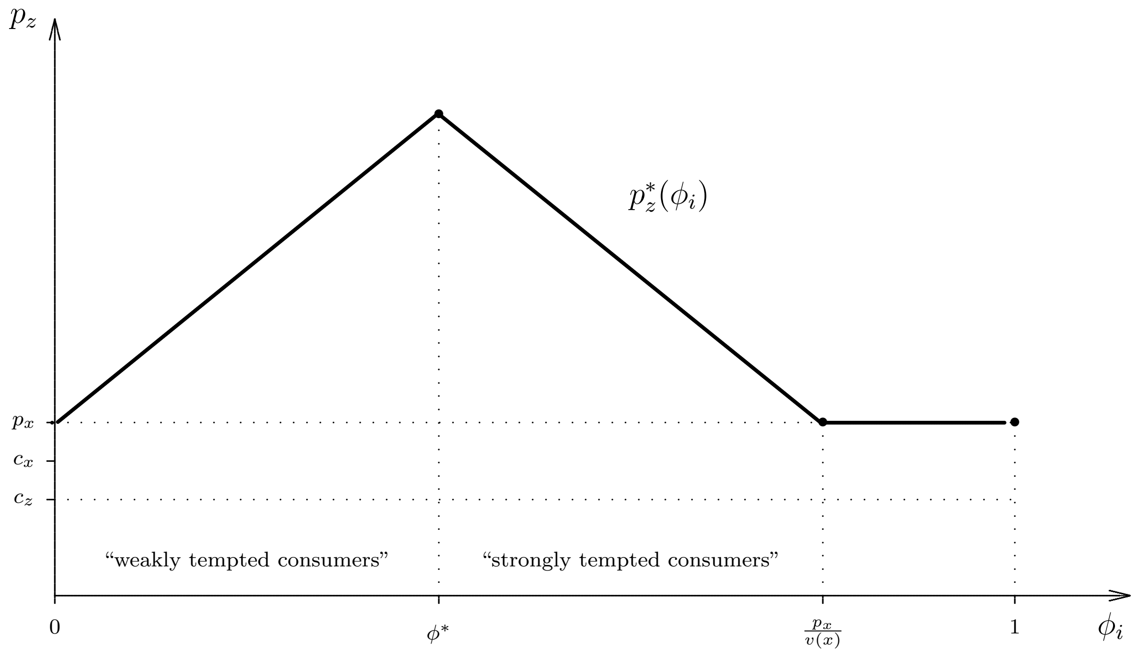

Figure 1. The thick curve in the figure shows the willingness to pay, $p_z^*(\phi)$, for each consumer. When the seller is able to perfectly observe ϕ_i, this corresponds to the price charged in equilibrium.

The loyalty scheme technology modelled in this paper has three features that I explain and motivate below.

Individual Pricing. By accepting a loyalty scheme, a consumer becomes identifiable. That is, the superstore can charge a specific price for him that is inaccessible to others. In reality, consumers with a 'loyalty card', a 'fidelity scheme', or a 'customer account' are assigned a serial number that is immediately recognisable. Hence companies can issue specific offers and discounts on specific products targeted at individual consumers or group of consumers. This is even more the case for online stores and grocery delivery systems where purchases can be made only by registered customers.[15]

Observability. By accepting a loyalty scheme, a consumer grants the superstore information about his preferences. In reality, whenever consumers 'scan their card' at the till when paying, or make a purchase using their customer account, they generate a large amount of data. This data is stored in the superstore's servers and can be studied by firms in order to elicit consumer preferences. This helps the firm creating individual pricing schemes tailored to the preferences of each customer. There is growing interest on whether the use of such data in order to carry out price discrimination can be harmful or not to consumers.[16]

[15]Some examples are the 'just for you offers' in the newsletters of many online retailers, offers on 'wish-lists' like Amazon.com's, and 'clubcard coupons' from Tesco.

[16]For discussions about Big Data and price discrimination in the media see: "Will Big Data Bring More Price Discrimination?", A. Ozimek, *Forbes*, September 1^{st}, 2013; "Hiding from Big Data", Technology

No Deception/Hard Evidence. While my model lasts for two periods, in reality consumer-retailer relationships can last for years. Throughout these years, using his loyalty card, the consumer often acquires 'points' that he can then turn into gifts, vouchers or (personalised) discounts — often called 'loyalty prices'. In addition, the retailer may induce special offers only for consumers that hold a fidelity card and who use it frequently. In other words, sellers create plenty of incentives for consumers to always scan their card or use their account for every single purchase they make. Years of data on purchases are stored and studied by firms in order to elicit consumer preferences. This makes it implausible to think of a consumer who carefully manipulates his purchases over the course of a long period of time in order to deceive the retailer and make her believe that his preferences are something different from the truth. Hence, in this model, accepting a loyalty scheme is equivalent to acquiring 'hard evidence'.[17]

Examples of loyalty schemes that satisfy the three features above are: 'Kroger Card' and 'Clubcard' offered by Kroger, the largest grocery retailer in the USA by revenue[18] and Tesco, the third largest grocery retailer in the world by revenue[19]. A non-grocery seller example is 'My Best Buy Reward Program' by the American electronic retailer Best Buy. The model also applies to online retailers such as Amazon.com. In order to make a purchase on the sellers' website, consumers have to be registered. By registering they allow the firm to observe all their purchases and to offer them personally tailored prices.

Taking into account the examples and the features mentioned above, I now present the way I model loyalty schemes in this paper. As mentioned in section 3, if consumer i accepts (rejects) the scheme he faces price p_A (p_R) for good z. These prices are set by the superstore in period 1.

On top of facing a new price, when a consumer accepts the scheme, he reveals an exogenous amount of information about his willingness to pay to the seller. I model this by assuming that the loyalty scheme technology applies an exogenous partition $\mathcal{P}$ to the type space, with each element being an interval $\left[\phi^l, \phi^r\right] \subseteq [0,1]$. By accepting the loyalty scheme, consumer i reveals the belonging of his ϕ_i to an element of $\mathcal{P}$, without the possibility of lying. Hence, the seller can set a specific value of p_A for each element of $\mathcal{P}$. I consider partitions with distinct and ordered elements. This captures the idea that the seller can position the consumer on the $[0,1]$ spectrum, by exploiting the

Quarterly, *The Economist*, July 7^{th}, 2014; "Big Data, Financial Services and Privacy", Finance and Economics, *The Economist*, February 9^{th}, 2017. For references on individual pricing see Shiller (2015), Waldfogel (2015) and Shiller and Waldfogel (2011).

[17]By hard evidence is intended a piece of information that would be accepted as *true* by a judge in court; information which is true, or at least costly to falsify.

[18]"2013 Top 100 Retailers", *STORES Media*, 2013.

[19]"Tesco to outpace growth at global rivals" M. Potter, *Reuters*, 16 February 2011.

information granted by the loyalty scheme, and divide her customer base into groups: "non-tempted", "tempted", "very tempted" and so on...

I consider all relevant levels of fineness of $\mathcal{P}$, starting from the trivial partition $\mathcal{P}_1 = \{[0,1]\}$. In the latter case, the seller observes only that a consumer who accepts the loyalty scheme is *willing* to do so. Hence, I call this type of scheme *ϕ-uninformative*. I show below how this type of loyalty scheme always improves profits at the expense of consumer welfare. Hence, a social planner who values consumer welfare should prohibit these sorts of loyalty schemes.

The other extreme case is to assume that once a consumer accepts the loyalty scheme he fully discloses his type to the seller. In this case $\mathcal{P}$ is composed of uncountably many singletons elements, each containing a single ϕ_i, i.e. a single consumer. I show how, in this case, in equilibrium every consumer is willing to accept a loyalty scheme. While the superstore profits are increased, the aggregate consumer welfare does not necessarily decrease.

In between these two cases there are many possible refinements of the trivial partition $\mathcal{P}_1$.[20] As $\mathcal{P}$ becomes finer, the number of subintervals of $[0,1]$ (its elements) increases. That is, acceptance of the scheme reveals more precise information about ϕ_i and the superstore divides consumers into more groups. When consumer i accepts a loyalty scheme, the seller updates her beliefs. She knows that the consumer's ϕ_i distributes as a uniform in the element of the partition to which it belongs.

With this model in mind, I derive the equilibrium prices and profits for any possible relevant partition of fineness n.

Let $\mathcal{P}_n = \{[0,\phi^1), [\phi^1,\phi^2), ..., [\phi^{n-2},\phi^{n-1}), [\phi^{n-1},1]\}$ be a partition of fineness n (with n elements), then the seller sets $n+1$ prices for z. Namely: p_R, the price of rejection, and the n values of p_A, one for each element of $\mathcal{P}_n$. Formally:

$$p_z = \begin{cases} p_A = \left\{ \begin{array}{ll} p_n & \text{if } \phi_i \geq \phi^{n-1} \\ p_{n-1} & \text{if } \phi_i \in [\phi^{n-2},\phi^{n-1}) \\ p_{n-2} & \text{if } \phi_i \in [\phi^{n-3},\phi^{n-2}) \\ \dots & \\ p_2 & \text{if } \phi_i \in [\phi^1,\phi^2) \\ p_1 & \text{if } \phi_i < \phi^1 \end{array} \right\} & \text{if } i \text{ accepts} \\ p_R & \text{if } i \text{ rejects.} \end{cases} \tag{8}$$

The equilibrium price derivation for the ϕ-uninformative case is presented first.

[20] Partition X of a set is a refinement of partition Y if every element of X is a subset of some element of Y.

7. Uninformative Loyalty Schemes

When the seller has access to ϕ-uninformative loyalty schemes, she sets:

$$p_z = \begin{cases} p_A & \text{if } i \text{ accepts} \\ p_R & \text{if } i \text{ rejects} \end{cases} \quad \text{for all } i.$$

As Figure 1 shows in Section 5, strongly tempted consumers are tempted by the first best offer set for consumers to their left. To see this, consider consumer ϕ^j and ϕ^{j+h} such that $\phi^* < \phi^j < \phi^{j+h} \leq \frac{p_x}{v(x)}$. First of all, for ϕ^{j+h} it is true that $(z, p_z^*(\phi^j)) \succ_T (x, p_x)$. Since $W_{j+h}\left(\left\{(x,p_x), \left(z, p_z^*(\phi^{j+h})\right), \underline{0}\right\}\right) = 0$, then

$$W_{j+h}\left(\left\{(x,p_x), \left(z, p_z^*(\phi^{j})\right), \underline{0}\right\}\right) < W_{j+h}\left(\left\{(x,p_x), \underline{0}\right\}\right).$$

Were $p_z^*(\phi^j)$ charged to consumer ϕ^{j+h}, the latter would strictly prefer not to enter the superstore, not even to buy x.

Now, let $p_{\max}$ be the smallest price satisfying $U(z, p_{\max}) + v(z) - p_{\max} < 0$, that is, the smallest price larger than the period 2 willingness to pay of the most tempted consumer. When the superstore sets $p_{\max}$ as the price of z no consumer is tempted and they are all willing to enter the superstore in order to purchase x. Obviously in this case the superstore is giving up on a lot of potential profits, by not selling the least costly and most valuable product on the market. However, by setting $p_{\max}$ as the price of rejection, the superstore is now "free" of charging any price p_A to consumers who accept the loyalty scheme. Strongly tempted consumers who find p_A tempting, but not worth a purchase according to their self-control preferences, can reject the scheme and face menu M_R, avoiding temptation all together. In this way, loyalty schemes work as a *commitment device* for tempted consumers.

Before showing how the above shapes the equilibrium, let me define $\Phi(p_z)$ as the set of consumers willing to purchase good z at price p_z according to their period 1 self-control preferences.

Lemma 1. *For any price $p_z > p_x$ offered by the seller for good z, only consumers in*

$$\Phi(p_z) \equiv \left[\underline{\phi}(p_z), \overline{\phi}(p_z)\right] \equiv \left[\frac{2(p_z - p_x)}{v(z) - v(x)}, \frac{2p_x - p_z}{v(x)}\right] \tag{9}$$

are willing to enter the superstore and purchase z.

Proposition 2. *When loyalty schemes are ϕ-uninformative, if goods x and z are sufficiently* far *substitutes, in equilibrium the seller sets:*

$$p_z = \begin{cases} p_A = p_z^\dagger & \text{if } i \text{ accepts} \\ p_R \geq p_{\max} & \text{if } i \text{ rejects} \end{cases} \tag{10}$$

where

$$p_z^\dagger = \tfrac{1}{2}(\phi^* v(z) + p_x + c_z - c_x).$$

Consumers in $\Phi(p_z^\dagger)$ accept the loyalty scheme and enter the superstore to buy z. Consumers outside $\Phi(p_z^\dagger)$ have the (weak) incentive to reject the scheme and enter the superstore to buy x.[21]

Following the discussion at the end of Section 4, goods x and z are relatively far substitutes when $v(z)$ is sufficiently larger than $v(x)$. In particular the condition for price (10) to apply (derived in the proof) is given by:

$$v(z) \geq v(x)\frac{(p_x + c_x - c_z)}{(p_x + c_z - c_x)}, \tag{11}$$

where the RHS is larger than $v(x)$.

To understand the condition better, notice that the superstore always has the option to sell good z to the whole market by offering either p_A or p_R equal to p_x. Intuitively, the more the goods differ to the eyes of the tempted consumers, the more attractive it becomes to exclude some consumers from the purchase of good z in order to sell at a higher price. That is, when (11) holds, the difference in willingness to pay for z between two consumers ϕ^j and ϕ^h, with $\phi^h > \phi^j$, is large enough for the superstore to be interested in price discriminating among them.

Figure 2 below shows the equilibrium p_A and the profits the superstore obtains from the sale of z when the two goods are far enough substitutes. It also represents the set of consumers who are willing to enter the superstore to buy z.

Because of their self-control and temptation, contrary to classical price discrimination models, raising the price of the good excludes consumers both at the start of the valuation distribution and at its end. As anticipated, some strongly tempted consumers, i.e. consumers beyond $\overline{\phi}(p_z^\dagger)$, reject the loyalty scheme. By doing so, they have the chance to commit themselves to face menu M_R — which features no tempting offers — once in the superstore, using the rejection of the loyalty scheme as a commitment device. In this way, the superstore is *optimally* inducing some consumers to reject the loyalty scheme. As I shall now show, however, the message beyond Proposition 2 is far more powerful than it initially seems.

Deriving welfare results in models with time-inconsistencies is generally very difficult. The hardest question to tackle is: "which among the utility levels should be given more weight? the ex-ante or the ex-post?" My results, however, are derived studying changes in price. When a price changes, the direction of the change in utility is the same ex-ante and ex-post. Hence, the question becomes mute as the change in the overall utility level has a clear sign.

Keeping the assumption on private information of consumers, suppose the superstore did not have access to the loyalty scheme technology. A key difference arises: if the

[21]Notice that the reversed case $p_A \geq p_{\max}$, $p_R = p_z^\dagger$ can also be an equilibrium, as shown in the proof. This case not only does not survive an ϵ increase in informativeness granted by the loyalty schemes, but it is also counter intuitive since in reality consumers never face a higher price when accepting the loyalty scheme. Hence, I rule out this case.

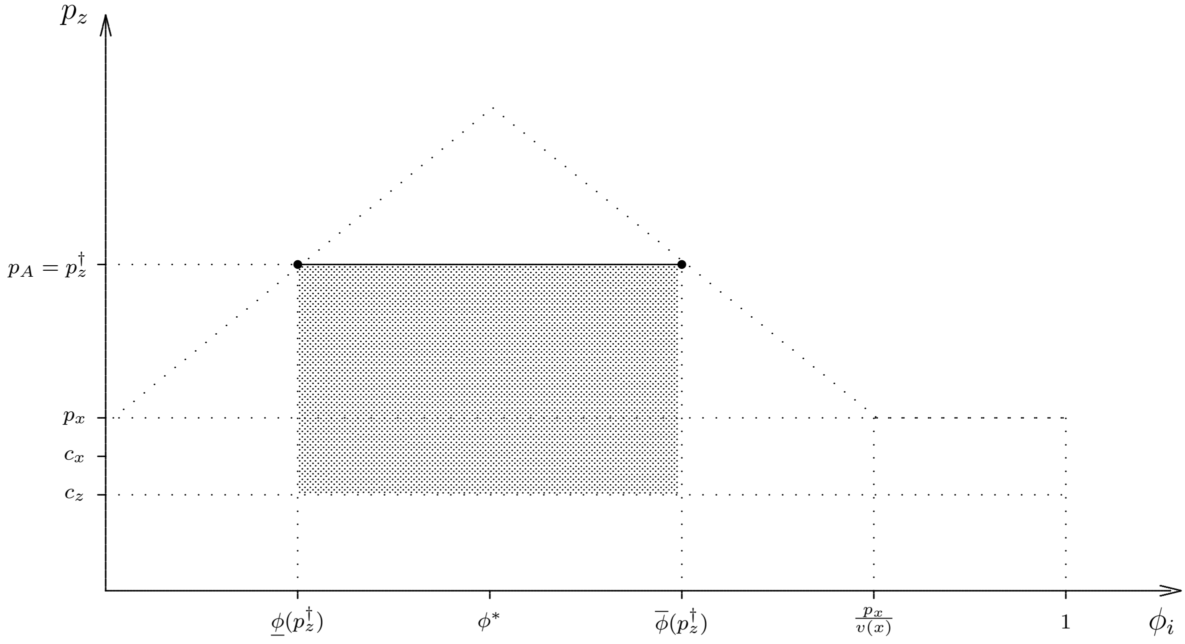

FIGURE 2. The figure shows the set of consumers that accept the loyalty scheme, $\left[\underline{\phi}(p_z^\dagger), \overline{\phi}(p_z^\dagger)\right]$, and enter the superstore to purchase z when $p_a = p_z^\dagger$ and $p_R \geq p_{\max}$. The shaded area represents the profits obtained by the superstore from selling good z.

superstore were to sell z at any price $p > p_x$, in the absence of loyalty schemes this price would be faced by the entire market. Especially, this price would be faced by strongly tempted consumers beyond $\overline{\phi}(p)$ that would be tempted by (z, p) and would therefore prefer to make their purchase of x somewhere else.[22] The lack of the commitment device aspect of the loyalty scheme generates the welfare results presented in the following Proposition.

Proposition 3. *If goods x and z are sufficiently* far *substitutes, i.e. condition* (11) *holds, when loyalty schemes are ϕ-uninformative, their use:*

i) *raises the price at which consumers buy good z in equilibrium;*
ii) *increases the superstore's profit;*
iii) *decreases aggregate consumer surplus.*

Point i) is key to the rest and it requires the most attention. First of all, for Proposition 3 to take place condition (11) must hold. This is quite straight forward: if the superstore were to set $p_A = p_x$, of course the loyalty scheme would not hurt any consumer. In the proof of the Proposition, I show how, when the superstore sets a price larger than p_x in the absence of loyalty schemes, she does so also in their presence, but the reverse is not true. This implies that under no parameter configuration the superstore sets $p_z = p_x$ in the presence of loyalty schemes and $p_z > p_x$ in their absence.

[22]I fully solve the case of private information and no loyalty schemes in the proof of Proposition 3.

In order to prove Point i) then, it is enough to show that if the principal sets price $p > p_x$ in the absence of loyalty schemes, it is always true that $p_z^\dagger > p$. The intuition behind this is that, while the benefits of increasing the price of z are the same regardless of whether loyalty schemes are available or not, the costs of doing so are different and larger when loyalty schemes are not in place.

To see this, let loyalty schemes be unavailable, and suppose the superstore wants to understand the effect on her profits from selling z at a $p > p_x$ instead of p_x. As she increases the price of z above p_x some weakly tempted consumers become uninterested in the good, but are willing to purchase x in the superstore. From these types, the superstore "loses" $\pi(z, p_z) - \pi(x, p_x)$. At the same time, some strongly tempted consumers now find it sub-optimal to enter the superstore in the first place. This is because, on the one hand, the price of z is too large for them to buy it optimally according to their self-control preferences. On the other, it is small enough to tempt them once they are in period 2. This generates a self-control cost that they can avoid by simply stepping into one of the other stores. Hence from these consumers, the superstore loses the entire $\pi(z, p_z)$.

As shown in Proposition 2, this is not the case when uninformative loyalty schemes are in place. By offering a very large price for z after rejection, the superstore ensures that strongly tempted consumers outside $\Phi(p_A)$ do not have the (strict) incentive to make their purchases somewhere else. Hence, when deciding *how much* to raise the price of z above p_x, the superstore takes into account losing only $\pi(z, p_z) - \pi(x, p_x)$ from these consumers. This decreases the concerns of raising p_A above p_x compared to the case of no loyalty schemes.

Finally, notice that the gains of increasing the price of z above p_x are instead exactly the same and equal to $\pi(z, p_z) - \pi(x, p_x)$ for all consumers in $\Phi(p_z)$. These two aspects together result in $p_z^\dagger > p$.

Given the above, points ii) and iii) are quite intuitive. Not only does the use of loyalty schemes raises the equilibrium price of z but it also decreases the number of consumers walking away from the superstore. Hence, it comes as no surprise that the superstore is better off using the schemes. On the other hand, consumers that would buy good z in the absence of loyalty schemes now face either a larger price, $p_z^\dagger$, or they end up purchasing a good they value less, i.e. good x. Hence, aggregate consumer surplus has decreased.

As I show in the next section, Proposition 3 carries over to the case of ϕ-informative schemes only under specific conditions. A social planner concerned with consumer protection and consumer welfare should, therefore, always prohibit loyalty schemes in markets where temptation and self-control are an issue if their use is purely to carry out second-degree price discrimination, without any study of consumer preferences. When loyalty schemes are ϕ-informative, some consumers with a low valuation may end up buying good z at a relatively low price in equilibrium. This creates potential for an overall increase in the aggregate consumer surplus, as I show below.

8. Informative Loyalty Schemes

So far, the loyalty scheme has worked only as a means for the superstore to offer different menus to consumers. Price discrimination has taken place but only in order to offer a commitment device to strongly tempted consumers. Let loyalty schemes now grant some information to the superstore about the temptation level of consumers. Consider a partition $\mathcal{P}_n$ of fineness $n > 1$. As opposed to $\mathcal{P}_1$, loyalty schemes are now *ϕ-informative* and the seller can observe to which element of $\mathcal{P}_n$ the ϕ_i of each accepting consumer belongs. Figure 3 shows an example for a partition $\mathcal{P}_6$ of fineness 6. The example highlights how this model has features in common with the literature on 'hard evidence', where consumers can signal their belonging to a certain subset of the type space without the possibility of lying. I review this literature in Section 9.

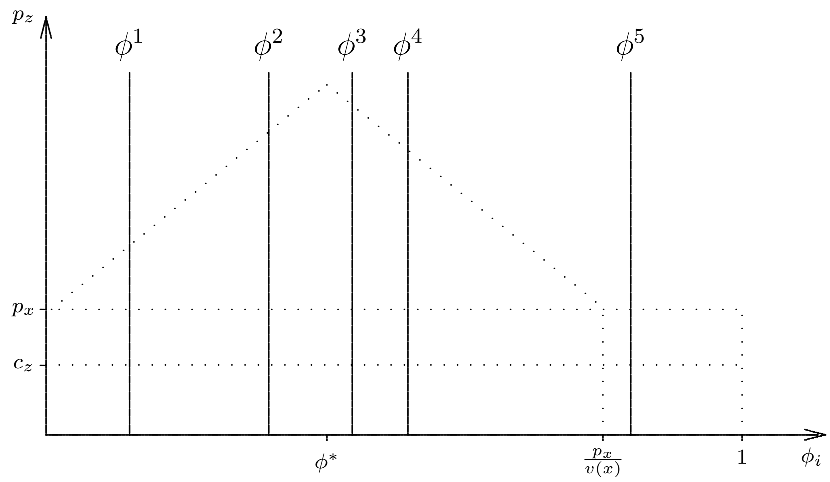

Figure 3. The graph shows an example for of $\mathcal{P}_6$. If a consumer accepts the loyalty scheme, the seller can observe if his ϕ_i is in $[0, \phi^1]$, $(\phi^1, \phi^2]$, $(\phi^2, \phi^3]$, $(\phi^3, \phi^4]$, $(\phi^4, \phi^5]$ or $(\phi^5, 1]$.

In order to define the equilibrium for all relevant refinements of $\mathcal{P}_1$, in Appendix B I derive an algorithm that identifies p_A for every level of fineness $n \in [2, \infty)$. The optimal pricing scheme set by the superstore is of the type described in (8). In equilibrium, p_A is a discontinuous function of ϕ_i and features n different prices, one for each element of the partition. An equilibrium is therefore defined by a function p_A, a price p_R and consumer behaviour.

To identify the algorithm, I divide the problem at hand into n smaller subproblems, one for each interval. The superstore sets a value for p_A for each subproblem and a value for p_R for all of them. Since consumers cannot lie about their ϕ_i, the value for p_A set in any interval is irrelevant for consumers in other intervals, while the value of p_R affects all of them.

In order to price discriminate among consumers, the superstore has the incentive to learn something about their level of temptation and, therefore, to induce them to accept the loyalty scheme. This happens under two circumstances: either $p_A > p_R$, and they are tempted by (z, p_R), but it is ex-ante suboptimal to choose this offer ex-post, or $p_R > p_A$, and they would like to choose offer (z, p_A) ex-post. Since the superstore wants *different* consumers to buy at *different* prices, I show that in equilibrium $p_R \geq p_{\max} > p_A$ and only consumers that accept the loyalty scheme buy z.

While the full set of rules defining the algorithm is left for the most curious readers to discover in the appendix, in order to better present the welfare results in Section 8.1, I now present a numerical example and describe the basic features of the algorithm.

Numerical Example. Let $p_x = 2$, $c_x = 1$, $c_z = \frac{1}{2}$, $v(z) = 21$ and $v(x) = 3$ and consider an exogenous partition of the following type:

$$\tilde{\mathcal{P}}_n = \left\{[0, 0.01), [0.01, 0.12), [0.12, 0.25), [0.25, 0.45), [0.45, 0.55), \{j\}_{j\in[0.55,0.75)}, [0.75, 1]\right\}.$$

Such a partition is composed of uncountably many elements. If a consumer in

$$[0, 0.01), [0.01, 0.12), [0.12, 0.25), [0.25, 0.45), [0.45, 0.55) \text{ or } [0.75, 1]$$

accepts the loyalty scheme, the superstore only learns that his type is distributed uniformly in the element he belongs to. If, instead, a consumer in $[0.55, 0.75)$ accepts the loyalty scheme, then the superstore perfectly observes his type.

For the values set above, $\phi^* = 0.16$ and in the absence of loyalty schemes the superstore sets $p'_z = \frac{25}{12}$, which yields $\Phi(p'_z) = [0.01, 0.64]$. Given the rules derived in the appendix, this framework generates Figure 4 where the red line represents price function p_A. The function is plotted only for consumers that accept the loyalty scheme, enter the superstore and buy good z.

The figure shows the optimal price set for each element. The smaller are the elements of the partition, the more the equilibrium function p_A resembles the price of first-best. In particular, it is easy to see how, in the interval of the type space partitioned in singleton elements, the full information equilibrium is perfectly restored. This leads to the first result of this section.

Proposition 4. *When all consumers that differ in their ex-ante willingness to pay are able to perfectly disclose their type by accepting the loyalty scheme the full information outcome takes place.*

When the superstore perfectly observes the temptation level of accepting consumers she can offer them the full information price $p^*_z(\phi_i)$. Consumer i has the weak incentive to enter the store, accepting the loyalty scheme, in order to purchase good z. Notice that consumers beyond $\frac{p_x}{v(x)}$ do not differ in their willingness to pay. Hence, in order to replicate the full information equilibrium it is enough for the superstore to be able

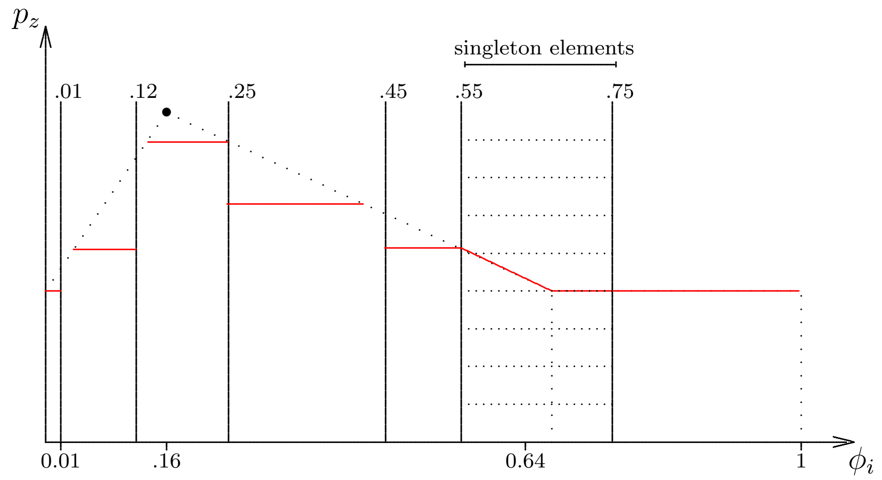

FIGURE 4. In the figure the equilibrium for the special case of $\tilde{\mathcal{P}}_n$ is shown. The partition features uncountably many singleton elements in the interval $[0.55, 0.75)$. Price p_A is described by the, discontinuous, function represented by the red line. The function is plotted only for consumers that accept the loyalty scheme, enter the superstore and buy good z.

to tell these types apart from consumers that do differ in their willingness to pay, i.e. those with $\phi_i < \frac{p_x}{v(x)}$.

I define loyalty scheme structures that satisfy the conditions of Proposition 4 as *fully informative schemes*.

Consider now all the other intervals in Figure 4 that are non-singletons. In each of these intervals only two things can happen. The superstore finds it optimal to set p_A either such that all consumers in an interval purchase the good or such that it induces only a subset of them to do so. The actual rules governing this decision, however, are more complicated than it seems. The optimal pricing rule for each interval depends on two things: the proportion of weakly vs. strongly tempted consumers in the interval and the willingness to pay of the consumers at the boundary of it.

The first and second element of $\tilde{\mathcal{P}}_n$ are composed of only weakly tempted consumers. By raising p_A in any of these intervals, the superstore is only loosing consumers that do not value the good enough to make the purchase. In other words, the only boundary that binds from Lemma 1 is the left one $\underline{\phi}(p_A)$.

Similarly, the fourth, fifth and last elements are composed of only strongly tempted consumers. By raising p_A in any of these intervals, the superstore is only losing consumers that are so tempted by z to find it suboptimal to purchase it ex-ante. In other words, the only boundary that binds from Lemma 1 is the right one $\overline{\phi}(p_A)$.

For elements like the third, instead, the composition of the interval is not enough to determine the binding boundary of $\Phi(p_A)$. Looking closely, it is easy to see that the interval is composed mostly of strongly tempted consumers. Nevertheless, the boundary that binds from Lemma 1 is actually the left one. This follows from the relative slope of the willingness to pay of weakly vs. strongly tempted consumers. In this specific case, the left limit consumer has a lower willingness to pay from the right limit consumer even though he is "closer" to the peak. In other words, what matters for intervals like the third one in Figure 4 is the elasticity of demand of weakly tempted consumers relative to that of strongly tempted consumers.

8.1. **Welfare Implications of Uninformative Loyalty Schemes.** In this section I present the results of Proposition 3 that carry over to the ϕ-informative loyalty schemes case, and under what conditions. Differently from the case of ϕ-uninformative schemes however, it is now impossible to make statements about welfare effects that hold generally. Hence, my approach is to identify those structure of loyalty schemes that are certain to hurt consumers.

To see why general statements are impossible to make, suppose that the superstore offers good z a price $p > p_x$ when loyalty schemes are not available. Consider a partition like the one in Figure 3, replicated below, and suppose that price p_5, the one set for the fifth element of the partition $[\phi^4, \phi^5]$, takes a value between p_x and p.

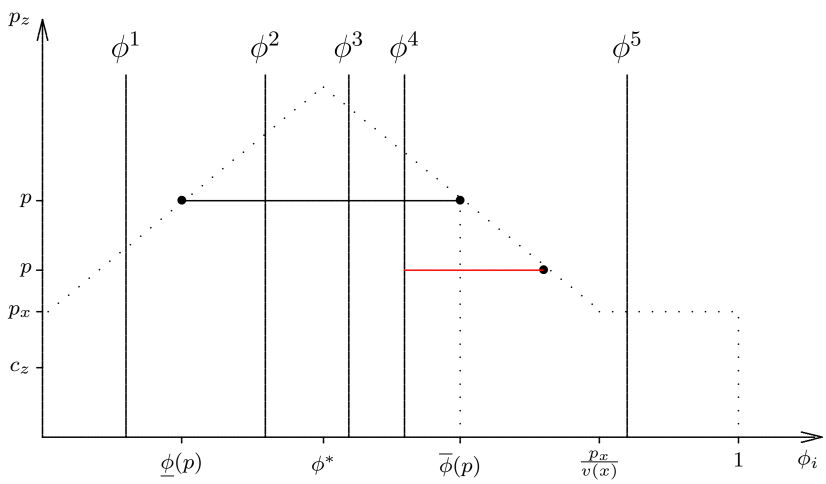

FIGURE 5. The example of Figure 3 is presented again. The figure presented here shows how the price offered to some consumers, namely $\left[\phi^a, \overline{\phi}(p)\right]$, for good z may decrease after the introduction of ϕ-informative schemes.

In the case of Figure 5, all consumers in $[\phi^4, \overline{\phi}(p)]$ face $p_A < p$ and are made *better off* by the introduction of loyalty schemes. Hence, a general statement over the welfare effects of loyalty schemes has to take this into account. I start from the case of fully informative schemes.

Proposition 5. *If goods x and z are sufficiently* close *substitutes, the introduction of fully informative schemes*

i) *raises the price at which consumers buy good z in equilibrium;*

ii) *increases the superstore's profit;*

iii) *decreases aggregate consumer surplus.*

First of all, while Proposition 5 may look the same of Proposition 3 with an additional requirement, it is not. The result now requires z and x to be sufficiently *close* substitutes instead of sufficiently *far* ones.

Intuitively, the use of loyalty schemes always hurts consumers whenever goods are so closely substitutable that the superstore would not find it optimal to discriminate were loyalty schemes unavailable. When this is the case, granting her more information about consumers allows her to discriminate among some of them, increasing the incentives to raise the price for a subset of these consumers. This ensures the superstore more profits at the expense of consumer surplus. However, this is also true for ϕ-uninformative loyalty schemes, so where does the difference come from? It lies in the information granted by the schemes and in the amount of consumers purchasing good z.

Introducing a ϕ-uninformative loyalty scheme structure never increases the amount of consumers purchasing the good. Fully informative schemes do. In fact, they allow the superstore to serve the whole market, charging each consumer their period 1 willingness to pay. Needless to say, this worsens the situation of consumers that buy good z in the absence of loyalty schemes. But now, suppose that without loyalty schemes there are some consumers who do not buy good z. When fully informative schemes are introduced they switch to purchasing good z. This yields them the same period 1 utility as they would obtain from purchasing x — this follows from the IC binding in (5). Their period 2 utility however must have increased because they are purchasing the most valuable product in the market.

Given the above, a general statement about fully informative schemes can only be made when the superstore serves the whole market in the absence of loyalty schemes. This happens when goods x and z are sufficiently *close* substitutes. That is, when $v(z)$ is sufficiently close to $v(x)$. From the proof of Proposition 2, we know that this happens when the following condition holds:

$$v(z) < v(x)\frac{(2c_x + c_z)}{c_z}. \tag{12}$$

Let me now relax the assumption of fully informative scheme to study the welfare implication of more complicated partition structures. Following the discussion of Figure 5, I will assume the case of partitions that do not include consumers that would buy the good in the absence of loyalty schemes. Formally, this implies that the set $\Phi(p)$ of consumers purchasing z at price p in the absence of loyalty schemes is a *proper* subset of an element of $\mathcal{P}_n$.[23]

Proposition 6. *Let loyalty schemes be ϕ-informative towards consumers who differ in ex-ante willingness to pay, but not towards those who buy good z in the absence of the*

[23]Subset A of a set B is called proper, or strict, if A is contained in B but it is not equal to B.

schemes. When goods z and x are sufficiently close *substitutes, i.e. when* (12) *holds, loyalty schemes:*

i) *increase the price at which some consumers buy good z;*
ii) *do not decrease the price of z for any consumer;*
iii) *increase the superstore's profit;*
iv) *decrease aggregate consumer surplus.*

The conditions for Proposition 6 are a combination of those already observed. The first requirement is for loyalty schemes to be ϕ-informative towards consumers who differ in ex-ante willingness to pay. As explained for Proposition 4, this requires that the partition cannot contain an element which is the superset of $[0, p_x/v(x)]$. To see why this is needed, notice that if this were not the case, then $p_z^\dagger > p_x$ would not necessarily hold and loyalty schemes could potentially bring no changes at all. In other words, this requirement rules out welfare-irrelevant loyalty scheme structures.

Second, condition (12) is key to this Proposition as well. The idea is very simple. Suppose the seller offers p for z when loyalty schemes are unavailable. Consider the simple partition $\mathcal{P}_3 = \{[0, \underline{\phi}(p)), [\underline{\phi}(p)), \overline{\phi}(p)], (\overline{\phi}(p), 1]\}$. Since the second element contains $\Phi(p_z^\dagger) \subsetneq \Phi(p)$, $p_2 = p_z^\dagger$. As for the first and third element, the discussion above for the numerical example applies. Surely p_1 and p_3 belong to $[p_x, p_z^\dagger)$ and some consumers in the first and third element now purchase z. Considering that some of them would buy good x in the absence of the loyalty schemes, and have now switched to purchasing z, their surplus must have increased.

Suppose now, instead, that the substitutability between the two goods is sufficiently low to induce the superstore to charge price $p \in (p_x, p_z^\dagger)$. Whenever loyalty schemes allow the superstore to tell apart a group of consumers outside $\Phi(p)$, their welfare effect is never unidirectional. Generally three effects take place.

Proposition 7. *If goods x and z are sufficiently* far *substitutes, i.e.* (12) *fails, the introduction of ϕ-informative loyalty schemes has three different effects on prices.*

1) *Consumers that buy good x in the absence of loyalty schemes, face a non-negative change in welfare.*
2) *Consumers that buy good z in the absence of loyalty schemes and switch to x when the latter are introduced, face a non-positive change in welfare.*
3) *Consumers that buy good z in both cases, face a change in welfare inversely proportional to the change in the price they pay for z.*

When informative loyalty schemes are introduced, if a consumer switches from one good to the other two things may have happened. First, he may find z more attractive than before because its price is now lower, as discussed in Figure 5. Second, he may find z less attractive than before because its price is now higher. In the first case he switches from x to z and his surplus cannot decrease since x is his outside option. In the second case he switches from z to x. Good x, however, was available when he choose z in the absence of loyalty schemes, and at the same price p_x. Hence, it must be that he is worse off when loyalty schemes are introduced, since he is choosing a previously dominated alternative.

If, on the other hand, a consumer does not change his purchase decision after the introduction of loyalty schemes he is still buying either x or z. In the first case, he faces no change in surplus since p_x is independent of the loyalty scheme. In the second case, depending on the structure of the partition, p_A can be both larger or smaller than the price of z in the absence of loyalty schemes. The change in price creates an opposite effect on the consumer's utility both in period 1 and period 2.

Given the above, the overall welfare effect of loyalty schemes depends crucially on the position and size of each element. While Proposition 5 and 6 identifies a set of consumer surplus-hurting partitions, given a more general partition, the positive effects at number 1) and 3) of Proposition 7 need to be compared to the negative effects of points 2) and 3). The sign of the resulting overall change defines the welfare effect of the use of loyalty schemes.

Even though the overall effect on consumer welfare of a general partition remains ambiguous, an important final result can be derived from the model.

Proposition 8. *There exists no partition such that the loyalty scheme technology Pareto Improves over a framework of private information with no schemes.*

This impossibility result contrasts with the general results on price discrimination with 'hard evidence' (Bergemann, Brooks, and Morris, 2015; Pram, 2017). The intuition behind it is that when self-control problems are disregarded, the commitment device function of the loyalty scheme is completely lost. No consumer is affected by the price of a good he does not buy. In fact, if the price for z in the absence of loyalty schemes is p, a ϕ-informative or uninfromative scheme that does not separate consumers in $\Phi(p)$ into different groups/elements would have no effect on price p. Hence, it would not affect and create no negative effects on the aggregate surplus of consumers in $\Phi(p)$. The superstore would have no incentive to introduce a ϕ-uninformative scheme at all, while such schemes are very valuable to her in the presence of self-control problems.

9. Related Literature

My paper contributes to three different branches of literature. First of all, is the very vast literature that studies loyalty schemes, from Marketing and Retailing to Industrial Organisation, both from a theoretical and from an empirical point of view.

Empirically, studies have focused on the effect that loyalty schemes have on consumer lifetime value, loyalty enhancement, duration and retention (among others, Byrom, 2001; Lewis, 2004; Meyer-Waarden, 2007; Gomez, Arranz, and Cillàn, 2006).

Theoretical studies on loyalty schemes contribute to the Competition Policy and Industrial Organisation literatures. To my knowledge, however, the literature lacks a study that models loyalty schemes as a tool for price discrimination. They have been studied as business-stealing tools (Caminal and Claici, 2007), as bundled loyalty discounts (Greenlee, Reitman, and Sibley, 2008), as "bribes" to agents that buy products with the principal's money (Basso, Clements, and Ross, 2009), as collusion tools (Ackermann, 2010) and in dynamic environments as tools to increase consumer participation and expenditure (Chen and Pearcy, 2010; Caminal, 2012). Ackermann (2010)'s loyalty card schemes offer discounts similar to those analysed in this paper. His findings focus on the competition aspect of loyalty cards. Caminal (2012) deals with a model closer to the one described in this paper, but focuses on the aspect of consumer preference dependance across two periods. The intuition and idea of loyalty schemes is fundamentally different from the one described in this paper. There, they are rewards and tools used to *continue* the purchasing relationship. Here, they are a means of *exchange* that the superstore uses to acquire information about consumer level of temptation.

In my model I use self-control preferences of the Gul and Pesendorfer (2001) type. These preferences have been shown to, among other things, violate the classical 'efficiency at the top' results of screening models (Esteban and Miyagawa, 2006; Esteban, Miyagawa, and Schum, 2007) and allow for the achievement of full information first best contracts, even in situations where consumers hold private information (Esteban and Miyagawa, 2005). I have applied these preferences in Foschi (2016) to online betting, gambling and the gaming industries to study the welfare effect of practices like the offer of entry-vouchers with no strings attached. Besides the application, the main difference between these papers and the present one is the assumption that the seller has no control over the quantity of good offered to each consumer. Each consumer is interested in the purchase of one unit and one unit only.

Finally, as mentioned already, my paper enters the literature on price discrimination in the presence of hard evidence. In particular, I show how one of the most recent welfare results that identifies evidence structures that Pareto Improve over a framework of private information (Bergemann, Brooks, and Morris, 2015; Pram, 2017) fails in the presence of consumers who suffer from self control problems.

10. Conclusions

I suggest a novel point of view on loyalty schemes used by retailers in markets where temptation and self-control are an issue. I have shown how, contrary to general opinion, loyalty schemes hurt aggregate consumer surplus whenever they are *not* used by the seller to study consumer preferences. This follows from two main factors.

First, if the retailers' prior on consumer preferences stays the same, regardless of whether they register with the scheme or not, she cannot price discriminate. Therefore, the potential social benefit of selling her product to consumers with lower valuation at a lower price than the market one is lost.

Second, overly tempted consumers who want to avoid impulse purchasing can now reject the loyalty scheme, face a high price for tempting goods and save their self-control cost. They are therefore happy to carry out the rest of their shopping from the retailer. This decreases the retailer concerns of increasing the price of the tempting good at which consumers engage in impulse purchasing. Now the seller is not afraid to lose consumers from the high end of the type space any longer. Hence, the equilibrium price at which consumers purchase the tempting good is higher under loyalty schemes compared to when the schemes are unavailable.

This opens up the question on the actual degree of observability on consumer preferences in industries like the ones considered in this paper. How much information do retailers observe when consumers register with the loyalty scheme? How do they divide their customer base? Empirical answers to these questions, combined with the theory above, would allow us to better understand in which markets consumers are endangered by the presence of loyalty schemes.

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APPENDIX A. ASSUMPTION 1 AND INCENTIVES TO ENTER

Assumption 1 implies that $p_x \leq \frac{u(x)+\phi_i v(x)}{2}$ for all ϕ_i. Hence, it implies that $p_x \leq \frac{u(x)}{2}$.

The condition for a consumer to enter in one of the stores that serve only x can be written as:

$$\begin{aligned} &W_i(M_s) \geq 0 \\ &u(x) + \phi_i v(x) - 2p_x - \max\{\phi_i v(x) - p_x, 0\} \geq 0 \end{aligned} \tag{13}$$

When $\underline{0} \succ_T (x, p_x)$, $\max\{\phi_i v(x) - p_x, 0\} = 0$. The above expression than boils down to $u(x) + \phi_i v(x) - 2p_x \geq 0$ that is solved by any $p_x \leq \frac{u(x)+v(x)}{2} > \frac{u(x)}{2}$. When $(x, p_x) \succ_T \underline{0}$, $\max\{\phi_i v(x) - p_x, 0\} = \phi_i v(x) - p_x$. The above becomes $u(x) - p_x \geq 0$, which is also solved by any $p_x \leq \frac{u(x)}{2}$.

Hence, Assumption 1 implies that each consumer has also the incentive to enter one of the stores that serves only x as opposed to enter no store. That is they prefer menu M_s to a menu composed of only the null offer.

Consider dropping Assumption 1. In other words, suppose there exist some types who have no interest in purchasing good x. These are consumers such that

$$\phi_i \in \left[0, \frac{2p_x - u(x)}{v(x)}\right).$$

First of all, notice that this does not imply that there exist no types tempted from offer (x, p_x). From Proposition 1 we know that consumers with $\phi_i > \frac{p_x}{v(x)}$ have preferences such that $(x, p_x) \succ_T \underline{0}$. It is easy to show that

$$\frac{p_x}{v(x)} < \frac{2p_x - u(x)}{v(x)} \quad \Rightarrow \quad p_x > u(x).$$

As discussed in the main body of the paper, $p_x > u(x)$ implies $U(x, p_x) < 0$ a case that falls outside the applications considered in this paper. Therefore I rule it out and only consider $p_x \in \left[\frac{u(x)}{2}, u(x)\right]$ as a potential deviation from Assumption 1. Consumers who are not interested in purchasing x are not tempted by it either.

Consider the case of full information and the problem faced by the seller for consumers in $\left[0, \frac{2p_x - u(x)}{v(x)}\right)$:

$$\begin{aligned} &\max_{p_z} [p_z - c_z] \\ \text{s.t.} \quad &W_i(M_m(p_z)) \geq 0 \\ &U(z, p_z) + V_i(z, p_z) \geq 0. \end{aligned}$$

The solution to the above is given by $\tilde{p}_z = \frac{u(z)+\phi_i v(z)}{2}$. It remains to check whether selling at this price yields profits larger than $p_x - c_x$. Otherwise the superstore would not sell good z in the first place. This happens when

$$\frac{u(z)+\phi_i v(z)}{2} > p_x + c_z - c_x \quad \Rightarrow \quad \phi_i \geq \tilde{\phi} \equiv \frac{2(p_x + c_z - c_x) - u}{v(z)}.$$

Consumers with $\phi_i < \tilde{\phi}$ are irrelevant to the superstore and the market in general and can therefore be disregarded entirely. To see why this is enough to show that my results would hold in this case as well consider the following.

The key underlying requirements for Propositions 4 to 8 to hold are two: the non-monotonic, bell shaped full information price and the commitment device function of loyalty schemes. For the second to be true, it is enough to show that there exist consumers tempted by the full information price charged to those with a lower ϕ_i — and therefore potentially tempted by the price set by the superstore under asymmetric information — to the point of not entering the superstore. That is, the existence of a downward sloping portion of the full information price curve.

In order to see that all the above holds for the case of $p_x \in \left[\frac{u(x)}{2}, u(x)\right]$ notice that (6) is charged to consumers beyond $\tilde{\phi}$. Since the slope of $\tilde{p}_z$ is positive, and (6) is bell shaped, then regardless of whether ϕ^* is smaller equal or larger than $\tilde{\phi}$, the bell shape is preserved and the full information price curve features a downward sloping portion.

When $p_x > u(x)$, it is possible to identify parameter conditions under which the full information price function approximates a u-shaped function.

Appendix B. Optimal Pricing Algorithm for ϕ-informative Schemes

I start my derivations by explaining why, as mentioned in Section 8, in equilibrium $p_R \geq p_{\max} > p_A$ and only consumers that accept the loyalty scheme buy z.

To see this, consider the case of consumer ϕ_a belonging to element Φ^a of a general partition $\mathcal{P}_n$. Suppose this consumer buys at p_R, after rejecting the scheme. Notice that (z, p_R) is tempting for all types $\phi_i > \phi_a$. Take, now, consumer $\phi_b > \phi_a$ belonging to element b. If he does not buy z at the p_A set for element b, but is tempted by (z, p_A), he would like to reject the scheme and face a high p_R, in order to enter the superstore and buy x free of self-control cost. Since, however, p_R is such that he is tempted also by (z, p_R) he has the incentive to enter one of the smaller stores. The superstore, therefore, "loses" type ϕ_b.

Alternatively, it may be that p_R is so low that ϕ_b is actually willing to reject the loyalty scheme in order to enter the superstore and choose (z, p_R). If this were to happen, however, the superstore would be selling z at the same price to consumers belonging to different elements of $\mathcal{P}_n$, losing the opportunity to price discriminate between them.

Setting $p_R \geq p_{\max}$ solves both the issues above and does not constrain the superstore to a suboptimal situation. Consumers in any interval that buy good z accept the loyalty scheme while consumers that do not buy z, but are tempted by (z, p_A), can reject the scheme to save their self-control cost.

Given this, I now discuss the equilibrium for a general interval of a general $\mathcal{P}_n$. For now, I rule out the possibility of the interval to be a singleton. I provide the rule for singleton elements in Lemma 7.

Consider element $\left[\phi^l, \phi^r\right]$ of $\mathcal{P}_n$ where ϕ^l, for "left", and ϕ^r, for "right", represent its endpoints. For each of these elements, the seller faces a problem analogous to (34) but constrained to $\left[\phi^l, \phi^r\right]$. Hence, she first identifies the subinterval of types that enter the store for a given p_A, $\Phi(p_A)$, and then sets the optimal p_A accordingly.

Consumers too face the same decision of the case of ϕ-uninformative loyalty schemes. If they reject the loyalty scheme, the incentive to enter any store in the market is the same since the presence of $(z, p_{\max})$ does not affect their ex-ante utility, from the definition of $p_{\max}$. If they accept and enter the superstore, instead, they face menu $M_m(p_A) = \{(x, p_x), (z, p_A), 0\}$. Their decision depends on whether

$$W_i(\{(x, p_x), (z, p_A), 0\}) \geq W_i(\{(x, p_x), 0\})$$

or not.

Consider $\phi_i \in \Phi(p_A)$. Since the superstore sets p_A to sell z to these consumers only, for every $\phi_i \in \Phi(p_A)$, it must be that $W_i(\{(x, p_x), (z, p_A), 0\}) \geq W_i(\{(x, p_x), 0\})$. The latter holds with strict inequality for consumers in the interior of $\Phi(p_A)$. They have a strong incentive to accept the scheme and enter the superstore to buy z. Notice that these types always exist, since profits from (z, p_A) are zero for any p_A such that $\Phi(p_A)$ is a singleton.

Consumers in $\left[\phi^l, \phi^r\right] \setminus \Phi(p_A)$, instead, have a lower willingness to pay and are of "no interest" for the superstore. Their equilibrium decision changes depending on whether they are strongly or weakly tempted. To understand why, consider Figure 6 below.

Take the case of $\left[\phi^l, \phi^r\right]$ composed of weakly tempted consumers in the figure. The willingness to pay for z is increasing in ϕ_i in the interval considered. If the superstore wants to attract a subset of these consumers into the store, she will set a price p_A in the range $\left[p_z^*(\phi^l), p_z^*(\phi^r)\right]$. From the willingness to pay curve, the least tempted consumer to enter the store accepting the loyalty scheme is the ϕ_i such that $p_z^*(\phi_i) = p_A$. All consumers to his right accept the loyalty scheme and buy z. All consumers to his left are indifferent between accepting or rejecting the loyalty scheme and have a (weak) incentive to enter the retailer's superstore — exactly as in Proposition 2. Since the seller has no information inside the interval to discriminate between these consumers and those who buy z at p_A, she can only screen them out of the market for z.

Consider now the case of interval $\left[\phi^{l'}, \phi^{r'}\right]$ of strongly tempted consumers. The same logic explained above applies. However this time, given a price p_A, the *most* tempted

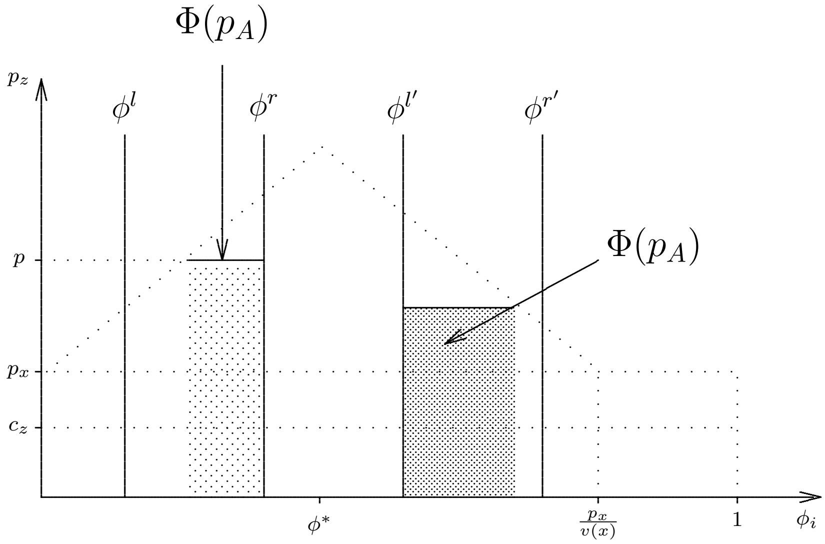

FIGURE 6. Two examples of behaviour of consumers of a general element of $\mathcal{P}_n$. When the interval is composed of only weakly tempted consumers, e.g. $[\phi^l, \phi^r]$, only those below the light shaded area accept the loyalty scheme and enter the superstore. Consumers to the left of the area are indifferent between accepting and rejecting the loyalty scheme. When the interval is composed only of strongly tempted consumers, e.g. $[\phi^{l'}, \phi^{r'}]$, only those below the dark shaded area accept the loyalty scheme and enter the superstore. Consumers to the right of the area strictly prefer to reject the loyal scheme.

consumer to accept the scheme is the ϕ_i such that $p_z^*(\phi_i) = p_A$. All consumers to his left accept the scheme and buy z. All consumers to his right, instead, are tempted by (z, p_A). Hence, as in the trivial partition case, they use the rejection of the loyalty scheme as a commitment device, not to be tempted ex-post.

To summarise, the solutions to each of the n subproblems of intervals $\left[\phi^l, \phi^r\right]$ have a common structure. The seller sets $p_R \geq p_{\max}$ and p_A according to set $\Phi(p_A) \subseteq \left[\phi^l, \phi^r\right]$. Consumers in $\Phi(p_A)$, indeed, accept the loyalty scheme and enter the superstore to buy z. Consumers in $\left[\phi^l, \phi^r\right] \setminus \Phi(p_A)$ behave as explained above. The combination of all the n solutions describes the equilibrium of the game for a general partition $\mathcal{P}_n$. The next set of Lemmas provides an algorithm that identifies function p_A. Define $p_1, p_2, ..., p_n$ where p_1 is the value for p_A in the first element of $\mathcal{P}_n$, $[0, \phi^1)$, p_2 the one for the second, $[\phi^1, \phi^2)$, and so on. The algorithm changes depending on the position and size of the n elements of $\mathcal{P}_n$.

Consider a general element $\Phi_i = \left[\phi^l, \phi^r\right]$, where $\phi^l, \phi^r \in [0, 1]$. First of all, if $\Phi(p_z^\dagger) \subseteq \Phi_i$, the seller sets $p_z = p_z^\dagger$ as in the case of ϕ-uninformative schemes.

Lemma 2. *If an element of the partition is composed of at least all consumers that buy z in the case of ϕ-uninformative loyalty schemes, then the price of acceptance is set to $p_A = p_z^\dagger$.*

Proof. Simply notice that:

$$
\begin{aligned}
& arg \max_{p_z} \int_{\Phi(p_z)} \pi_z d\phi + \int_{\phi^l}^{\underline{\phi}(p_z)} \pi_x d\phi + \int_{\overline{\phi}(p_z)}^{\phi^r} \pi_x d\phi \\
&= arg \max_{p_z} \left(\underline{\phi}(p_z) - \phi^l\right)(p_x - c_x) + \left(\overline{\phi}(p_z) - \underline{\phi}(p_z)\right)(p_z - c_z) + \left(\phi^r - \overline{\phi}(p_z)\right)(p_x - c_x) \\
&= arg \max_{p_z} \left(\overline{\phi}(p_z) - \underline{\phi}(p_z)\right)(p_z - c_z - p_x + c_x) \quad \text{if} \quad [\phi^l, \phi^r] \supseteq \Phi(p_z^\dagger)
\end{aligned}
$$

Hence, as long as $[\phi^l, \phi^r] \supseteq \Phi(p_z^\dagger)$, the equilibrium price for p_z is $p_z^\dagger$. □

The intuition behind this is that consumers outside $\Phi(p_z^\dagger)$ are excluded from the market for z when the price is set for the entire $[0, 1]$ interval. Hence, restricting the set of consumers to Φ_i with $\Phi(p_z^\dagger) \subseteq \Phi_i \subseteq [0, 1]$, adds no relevant information to the seller's optimization.

The rest of the algorithm describes the rules for the case of $\Phi(p_z^\dagger) \nsubseteq \Phi_i$. For simplicity, I being by studying the first, p_1, and the last, p_n, prices of the algorithm and then move to all the ones in the middle. I start with Lemma 3, that describes the value of p_1.

Lemma 3. *If $\Phi(p_z^\dagger) \nsubseteq \Phi_i$, the value of p_1 depends on the composition of the first element of $\mathcal{P}_n$, $\Phi_1 = [0, \phi^1)$.*
When element Φ_1 is composed of only weakly tempted consumers, i.e. $\phi^1 < \phi^*$, the value of p_1 is given by:

$$
p_1 = \max\left\{p_x, \arg\max_{p_z} \int_{\underline{\phi}(p_z)}^{\phi^1} \pi_z d\phi + \int_0^{\underline{\phi}(p_z)} \pi_x d\phi\right\}, \tag{14}
$$

When element Φ_1 is composed of both weakly and strongly tempted consumers, i.e. $\phi^1 \in [\phi^*, \overline{\phi}(p_z^\dagger))$, the value of p_1 is given by:

$$
p_1 = \max\left\{p_x, \min\left\{p_z^*(\phi^1), \arg\max_{p_z} \int_{\underline{\phi}(p_z)}^{\phi^1} \pi_z d\phi + \int_0^{\underline{\phi}(p_z)} \pi_x d\phi\right\}\right\}, \tag{15}
$$

Proof. Consider the maximisation problem in (14). Element Φ_1 is composed of only weakly tempted consumers. Hence, the seller knows that by setting a price level p_1 she excludes from the market for z only consumers that do not value the good enough to pay p_1, i.e., $[0, \underline{\phi}(p_1))$. The latter, however, are still willing to enter the superstore to buy x. Of course this maximization ignores the cutoff $\underline{\phi}(p_1) \geq 0$. Hence, if the solution is smaller than p_x, the seller sets $p_1 = p_x$.

When Φ_1 is also composed of some strongly tempted consumers, the maximisation problem is ignoring the portion of negatively sloped willingness to pay that goes from ϕ^* to ϕ^1. This is the reason for the extra cutoff at $p_z^*(\phi^1)$. □

Before moving to the more complicated derivation of $p_2, ..., p_{n-1}$, I identify p_n, the price the superstore charges to consumers that prove to be in the last element of $\mathcal{P}_n$, $\Phi_n = [\phi^{n-1}, 1]$.

Lemma 4. *If $\Phi(p_z^\dagger) \nsubseteq \Phi_i$, the value of p_n depends on the composition of the last element of $\mathcal{P}_n$, $\Phi_n = [\phi^{n-1}, 1]$.*
When element Φ_n is composed of only strongly tempted consumers, i.e. $\phi^{n-1} > \phi^*$, the value of p_n is given by:

$$\Rightarrow p_n = \max \left\{ p_x, \arg\max_{p_z} \int_{\phi^{n-1}}^{\overline{\phi}(p_z)} \pi_z d\phi + \int_{\overline{\phi}(p_z)}^{1} \pi_x d\phi \right\}, \tag{16}$$

When element Φ_n is composed of both weakly and strongly tempted consumers, i.e. $\phi^{n-1} \in [\underline{\phi}(p_z^\dagger), \phi^*)$, the value of p_1 is given by:

$$\Rightarrow p_n = \max \left\{ p_x, \min \left\{ p_z^*(\phi^{n-1}), \arg\max_{p_z} \int_{\phi^{n-1}}^{\overline{\phi}(p_z)} \pi_z d\phi + \int_{\overline{\phi}(p_z)}^{1} \pi_x d\phi \right\} \right\}, \tag{17}$$

The intuition and proof for Lemma 4 are analogous to the ones described for Lemma 3 and therefore omitted.

I now identify $p_2, ..., p_{n-1}$. I look at the equilibrium p_i for the general element $\Phi_i = \left[\phi^l, \phi^r\right]$ of $\mathcal{P}_n$, where this time $\phi^l, \phi^r \in (0,1)$. The value of p_i is affected, in particular, by the composition of Φ_i, i.e. on what is the proportion of strongly vs weakly tempted consumers in the interval. Lemma 5 describes the case of Φ_i to be composed of only weakly or strongly tempted consumers, while in Lemma 6 both types of tempted consumers can be part of Φ_i.

Lemma 5. *When element Φ_i is composed only of weakly tempted consumers (i.e. $\phi^r < \phi^*$), the value of p_i is given by:*

$$p_i = \max \left\{ p_z^*(\phi^l), arg \max_{p_z} \int_{\underline{\phi}(p_z)}^{\phi^r} \pi_z d\phi + \int_{\phi^l}^{\underline{\phi}(p_z)} \pi_x d\phi \right\} \tag{18}$$

When element Φ_i is composed only of strongly tempted consumers (i.e. $\phi^l > \phi^*$), the value of p_i is given by:

$$p_i = \max \left\{ p_z^*(\phi^r), arg \max_{p_z} \int_{\phi^l}^{\overline{\phi}(p_z)} \pi_z d\phi + \int_{\overline{\phi}(p_z)}^{\phi^r} \pi_x d\phi \right\} \tag{19}$$

Proof. The proof follows the one for the previous Lemmas. Notice that p_x plays no role in the maximisation unless $\phi^r > \frac{p_x}{v(x)}$. While $p_n = p_x$ always when $\frac{p_x}{v(x)}$. □

Notice that for the case of only weakly tempted consumers, p_x is never the price of equilibrium for z in the interval, since all consumers in it have a willingness to pay strictly larger than p_x. Hence, the lower bound for the optimal price is not p_x anymore but rather $p_z^*(\phi^l)$.

In the case of an interval with only strongly tempted consumers, instead, p_i can indeed be equal to p_x. Setting $p_i = p_x$ may become optimal if there are consumers in Φ_i that, ex-ante, value good z as good x (i.e. $\phi^r > \frac{p_x}{v(x)}$).

Lemma 6 analyses the case of an interval that is not the superset of $\Phi(p_z^\dagger)$, but it is composed of both weakly and strongly tempted consumers, i.e. $\phi^* \in \Phi_i$. Notice that in this case knowing the proportion of strongly vs. weakly tempted consumers in the interval is not enough to identify the maximisation problem that provides the optimal price. To see why, let the decreasing portion of the willingness to pay curve be flatter than the increasing portion. Assume the case of an interval composed mostly of strongly tempted consumers, i.e., $(\phi^r - \phi^*) \geq (\phi^* - \phi^l)$. If the decreasing portion of the curve is flat enough, the willingness to pay of ϕ^r is larger than the one of ϕ^l. Hence the proportion of weakly vs strongly tempted consumers is not enough to identify the optimal rule for p_A.

Lemma 6. *Let $\Phi(p_z^\dagger) \nsubseteq \Phi_i$ where element Φ_i is composed of both strongly and weakly tempted consumers (i.e. $\phi^* \in \Phi_i$). If the least tempted consumer in Φ_i has the lowest willingness to pay then:*

$$\Rightarrow p_i = \max\left\{p_z^*(\phi^l), \min\left\{p_z^*(\phi^r), arg\max_{p_z} \int_{\underline{\phi}(p_z)}^{\phi^r} \pi_z d\phi + \int_{\phi^l}^{\underline{\phi}(p_z)} \pi_x d\phi\right\}\right\} \tag{20}$$

If the most tempted consumer in Φ_i has the lowest willingness to pay then:

$$\Rightarrow p_i = \max\left\{p_z^*(\phi^r), \min\left\{p_z^*(\phi^l), arg\max_{p_z} \int_{\phi^l}^{\overline{\phi}(p_z)} \pi_z d\phi + \int_{\overline{\phi}(p_z)}^{\phi^r} \pi_x d\phi\right\}\right\} \tag{21}$$

Lemma 6 is a combination of the previous ones and completes the algorithm. Its proof follows from the discussion above and the proofs of the other Lemmas.

To conclude the algorithm consider the case of a singleton element. If consumers with types belonging to such an element accept the loyalty scheme, they perfectly disclose their temptation level to the superstore.

Lemma 7. *If by accepting the loyalty scheme, consumer i perfectly disclose his type to the superstore, the equilibrium value of p_A coincides with the first best price:*

$$p_A = p_z^*(\phi_i). \tag{22}$$

Each consumer in a singleton element of $\mathcal{P}_n$, accepts the loyalty scheme and enters the store to buy good z.

Appendix C. The Case of $c_z > c_x$

In this appendix I analyse the case of $c_z > c_x$. I present the modified results for this section as Corollaries to the Propositions of the paper. The intuitions and mechanics of the equilibrium do not change. However, more consumers are now excluded from the market for good z. Furthermore, there are conditions over c_z for the superstore to find it profitable to sell good z in the first place.

C.1. **Full Information.** As mentioned above, selling z is not necessarily more profitable for the superstore anymore. She sells z if and only if she can charge:

$$p_z \geq p_x + c_z - c_x. \tag{23}$$

Consider equation (6). It is easy to see that condition (23) is not satisfied for all ϕ_i.

Corollary 1. *When $c_z \in [c_x, p_x + c_x)$ and*

$$v(z) \geq v(x)\frac{p_x + c_z - c_x}{p_x + c_x - c_z}, \tag{24}$$

Proposition 1 holds but good z is offered only to consumers with

$$\phi_i \in \Phi_0 \equiv \left[\tfrac{2(c_z - c_x)}{v(z)-v(x)}, \tfrac{(p_x+c_x)-c_z}{v(x)}\right].$$

Proof. As described above, the superstore is willing to sell good z to consumer i if and only if condition (23) holds. It is easy to show how this condition holds only for specific values of ϕ_i. First consider the case of $\phi_i < \phi^*$. In this case $\frac{1}{2}\phi_i(v(z)-v(x))+p_x \geq p_x + c_z - c_x$ must hold. That happens if and only if $\phi_i \geq \frac{2(c_z-c_x)}{v(z)-v(x)}$. If, instead, $\phi_i \in \left[\phi^*, \frac{p_x}{v(x)}\right]$ then $2p_x - \phi_i v(x) \geq (p_x + c_z) - c_x$ must hold. That happens if and only if $\phi_i \leq \frac{(p_x+c_x)-c_z}{v(x)}$. for the case of $\phi_i > \frac{p_x}{v(x)}$, it is obvious to see that $p_z = p_x$ cannot satisfy (23). Finally, notice that the right boundary of Φ_0 does not exist when $p_x + c_x < c_z$. □

Since when $p_x + c_x < c_z$ set Φ_0 becomes 'degenerate' I only consider $c_z \in [c_x, p_x + c_x)$ as an alternative to $c_x < c_z$.

C.2. **Asymmetric Information.** The following is a Corollary to Result 1 in Appendix E.

Corollary 2. *When $c_z \in [c_x, 2c_x]$ and Result 1 holds with*

$$p_z = \begin{cases} p_z' & \text{if } v(z) \geq \frac{v(x)(4c_x - c_z)}{2c_x - c_z} \\ p_x & \text{otherwise} \end{cases}. \tag{25}$$

When $c_z > 2c_x$, the price set under asymmetric information is $p_x + c_z - c_x$ and all consumers in Φ_0 purchase the good.

Proof. To prove the Corollary simply notice that the superstore still solves

$$p_z' = arg\max_{p_z} \int_{\Phi(p_z)} \pi_z d\phi + \int_0^{\underline{\phi}(p_z)} \pi_x d\phi = p_x + \tfrac{1}{2}c_z - \frac{c_x v(x)}{v(z) - v(x)}.$$

Hence, the only change follows from p_z' to be greater than $p_x + c_z - c_x$ instead of simply p_x. This happens when

$$p_x + \tfrac{1}{2}c_z - \frac{c_x v(x)}{v(z) - v(x)} \geq p_x + c_z - c_x$$
$$v(z)(2c_x - c_z) \geq v(x)(4c_x - c_z)$$

which never holds for $c_z > 2c_x$ and yields the condition in the Corollary otherwise. □

C.3. **ϕ-uninformative Loyalty Schemes.** As for the case of private information, $p_z^\dagger$ is unchanged when $c_z > c_x$. Now, however, we need a new condition on the substitutability between z and x.

Corollary 3. *When $c_z \in [c_x, p_x + c_x)$, Proposition 2 holds. The new requirement on substitutability is given by* (24)*.*

Proof. As for the case of private information, the new condition on $p_z^\dagger$ to be larger than $p_x + c_z - c_x$ is given by

$$\tfrac{1}{2}\left(\frac{2p_x v(z)}{v(z)+v(x)} + p_x + c_z - c_x\right) \geq p_x + c_z - c_x$$

which yields (24) when $c_z < p_x + c_x$. □

Given the above, Proposition 3 is unchanged.

C.4. **ϕ-informative Loyalty Schemes.** The case of informative loyalty schemes requires a bit of attention. The Lemmas in Appendix B hold with a slight modification:

Lemma 8. *When $c_z > c_x$. the rules governing the optimal pricing algorithm are as follows:*

(i) $p_R = p_{\max}$.
(ii) p_A *as described by (14)-(22) for elements of* $\mathcal{P}_n$ *that are a subset of* Φ_0.
(iii) $p_A = p_R$ *for elements of* $\mathcal{P}_n$ *that are outside of* Φ_0.
(iv) p_A *as described by (18)-(21) with the* $p_z^*(\phi^j)$*, for* $j = l, r$*, cutoffs replaced by:*

$$\max\left\{p_z^*(\phi^l), p_x + c_z - c_x\right\} \quad \textit{and} \tag{26}$$

$$\max\left\{p_x + c_z - c_x, p_z^*(\phi^r)\right\}, \tag{27}$$

for elements of $\mathcal{P}_n$ *that intersect with* Φ_0.

Proof. Points (i) and (ii) are straight forward and follow from the fact that the maximisation problem of the superstore is unchanged in those cases. Points (iii) and (iv) ensure that the superstore does not sell to consumers outside of Φ_0. □

Given the above, I present Corollary 4 to 6 to Propositions 4 to 6. Of course, Proposition 7 holds.

Corollary 4. *When $c_z > c_x$, for loyalty schemes to be fully informative only consumers in Φ_0 have to be able to fully disclose their type upon acceptance of the scheme.*

Proof. The proof is straightforward and it follows from the fact that the superstore has no interest in selling z to consumers outside Φ_0. □

Corollary 5. *When $c_z > 2c_x$ Proposition 5 holds without the need for enough substitutability between z and x.*

When $c_z < 2c_x$ *Proposition 5 holds with the new requirement on substitutability being*

$$v(z) \geq \frac{v(x)(4c_x - c_z)}{2c_x - c_z}.$$

Proof. When $c_z > 2c_x$ the superstore always serves all consumers in Φ_0 in the absence of loyalty schemes. Hence, whatever the substitutability between goods, fully informative schemes always hurt consumers. When $c_z < 2c_x$ the new condition follows from Corollary 2. □

Corollary 6. *When* $c_z > 2c_x$ *Proposition 6 holds without the need for enough substitutability between* z *and* x*.*

When $c_z < 2c_x$ *Proposition 6 holds with the new requirement on substitutability being*

$$v(z) \geq \frac{v(x)(4c_x - c_z)}{2c_x - c_z}.$$

Loyalty schemes can now also be informative towards consumers who do not differ in their ex-ante willingness to pay.

Proof. When $c_z > 2c_x$ the superstore always serves all consumers in Φ_0 in the absence of loyalty schemes. Hence, whatever the substitutability between goods, fully informative schemes always hurt consumers. When $c_z < 2c_x$ the new condition follows from Corollary 2. □

Appendix D. Consumers Without Self-Control Problems

In the paper, I have analysed the market equilibrium for the case of consumers that suffer from self-control problems. In this Appendix, I describe the results of a "classical" model that fails to account for this aspect of consumer preferences. I show how such a model would miss important qualitative characteristics captured by this paper.

First of all, I need to clarify a controversial point. The fact that consumers are free of self-control problems does not mean that they are not tempted by good z. Their temptation utility function $V_i(\cdot)$, and so their valuation of the good, is unchanged. What does change, is their ex-ante utility, since now consumers are assumed not to care for the self-control cost of resisting temptation.

In other words, the model becomes one with classical consumers that, considering their preferences, buy good $a = z, x$ solving:

$$\max_{a \in M_j} [U(a, p_a) + V_i(a, p_a)]. \tag{28}$$

This new and simpler model has no ex-ante nor ex-post stage. Consumers simply evaluate $U(a, p_a) + V_i(a, p_a)$ at $a = x, z, 0$ and enter the store that sells the offer that maximises their utility.

D.1. **First-best.** If the superstore can perfectly observe consumers' level of temptation, she solves:

$$\max_{p_z}[p_z - c_z] \qquad \text{s.t.}$$
$$u(z) + \phi_i v(z) - 2p_z \geq u(x) + \phi_i v(x) - 2p_x$$
$$\Longrightarrow p_z^*(\phi_i) = \tfrac{1}{2}\phi_i(v(z) - v(x)) + p_x$$
$$\text{for all } \phi_i \in [0,1]\,.$$

D.2. **Asymmetric Information.** If the superstore cannot observe temptation levels and has no access to any loyalty schemes technology, she compares the expected profits of attracting all consumers into the store ($p_z = p_x$) with the ones of excluding consumers with a low valuation of the good.[24] She understands that, setting price p for good z only consumers with $\phi_i \geq \frac{2(p_z - p_x)}{v(z)-v(x)}$ enter her superstore. That is: $\Phi(p) \equiv \left[\underline{\phi}(p), 1\right]$. Hence, the optimal price to set if she wants to exclude less tempted consumers from the market is given by:

$$\begin{aligned} p_z' &= \arg\max_{p_z} \int_{\Phi(p_z)} (p_z - c_z)d\phi + \int_0^{\underline{\phi}(p_z)} (p_x - c_x)d\phi \\ &= \tfrac{1}{4}\left(v(z) - v(x)\right) + \tfrac{1}{2}(p_x + c_z - c_x) \end{aligned} \tag{29}$$

which only attracts consumers with $\phi_i \geq \frac{1}{2} + \frac{c_z - c_x}{v(z)-v(x)} = \underline{\phi}(p_z')$. This case is represented, together with first best, in Figure 7.

D.3. **Loyalty Schemes.** Suppose now that the superstore has access to loyalty schemes. By looking at Figure 4 it is easy to see how the algorithm for p_A described in Appendix B works for this case as well with some exemptions. First of all, notice that if consumers do not suffer from self-control problems, none of them finds an offer "too tempting not to buy it in period 2, but too costly to find it optimal to fall to temptation". That is to say, there is no downward sloping portion of the willingness to pay curve. Hence, to find the equilibrium prices, one should use the algorithm I derive considering elements composed of only weakly tempted consumers.[25] This implies that, unlike a model with individuals that suffer from self control problems, the equilibrium price of asymmetric information and the one for ϕ-uninformative loyalty schemes coincide.

This eliminates the key aspect behind all the results of the paper. A model without consumers with self-control problems would misreport completely the situation of highly tempted consumers. Here, they are never excluded from the market. No consumer in the market is strictly willing to reject the loyalty scheme for "commitment" reasons. There is no reason for them to reject a loyalty scheme since when they are *too*

[24]Price p_x attracts all consumers in since it is the highest price that type $\phi = 0$ is willing to pay for z. All other consumers have higher willingness to pay.

[25]Notice that in the case of consumers without self-control problems, $\phi^* = 1$, there is no $\overline{\phi}(p_z')$ and the fraction $\frac{p_x}{v(x)}$ looses importance.

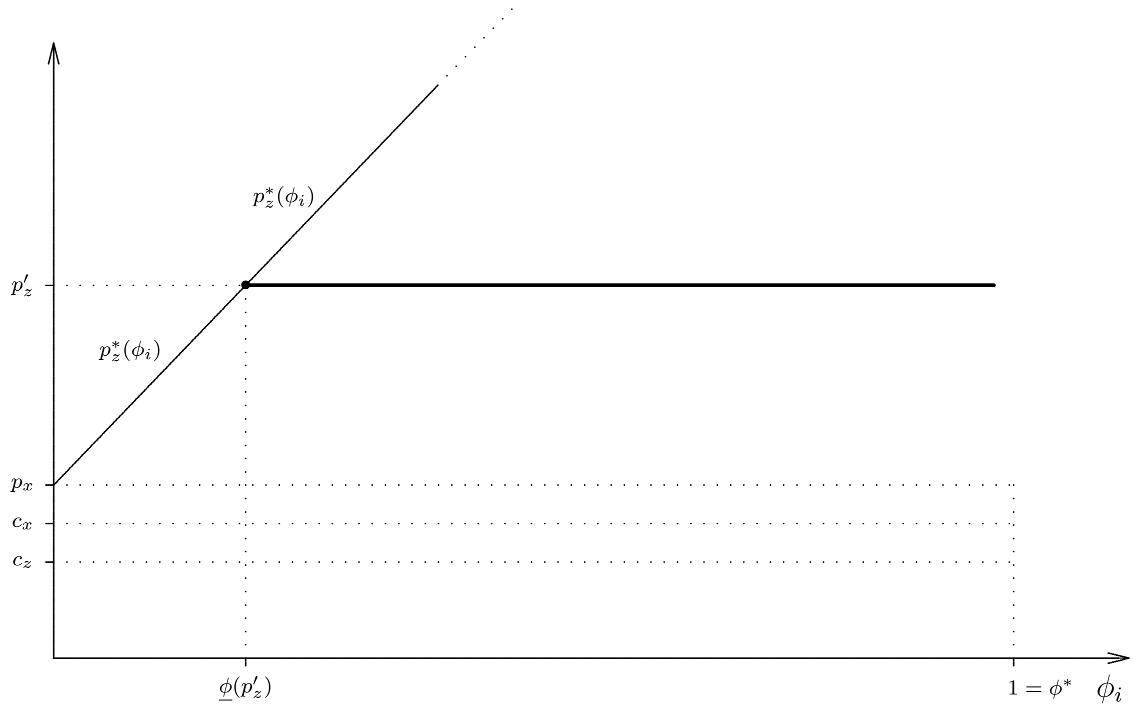

FIGURE 7. In the figure, I show the case of a model with consumers free from self-control problem. The first-best price is (partially) represented by the $p_z^*(\phi_i)$ line. The thick curve represents the price set when types are private information. It is plotted only for customers willing to purchase z at p'_z.

tempted they are "happy" to fall to temptation. Hence, the asymmetric information and ϕ-uninformative loyalty scheme equilibria coincide. The welfare results and policy implications of the paper would not apply any longer.

APPENDIX E. PROOFS

PROOF OF PROPOSITION 1

Assumption 1 ensures that each consumer is willing to purchase x as opposed to nothing and enter one of the small stores as opposed to entering nowhere. Hence, (5) can be rewritten as:

$$\max_{p_z}[p_z - c_z] \qquad \text{s.t.}$$

$$\max_{a \in M_m}[u(a) + \phi_i v(a) - 2p_a] - \max_{a \in M_m}[\phi_i v(a) - p_a] \geq u(x) + \phi_i v(x) - 2p_x - \max_{a \in M_s}[\phi_i v(a) - p_a]$$

$$u(z) + \phi_i v(z) - 2p_z \geq u(x) + \phi_i v(x) - 2p_x.$$

The IC holds if and only if

$$p_z \leq \tfrac{1}{2}\phi_i (v(z) - v(x)) + p_x \tag{30}$$

where I used $u(x) = u(z)$.

Consider now the most tempting offer as defined in Section 4. Notice that, for i:

$$(x, p_x) \succ_T 0 \text{ if } \quad \phi_i > \frac{p_x}{v(x)} \tag{31}$$

$$(z, p_z) \succ_T (x, p_x) \text{ if } \quad p_z \leq \phi_i \left(v(z) - v(x)\right) + p_x \tag{32}$$

$$(z, p_z) \succ_T 0 \text{ if } \quad p_z \leq \phi_i v(z). \tag{33}$$

It is easy to see then that every price that satisfies the IC, also makes $(z, p_z) \succ_T (x, p_x)$ for i. To understand why this is true, remember that the tempting features of good z are not hurting the consumer ($u(z) = u(x)$). Hence, if he values the good enough to find it optimal to fall to temptation and buy it, the temptation utility he obtains from offer (z, p_z) is also high enough to make (32) hold.

Start by assuming that **(31) holds**. Notice that conditions (30) and (32) are identical. Hence, a solution to (5) cannot fail to satisfy (32). I now solve (5) assuming that the solution satisfies (32) and then check that this is true afterwards. Notice that in this case (32) implies (33) and $W_i(M_s) = u(x) - p_x$. Hence the PC becomes:

$$u(z) + \phi_i v(z) - 2p_z - \phi_i v(z) + p_z \geq u(x) - p_x$$
$$\Rightarrow p_z^* = p_x$$

where p_z^* clearly satisfies (30) and, therefore, (32).

Assume now that **(31) does not hold**. Given this, then (33) implies (32) and $W_i(M_s) = u(x) + \phi_i v(x) - 2p_x$, since (x, p_x) does not tempt 0. As above I solve (5) assuming (33) holds and then check for it with the solution found. When (33) holds the PC becomes:

$$u(z) + \phi_i v(z) - 2p_z - \phi_i v(z) + p_z \geq u(x) + \phi_i v(x) - 2p_x$$
$$\Rightarrow p_z^*(\phi_i) = 2p_x - \phi_i v(x).$$

Notice, however, that this $p_z^*(\phi_i)$ is compatible with (33) only for $\phi_i \geq \frac{2p_x}{v(z)+v(x)} < \frac{p_x}{v(x)}$. For $\phi_i < \frac{2p_x}{v(z)+v(x)}$, (33) fails at the $p_z^*(\phi_i)$ derived which brings to a contradiction.

When (33) and (31) both fail, IC and PC coincide. Hence,

$$p_z^*(\phi_i) = \tfrac{1}{2}\phi_i \left(v(z) - v(x)\right) + p_x$$

from (30). This solution violates (33) for all $\phi_i < \frac{2p_x}{v(z)+v(x)}$.

Proof of Lemma 1

It is obvious that no consumer for which $(x, p_x) \succ_T 0$ will enter the superstore for $p_z > p_x$. Hence the only relevant interval of ϕ_i is $\left[0, \frac{p_x}{v(x)}\right)$. Notice, from the derivations of Proposition 1, that consumers in $[0, \phi^*)$ enter the superstore if and only if $p_z \leq \frac{1}{2}\phi_i \left(v(z) - v(x)\right) + p_x$. Solving this inequality for ϕ_i, I find the subset of $[0, \phi^*)$

of consumers that enter the superstore for a given p_z. This is given by $\left[\frac{2(p_z-p_x)}{v(z)-v(x)}, \phi^*\right)$. A similar reasoning can be applied to consumers in $\left[\phi^*, \frac{p_x}{v(x)}\right)$. They enter the superstore if and only if $p_z \leq 2p_x - \phi_i v(x)$. Solving this for ϕ_i I find the subset of $\left[\phi^*, \frac{p_x}{v(x)}\right)$ consumers the enter at a given p_z. This is given by $\left[\phi^*, \frac{2p_x-p_z}{v(x)}\right]$. To conclude the proof notice that the union of the two subsets is the continuous interval $\Phi(p_z)$.

Proof of Proposition 2

First of all notice that, since loyalty schemes are ϕ-uninformative, in equilibrium, there cannot be two different consumers buying good z at different prices. The reason for this is that the superstore has no information to discriminate between two different consumers. Hence, she cannot set up an incentive compatible pricing scheme that makes consumers self-select.[26] Every consumer that buys z does so at the lowest possible price available between p_A and p_R.

Given this, the superstore sets either p_A or p_R equal to a price that induces the optimal interval of consumers, $\Phi(\cdot)$, to enter the store. As discussed in Section 7 by setting the price of rejection equal to $p_{\max}$ the superstore can avoid the risk of tempting strongly tempted consumers and set p_A such that the optimal subset of consumers purchases z. From Lemma 1, we have the set of consumers willing to enter the superstore as a function of p_A. Hence, the maximisation of the superstore is given by:

$$\begin{aligned} arg \max_{p_z} \int_{\Phi(p_z)} \pi_z d\phi + \int_0^{\underline{\phi}(p_z)} \pi_x d\phi + \int_{\overline{\phi}(p_z)}^1 \pi_x d\phi \\ \Rightarrow p_z^\dagger = \tfrac{1}{2}(\phi^* v(z) + p_x + c_z - c_x) \end{aligned} \tag{34}$$

As for consumers' equilibrium strategy, notice that those in $\left[0, \underline{\phi}(p_z^\dagger)\right]$ have no interest is buying good z at $p_z^\dagger$ nor are they tempted by it. Hence, they obtain the same ex-ante utility no matter what they do.

Consumers in $\Phi(p_z^\dagger)$ obtain a strictly larger $W_i(\cdot)$ if they accept the scheme, enter the superstore and buy good z, with respect to any other action at their disposal. To see why this is true, notice that every consumer in $\Phi(p_z^\dagger)$ has a strictly larger willingness to pay than consumers $\underline{\phi}(p_z^\dagger)$ and $\overline{\phi}(p_z^\dagger)$. Also, notice that $p_z^\dagger = p_z^*(\underline{\phi}(p_z^\dagger)) = p_z^*(\overline{\phi}(p_z^\dagger))$. Hence, consumers inside the interval obtain $W_i\left(\left\{(x,p_x),(z,p_z^\dagger),0\right\}\right) > W_i\left(\{(x,p_x),0\}\right) = W_i\left(\{(x,p_x),(z,p_R),0\}\right)$.

Finally, consider consumers in $\left[\overline{\phi}(p_z^\dagger), 1\right]$. They are tempted by $(z, p_z^\dagger)$ so much that they would choose it, were they to accept the scheme and enter the superstore. However, it is easy to check that their ex-ante utility obtained by doing so is $W_i\left(\left\{(x,p_x),(z,p_z^\dagger),0\right\}\right) < W_i\left(\{(x,p_x),0\}\right) = W_i\left(\{(x,p_x),(z,p_R),0\}\right)$. Hence, they

[26]Unless of course, one where some consumers "self-exclude" from the market, as the one described here.

strictly prefer to reject the loyalty scheme. The utility from entering any store becomes then $W_i\left(\{(x,p_x),0\}\right)$. Hence, they have a weak incentive to enter the superstore.

To conclude the proof I derive condition (11). Notice that (34) ignores the fact that the superstore has the option of serving the entire market at p_x. Hence, the result could very well be smaller than p_x. A choice that would be irrational for the superstore. To ensure that this does not happen I calculate

$$\begin{aligned}
\frac{p_x v(z)}{v(z)+v(x)} &+ \tfrac{1}{2}p_x + \tfrac{1}{2}(c_z - c_x) \geq p_x \\
\frac{p_x v(z)}{v(z)+v(x)} &- \tfrac{1}{2}p_x + \tfrac{1}{2}(c_z - c_x) \geq 0 \\
v(z)(p_x + c_z - c_x) &- v(x)(p_x - c_z + c_x) \geq 0
\end{aligned}$$

which generates (11).

Finally, as mentioned in the main body of the paper, I rule out the case of $p_A > p_R$. There technically exist, however, such an equilibrium for the uninformative loyalty schemes case. Since p_A is constant over ϕ_i:

Corollary 7. *When loyalty schemes are ϕ-uninformative, there exist a second equilibrium where the superstore sets*

$$p_z = \begin{cases} p_R = p_z^\dagger & \textit{if } i \textit{ accepts} \\ p_A \geq p_{\max} & \textit{if } i \textit{ rejects} \end{cases} \tag{35}$$

Consumers in $\Phi(p_z^\dagger)$ reject the loyalty scheme and enter the superstore to buy z. Consumers outside $\Phi(p_z^\dagger)$ have the (weak) incentive to accept the scheme and enter the superstore to buy x. This equilibrium is ruled out in the model.

Proof of Proposition 3

No Loyalty Schemes. Suppose that consumers hold private information on their temptation levels but the superstore does not make use of the loyalty scheme technology. It is easy to see how Lemma 1 still holds. The difference with the case of uninformative loyalty schemes lies in the fact that for any $p_z > p_x$ consumers beyond $\overline{\phi}(p_z)$ now strictly prefer to enter one of the smaller stores instead of the superstore (as discussed in Section 7). The superstore has two options: sell good z to the entire market by setting $p_z = p_x$ or setting a larger price and sell good z only to a subset of consumers. This is summarised in what follows.

Result 1. *When the seller cannot observe consumers' temptation levels and does not have access to loyalty schemes, the equilibrium price for z is given by:*

$$p_z = \begin{cases} p'_z & \textit{if } v(z) \geq \frac{v(x)(2c_x + c_z)}{c_z} \\ p_x & \textit{otherwise,} \end{cases} \tag{36}$$

$$\textit{where} \quad p'_z = p_x + \tfrac{1}{2}c_z - \frac{c_x v(x)}{v(z) - v(x)}.$$

If $p_z = p_x$ all consumers enter the superstore and buy z. Otherwise only consumers in $\Phi(p'_z)$ do so while consumers in $[\overline{\phi}(p'_z), 1]$ purchase x from one of the smaller stores.

Proof. For p'_z, simply notice that

$$p'_z = arg \max_{p_z} \int_{\Phi(p_z)} \pi_z d\phi + \int_0^{\underline{\phi}(p_z)} \pi_x d\phi = p_x + \tfrac{1}{2}c_z - \frac{c_x v(x)}{v(z) - v(x)}.$$

□

Intuitively, the more valuable are the tempting features of z to consumers, the more the superstore is inclined to exclude some consumers from its purchase.

Proof of Proposition 3. To prove point i), first I need to show that $v(z) \geq \frac{v(x)(2c_x + c_z)}{c_z}$ is implied by (11). Notice that:

$$\frac{(2c_x + c_z)}{c_z} \geq \frac{(p_x + c_x - c_z)}{(p_x + c_z - c_x)} \Rightarrow 2c_x(p_x - c_x) + 2c_z^2 \geq 0$$

is always true. Hence, the proof requires

$$\begin{gathered} p_x + \tfrac{1}{2}c_z - \frac{c_x v(x)}{v(z) - v(x)} < \tfrac{1}{2}(\phi^* v(z) + p_x + c_z - c_x) \\ p_x \left(1 - \frac{1}{2} - \frac{v(z)}{v(z) + v(x)}\right) + c_x \left(\frac{1}{2} - \frac{v(x)}{v(z) - v(x)}\right) < 0 \\ c_x \frac{v(z) - 3v(x)}{v(z) - v(x)} - p_x \frac{v(z) - v(x)}{v(z) + v(x)} < 0. \end{gathered}$$

Since $p_x > c_x$ we just need to show that

$$\frac{v(z) - v(x)}{v(z) + v(x)} > \frac{v(z) - 3v(x)}{v(z) - v(x)},$$

which rearranged results in

$$(v(z) - v(x))^2 > v(z)^2 - 2v(x)v(z) - 3v(x)^2,$$

which is trivially true.

To prove point ii), notice that both p'_z and p_x are an available solution to (34). Hence, it must be that the superstore is better off setting $p_z^\dagger$.

Finally to prove point iii), notice that consumers who buy good z both in the absence and presence of loyalty schemes now face a higher price, and therefore a lower surplus. Now consider consumers who switched from buying z in the absence of loyalty schemes to buying x in their presence. These consumers had the option of buying x at the same price when schemes were not in place and they chose not too. This means that buying z at p'_z (or p_x) grants them a larger surplus than purchasing x. Hence, also these consumers are worse off and the aggregate consumer welfare has decreased.

Proof of Proposition 4

The proof is straightforward. If the consumer discloses his ϕ_i completely, he is charged $\phi_i^*(\phi_i)$, which satisfies both his PC and IC. Therefore, he accepts the scheme, enters the superstore and purchase z.

Proof of Proposition 5

The Proposition is proven in the main body of the paper.

Proof of Proposition 6

First of all, notice that whenever the loyalty scheme partition features an element which is the superset of $\Phi(p_z^\dagger)$ the optimal p_A for that element is $p_z^\dagger$. To see this, let the boundaries of the superset being $\left[\phi^l, \phi^r\right]$. The maximisation problem solved by the superstore for this interval is given by:

$$\begin{aligned} & arg \max_{p_z} \int_{\Phi(p_z)} \pi_z d\phi + \int_{\phi^l}^{\underline{\phi}(p_z)} \pi_x d\phi + \int_{\overline{\phi}(p_z)}^{\phi^r} \pi_x d\phi \\ & = arg \max_{p_z} \int_{\Phi(p_z)} \pi_z d\phi + \left(\underline{\phi}(p_z) - \phi^l\right) \pi_x + \left(\phi^r - \overline{\phi}(p_z)\right) \pi_x. \end{aligned}$$

Notice that this problem is equivalent to (34) with the only difference being in the terms

$$-\phi^l \pi_x + \phi^r \pi_x.$$

But these terms are independent of p_z and therefore do not affect the solution of the problem.

Now consider condition (11). Recall that this condition ensures that the superstore finds it optimal to charge $p_z^\dagger$ instead of p_x and serve the entire market. Suppose that the partition features three elements $[0, \phi^1), [\phi^1, \phi^2), [\phi^2, \phi^3]$, where $\overline{\phi}(p_z^\dagger) < \frac{p_x}{v(x)} < \phi^1 < \phi^2 < \phi^3$, and suppose the superstore is looking for the optimal p_1. Not only due to the argument above $p_1 = p_z^\dagger$, but we also know that the condition for the superstore to prefer $p_z^\dagger$ to p_x is identical to (11). To see this, notice that the maximisation problem faced by the superstore is:

$$\begin{aligned} & arg \max_{p_z} \int_{\Phi(p_z)} \pi_z d\phi + \int_{0}^{\underline{\phi}(p_z)} \pi_x d\phi + \int_{\overline{\phi}(p_z)}^{\phi^1} \pi_x d\phi + \int_{\phi^1}^{\phi^2} \pi_x d\phi \\ & = arg \max_{p_z} \int_{\Phi(p_z)} \pi_z d\phi + \int_{0}^{\underline{\phi}(p_z)} \pi_x d\phi + \int_{\overline{\phi}(p_z)}^{\phi^1} \pi_x d\phi \\ & = arg \max_{p_z} \int_{\Phi(p_z)} \pi_z d\phi + \int_{0}^{\underline{\phi}(p_z)} \pi_x d\phi + \int_{\overline{\phi}(p_z)}^{1} \pi_x d\phi \\ & \Rightarrow p_z^\dagger = \tfrac{1}{2}(\phi^* v(z) + p_x + c_z - c_x). \end{aligned}$$

Hence, whenever $\Phi(p_z^\dagger)$ is a subset of an element $\Phi_i \in \mathcal{P}_n$, the optimal price p_A for its superset is $p_z^\dagger$. Furthermore, if parameters are such that

$$\Phi(p_z^\dagger) \subset \left[0, \frac{p_x}{v(x)}\right] \subset \Phi_i$$

then the principal sets p_x when (11) fails and $p_z^\dagger$ otherwise.

Finally, recall from the proof of Proposition 3 that (11) is looser than

$$v(z) \geq v(x)\frac{2c_x + c_z}{c_x},$$

the equivalent condition for the case of no loyalty schemes. With this in mind, I can go on to the proof of point i).

Point i) & ii). The statement "goods z and x are sufficiently close substitutes" in the Proposition refers to $v(z) < v(x)\frac{2c_x+c_z}{c_x}$, that is, to the case where the superstore sells good z at p_x in the absence of loyalty schemes. It is straightforward then to see how loyalty schemes cannot decrease the price of z. When the partition features at least one element to the left of $\frac{p_x}{v(x)}$, i.e. when loyalty schemes are ϕ-informative towards consumers who differ in ex-ante willingness to pay, consumers in those elements are paying more than p_x.

Point iii) and iv). The proof follows the same steps of Proposition 3.

Proof of Proposition 7

The Proposition is proven in the main body of the paper.

Proof of Proposition 8

Notice that when loyalty schemes are introduced either the price of z stays the same (p_x) for all consumers, or, if it decreases for some, it must have increased for others. This happens at least for consumers in $\Phi(p_z')$, where the price rises to $p_z^\dagger$ or higher.