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Strategic Disclosure in Networks*

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Abstract

I study strategic transmission of verifiable information through intermediaries, and find that equilibrium full disclosure requires that all players are biased in the same direction relative to the decision maker. By embedding this strategic disclosure game into networks, I explore the intersection of information transmission in networks and strategic communication—two major economic theory research strands. When each networked player may hold information useful for any other's decision, I find that the unique ex-ante optimal network is a line where players are ordered by their bliss points. This is also the unique network immune to coalitional deviations.

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1 Introduction

This paper develops a model of strategic disclosure of verifiable information through intermediaries. Further, by embedding the disclosure game in a network framework, I explore the intersection of two key areas in economic theory: information transmission in networks and strategic communication.

In a basic strategic disclosure game à la Milgrom (1981), there is an expert who knows the state of the world—an element of the real line—and an uninformed decision maker. Before the decision is made, the expert may disclose her information. The decision maker seeks to match her choice to the true state, while the expert prefers a biased outcome. Although the expert cannot lie, she can withhold information. Nevertheless, full disclosure occurs in equilibrium. This outcome is sustained by "worst-case" beliefs off the equilibrium path: if the expert were to withhold any information, the decision maker would presume the state most unfavorable to the biased expert, consistent with the information disclosed.

Section 2 of this paper studies the strategic disclosure of verifiable information through intermediaries. Disclosure occurs sequentially, beginning with the expert and proceeding along a path of intermediaries until it reaches the decision maker. Players have misaligned preferences, in the sense that each seeks to persuade the decision maker to choose a different action, given the same state of the world.[1]

My main result, Proposition 1, shows that the expert's information reaches the decision maker in equilibrium if and only if the expert and all intermediaries are biased in the same direction relative to the decision maker. Only then do all players precisely transmit the expert's information along the path, and Milgrom's (1981) logic extends to communication through intermediaries. The reason is straightforward: no off-path beliefs can simultaneously punish both the expert and all intermediaries for withholding information when some are biased to the right and others to the left. To punish the former, the decision maker's choice would need to be as left-leaning as possible, given the information received, while also being as right-leaning as possible to punish the latter. But, of course, this is

[1] To simplify the exposition, I rule out the possibility of partial information disclosure. Further, I assume that players' payoffs follow a quadratic loss function with misaligned bliss points, as is common in strategic communication games. I relax these assumptions in Section 4 and show that my results remain qualitatively unchanged when generalizing players' payoffs, when partial disclosure is allowed, and when the expert lacks direct knowledge of the state and relies on noisy signals.

impossible.

In Section 3, I embed the above analysis of indirect strategic disclosure in a network framework, and provide a model of strategic communication in networks. An expert and a decision maker are randomly selected from among the network's players, each pair with positive probability. Once chosen, their identities become common knowledge, and a disclosure game is played within the network. The expert's information is transmitted to the decision maker through any connecting path of players.[2]

For each network and realized expert–decision maker pair, I determine the value of the most informative equilibrium in the resulting disclosure game. The average value of these equilibria, weighted by the ex-ante probability of each pair, determines the value that each player assigns to the network. These values are then used to assess the network's optimality and endogenous formation. This simple, micro-founded construction captures the key features of both communication in networks—where each player benefits from the information she may receive from any other player—and strategic transmission of verifiable information, where players may withhold information to influence decision makers' choices.

Given any network and expert–decision maker pair, Proposition 1 applies. If the expert is connected to the decision maker through a unique path of players in the network, the state is disclosed if and only if all the players on the path are biased in the same direction, relative to the decision maker. This result has important implications for the study of communication in networks. Existing analyses based on information diffusion models assume that players learn information from those they are linked with, but abstract from the possibility that information is strategically misrepresented.

These implications are underscored when examining network optimality and endogenous formation. I find that the unique optimal network is a line in which each player forms links only with those who have the closest bliss points. This ordered line also arises as the unique equilibrium of a bilateral link sponsorship game à la Myerson (1991) that is immune to coalitional deviations. These findings complement fundamental results in network theory (e.g., Jackson and Wolinsky, 1996). If communication is non-strategic and subject to

[2] To simplify the exposition, my analysis abstracts from non-strategic communication constraints and assumes a single expert and a single decision maker. I relax these assumptions in Section 4 and show that my results remain qualitatively unchanged in settings with multiple experts and decision makers, and also when network transmission decays independently of communication choices, as long as the decay probability is sufficiently small relative to the misalignment of players' bliss points.

technological constraints such as information decay, it is well established that the optimal networks are stars. This difference is especially pronounced in terms of network centrality: while the star is the most centralized minimally connected network, the line is the most decentralized.[3]

The complementarity with models of non-strategic information diffusion in networks is also reflected in different applications. Information diffusion models naturally apply to economic settings where there are no action externalities or where agents share similar goals.[4] Instead, my paper examines networks in which agents have a strategic incentive to mislead one another because each is affected by everyone else's decisions, and their preferences are not aligned. The primary motivation and application of my work are networks of political decision makers. Social connections are widely recognized as an important, if not fundamental, feature of politics.[5] While a large empirical literature on political networks has emerged, systematic theoretical modeling—such as that developed through network economics—remains in its early stages.[6]

A key feature of politics is the divergence in preferences among agents, driven by ideological differences that often lead them to try and mislead one another. Furthermore, career politicians are typically well acquainted with each other's personal ideological views and committed platforms.[7] Political agents also tend to form alliances with those who are ideologically closest to them, as predicted by my analysis. My findings suggest that—perhaps unexpectedly—this network structure is optimal for ensuring the transmission of information, at least when such information is verifiable. Only then can a political decision maker identify who is to blame if information is withheld. Information from left- (or right-) leaning politicians travels through like-minded agents. If it fails to arrive, the decision maker assumes it was opposing evidence and responds by choosing the most extreme posi-

[3] Further, my results hold even though, unlike in Galeotti, Goyal, and Kamphorst (2006), and Calvó, de Martí, and Prat (2015), for example, the cost of linking is the same across all pairs, and each player's information holds equal value for others. The optimality of linking with those closest in bliss points is not due to homophily (e.g., McPherson, Miller, and Cook, 2001). Rather, it minimizes aggregate connection costs while ensuring that Milgrom's (1981) equilibrium disclosure results apply.

[4] For example, they have been used to study firms and other organizations in business economics, where different groups within an organization pursue the same objectives (e.g., profit maximization).

[5] This has been acknowledged as early as Routt (1938).

[6] A comprehensive review is provided in the handbook edited by Victor, Montgomery, and Lubell (2017).

[7] This assumption is supported by a vast empirical literature dedicated to estimating such political views and platforms, following the foundational work of Poole and Rosenthal (1996) and the Manifesto Project by Budge et al. (2001). It is also standard in political economy models, such as those studying information aggregation in committees or legislatures.

tion consistent with what she knows. This discourages other politicians from withholding information.[8]

1.1 Related Literature

Within the extensive literature developed over the years in network theoretical economics, one important focus is information transmission in networks.[9] The diffusion approach commonly adopted applies to communication among agents who have no incentive to strategically mislead one another. This is also the case in 'Bayesian learning' models, where each player learns through equilibrium inference based on their neighbors' choices, either via explicit communication or by observing actions. These models assume there are no externalities across agents' decisions and that their preferences are aligned.[10]

Specifically, Bala and Goyal (1998) studied a pioneering model in which there is no explicit information transmission; players observe their neighbors' choices and thus indirectly learn. Acemoglu, Bimpikis, and Ozdaglar (2014) introduced a model with explicit information transmission.[11] In all these models, there are no externalities across players' choices, and their preferences are aligned. Consequently, players have no incentive to strategically manipulate information or mislead one another—precisely the focus of this paper.

Most importantly, a key assumption in existing network models is that the value of connecting two agents, i and j, is independent of the characteristics of other players along the connecting path. However, this assumption does not hold in the full-fledged analysis of strategic communication presented in this paper. While this result is established here for verifiable information transmission, Ambrus, Azevedo, and Kamada (2013) and Mahzoon (2025) prove an analogous finding in the cases of cheap talk and Bayesian persuasion, respectively. These findings clarify the limitations of existing network models in accounting for strategic information transmission and underscore the opportunity to develop a new

[8] The insights provided here about political networks are further developed in a previous version of this paper, Squintani (2018).

[9] This literature is so vast that a comprehensive survey is beyond the scope of this paper. For a detailed review, see the handbook edited by Bramoullé, Galeotti, and Rogers (2017).

[10] A fortiori, strategic information transmission in networks is also not addressed in 'naive learning' models, where players' learning is not based on equilibrium beliefs. For example, Golub and Jackson (2010) study a model in which players' beliefs are assumed to be weighted averages of their neighbors'.

[11] While in these papers information originates solely among players in the network, Galperti and Perego (2025) examine a model in which a biased outside sender may also transmit a signal to the players. Still, there is no strategic information transmission within the network, as preferences remain aligned.

class of models.

Building on the seminal works of Milgrom (1981) and Crawford and Sobel (1982), strategic information transmission has become one of the central topics in the economics of information. While communication of unverifiable information (cheap talk) generally leads to imprecise decisions, verifiable information can be fully disclosed in equilibrium. My analysis offers a different perspective on this insight. When an expert communicates with a decision maker through intermediaries, verifiable information is fully disclosed in equilibrium if and only if all players on the communication path are biased in the same direction relative to the decision maker.[12]

While the literature on strategic information transmission has branched out theoretically in several directions, the study of indirect communication through intermediaries remains underdeveloped.[13] Ambrus, Azevedo, and Kamada (2013) study the case of cheap talk and show that the analysis is significantly more complex than in the case of verifiable information disclosure considered here. The result that intermediation cannot improve information transmission holds only for pure strategy equilibria. They provide a partial characterization of mixed strategy equilibria, show instances in which intermediation improves upon direct communication, and provide necessary conditions.[14] For the case of verifiable information disclosure I consider here, instead, intermediation cannot ever improve upon direct communication, in equilibrium.

Closer to my work, Gieczewski (2022) studies a model of learning in networks with the transmission of verifiable information among agents with misaligned preferences. Unlike my paper, he does not consider network formation, or optimality. Experts learn the state

[12] Here, each player's bias relative to the decision maker is independent of the state of the world, and equilibrium full disclosure fails due to communication through intermediaries. Instead, Giovannoni and Seidmann (1997) study disclosure games with one expert and one decision maker, where the expert's bias is state-dependent. They show that full disclosure occurs in equilibrium if and only if the expert is biased in the same direction for all states of the world. Onuchic and Ramos (2025) study multi-agent disclosure protocols ranging from unilateral to consensual, finding that full disclosure occurs only when members can unilaterally disclose output. None of these papers study strategic disclosure in networks, or through intermediaries.

[13] Most studies of communication are staged in two-player models with one expert and one decision maker. Exceptions include Battaglini (2002) on many-to-one communication, Farrell and Gibbons (1989) on one-to-many communication, and Galeotti, Ghiglino, and Squintani (2013) on many-to-many communication. None of these papers address indirect communication through intermediaries or communication in networks.

[14] Further, Ivanov (2010) studies cheap talk via a strategic mediator; Bloch, Demange, and Kranton (2018) examine the spread of possibly false information by agents with diverse motives, without addressing network optimality; and Migrow (2019) explores optimal hierarchies of biased agents reporting unverifiable information to a single decision maker.

with probability less than one, which prevents full information disclosure. He finds that full learning requires sufficiently dense networks. When agents are forward-looking, concerns about learning cascades lead players to divide into like-minded, non-communicating groups.

2 Strategic Disclosure through Intermediaries

This section formulates and solves a simple model of strategic disclosure through intermediaries, based on the seminal work by Milgrom (1981) on direct strategic disclosure. I show that the state of the world is relayed to the decision maker if and only if all the other players wish to bias her decision in the same direction.

The Model An expert e knows the state of the world $x \in X = [\underline{x}, \overline{x}] \subset \mathbb{R}$.[15] Every other player only knows the distribution F of x, which I assume has a continuous density f that is strictly positive on X.

The state x can be disclosed along a finite ordered sequence, or directed path, $p = (e, ..., d)$ of players, of length $l \geq 1$.[16] The path p starts from the expert e, terminates with a decision maker d, and (possibly) includes $l-1$ intermediaries. In each period $t = 0, ..., l-1$, the $t+1$-th player i on p is called to act. If i knows x, then she may disclose x to the $t+2$-th player on p, her immediate successor. If i does not disclose x, then none of her successors will learn anything about it.

Formally, let $\omega_i(h^t) \in \{\{x\}, X\}$ be the information held by i at any history h^t.[17] In period $t = 0$, given the null history h^0, the expert's information is $\omega_e(h^0) = \{x\}$, while every other player i has $\omega_i(h^0) = X$. For any period $t = 0, ..., l-1$ and history h^t, the $t+1$-th player i on p sends a message $\hat{m}^t_{ij} \in \{\{x\}, X\}$ to her successor j on p, subject to the restriction that if $\omega_i(h^t) = X$, then $\hat{m}^t_{ij} = X$.[18] As a result, for any period $t = 1, \ldots, l$ and history h^t, the information of the $t+1$-th player i on p is $\omega_i(h^t) = X$ unless, in history h^t, all of i's predecessors j on the path p played $\hat{m}^\tau_{jk} = \{x\}$ for all $\tau = 0, \ldots, t-1$.

[15] In Milgrom (1981), the state space X consists of the positive reals. Following Seidman and Winter (1997), I assume X to be a closed interval as this simplifies subsequent analysis.

[16] For any path p, its length $l(p)$ is the number of elements of p minus 1.

[17] As is standard, $\omega_i(h^t)$ refers to what player i knows at h^t through direct observation. This is distinct from equilibrium beliefs, which are based on knowledge of the equilibrium strategies.

[18] Milgrom (1981) also allows for partial disclosure, i.e., a player i who knows x at a history h^t, $\omega_i(h^t) = \{x\}$, may send messages $\hat{m}^t_{ij} \subseteq X$ such that $x \in \hat{m}^t_{ij} \neq X$. To simplify the exposition, I do not consider partial disclosure here, and defer it to Section 4.

At the terminal period $T = l$, after receiving message $\hat{m}_{id}^l$ from her predecessor i on p, the decision maker d chooses an action $\hat{y} \in \mathbb{R}$, based on her information $\omega_d(h^T)$ at any history h^T, and her equilibrium beliefs. As is standard, players' strategies are measurable functions of the game's histories. I denote player d's pure strategy by y_d and each other player i's disclosure strategy by m_i. Mixed strategies are defined in the usual way, and I denote mixed disclosure strategy profiles by μ.

Following the strategic disclosure game specification of Seidmann and Winter (1997), each player i's payoff is maximized if the decision $\hat{y}$ matches a bliss point $x + b_i$. The "relative bliss point" $b_i \in \mathbb{R}$ represents player i's idiosyncratic preference relative to the common state x. Player i's payoff from decision $\hat{y}$ is given by:

$$L_i(\hat{y}, x) = -(\hat{y} - x - b_i)^2.$$

This description of the game is common knowledge, and I study (pure and mixed strategy) Perfect Bayesian Equilibrium.

Equilibrium in the Disclosure Game I begin with two simple results that follow directly from the players' quadratic loss functions. In every pure or mixed strategy equilibrium, at any history h^T, the decision maker d plays:

$$y_d(h^T) = \mathbb{E}[x|\omega_d(h^T), \mu] + b_d,$$

and thus each player i's ex-ante expected equilibrium payoff is:

$$-E(y_d(h^T) - x - b_i)^2 = -\mathbb{E}[\mathrm{Var}(x|\omega_d(h^T), \mu)] + (b_i - b_d)^2. \tag{1}$$

The decision d equals the expected value $\mathbb{E}[x|\omega_d(h^T), \mu] + b_d$ of the decision maker's bliss point $x + b_d$, conditional on her information $\omega_d(h^T)$ and on knowledge of equilibrium strategies μ. Of course, if d knows x at history h^T, i.e., $\omega_d(h^T) = \{x\}$, her decision is $y_d(h^T) = x + b_d$. If instead d knows nothing, i.e., $\omega_d(h^T) = X$, then she chooses $y_d(h^T) = \hat{x} + b_d$, where $\hat{x}$ denotes her equilibrium expectation $\mathbb{E}[x|X, \mu]$ upon not being disclosed x.

Further, each player i's expected loss $E(y_d(h^T) - x - b_i)^2$ can be decomposed into the expected residual variance $\mathbb{E}[\mathrm{Var}(x|\omega_d(h^T), \mu)]$, and the squared bias $(b_i - b_d)^2$ of player i relative to d. As a consequence, the players' ex-ante payoffs are aligned in every disclosure game. They all wish to minimize d's expected residual variance, meaning they prefer that x be disclosed to d so that she chooses $\hat{y}$ with the most precise information possible.

I now introduce the following concept: a path p from an expert e to a decision maker d has *bias reversals* if there exist players i, j on p such that $b_i < b_d < b_j$.

The next result shows that the state x is disclosed through the path p connecting e to d if and only if p has no bias reversals. Further, if the bias reversals are sufficiently large,[19] then no information reaches the decision maker—completely overturning full-disclosure results. Finally, we characterize the equilibrium when the path p has bias reversals. Let ℓ be the player i with the minimal bliss point b_i on p, and h be the player i with the maximal b_i. Define the mapping $U : X \to \mathcal{P}(X)$ as: $U(x) = (\max\{\underline{x}, x - 2(b_h - b_d)\}, \min\{x + 2(b_d - b_\ell), \overline{x}\})$.[20]

Proposition 1 *In any equilibrium (μ, y_d) of the disclosure game through intermediaries played on a path p from expert e to decision maker d:*

a. If p has no bias reversals, then every state x is disclosed to d, who then plays $\hat{y}_d = x + b_d$, and thus $\mathbb{E}[\text{Var}(x|\omega_d(h^T), \mu)] = 0$;

b. If p has bias reversals, then no state $x \in U(\hat{x})$ is disclosed to d, who plays $\hat{y}_d = \hat{x} + b_d$ with $\hat{x} = \mathbb{E}[x|U(\hat{x})]$, and hence $\mathbb{E}[\text{Var}(x|\omega_d(h^T), \mu)] > 0$;

c. If there exist players i, j such that $b_i - b_d > 0$ and $b_d - b_j > 0$ are sufficiently large, then d never learns x and plays $\hat{y}_d = \mathbb{E}[x] + b_d$, so that $\mathbb{E}[\text{Var}(x|\omega_d(h^T), \mu)] = \text{Var}(x)$.

These result are intuitive. Suppose the path p has no bias reversals, for instance, $b_i > b_d$ for all $i \neq d$. Then, as in Milgrom (1981), there exists a Perfect Bayesian Equilibrium in which every player i transmits x along the path p, i.e., $\hat{m}_{ij} = \omega_i(h^t)$ for all $i \neq d$ and histories h^t at which i is called to play. As a result, the decision maker d learns x precisely and chooses $\hat{y} = x + b_d$, so that $\mathbb{E}[\text{Var}(x|\omega_d(h^T), \mu)] = 0$.

This equilibrium is supported by 'worst-case' off-path beliefs, which interpret withheld information as evidence contrary to the players' biases, thereby deterring information withholding. Formally, these beliefs assign probability one to $x = \min \omega_d(h^t)$ for any terminal

[19] This situation corresponds to 'transparent motives' as defined in Lipnowski and Ravid (2020), and originally studied by Milgrom (1981).

[20] $\mathcal{P}(X)$ denotes the set of subsets of X.

history h^T. Due to these off-path beliefs, any player $i \neq d$ knows that withholding x can only shift d's choice $\hat{y}$ leftward, against her bias $b_i - b_d > 0$. So she discloses x in equilibrium.

Further, it cannot be that $\mathbb{E}[\text{Var}(x|\omega_d(h^T), \mu)] > 0$ in equilibrium, unless there are bias reversals. This follows from a generalized 'unraveling' argument similar to that in Milgrom (1981). If such an equilibrium existed, then there would be histories h^T where the decision maker d does not know x. Formally, the set $\hat{X}$ of states for which $\omega_d(h^T) = X$ for some h^T would have strictly positive measure. Hence, we would have $\hat{x} < \sup \hat{X}$. For states x close to the upper bound of $\hat{X}$, all players $i \neq d$ would strictly prefer to reveal x rather than withhold it—since doing so would move d's action closer to their bliss points. But this contradicts the definition of $\hat{X}$.

However, both the worst-case beliefs and unraveling arguments break down when the path p has bias reversals. Plainly, there cannot exist off-path beliefs and corresponding $\hat{y}$ decisions that simultaneously punish both players biased to the right ($b_i > b_d$) and players biased to the left ($b_j < b_d$) for withholding information. As a result, there exists no equilibrium in which d learns x precisely. That is, $\mathbb{E}[Var(x|\omega_d(h^T), \mu)] > 0$.[21]

To illustrate this, consider the following three-player example.

Example 1 (Three Players) There are three players: an expert e, an intermediary i, and a decision maker d, with bliss points $b_i < b_d < b_e$. That is, e is right-biased and i is left-biased. At time $t = 1$, the expert e may disclose x to i or send $\hat{m}_{ei}^1 = X$. If i receives x, she may disclose it to d at time $t = 2$, or instead send $\hat{m}_{id}^2 = X$. If i is not informed by e, she can only send X. At time $t = 3$, player d chooses $y \in \mathbb{R}$.

Suppose i knows x at history h^2—i.e., $\omega_i(h^2) = \{x\}$. If she withholds x and sends X, then d plays $y_d(X) = \hat{x} + b_d$, and i's payoff is $u_i = -(\hat{x} + b_d - x - b_i)^2$. If i discloses x, then d plays $y_d(x) = x + b_d$, and i's payoff is $u_i = -(b_d - b_i)^2$. Therefore, i withholds x whenever $x \in (\hat{x}, \hat{x} - 2(b_d - b_i))$, ignoring ties for simplicity. Intuitively, the left-biased i withholds the state if (i) withholding results in a more favorable (i.e., leftward) decision, and (ii) that leftward decision is closer to her bliss point than the full disclosure decision would be.

Now consider the expert e. If she withholds x and sends X, player d ultimately plays $\hat{x} + b_d$, giving e a payoff of $-(\hat{x} + b_d - x - b_e)^2$. If e discloses x and i relays it to d, then d plays $x + b_d$, and e's payoff is $-(b_d - b_e)^2$. Thus, e withholds x when $x \in (\hat{x} - 2(b_e - b_d), \hat{x})$, if

[21] See the proof of Proposition 1 in Appendix A for the exact bound.

she anticipates that i will relay it to d in the considered equilibrium. Of course, the expert e's choice is irrelevant for any state she anticipates i will withhold from d. In any case, d will not receive x for any $x \in (\hat{x} - 2(b_e - b_d), \hat{x})$.

By joining the sets of states withheld by either i or e, we find that d will not be informed of any $x \in U(\hat{x})$, except possibly $\hat{x}$ itself. To complete the equilibrium construction, d's belief $\hat{x}$ must be consistent with this set: $\hat{x} = E\left[x|U(\hat{x})\right]$. For all $x \in U(x)$, the decision maker plays $\hat{y} = \hat{x} + b_d$. (Hence, it is immaterial whether the state $\hat{x}$ is conveyed to d or withheld.) ◇

Beyond the three-player case, the analysis mirrors Example 1. Consider any player i with $b_i > b_d$ who knows the state x at a history h^t where she is called to play. Pick any state x that she expects will be relayed to the decision maker d by her successors j on the path p, in equilibrium, if she discloses it. Player i withholds x when $x \in (\hat{x} - 2(b_i - b_d), \hat{x})$. For any x that i expects at least one of her successors will withhold, her own decision is irrelevant. Hence, none of the states in $x \in (\hat{x} - 2(b_i - b_d), \hat{x})$ will be disclosed to d. Likewise, for every i with $b_i < b_d$, the states $x \in (\hat{x}, \hat{x} - 2(b_d - b_i))$ will be withheld from d. Applying this reasoning to all players $i \neq d$, we conclude that the states $x \in U(\hat{x})$ are not disclosed to d in equilibrium. Consequently, d chooses $\hat{y} = E\left[x|U(\hat{x})\right] + b_d$, and the expected residual variance is $\mathbb{E}[\text{Var}(x|\omega_d(h^T), \mu)] > 0$.

If the biases $b_i - b_d > 0$ and $b_d - b_j > 0$ are sufficiently large, then i attempts to push $\hat{y}$ rightward and j leftward, regardless of the state x. In this case, the set of states $U(\hat{x})$ covers the entire state space X, and the expected residual variance becomes $\mathbb{E}[\text{Var}(x|\omega_d(h^T), \mu)] = \text{Var}(x)$.

To provide a concrete illustration, I conclude the section with the case of a uniformly distributed state, where the equilibria are calculated in closed form.

Example 2 (Uniform State Distribution) Suppose that $X = [0,1]$, $x \sim U([0,1])$ and the path p has bias reversals. Then, the equilibrium is generically as follows:

a. If $b_d - b_\ell < \min\{1/4, b_h - b_d\}$, then $\hat{x} = 2(b_d - b_\ell)$, the set of undisclosed states is $U(\hat{x}) = [0, 4(b_d - b_\ell)]$ and d plays $\hat{y} = 3b_d - 2b_\ell$ for $x \in U(\hat{x})$;

b. If $b_h - b_d < \min\{1/4, b_d - b_\ell\}$, then $\hat{x} = 1 - 2(b_h - b_d)$, $U(\hat{x}) = [1 - 4(b_h - b_d), 1]$ and d plays $\hat{y} = 1 - 2b_h + 3b_d$ for $x \in U(\hat{x})$;

c. Else, if $\min\{b_d - b_\ell, b_h - b_d\} > 1/4$, then $\hat{x} = 1/2$, $U(\hat{x}) = [0,1] = X$ and d plays $\hat{y} = 1/2 + b_d$ for all x.

This follows from Part b of Proposition 1, together with the following arguments.

There can be no generic equilibrium where $0 < \hat{x} - 2(b_h - b_d)$ and $\hat{x} + 2(b_d - b_\ell) < 1$, because the condition $\hat{x} = E[x|U(\hat{x})]$ becomes $\hat{x} = [\hat{x} - 2(b_h - b_d) + \hat{x} + 2(b_d - b_\ell)]/2$, i.e., $b_h - b_d = b_d - b_\ell$: a knife-hedge condition on b_ℓ, b_d and b_h.

When $\hat{x} - 2(b_h - b_d) < 0$ and $\hat{x} + 2(b_d - b_l) < 1$, the condition $\hat{x} = E[x|U(\hat{x})]$ yields $\hat{x} = 2(b_d - b_\ell)$. Consistency thus requires that $\hat{x} - 2(b_h - b_d) = 2(b_d - b_\ell) - 2(b_h - b_d) < 0$, i.e., that $b_h - b_d > b_d - b_\ell$. Player h must be more biased to the right than l is biased to the left. The other consistency requirement is that $\hat{x} + 2(b_d - b_\ell) < 1$, i.e., that $b_d - b_\ell < 1/4$.

Conversely, when $\hat{x} - 2(b_h - b_d) > 0$ and $\hat{x} + 2(b_d - b_\ell) > 1$, the condition $\hat{x} = E[x|U(\hat{x})]$ yields $\hat{x} = 1 - 2(b_h - b_d)$. Consistency then requires that $b_h - b_d < b_d - b_\ell$ and $b_h - b_d < 1/4$.

Finally, when $\hat{x} - 2(b_h - b_d) < 0$ and $\hat{x} + 2(b_d - b_\ell) > 1$, then $U(\hat{x}) = [0,1]$ and $\hat{x} = 1/2$, so that consistency requires $b_h - b_d > 1/4$ and $b_d - b_\ell > 1/4$. ◇

3 Strategic Disclosure in Networks

In order to explore strategic information transmission in networks, I embed the strategic disclosure framework presented in Section 2 into a network setting inspired by the model of 'social communication' by Jackson and Wolinsky (1996).[22] The analysis shows that the unique ex-ante optimal network—once strategic incentives to withhold information are accounted for—is a line in which players are ordered by their bliss points. This is also the only network that is immune to coalitional deviations in a bilateral sponsorship game of network formation.

The Model Suppose that a set $\mathcal{N}$ of n players is embedded in an undirected network N, a symmetric $n \times n$ matrix, with $N_{ij} \in \{0,1\}$ and $N_{ii} = 1$ for all $i, j \in \mathcal{N}$. For any $i \neq j$, the notation $N_{ij} = 1$ indicates that i is linked to j, meaning i can disclose x to j if she knows it. After network N is formed, a pair of players d and e is randomly selected

[22] A key aspect of their model is communication decay: even if a player attempts to transmit her information, it may be lost with positive probability. To simplify the exposition, I assume no decay here and defer its consideration to Section 4.

from $\mathcal{N}$ according to a full-support probability distribution P. Player d takes the role of the decision maker, while e is the informed expert—the only player who knows the state of the world $x \in X$. Their identities become common knowledge among all players in $\mathcal{N}$ after selection.

Player e's information may travel to d through the network N along any (directed) path p that connects e and d.[23] If e and d are not connected in N, then e's signal cannot reach d. As will become clear later, my results hold for any communication protocol where, if e and d are connected by a unique path p, communication proceeds step by step along p as in the model of Section 2.

A simple protocol with these properties is as follows. Letting $\bar{l}$ denote the length of the longest path p from e to d, there are $T = \bar{l}$ periods of information transmission. At time $t = 0$, the expert e's information is $\omega_e(h^0) = \{x\}$, while every other player i has $\omega_i(h^0) = X$. Fix any path p from e to d. At time $t = 0, \ldots, l(p) - 1$, the $t + 1$-th player i on p sends a private message $\hat{m}_{ij}^t \in \{\{x\}, X\}$ to the $t + 2$-th player j.[24] As in Section 2, each message $\hat{m}_{ij}^t$ is subject to the restriction that, if $\omega_i(h^t) = X$, then $\hat{m}_{ij}^t = X$, for any i, j and h^t.

At time T, given any terminal history h^T, player d chooses $\hat{y}_d \in \mathbb{R}$ based on her information $\omega_d(h^T)$ and her equilibrium beliefs.[25] In line with payoff specifications in Section 2, each player i suffers a quadratic loss if player d's choice $\hat{y}_d$ diverges from her realized bliss point $x + b_i$. To capture more realistic preferences, I allow each player i to weight some decision makers' choices more heavily than others. Player i's payoff for decision $\hat{y}_d$ is given by:

$$L_i(\hat{y}_d, x) = -\alpha_{id}(\hat{y}_d - x - b_i)^2,$$

where the utility weights α_{id} satisfy $\alpha_{id} > 0$ for all i, d and $\sum_{d \in \mathcal{N}} \alpha_{id} P(d) = 1$ for all i. I assume that the ex-ante bliss points are ordered as $b_1 < \cdots < b_n$ and that they are common knowledge, consistent with the motivating applications presented in the Introduction.

Given a network N, an expert e, and a decision maker d, let μ_{edN} denote a (possibly

[23] Two players i and j are linked by the path $p = (i, h_1, \ldots, h_{l-1}, j)$ of length l in network N if i is linked to h_1, h_k is linked to h_{k+1} for every $k = 1, \ldots, l - 2$, and h_{l-1} is linked to j.

[24] Under this protocol, players may need to send or receive messages multiple times if there are multiple paths from e to p in network N. Nevertheless, information updating along histories and strategy definitions follow the standard rules of extensive-form games.

[25] As in the model of Section 2, we abstract from the possibility that players could further communicate by cheap talk to players that are not neighbors in N, or to immediate predecessors in a path p from e to d.

mixed) equilibrium strategy profile of the players $i \neq d$ along any path p from e to d, and let y_d denote the associated pure equilibrium strategy of player d. Hence, each player i's expected payoff in the strategic disclosure game, given N, e, and d, under the equilibrium (μ_{edN}, y_d), is

$$u_i(\mu_{edN}, y_d; e, d, N) = -\alpha_{id}\mathbb{E}[(y_d(h^T; \mu_{edN}, N) - x - b_i)^2],$$

where the expectation is taken over h^T and x. If multiple equilibria (μ_{edN}, y_d) exist, I select the one that yields the highest expected payoff $u_i(\mu_{edN}, y_d; e, d, N)$ for all players i. (I will later show that the equilibria (μ_{edN}, y_d) are Pareto-ranked ex ante for all e, d, N.) I refer to this equilibrium as the 'most informative equilibrium' and denote it by (μ^*_{edN}, y^*_d).

Aggregating across all possible realizations of expert–decision maker pairs (e, d), we obtain each player i's ex-ante value of network N, including the costs of the links in N:

$$U_i(N) = -\sum_{(e,d)\in\mathcal{N}^2:e\neq d} \alpha_{id}\mathbb{E}[(y^*_d(h^T; \mu^*_{edN}, N) - x - b_i)^2]P(e, d) - \sum_{j\in\mathcal{N}\setminus\{i\}} N_{ij}c.$$

As in the social communication game introduced by Jackson and Wolinsky (1996), I define the welfare of each network N as the sum of the players' ex-ante values: $W(N) = \sum_i U_i(N)$. Later, I will relate this utilitarian welfare concept to network information transmission efficiency and aggregate link costs. Indeed, we will find that the optimal network here achieves full disclosure—that is, the state x is disclosed from every expert e to any decision maker d for every $x \in X$—at minimal aggregate link cost $c(N) = \sum_{(i,j)\in\mathcal{N}^2:j\neq i} N_{ij}c$.

Network Optimality Proposition 1 has important implications for the study of information transmission in networks. With few exceptions (see Section 1.1), existing models of information diffusion in networks abstract from strategic communication incentives. In these models, the value of connecting players in a network N is determined by network-theoretical characteristics. The value of a path connecting a pair of players i and j is typically assumed to depend on its length, the number of connecting links, or the sum of the weights of those links, as in the learning model by De Groot (1974). In some cases, the network communication model is extended to include characteristics of the connected players i and j when determining the value of their connection (see, for example, Galeotti, Goyal, and Kamphorst, 2006).

The analysis of the full model of strategic disclosure in Section 2 shows that the value of a path p connecting e and d also depends on the characteristics of all intermediate players $i \neq e, d$ along path p. This highlights that models of communication in networks that do not explicitly incorporate strategic communication cannot adequately capture such strategic incentives. The model of network communication must be further augmented to include the biases of all players $i \neq e, d$ along path p, and to account for whether p exhibits bias reversals.

The next part of this section builds on the results of Section 2 to study optimal networks for the strategic disclosure of information. I show that the unique optimal network N is the line in which players are ordered according to their bliss points, which I define as the *ordered line*. Formally, this is the network N such that $N_{ij} = 1$ if and only if $|i - j| = 1$. Proposition 2 below holds for all meaningful link costs c—specifically, costs c that are (i) strictly positive, and (ii) not so large that the optimal network would not be connected even if information flowed freely and strategic disclosure were irrelevant.[26] Specifically, I define the cost threshold $\bar{c}$ as the largest value of c, as a function of P and α, such that the optimal network N would be connected if signal x were disclosed along any path p connecting any pair of players e and d.

Proposition 2 *For any link cost $c \in (0, \bar{c})$, utility weights $\boldsymbol{\alpha} > \mathbf{0}$, and full-support selection probability P, the unique optimal network N is the ordered line: It achieves disclosure of every signal x to every decision maker d from every expert e, at minimal aggregate link cost $c(N)$.*

To build intuition for this result, consider minimally connected networks (i.e., trees): networks where every pair of players is connected by a unique path. The proof of Proposition 2 shows that the ordered line is the unique tree to establish full disclosure—that is, the state x of every expert e is relayed to every decision maker d via the unique path p that connects them. In any other tree N, there exists at least one pair e, d connected through a path p with bias reversals. Proposition 1 then implies that x will not be disclosed to d. As a result, the ordered line is the unique optimal tree.

Next, consider any network that is not a tree. Since the link cost satisfies $c < \bar{c}$, disconnected networks (i.e., networks with pairs of players not connected by any path) are suboptimal, by definition of $\bar{c}$. Further, because $c > 0$, non-minimally connected networks

[26] A network N is connected if every pair i, j is connected by some path in N.

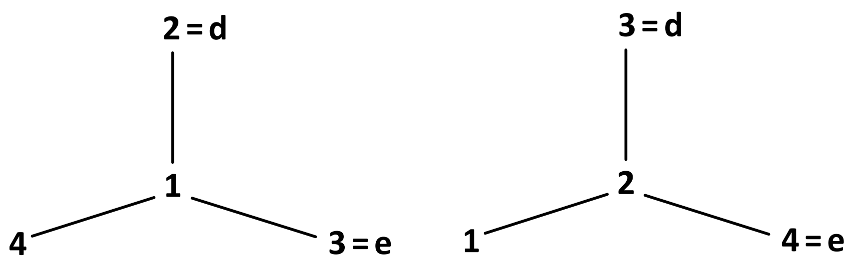

Figure 1: Four-player stars

(i.e., connected networks with loops) are also suboptimal. While they may also yield full disclosure in equilibrium, they do so at a higher aggregate link cost.

Although intuitive, Proposition 2 is not an immediate consequence of the notion of bias reversal. For example, one can construct paths such as $p = (2, 1, 3)$, in which players are not ordered by bliss points and yet no bias reversal occurs, since both players 2 and 1 are biased leftwards relative to the decision maker, player 3.

I present the proof in the main text, as it is both informative and fairly straightforward.

Proof of Proposition 2. Suppose for the moment that the optimal network is a tree. The ordered line clearly has no bias-reversal paths. For any pair of realized decision maker d and expert e with $b_e < b_d$, it must be that $b_i < b_d$ for each player i on the path p from e to d, and vice versa when $b_e > b_d$. By Proposition 1, the state x is always relayed along path p from e to d regardless of their realized identities.

Now consider any other tree N and focus momentarily on the case with $n = 4$ players. Up to relabeling of players, there are only two classes of trees: lines and stars. Of course, any non-ordered line contains paths with bias reversals. Moreover, every 4-player star also contains at least one bias-reversal path. If the center of the star is player $i = 1$ or $i = 2$, then the path p from $e = i+2$ to $d = i+1$ has a bias reversal (see Figure 1). Symmetrically, if the center is $i = 3$ or $i = 4$, the path p from $e = i - 2$ to $d = i - 1$ has a bias reversal.

For any number of players $n \geq 4$, the only tree that contains neither a 4-player star nor a 'non-ordered line' is the n-player ordered line. (The same is trivially true for $n = 2$, and also for $n = 3$, where stars and lines coincide.) So, for every n, because $\boldsymbol{\alpha} > 0$, the unique tree N in which x is transmitted for every realized e, d is the ordered line. By Proposition 1, this is the unique tree N such that for all e, d, the expected variance of d's decision is

$\mathbb{E}[\text{Var}(x|\omega_d(h^T);\mu^*_{edN},N)]=0$.

Using the mean-variance decomposition (1), welfare can be written as

$$W(N)=-n\sum_{(e,d):e\neq d}\mathbb{E}[\text{Var}(x|\omega_d(h^T);\mu^*_{edN},N)]P(e,d)-\sum_{i\in\mathcal{N}}\sum_{(e,d):e\neq d}(b_i-b_d)^2P(e,d)-\sum_{(i,j):j\neq i}N_{ij}c.$$

The aggregate link cost is $c(N)=(n-1)c$ in every tree N. Hence, the tree N that maximizes welfare $W(N)$ is the one that minimizes the sum of expected residual variances $EV(N)=n\sum_{(e,d):e\neq d}\mathbb{E}[\text{Var}(x|\omega_d(h^T);\mu^*_{edN},N)]P(e,d)$. Since the ordered line is the unique tree for which $\mathbb{E}[\text{Var}(x|\omega_d(h^T);\mu^*_{edN},N)]=0$ for all e and d, it is the unique optimal tree for any full-support probability distribution P.

Now, consider networks that are not minimally connected. If a network N is connected but not minimally connected, then its aggregate link cost is $c(N)>(n-1)c$. Because $c>0$, such a network is dominated by the ordered line, even if it achieves the same sum of expected residual variances $EV(N)=0$.

If the network N is not connected, then it is dominated by the ordered line for any $c<\bar{c}$, by the definition of $\bar{c}$ as the largest link cost such that the optimal network would be connected if x were disclosed along any path p connecting any pair of players e and d.

This establishes that the ordered line is the unique optimal network. ■

Endogenous Network Formation I now turn to analyzing which networks arise endogenously in a game where individual players pay the cost of their links. As in the welfare analysis, the network formation game is formulated *ex ante*, i.e., before the identities of the expert e and decision maker d are drawn and before the state x is realized.

Specifically, I model network formation as a 'bilateral sponsorship' game à la Myerson (1991), in which both linked players i and j must pay the cost c for any link to form. For any fixed selection probability P and utility weights $\boldsymbol{\alpha}$, the network N is formed as follows. Each player $i\in\mathcal{N}$ simultaneously submits a list $\ell_i\in\{0,1\}^{\mathcal{N}\setminus\{i\}}$ of players $j\neq i$ she is willing to link to at cost c, where $\ell_{ij}=1$ indicates that i commits to paying. A link between players i and j forms ($N_{ij}=N_{ji}=1$) if and only if both players commit to pay, i.e., $\ell_{ij}=1=\ell_{ji}$.

Once the network N is formed, an expert e and a decision maker d are drawn, the state x is realized and revealed to e, and the disclosure game is played on N. It is evident that the bilateral sponsorship game permits miscoordination in Nash equilibrium: a player i may choose $\ell_{ij} = 0$ solely because she anticipates that j will also play $\ell_{ji} = 0$, even though connecting would benefit both in the disclosure game. While Nash equilibrium identifies strategy profiles $\boldsymbol{\ell}$ that are immune only to individual deviations, I consider profiles $\boldsymbol{\ell}$ that are also immune to coalitional deviations, in line with the concept of core in cooperative games (Gillies, 1959). Specifically, I require that there does not exist any subset $\mathcal{N}' \subseteq \mathcal{N}$ and list profile $\boldsymbol{\ell}'_{\mathcal{N}'} = (\ell'_i)_{i \in \mathcal{N}'}$ such that the network N' induced by $(\boldsymbol{\ell}'_{\mathcal{N}'}, \boldsymbol{\ell}_{\mathcal{N} \setminus \mathcal{N}'})$ satisfies $\sum_{i \in \mathcal{N}'} U_i(N') > \sum_{i \in \mathcal{N}'} U_i(N)$. In other words, no coalition of players $\mathcal{N}'$ should be able to change their sponsored links in a way that makes all its members strictly better off, provided that transfers among them are possible.

With these definitions in place, I can now state my main result on endogenous network formation.

Proposition 3 *For any utility weights $\boldsymbol{\alpha} > 0$ and full-support selection probability P, there exists a cost threshold $\hat{c}$ such that for all $c \in (0, \hat{c})$, the ordered line is the unique network induced by a Nash equilibrium $\boldsymbol{\ell}$ of the bilateral sponsorship game that is immune to coalitional deviations.*

Qualitatively, this result parallels the welfare findings in Proposition 2, and the proof follows a similar logic. Because the ordered line guarantees that every decision maker d receives the signal x from every possible expert e, no player or coalition of players is willing to pay for additional links, given that $c > 0$; nor is any player or coalition willing to delete links when c is smaller than a certain threshold $\tilde{c}$. Further, because the ordered line is the unique optimal network for $c \in (0, \bar{c})$, any other network would be overturned by the coalition of all players deviating to form the ordered line. Letting $\hat{c}$ be the minimum between $\tilde{c}$ and $\bar{c}$ yields the desired result.

The range of link costs $(0, \hat{c})$ for which the ordered line arises in the bilateral sponsorship game as the unique network immune to coalitional deviations can be significantly smaller than the range $(0, \bar{c})$ for which it is uniquely optimal. There, $\bar{c}$ is the highest cost at which the aggregate cost $c(N)$ of a connected network N is offset by the collective benefit of full disclosure—the transmission of x between every e and d. Instead, $\tilde{c}$ is the highest cost $\tilde{c}$ at which each adjacent pair i and $i+1$, for $i = 1, \ldots, n-1$, is willing to pay to form a link ensuring full disclosure in the resulting ordered line. Players do not necessarily value each decision maker's action equally, i.e., α_{id} is not uniform across i and d. Some players may be significantly less willing to pay for ensuring that a particular d receives signals, and in such a case it will be that $\hat{c} = \tilde{c} < \bar{c}$.[27]

4 General Results

Having presented the main results in a streamlined version of the model, this section turns to generalizations.

4.1 Partial Disclosure and Decay

I develop a full-fledged model of verifiable information transmission in networks, in which players may choose to partially disclose their information. I also allow for transmission decay—that is, information may be lost independently of the message sent. This extends the model of Section 3 to include these features.

An expert e and decision maker d are randomly drawn from the players in a network N. Player e observes the state $x \in X$ and may disclose it to d along any path in N. With a unique path between e and d, the framework reduces to the disclosure game of Section 2, augmented for partial disclosure and decay. Let T be the length of the longest path p from e to d in N. At $t = 0$, the information is $\omega_e(h^0) = \{x\}$ and $\omega_i(h^0) = X$ for any $i \neq e$. For any path p from e to d, at time $t = 0, \ldots, l(p) - 1$, the $t+1$-th player i on p sends a message $\tilde{m}^t_{ij} \subseteq X$ to the $t+2$-th player j. Messages $\tilde{m}^t_{ij}$ are non-empty closed sets and are verifiable, i.e., $\omega_i(h^t) \subseteq \tilde{m}^t_{ij}$. At time T, the decision maker d chooses $\hat{y}_d \in \mathbb{R}$.

Decay is modeled as follows. With probability $1 - \delta$, independently across periods t, histories, and player pairs i, j, the message $\tilde{m}^t_{ij}$ is lost in transmission, and player j learns nothing about x. That is, she observes $\hat{m}^t_{ij} = X$, as if i had sent no information. Otherwise, j observes $\hat{m}^t_{ij} = \tilde{m}^t_{ij}$.

At every history h^t, the information $\omega_i(h^t)$ of any player $i \neq e$ is updated in the standard way. Suppose player i receives the vector of messages $\hat{\mathbf{m}}_i = (\hat{m}^t_{ji})$ at history h^t from her immediate predecessors j on paths p from e to d. Then, she updates her information at

[27]For example, suppose that α_{21} is small. Then, player 2 will only be willing to form the link with 1 if the cosy c is small, thus making also $\hat{c} = \tilde{c}$ small.

h^{t+1} according to the rule:

$$\omega_i(h^{t+1}) = \omega_i(h^t) \cap_j \hat{m}^t_{ji}.$$

In words, at history h^t, player i knows that x cannot be outside $\omega_i(h^t)$. Because each received message $\hat{m}^t_{ji}$ is verifiable, she also learns at h^{t+1} that x cannot be outside any set $\hat{m}^t_{ji}$. As a result, any message $\tilde{m}^t_{ij}$ disclosed by player $i \neq e, d$ at history h^t must be a (possibly weak) superset of the intersection of the messages $\hat{m}^\tau_{ji}$ received at times $\tau < t$ in the history h^t.

The disclosure results in Proposition 1 and the optimal network results in Proposition 2 continue to hold in this setting—with some qualifications. Most importantly, decay must be small, meaning δ must be close to one. Formal statements are given in Propositions 5 and 6, in Appendix B.

The analysis underlying Proposition 5 differs qualitatively from that in Section 2. To illustrate this, first assume there is no decay, but partial disclosure is possible. This makes equilibrium analysis significantly more complex for several reasons. First, even a player i who knows x at history h^t, i.e., $\omega_i(h^t) = \{x\}$, may choose to send a neighbor j a message $\tilde{m}^t_{ij}$ that is neither $\{x\}$ nor X. This flexibility gives players a continuum of messages to use, potentially enhancing their ability to convey information in equilibrium by using different messages to signal different states. Second, each intermediary $i \neq e, d$ may possess partial information about x, i.e., $\{x\} \subsetneq \omega_i(h^t) \subsetneq X$, which significantly enriches the message strategy space. Third, players other than d and her immediate predecessors must anticipate how their messages will influence subsequent players' disclosure decisions along the unique path p from e to d considered in Propositions 1 and 5.

These distinctions have no bearing on the proof that if p has no bias reversals, then there exists an equilibrium (μ, y_d) in which all players $i \neq d$ disclose their full information, i.e., $\tilde{m}^t_{ij}(h^t) = \omega_i(h^t)$, at every history h^t where they act, so that every state x is disclosed to d and $\mathbb{E}[\text{Var}(x|\omega_d(h^T), \mu)] = 0$.[28] The analysis becomes more subtle when the path p has bias reversals. In that case, for any belief $\hat{x}$ held by the decision maker upon receiving no information, full disclosure fails whenever $x \in U(\hat{x})$. However, partial disclosure may still occur, and proving that $\mathbb{E}[\text{Var}(x|\omega_d(h^T), \mu)] > 0$ requires accounting for this.

The issue is addressed by considering two cases. Pick any player i on p such that $b_i > b_d$

[28] As in the proof of Proposition 1, the equilibrium is supported by worst-case beliefs: if $b_i > (<)\, b_d$, then d believes $x = \min \omega_d(h^T)$—respectively, $x = \max \omega_d(h^T)$—at every terminal history h^T.

(the case $b_i < b_d$ is symmetric). First, suppose there exists a subset $\tilde{X}_i \subseteq (\hat{x} - b_i + b_d, \hat{x})$ of positive measure such that, for all $x' \in \tilde{X}_i$, player i learns at some h^t that $x \in (\hat{x} - b_i + b_d, \hat{x})$. Then, as in Proposition 1, player i is better off if her information is withheld from player d. Hence, d cannot be disclosed x at any terminal h^T that includes such histories h^t, and $\text{Var}(x|\omega_d(h^T), \mu) > 0$. In the second case, for almost all $x \in (\hat{x} - b_i + b_d, \hat{x})$ and histories h^t on the equilibrium path, player i does not learn that $x \in (\hat{x} - b_i + b_d, \hat{x})$ when this is the case. As a result, i cannot disclose x, player d will not learn it at any subsequent history h^T, and again $\text{Var}(x|\omega_d(h^T), \mu) > 0$. Integrating over all possible histories yields $\mathbb{E}[\text{Var}(x|\omega_d(h^T), \mu)] > 0$.

While partial disclosure introduces only technical complexity, decay leads to structural changes in the equilibrium, even when the path p has no bias reversals. The analysis draws on Dye (1985), who studied strategic direct disclosure to a decision maker by an expert who may be uninformed about the state. In fact, the equilibrium updating by d is analogous to our setting, in which the expert's message may be lost in transmission.

Suppose $b_i > b_d$ for all $i \neq d$ on the path p, and let $\hat{x}(\delta)$ solve

$$(1 - \delta^T)\mathbb{E}[x] + \delta^T \mathbb{E}[x|x \leq \hat{x}(\delta)] = \hat{x}(\delta). \tag{2}$$

Because $\delta < 1$, there exist terminal histories h^T such that $\omega_d(h^T) = X$. For every δ sufficiently close to one, an equilibrium (μ^δ, y_d^δ) is shown to exist in which all players $i \neq d$ disclose x if and only if $x > \hat{x}(\delta)$, and otherwise send $\tilde{m}_{ij}^t = X$. Player d's expectation of x upon receiving no information equals the left-hand side of (2), so she chooses $\hat{y} = \hat{x}(\delta) + b_d$. This gives each player $i \neq d$ a strict incentive to disclose x if and only if $x > \hat{x}(\delta)$, because $b_i > b_d$, thereby confirming the proposed strategy profile is an equilibrium. Further, it is immediate that $\lim_{\delta \to 1} \hat{x}(\delta) = \underline{x}$, and thus $\lim_{\delta \to 1} \mathbb{E}[\text{Var}(x|\omega_d(h^T), \mu)] = 0$.

Instead, if p has bias reversals, it is still the case that $\lim_{\delta \to 1} \mathbb{E}[\text{Var}(x|\omega_d(h^T), \mu^\delta)] > 0$ for any sequence of equilibria (μ^δ, y_d^δ), as when there is no decay. This completes the analysis leading to Proposition 5.

With Proposition 5 in place, Proposition 6 follows by arguments parallel to those in Section 3 used to establish Proposition 2. The conclusions, however, no longer hold across the full cost range $(0, \bar{c})$ when decay is present. Because $\delta < 1$, networks that are not minimally connected can outperform the ordered line when c is very small. Also, when c is near the upper bound $\bar{c}$, optimal networks may be disconnected. Nonetheless, for δ

close to 1, there exists an "intermediate" cost range (c_-, c^+) with $0 < c_- < c^+ < 1$ such that the ordered line is the unique optimal network. As decay vanishes, i.e., as $\delta \to 1$, this intermediate range expands to encompass the entire interval $(0, \bar{c})$. Similar logic applies when generalizing Proposition 3.

4.2 General Functional Forms

The analysis in Section 3 assumed that the expert e knew the state x precisely, and that players' loss functions L_i took the quadratic form $L_i(y, x) = -(y - x - b_i)^2$, with a state-independent bias b_i. I now show how to relax these assumptions and generalize the main findings.

Suppose that e observes a signal $s \in S = [\underline{s}, \overline{s}] \subset \mathbb{R}$. The distribution of s given x is determined by the density $g(s|x)$, which is assumed to be strictly positive on S. The signal s is informative about x in the sense that it satisfies the monotone likelihood ratio property: if $s' > s$ and $x' > x$, then $g(s'|x')/g(s|x') > g(s'|x)/g(s|x)$.

Further, consider a general loss function $L_i(y, x)$ that is twice continuously differentiable and satisfies two properties: (i) concavity, so that $\partial^2 L_i / \partial y^2 < 0$; and (ii) supermodularity, meaning $\partial^2 L_i / \partial y \partial x > 0$ and $\partial L_{i+1}/\partial y > \partial L_i / \partial y$. Each player i's expected value from any equilibrium (μ_{edN}, y_d) of the disclosure game—given network N, expert e, and decision maker d—is:

$$u_i(\mu_{edN}, y_d; e, d, N) = \alpha_{id}\, \mathbb{E}\left[L_i\left(y_d(h^T; \mu_{edN}, e), x\right)\right].$$

The concavity of L_i and the monotone likelihood ratio property ensure that, for any signal $s \in S$, there exists a unique decision $y_i(s)$ that maximizes player i's expected payoff $\mathbb{E}[L_i(y, x)|s]$. Together with supermodularity, these properties imply that $\mathbb{E}[L_i(y, x)|s]$ is strictly increasing (decreasing) in y for all $y \leq (\geq) y_d(s)$ if $i > (<)\, d$. So, for any signal s, player $i > d$ prefers to bias d's decision to the right, and vice versa. A path p from e to d has bias reversals if there exist i, j on p such that $i < d < j$.

To capture the trade-off between the overall quality of information disclosure and the aggregate cost of links, I earlier measured a network's efficiency as the probability-weighted sum of players' optimal equilibrium payoffs $u_i(\mu^*_{edN}, y^*_d; e, d, N)$ over expert–decision maker pairs (e, d). Unlike in Section 3 however, the ex-ante equilibrium payoffs $u_i(\mu_{edN}, y_d; e, d, N)$ can no longer be decomposed into a common component—such as the residual variance

$\mathbb{E}[\text{Var}(x|\omega_d(h^T), \mu)]$—and an idiosyncratic loss term $(b_i - b_d)^2$ independent of equilibrium choices. This lack of alignment in payoffs across players makes the earlier efficiency measure unsuitable. Here, I directly define as *optimal* any network N that achieves disclosure of each expert e's signal s to every decision maker d while minimizing the aggregate link cost $c(N)$.[29]

The equilibrium disclosure results of Section 2, and the optimality results of Section 3 extend to this more general setting. The formal statements appear as Propositions 7 and 8 in Appendix B.

4.3 Many Experts and Decision Makers

I consider the case in which multiple players may have information about x and may be called to make decisions. A non-empty set $E \subseteq \mathcal{N}$ of experts and a non-empty set $D \subseteq \mathcal{N}$ of decision makers are randomly drawn from a full-support distribution P. Each expert $e \in E$ holds a signal $s_e \in S$, and each decision maker $d \in D$ chooses an action $\hat{y}_d \in \mathbb{R}$ after communication occurs over network N. Each player i's loss function L_i takes the quadratic form of Section 3: $L_i(\hat{\mathbf{y}}_D, x) = -\sum_{d \in D} \alpha_{id}(\hat{y}_d - x - b_d)^2$.

In line with the ideas of Jackson and Wolinsky (1996), I assume that the signal s_e of each e is valuable to every d and cannot be replicated by the information of others in the network. Specifically, for any set of experts E, I first assume that the distribution g_E of the signal profile $\mathbf{s}_E$ satisfies the monotone likelihood ratio property. Furthermore, for any signal profile $\mathbf{s}_E$, player $e \in E$ and signal s_e, and any set S_e such that $\{e\} \subsetneq S_e \subseteq S$, I assume that knowing s_e is more informative than knowing only S_e—in the sense that it leads to a lower expected quadratic loss. This holds, for example, when the signals s_e are i.i.d. conditional on x and the inference problem is not trivial.

Assumption 1 *For any set of experts $E \subseteq \mathcal{N}$, the distribution g_E satisfies:*

a. if $\mathbf{s}'_E \geq \mathbf{s}_E$ and $x' > x$, then $g_E(\mathbf{s}'_E|x')/g_E(\mathbf{s}_E|x') > g_E(\mathbf{s}'_E|x)/g_E(\mathbf{s}_E|x)$,

b. for any profile $\mathbf{s}_E$, any player $e \in E$ and signal s_e and any set $S_e : \{s_e\} \subsetneq S_e \subseteq S$,

$$\mathbb{E}[\text{Var}(x|\{\mathbf{s}_{E\setminus\{e\}}\} \times S_e)] > \mathbb{E}[Var(x|\{\mathbf{s}_E\})]. \tag{3}$$

[29] This definition is meaningful only if c is not so high that a disconnected network becomes preferable despite the loss of disclosure.

Given a network N with realized sets E and D, the transmission of signals s_e from experts $e \in E$ to decision makers $d \in D$ follows the protocol considered earlier in this section, with the qualification that each player i's information set $\omega_i(h^t)$ at any history h^t is a possibly proper subset of S^E.[30] Let T denote the length of the longest path p from any $e \in E$ to any $d \in D$. At $t = 0$, the information sets $\omega_i(h^0)$ satisfy $\omega_e(h^0)|_e = \{s_e\}$ for all $e \in E$, and $\omega_i(h^0)|_j = S$ for all other i, j, where the index j refers to information about signal s_j. For every pair (e, d) with $e \in E$ and $d \in D$, and for any path p from e to d, at each time $t = 0, \ldots, l(p) - 1$, the $t + 1$-th player i on p sends a verifiable message $\hat{m}_{ij} \subseteq S^E$—that is, $\omega_i(h^t) \subseteq \hat{m}_{ij}$—to the $t+2$-th player j. This allows for partial disclosure, for generality. At every history h^t, each player's information set $\omega_i(h^t)$ is updated in the standard way. At time T, every decision maker $d \in D$ chooses her $\hat{y}_d$.

The next result restates Proposition 2 in this generalized environment.

Proposition 4 *Suppose that $\boldsymbol{\alpha} > 0$, non-empty sets E and D of experts and decision makers are chosen with a full-support distribution P, and Assumption 1 holds: all signals s_e are informative about x and none is redundant. Then, for any link cost $c \in (0, \bar{c})$, the ordered line is the unique optimal network.*[31]

The core of the proof is to show that, for all realized sets E and D, the signals of all experts reach all decision makers when the network N is the ordered line. The logic is similar to that of Proposition 1. Each decision maker d knows how to interpret the withholding of a signal s_e. Signals from experts $e > d$, who are biased to the right, are transmitted to d through intermediaries i with $e \geq i > d$, who are also biased to the right. Suppose one such signal s_e is not disclosed to d. Then d presumes that the withheld information supports a more leftward action—she forms an off-path belief that $s_e = \min \omega_d(h^T)|_e$. This leads her to choose the most rightward decision consistent with $\omega_d(h^T)|_e$, deterring all players $i > d$ from withholding any s_e with $e \geq i > d$ on the path from e to d. A symmetric argument ensures that signals from experts $e < d$, who are biased to the left, are also disclosed to d.

These arguments imply that the signals s_e of all experts e reach all decision makers d when N is the ordered line. The suboptimality of any other network, and hence the result

[30] This construction implies that both the content of each signal s_e and the identity of the expert e who originated it are verifiable. Proposition 4 extends qualitatively to the case in which only the content of each signal s_e is verifiable.

[31] As in Proposition 2, $\bar{c}$ denotes the maximum link cost such that full disclosure of all signals s_e to every decision maker justifies the aggregate cost of a connected network.

that only the ordered line is the unique network immune to coalitional deviations, follow from the same arguments used in Section 3. These conclusions rely on the fact that the distribution P has full support. In other words, any pair (E, D) with $E = \{e\}$ and $D = \{d\}$ can be selected with positive probability. By Proposition 2, the ordered line is the unique tree that guarantees that the signal s_e of expert e reaches decision maker d regardless of their realized identities. As a result, also here, the ordered line is the unique optimal tree. And again, non-tree networks are suboptimal as long as $c \in (0, \bar{c})$, link costs are positive, and not too large.

5 Conclusion

This paper has studied the strategic transmission of verifiable information through intermediaries. I have shown that full disclosure occurs in equilibrium if and only if the expert and all intermediaries along the information transmission path are biased in the same direction relative to the decision maker.

By embedding this strategic disclosure framework into a network setting, this paper bridges two major strands of economic theory: information transmission in networks and strategic communication. When each player in the network may hold information relevant for others' decisions, I have found that the unique ex-ante optimal network is the *ordered line:* the network where each player connects only to those with the most similar preferences. This network structure prevents strategic information withholding, and thus ensures efficient information flow, at minimal aggregate link cost. Finally, the paper has shown that the ordered line also arises endogenously as the unique network that is immune to coalitional deviations in a bilateral link formation game à la Myerson (1991).

These findings have important implications for political economy, where strategic communication often shapes interactions between political decision makers. When political agents connect with those who share similar views, no verifiable information can be withheld in equilibrium. If any information were withheld, decision makers would be able to identify which political side was responsible and adjust their actions accordingly. By contrast, if agents are connected in any other configuration within a minimally connected network, full information disclosure cannot be sustained.

This study opens several avenues for future research. One promising direction concerns the optimality of networks under uncertainty about player preferences. Two cases are of particular interest: (i) preferences are unknown ex-ante to the social planner but are common knowledge among players in the disclosure game, and (ii) preferences are unknown ex-ante and remain privately known to the individual players engaged in the disclosure game.

The first case is simpler. Unless the link cost c is too large, if players' bliss points b_i are unknown ex-ante, the optimal network N is complete (i.e., $N_{ij} = 1$ for all pairs i, j). This is the only network that guarantees full disclosure from every expert to every decision maker. If $N_{ij} = 0$ for any pair i, j, it is ex-ante possible that i is the most left-leaning (or right-leaning) player and j is the second most left-leaning (or right-leaning). Since they are not connected, any path from j to i would involve a bias reversal, preventing j's information from reaching i. I leave the analysis of the second case for future work.

Another promising direction is to investigate the implications of multidimensional states and decisions. A more ambitious avenue would be to incorporate repeated models of strategic communication and examine how networks evolve over time.

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Appendix A: Omitted Proofs

Proof of Proposition 1. Part a. Suppose the path p has no bias reversals, e.g., $b_i > b_d$ for all $i \neq d$. (The proof for $b_i < b_d$ for all $i \neq d$ is the mirror-like image.)

We construct the following strategy profile and verify that it constitutes a Perfect Bayesian Equilibrium. The disclosure strategies μ are as follows. Every player $i \neq d$ who knows x—i.e., $\omega_i(h^t) = \{x\}$—at a history h^t at which she is called to play discloses x to her successor j on p, i.e., she plays $m_{ij}(h^t) = \{x\}$. Of course, if i does not know x, so that $\omega_i(h^t) = X$, then she cannot disclose anything and must send $m_{ij}(h^t) = X$, her beliefs are irrelevant. The decision maker d adopts the strategy $y_d(h^T) = \min \omega_d(h^T) + b_d$ for every history h^T.

Under this profile, the message $\hat{m}_{ij} = \{x\}$ travels along the path p from the expert e to the decision maker d on the path of play. Consequently, d chooses $y_d(h^T) = x + b_d$, and thus the expected residual variance satisfies $\mathbb{E}[\mathrm{Var}(x|\omega_d(h^T), \mu)] = 0$.

To verify this is an equilibrium, first note that all beliefs are admissible. Perfect Bayesian Equilibrium requires beliefs to be consistent with Bayes rule only on information sets $\omega_j(h^t)$ on the equilibrium path. For every state x, the unique terminal information sets on path is $\omega_d(h^T) = \{x\}$ and $\min \omega_j(h^t) = x$, consistently. Further, d's strategy y_d is clearly optimal given her beliefs.

Now, note that by construction any player $i \neq e$ does not know x, i.e., $\omega_i(h^\tau) = X$, at any history h^τ before she is called to play. Thus i's information at any history h^t she is active coincides with the message $\hat{m}_{ji}^{t-1}$ received in the history h^t at time $t-1$ from her immediate predecessor j on path p: $\omega_i(h^t) = \hat{m}_{ji}^{t-1}$. For the same reason, i knows x at a history h^t if and only if all her predecessors j on p disclosed x along that history h^t—that is, each sent $\hat{m}_{jk}^{\tau} = \{x\}$ to her immediate successor k on p at the time τ when she was active. Hence, there exists a unique history h^t in which i knows x.

Next, we verify that each player $i \neq d$ has no incentive to deviate from the stipulated strategy μ at any history h^t she is active. Of course, if $\omega_j(h^t) = X$, then she can only send $\hat{m}_{ij}^t = X$ to her immediate successor j on p, consistently with μ. So, consider the history h^t such that $\omega_i(h^t) = \{x\}$. Player i anticipates that all her successors $j \neq d$ on p will play $\hat{m}_{jk}^\tau = \omega_j(h^\tau)$ at any history h^τ that extends h^t, by the imputation that they adhere to μ. This, together with the results in the previous paragraph, implies that i anticipates that d's information at any terminal history h^T that extends h^t will coincide with the message she sends to her immediate successor j on p, i.e., $\omega_d(h^T) = \hat{m}_{ij}^t$.

If i sends $\hat{m}_{ij}^t = X$, then d plays $y_d(h^T) = \underline{x} + b_d$. If instead i sends $\hat{m}_{ij}^t = \{x\}$, then d chooses $y_d(h^T) = x + b_d$. Because i knows x at history h^t, her payoff is $L_i(\hat{y}, x) = -(\hat{y} - x - b_i)^2$, which increases in $\hat{y}$ for $\hat{y} < x + b_i$. As $b_i > b_d$, player i prefers the outcome $\hat{y} = x + b_d$ to the outcome $\hat{y} = \underline{x} + b_d$. Hence, she does not deviate from sending $\hat{m}_{ij}^t = \{x\}$. As the argument holds for all $x \in X$, it concludes the verification that (μ, y_d) is a Perfect Bayesian Equilibrium.

I now show that in every equilibrium (μ, y_d), the message $\hat{m}_{ij} = \{x\}$ travels from e to d along path p, and d chooses $\hat{y} = x + b_d$, so that $\mathbb{E}[\mathrm{Var}(x|\omega_d(h^T), \mu)] = 0$. Suppose not. Then there exist a non-null subset $\hat{X} \subseteq X$ such that $y_d(h^T) \neq x + b_d$ and hence that $\omega_d(h^T) \neq \{x\}$, for all $x \in \hat{X}$ and some terminal histories h^T containing x on the equilibrium path. For all such x and h^T, player d observes $\omega_d(h^T) = X$ and chooses $y(h^T) = \hat{x} + b_d$, where $\hat{x} = \mathbb{E}[x|X, \mu]$.

By the intermediate value theorem, there is a non-null subset $\tilde{X} \subseteq \hat{X}$ such that $x > \hat{x}$ for all $x \in \tilde{X}$. Pick any such an x, and proceed by backward induction, labelling the players on p as $i_0 = e$, $i_1, \ldots, i_{T-1}$, $i_T = d$.

Start with i_{T-1} at the history h^{T-1} where $\omega_{i_{T-1}}(h^{T-1}) = \{x\}$. Because $x > \hat{x}$ and $b_{i_{T-1}} > b_d$, player i_{T-1} strictly prefers that d chooses $x + b_d$ rather than $\hat{x} + b_d$. Thus, she strictly prefers sending $\hat{m}_{i_{T-1}, i_T} = \{x\}$ over $\hat{m}_{i_{T-1}, i_T} = X$. Of course, at any other history h^{T-1}, player i_{T-1} sends $\hat{m}_{i_{T-1}, i_T} = X = \omega_i(h^{T-1})$.

Now consider any player i_t with $t < T-1$ at the history h^t such that $\omega_i(h^t) = \{x\}$. Let the induction hypothesis be that for any $\tau = 1, \ldots, T-1-t$, player $i_{t+\tau}$ sends $\hat{m}_{i_{t+\tau} i_{t+\tau+1}} = \omega_{i_{t+\tau}}(h^{t+\tau})$ to her immediate successor $i_{t+\tau+1}$. Then, i_t anticipates that for any terminal history h^T that extends h^t, player d will observe $\omega_d(h^T) = \hat{m}_{i_t i_{t+1}}$: player d will know x if and only if i_t discloses it to her immediate successor i_{t+1}. As is the case for i_{T-1}, because $x > \hat{x}$ and $b_{i_t} > b_d$, player i_t strictly prefers to send $\hat{m}_{i_t, i_{t+1}} = \{x\}$ over $\hat{m}_{i_t, i_{t+1}} = X$.

By induction, we conclude that all players $i \neq d$ strictly prefer to send $\hat{m}_{i,j} = \{x\}$ to their immediate successor j on p, starting from from the expert e, who knows x. This concludes that d cannot observe $\omega_d(h^T) = X$ at any terminal history h^T on the equilibrium path when $x \in \tilde{X}$, thereby establishing the desired contradiction.

Part b. Suppose path p has bias reversals. Consider terminal histories h^T with $\omega_d(h^T) = X$. Let the belief $\hat{x} = \mathbb{E}[x|X; \mu]$ be arbitrary for now (the proof will show that $\omega_d(h^T) = X$ must be on the equilibrium path, and hence that $\hat{x}$ is determined by Bayes rule.)

Pick any player i with $b_i > b_d$. For any history h^t at which i is called to play, she may hold either $\omega_i(h^t) = \{x\}$ or X. If $\omega_i(h^t) = X$, then she cannot disclose x. Consider any state $x \in X$ such that $\hat{x} + 2(b_d - b_i) < x < \hat{x}$, and suppose that $\omega_i(h^t) = \{x\}$. In the terminal history h^T such that $\omega_d(h^T) = \{x\}$, player d's decision is $\hat{y} = x + b_d$. Because $2(b_i - b_d) > \hat{x} - x > 0$, it follows that

$$
\begin{aligned}
u_{id}(\hat{x} + b_d, x) - u_{id}(x + b_d, x) &= -(\hat{x} + b_d - x - b_i)^2 + (b_d - b_i)^2 \\
&= -(\hat{x} - x)\left[(\hat{x} - x) - 2\left(b_i - b_d\right)\right] > 0.
\end{aligned}
$$

Player i strictly prefers that d is not disclosed x and plays $\hat{y} = \hat{x} + b_d$, rather than d learns x and plays $\hat{y} = x + b_d$.

Suppose that, in the considered equilibrium, i's successors j on p relay x to their immediate successors k, i.e., play $\hat{m}^{\tau}_{jk} = \{x\}$ at the history h^{τ} in which they know x. Then, if

i sends $\hat{m}_{ij}^t = \{x\}$ to her immediate successor j, player d will eventually learn x: it will be that $\omega_d(h^T) = \{x\}$ for the equilibrium terminal history h^T that contain h^t and $\hat{m}_{ij}^t = \{x\}$. Hence, i strictly prefers sending $\hat{m}_{ij}^t = X$, as this implies that $\omega_d(h^T) = X$ at every subsequent history h^T. Of course, if in equilibrium some successors j on p play $\hat{m}_{jk}^\tau = X$ at the history h^τ in which they know x, then i's choice at h^t is irrelevant. In either case, no state $x \in (\hat{x}, \hat{x} + 2\max_{b_i > b_d}(b_i - b_d))$ can be disclosed to d and lead to $\omega_d(h^T) = \{x\}$ at any history h^T on the equilibrium path.

The converse—namely, that all states $x \notin [\hat{x}, \hat{x} + 2\max_{b_i > b_d}(b_i - b_d)]$ must be disclosed to d on the equilibrium path—is established via an induction argument analogous to the one in Part a.

A symmetric argument shows that no state $x \in (\hat{x} - 2\min_{b_i < b_d}(b_i - b_d), \hat{x})$ can lead to $\omega_d(h^T) = \{x\}$ in equilibrium, and that all states $x \notin [\hat{x} - 2\min_{b_i < b_d}(b_i - b_d), \hat{x}]$ must be be disclosed to d on the equilibrium path.

As a consequence, d plays $\hat{y} = \hat{x} + b_d$ if $x \in U(\hat{x}) = (\hat{x} - 2(b_h - b_d), \hat{x} + 2(b_d - b_\ell))$ and $\hat{y} = x + b_d$ if $x \notin [\hat{x} - 2(b_h - b_d), \hat{x} + 2(b_d - b_\ell)]$. Thus, the expected residual variance of equilibrium (μ, y_d) is $\mathbb{E}[\text{Var}(x|\omega_d(h^T), \mu)] = \mathbb{E}[(\hat{x} - x)^2 | U(\hat{x})] = \text{Var}(x|U(\hat{x}))$, where, by consistency of the decision maker d's beliefs on $U(\hat{x})$, it must be that $\hat{x} = \mathbb{E}[x|U(\hat{x})]$. As a result, the equilibrium expected residual variance can be bounded as follows:

$$\min_{(\mu, y_d)} \mathbb{E}[\text{Var}(x|\omega_d(h^T), \mu)] \geq \min_{\hat{x} \in X} \int_{\max\{\underline{x}, \hat{x} - 2(b_h - b_d)\}}^{\min\{\hat{x} + 2(b_d - b_\ell), \overline{x}\}} (\hat{x} - x)^2 f(x) dx > 0.$$

Part c. The results follows from Part b as corollaries. If either $b_h - b_d > 0$ or $b_d - b_\ell > 0$ is sufficiently large in absolute value, then the state space X coincides with $U(\hat{x})$ for all $\hat{x} \in X$. Hence, $\hat{x} = \mathbb{E}[x]$, and player d's decision is $y(h^T) = \mathbb{E}[x] + b_d$ for almost every h^T. The equilibrium expected residual variance is

$$\mathbb{E}[\text{Var}(x|\omega_d(h^T), \mu)] = \int_X (\mathbb{E}[x] - x)^2 f(x) dx = \text{Var}(x).$$

■

Proof of Proposition 3. The proof that the ordered line is immune from coalitional deviations for any c small enough is immediate. Because the ordered line has no bias-reversal paths, Proposition 1 implies that every state $x \in X$ is transmitted from e to d regardless of their realized identities. Given that $P(e, d) > 0$ for all e, d, and $\boldsymbol{\alpha} > \mathbf{0}$, there exists a threshold $\tilde{c} > 0$ such that for all link costs $c \leq \tilde{c}$, deleting links from the ordered line is detrimental to each player, and thus to any coalition of players. Further, adding any link to the ordered line is costly, as $c > 0$, and does not improve disclosure of x. So it cannot be beneficial to any coalition.

The proof that the ordered line is the unique network immune from coalitional deviations is also immediate, as it is the unique optimal network for $c \in (0, \bar{c})$. Thus every other network N can be blocked by a deviation of the grand coalition (i.e., the set of all players) forming the ordered line.

Letting $\hat{c} = \min\{\tilde{c}, \bar{c}\}$ yields the desired result. ■

Proof of Proposition 4. The result is proved for general disclosure games that allow for partial disclosure. For clarity, I distinguish two parts.

Part a. Suppose that the network N is the ordered line. I show that there exists a Perfect Bayesian Equilibrium such that, for all sets E of experts and D of decision makers, every $d \in D$ receives the signal s_e from every $e \in E$.

Define the sets $I_+(E, D) = \{i : d < i \leq e \text{ for some } e \in E, d \in D\}$ and $I_-(E, D) = \{i : e \leq i < d \text{ for some } e \in E, d \in D\}$. Because N is the ordered line, $I_+(E, D)$ identifies players who relay information from experts to decision makers to their left, and $I_-(E, D)$ those who relay information to decision makers to their right.

Consider a profile of strategies μ in which every player $i \in I_+(E, D)$ discloses all her information $\omega_i(h^t)|_e \subseteq S$ about the signals s_e of all experts $e \in E$ such that $e \geq i$ to her neighbor $i - 1$ at any history h^t she is active, i.e., she sends message $m_{i,i-1}(h^t)|_e = \omega_i(h^t)|_e$ for all $e \geq i$. The transmission of information about s_e of $e < i$ is irrelevant. Symmetrically, every player $i \in I_-(E, D)$ discloses all her information $\omega_i(h^t)|_e$ about the signals s_e of all $e \in E$ such that $e \leq i$ to her neighbor $i + 1$ at all histories h^t she is active, sending $m_{i,i+1}(h^t)|_e = \omega_i(h^t)|_e$ for all $e \leq i$.

To complete the construction of profile $(\mu, \mathbf{y}_D)$ and associated beliefs, suppose that every player $d \in D$ at any terminal history h^T believes that $s_e = \min \omega_d(h^T)|_e$ with probability one for every $e \in E : e > d$, and $s_e = \max \omega_d(h^T)|_e$ with probability one for every $e < d$, and that player $d \in D$ chooses $y_d(h^T) = \mathbb{E}[x|\mathbf{s}_E] + b_d$ according to such a profile of signals $\mathbf{s}_E = (s_e)$.

Because N is the ordered line, note that on the path induced by strategy profile μ, the signal s_e of every expert $e \in E$ travels to every $d \in D$, and d chooses $y_d(h^T) = \mathbb{E}[x|\mathbf{s}_E] + b_d$, implying that $\mathbb{E}[\text{Var}(x|\omega_d(h^T), \mu)] = \mathbb{E}[\text{Var}(x|\mathbf{s}_E)]$.

To verify that this is a Perfect Bayesian Equilibrium, first note that the beliefs are consistent. For any $d \in D$ and signal s_e of any $e \in E$, the information sets $\omega_d(h^T)$ at the terminal histories h^T on the path induced by μ satisfy $\omega_d(h^T)|_e = \{s_e\}$, and thus $\min \omega_d(h^T)|_e = \max \omega_d(h^T)|_e = s_e$, ensuring consistency. Further, the strategy y_d of each player $d \in D$ is sequentially rational given her beliefs, by construction.

To continue with the verification, I now show that no player $i \in I_+(E, D)$ gains by deviating from the strategy μ that prescribes $m_{i,i-1}(h^t)|_e = \omega_i(h^t)|_e$ for all $e \in E$ such that $e \geq i$, at any history h^t at which she is active.

Player i's expected payoff at history h^t as a function of her message $\hat{m}^t_{i,i-1}$ can be written as follows:

$$\mathbb{E}[L_i(\hat{\mathbf{y}}_D, x)|\omega_i(h^t), \mu; \hat{m}^t_{i,i-1}] = \sum_{d \in D} \alpha_{id} \mathbb{E}[L_i(\hat{y}_d, x)|\omega_i(h^t), \mu; \hat{m}^t_{i,i-1}],$$

and note that for any $d \in D$, the expression $\mathbb{E}[L_i(\hat{y}_d, x)|\omega_i(h^t), \mu; \hat{m}^t_{i,i-1}]$ is a quadratic loss function, as it is the integral of the quadratic loss functions $L_i(\hat{y}_d, x) = -(\hat{y}_d - x - b_i)^2$.

For any $d \in D : d > i$, the choice $\hat{m}^t_{i,i-1}$ is irrelevant provided that every player j (including possibly i) abide by the strategy μ at any history $h^\tau \neq h^t$. This is because N is the ordered line and $i - 1$ does not lie on the path from i to d.

Pick any $d \in D : d < i$. Because N is the ordered line, i does not lie on any path from e to d for any $e \in E : e < i$. So, i's choice $\hat{m}^t_{i,i-1}|_{e<1}$ is irrelevant for the disclosure of signals s_e from such e to d, and she anticipates that every such signals s_e will be disclosed to d precisely, under μ.

Hence consider only the signals s_e of $e \in E : e \geq i$. Player i knows that she conforms to strategy μ and plays $\hat{m}^\tau_{i,i-1}|_{e \geq i} = \omega_j(h^\tau)|_{e \geq i}$ at any history $h^\tau \neq h^t$ at which she is active. Further, player i presumes that every player $j \neq i : d < j$ conforms to strategy μ and hence anticipates that they send message $\hat{m}^\tau_{j,j-1}|_{e \geq j} = \omega_j(h^\tau)|_{e \geq j}$ to their neighbors $j - 1$ at any history h^τ at which they are active.

Hence, there are 3 possibilities with respect to the implications of i's choice $\hat{m}^t_{i,i-1}|_{e \geq j}$ at history h^t with respect to the information $\omega_d(h^T)|_e$ about s_e of $e \geq i$ held by d at any terminal histories h^T that extends h^t. First, it may be that $\omega_d(h^T)|_e = \{s_e\}$ for all such h^T

regardless of i's choice $\hat{m}^t_{i,i-1}$ at history h^t. This happens when s_e is disclosed to d along a sequence of play from e to d where i moves at a time $\tau \neq t$. Second, it may be that $\omega_d(h^T)|_e$ is independent of $\hat{m}^t_{i,i-1}$ because $d-i > T-t$: the message $\hat{m}^t_{i,i-1}$ does not reach d because the path from i to d is longer than the number of disclosure periods from t to T. Third, and final, it may be that $\omega_d(h^T)|_e = \hat{m}^t_{i,i-1}|_e$ for all h^T that extends h^t. This occurs when t is the unique time τ such that $d-i \leq T-\tau$ at which i is active. In this case, player i controls the information about s_e that d will eventually receive.

As a result, player i expectation about player d's decision $y_d(h^T)$ as a function of her message $\hat{m}^t_{i,i-1}|_{e\geq i}$ is

$$\mathbb{E}[y_d(h^T)|\omega_i(h^t), \mu; \hat{m}^t_{i,i-1}|_{e\geq i}] = \mathbb{E}[\mathbb{E}[x|\left(\underline{\mathbf{s}}_{e\geq i}(\omega_d(h^T)|_{e\geq i}), \mathbf{s}_{e<i}\right)]|\omega_i(h^t)] + b_d,$$

where for any $e \geq i$, either $\omega_d(h^T)|_e = \hat{m}^t_{i,i-1}|_e$, or $\omega_d(h^T)|_e$ is independent from $\hat{m}^t_{i,i-1}|_e$. In the expression on the right-hand side, the external expectation is taken with respect to any profile of signals $\mathbf{s}_{e<i}$, whereas $\underline{\mathbf{s}}_{e\geq i}(\omega_d(h^T)|_{e\geq i})$ denotes the profile of signals $\mathbf{s}_{e\geq i}$ such that $s_e = \min \omega_d(h^T)|_e$ for all $e \geq i$.

To continue, because N is the ordered line, for any terminal history h^T that extends h^t, it holds that $\omega_i(h^t)|_{e\geq i} \subseteq \omega_d(h^T)|_{e\geq i}$. That is, everything d learns about a signal s_e from an expert $e \geq i$ must also be known by i. As a consequence, $\min \omega_d(h^T)|_e \leq \min \omega_i(h^t)|_e$ by set inclusion for all $e \geq i$, so that

$$\mathbb{E}[x|\left(\underline{\mathbf{s}}_{e\geq i}(\omega_d(h^T)|_{e\geq i}), \mathbf{s}_{e<i}\right)] \leq \mathbb{E}[x|(\underline{\mathbf{s}}_{e\geq i}(\omega_i(h^t)|_{e\geq i}), \mathbf{s}_{e<i})] \leq \mathbb{E}[x|(\omega_i(h^t)|_{e\geq i}, \mathbf{s}_{e<i})],$$

where the first inequality follows from supermodularity, and the second one by definition of $\underline{\mathbf{s}}_{e\geq i}(\omega_i(h^t)|_{e\geq i})$ as the profile $\mathbf{s}_{e\geq i}$ such that $s_e = \min \omega_i(h^T)|_e$ for all $e \geq i$.

Integrating across $\mathbf{s}_{e<i}$, and using $b_i > b_d$, I obtain:

$$\begin{aligned}\mathbb{E}[y_d(h^T)|\omega_i(h^t), \mu; \hat{m}^t_{i,i-1}|_{e\geq i}] &= \mathbb{E}[\mathbb{E}[x|\left(\underline{\mathbf{s}}_{e\geq i}(\omega_d(h^T)|_{e\geq i}), \mathbf{s}_{e<i}\right)]|\omega_i(h^t)] + b_d \\ &< \mathbb{E}[\mathbb{E}[x|(\omega_i(h^t)|_{e\geq i}, \mathbf{s}_{e<i})]|\omega_i(h^t)] + b_i = \mathbb{E}[x|\omega_i(h^t), \mu] + b_i.\end{aligned}$$

Note that the latter is the bliss point of player i's expected utility $\mathbb{E}[L_i(\hat{y}_d, x)|\omega_i(h^t), \mu]$ from d's choice $\hat{y}_d$. Hence, i would like to induce the highest $\mathbb{E}[y_d(h^T)|\omega_i(h^t), \mu; \hat{m}^t_{i,i-1}|_{e\geq i}]$ possible, through her message $\hat{m}^t_{i,i-1}|_{e\geq i}$. By verifiability, it must be the case that $\omega_i(h^t)|_{e\geq i} \subseteq \hat{m}_{i,i-1}|_{e\geq i}$. By set inclusion and the definition of $\underline{\mathbf{s}}_{e\geq i}(\cdot)$, player i cannot increase $\mathbb{E}[y_d(h^T)|\omega_i(h^t), \mu; \hat{m}^t_{i,i-1}|_{e\geq i}]$ by sending any message $\hat{m}^t_{i,i-1}|_{e\geq i} \neq \omega_i(h^t)|_{e\geq i}$.

Because the above arguments hold for all $d < i$, we obtain that i does not gain from sending any message $\hat{m}^t_{i,i-1}|_{e\geq i} \neq \omega_i(h^t)|_{e\geq i}$ at history h^t.

We have shown that no player $i \in I_+(E, D)$ gains by deviating from the strategy μ that prescribes $m_{i,i-1}(h^t)|_{e\geq i} = \omega_i(h^t)|_{e\geq i}$ at any history h^t she is called to play. An analogous, symmetric argument, shows that no $i \in I_-(E, D)$ gains by deviating from $m_{i,i+1}(h^t)|_{e\leq i} = \omega_i(h^t)|_{e\leq i}$ at any h^t she is active.

This concludes the verification that μ is a Perfect Bayesian Equilibrium. As a result, the ordered line ensures that every signal s_e reaches every decision maker d, and hence minimizes the weighted sum of expected residual variances $\sum_{d\in D} \alpha_{id}\, \mathbb{E}[\mathrm{Var}(x|\omega_d(h^T), \mu); N] = \sum_{d\in D} \alpha_{id}\, \mathbb{E}[\mathrm{Var}(x|\mathbf{s}_E)]$, for every realized E and D.

Part b. The proof that the ordered line is the unique optimal network is concluded with the following arguments.

First, consider any tree N other than the ordered line. I showed in the proof of Proposition 2 that there exist singleton realizations $E = \{e\}$ and $D = \{d\}$ such that their only connecting path p has bias reversals. For all such E, D, the expected residual variance is $\mathbb{E}[\mathrm{Var}(x|\omega_d(h^T), \mu); N] > \mathbb{E}[\mathrm{Var}(x|s_E)]$. As the aggregate link cost of such trees is the same as the ordered line, they are all dominated.

Second, every connected network N with loops has a higher aggregate link cost $c(N)$ than the ordered line, and hence it is dominated, at least as long as $c > 0$.

Finally, every network N that is not connected causes some decision makers d to lose all information about some signals s_e. Hence, by definition of $\bar{c}$, any disconnected network is dominated by the ordered line when $c < \bar{c}$. ■

Appendix B: Omitted Results

Proposition 5 *Consider the general disclosure game on a path p from the expert e to the decision maker d that allows partial disclosure and decay defined in Section 4.1.*

a. *If the path p has no bias reversals, then for every δ such that $\hat{x}(\delta) - \underline{x} < \min\{b_i - b_d\}$, there exists an equilibrium $\left(\mu^\delta, y^\delta_d\right)$ as follows: if $b_i > (<) b_d$ for all $i \neq d$, then, for some threshold $\hat{x}(\delta) \in (\underline{x}, \overline{x})$, every $i \neq d$ on p discloses every $x > (<) \hat{x}(\delta)$; Further, $\lim_{\delta\to 1} \hat{x}(\delta) = \underline{x}$ (respectively, $\lim_{\delta\to 1} \hat{x}(\delta) = \overline{x}$), and hence $\mathbb{E}[\mathrm{Var}(x|\omega_d(h^T), \mu^\delta)] \to 0$*

for $\delta \to 1$.

b. *If* p *has bias reversals, then* $\mathbb{E}[\mathrm{Var}(x|\omega_d(h^T), \mu^\delta)] > 0$ *as* $\delta \to 1$ *for any equilibrium sequence* $\left(\mu^\delta, y_d^\delta\right)$.

Proof. Part a. Say p has no bias reversals, e.g. $b_i > b_d$ for all $i \neq d$. (The proof for $b_i < b_d$ for all $i \neq d$ is the mirror-like image.) For any δ, let $\hat{x}(\delta)$ solve

$$\left(1 - \delta^T\right) \mathbb{E}[x] + \delta^T \mathbb{E}[x|x \leq \hat{x}] = \hat{x}.$$

For $\delta \to 1$, note that it is the case that $\hat{x} \to \mathbb{E}[x|x \leq \hat{x}]$. Because $\mathbb{E}[x|x \leq \hat{x}] < \hat{x}$ for any $\hat{x} > \underline{x}$ by the intermediate value theorem, it must be that $\hat{x} \to \underline{x}$. Take δ such that $\hat{x}(\delta) - \underline{x} < \min\{b_i - b_d\}$.

Consider the profile of strategies μ^δ such that each player $i \neq d$, at any history h^t in which she is active, discloses her information $\omega_i(h^t)$ to her immediate successor j on p—i.e., she sends $m_{ij}^\delta(h^t) = \omega_i(h^t)$—if $\min \omega_i(h^t) > \hat{x}(\delta)$, and else she sends $m_{ij}^\delta(h^t) = X$. Every player $i \neq e$ at any history h^t believes that $x = \min \omega_i(h^t)$ with probability one, unless $\omega_i(h^t) = X$, in which case she believes that $x = \hat{x}$ with probability one. The decision maker d plays $y_d^\delta(h^T) = \hat{x} + b_d$ if $\omega_d(h^T) = X$, and $y_d^\delta(h^T) = \min \omega_d(h^T) + b_d$ for every other terminal history information set $\omega_d(h^T)$.

I now verify that the profile $\left(\mu^\delta, y_d^\delta\right)$ is a Perfect Bayesian Equilibrium for every δ such that $\hat{x}(\delta) - \underline{x} < \min\{b_i - b_d\}$.

As in the proof of Proposition 1, the players' beliefs are consistent because the only information sets on path are such that $\omega_i(h^t) \in \{\{x\}, X\}$ for all i and h^t. Further, d's strategy y_d is clearly sequentially rational.

Now consider any player $i \neq d$ at any history h^t she is active. If $\omega_i(h^t) = X$, then i can only send $\tilde{m}_{ij}^t = \omega_i(h^t)$. Suppose $\omega_i(h^t) \neq X$. Then, player i believes that $x = \min \omega_i(h^t)$, and she anticipates that, given the successors' strategies in the profile μ^δ, either $\omega_d(h^T) = X$ or $\omega_d(h^T) = \tilde{m}_{ij}^t$ for any terminal history h^T that extends h^t. If $\omega_d(h^T) = X$, player d plays $y_d(h^T) = \hat{x}(\delta) + b_d$, and if $\omega_d(h^T) \neq X$, she plays $y_d(h^T) = \min \omega_d(h^T) + b_d$.

For every state $x > \hat{x}(\delta)$, player i is obviously better off if d plays $\hat{y} = x + b_d$ rather than $\hat{y} = \hat{x}(\delta) + b_d$. So, if $\min \omega_i(h^t) > \hat{x}(\delta)$, then i's optimal decision is to send $\tilde{m}_{ij}^t = \omega_i(h^t)$, so that $\hat{y} = \min \omega_i(h^t) + b_d$ with probability δ—with probability $1 - \delta$, the d's decision is $\hat{y} = \hat{x}(\delta) + b_d$ regardless of what message i sends. Hence, player i has no incentive to deviate from μ^δ when $\min \omega_i(h^t) > \hat{x}(\delta)$.

For every state $x < \hat{x}(\delta)$, player i is better off if d plays $\hat{y} = \hat{x}(\delta) + b_d$ rather than any $\hat{y} < \hat{x}(\delta) + b_d$, because $x + b_i > \hat{x}(\delta) + b_d$ for all $x \in [\underline{x}, \hat{x}]$ as implied by $\hat{x}(\delta) - \underline{x} < \min\{b_i - b_d\}$. Hence, when $\min \omega_i(h^t) < \hat{x}(\delta)$, player i sends message $\tilde{m}^t_{ij} = X$ in line with μ^δ.

Having concluded that $\left(\mu^\delta, y^\delta_d\right)$ is an equilibrium, we see that, because $\lim_{\delta \to 1} \hat{x}(\delta) = \underline{x}$,

$$\begin{aligned}\mathbb{E}[\mathrm{Var}(x|\omega_d(h^T), \mu^\delta)] &= \mathbb{E}(\mathbb{E}[x|\omega_d(h^T), \mu^\delta] - x)^2 \\ &= \int_{x \leq \hat{x}(\delta)} (\mathbb{E}[x|\omega_d(h^T)] - x)^2 f(x)dx \to 0, \quad \text{for } \delta \to 1.\end{aligned}$$

Part b. Suppose now that the path p has bias reversals. Consider any equilibrium (μ, y_D).

Suppose that $\hat{x} > \underline{x}$, and pick any player i on p such that $b_i > b_d$. There are two possibilities to consider.

First, suppose that there exists a non-null measure set $\tilde{X}_i(\hat{x}) \subseteq (\hat{x} - b_i + b_d, \hat{x})$ such that for all $x' \in \tilde{X}_i(\hat{x})$, player i learns that $\hat{x} + b_d - b_i < x < \hat{x}$ at some histories h^t that follow x' on the equilibrium path, and at which i is active. That is, player i's information $\omega_i(h^t)$ at such histories h^t is such that there does not exist almost any state $x \notin (\hat{x} - b_i + b_d, \hat{x})$ from which a history $\hat{h}^t \in \omega_i(h^t)$ can be reached with positive probability given that all i's predecessors on p play according to μ. For any $x' \in \tilde{X}_i(\hat{x})$, let $H^t_i(x')$ be the set of such histories h^t where i learns that $\hat{x} + b_d - b_i < x < \hat{x}$, and $\mu(H^t_i(x')|x') > 0$ be the probability that they are reached under strategies μ.

For any $x' \in \tilde{X}_i(\hat{x})$, player i would prefer that d plays $\hat{y} = \hat{x} + b_d$ rather than any decision $\hat{y} < \hat{x} + b_d$. This is because the bliss point $\mathbb{E}[x|\omega_i(h^t), \mu] + b_i$ of i's expected utility $\mathbb{E}[u_i(\hat{y}, x)|\omega_i(h^t), \mu]$ given information $\omega_i(h^t)$ and beliefs induced by equilibrium (μ, y_D) is such that

$$\mathbb{E}[x|\omega_i(h^t), \mu] + b_i > \hat{x} + b_d - b_i + b_i = \hat{x} + b_d.$$

Indeed, i can secure that d plays $\hat{y} = \hat{x} + b_d$ by blocking the transmission of any information on x, i.e., by sending message $\tilde{m}^t_{ij} = X$ to her immediate successor j on p. By doing so, i makes it impossible for any of her successors $j \neq d$ on p to send any message $\tilde{m}_{jk}$ other than $\tilde{m}_{jk} = X$ to their immediate successors, thereby making sure that $\omega_d(h^T) = X$ for all terminal histories that contain h^t and $\tilde{m}^t_{ij} = X$. Thus, for every terminal history h^T on the equilibrium path that contain any history h^t where i learns that $\hat{x} + b_d - b_i < x < \hat{x}$, it must be that the decision maker d's action is $y_d(h^T) \geq \hat{x} + b_d$.

As a result, for all h^T that extend such histories h^t, the residual variance is

$$\mathrm{Var}(x|\omega_d(h^T), \mu) = (\mathbb{E}[x|\omega_d(h^T), \mu] - x)^2 \geq (\hat{x} - x)^2.$$

Integrating over $\tilde{X}_i$, and the histories h^t where i learns that $\hat{x} + b_d - b_i < x < \hat{x}$, the expected residual variance of equilibrium (μ, y_D) is such that

$$\mathbb{E}[\mathrm{Var}(x|\omega_d(h^T), \mu)] \geq \int_{\tilde{X}_i(\hat{x})} (\hat{x} - x)^2 \mu \left(H^t(x)\,|x\right) f(x)dx > 0$$

independently of δ, because $\tilde{X}_i$ has non-null measure and $\mu\left(H_i^t(x)\,|x\right) > 0$ for $x' \in \tilde{X}_i$.

The second possibility to consider is such that for almost all $x \in (\hat{x} - b_i + b_d, \hat{x})$, and subsequent histories h^t on the equilibrium path, player i does not learn that $\hat{x} + b_d - b_i < x < \hat{x}$ at h^t. In such a case, also player d will not learn that $\hat{x} + b_d - b_i < x < \hat{x}$ at any history h^T that extends any such history h^t, as everything that d learns must be known also to all players on the path p. At all such terminal histories h^T, the residual variance is

$$(\mathbb{E}[x|\omega_d(h^T)] - x)^2 \geq \min\{(\hat{x} + b_d - b_i - x)^2, (\hat{x} - x)^2\}.$$

Integrating over $x \in (\hat{x} - b_i + b_d, \hat{x})$, we obtain, because $\hat{x} > \underline{x}$,

$$\mathbb{E}[\mathrm{Var}(x|\omega_d(h^T), \mu)] \geq \int_{\max\{\underline{x}, \hat{x}+b_d-b_e\}}^{\hat{x}} \min\{(\hat{x} + b_d - b_i - x)^2, (\hat{x} - x)^2\} f(x)dx > 0,$$

independently of δ.

Now, suppose that $\hat{x} < \overline{x}$, and pick any i on p such that $b_i < b_d$. Again, there are two possibilities. The first one is that player i learns that $\hat{x} < x < \hat{x} + b_d - b_i$ on some non-null measure set $\tilde{X}_i(\hat{x}) \subseteq (\hat{x}, \hat{x} + b_d - b_i)$ and some histories h^t at which she is called to play on the equilibrium path. Then, arguments symmetric to the ones above imply that, independently of δ,

$$\mathbb{E}[\mathrm{Var}(x|\omega_d(h^T), \mu)] \geq \int_{\tilde{X}_i(\hat{x})} (\hat{x} - x)^2 \mu \left(H^t(x)\,|x\right) f(x)dx > 0.$$

The other possibility is that player i almost never learns that $\hat{x} < x < \hat{x} + b_d - b_i$ when this the case, and then, independently of δ,

$$\mathbb{E}[\mathrm{Var}(x|\omega_d(h^T), \mu)] \geq \int_{\hat{x}}^{\min\{\overline{x}, \hat{x}+b_d-b_e\}} \min\{(\hat{x} - x)^2, (\hat{x} + b_d - b_i - x)^2\} f(x)dx > 0.$$

Because it is impossible for $\hat{x} = \underline{x}$ and $\hat{x} = \overline{x}$ simultaneously, the above arguments imply that, regardless of the equilibrium value of $\hat{x}$, the expected residual variance $\mathbb{E}[\text{Var}(x|\omega_d(h^T), \mu)]$ remains strictly positive, independently of δ. Hence, for any sequence $\delta \to 1$ and corresponding equilibria (μ^δ, y_d^δ), we conclude that $\lim_{\delta \to 1} \mathbb{E}[\text{Var}(x|\omega_d(h^T), \mu^\delta)] > 0$. ■

Proposition 6 *Consider the model of strategic disclosure on networks defined in Section 4.1. For all* $\boldsymbol{\alpha} > \mathbf{0}$ *and full-support* P*, there exists a decay threshold* $\bar{\delta} < 1$*, as well as intermediate cost ranges* (c_-, c^+) *and* $(\hat{c}_-, \hat{c}^+)$ *that depend on* δ*, with* $\lim_{\delta \to 1} c_-(\delta) = \lim_{\delta \to 1} \hat{c}_-(\delta) = 0$*,* $\lim_{\delta \to 1} c^+(\delta) = \bar{c}$*, and* $\lim_{\delta \to 1} \hat{c}^+(\delta) = \hat{c}$*, such that:*

a. for all $\delta \in (\bar{\delta}, 1]$ *and* $c \in (c_-, c^+)$*, the ordered line is the unique optimal network,*

b. for all $\delta \in (\bar{\delta}, 1]$ *and* $c \in (\hat{c}_-, \hat{c}^+)$*, the ordered line is the unique network immune to coalitional deviations in the bilateral sponsorship game.*

Proof. Part a. By the proof of Proposition 2, the ordered line is the only tree in which every pair of players e and d are connected through a path p without bias reversals. Hence, by Proposition 5, the ordered line is the only tree such that $\mathbb{E}[\text{Var}(x|\omega_d(h^T), \mu)] \to 0$ as $\delta \to 1$, for all e and d. I conclude that for all $\boldsymbol{\alpha} > \mathbf{0}$ and full-support P, there exists a threshold $\bar{\delta}_1 < 1$ such that for all $\delta \in (\bar{\delta}_1, 1]$, the ordered line is the unique optimal tree.

Now, consider networks that are not minimally connected. Pick δ sufficiently close to 1 (i.e., $\delta > \bar{\delta}_2$ for some given threshold $\bar{\delta}_2 < 1$). Then, because $\mathbb{E}[\text{Var}(x|\omega_d(h^T), \mu)] \to 0$ as $\delta \to 1$ for all e and d with the ordered line, it cannot be dominated by any connected network N with loops unless the link cost c is too low—i.e., $c < c_-(\delta)$ for some given threshold $c_-(\delta) > 0$, as the aggregate link cost $c(N)$ is strictly higher. Likewise, no disconnected network can dominate the ordered line unless c is too high—that is, $c > c^+(\delta)$ for some given threshold $c^+(\delta) < \bar{c}$, as some decision maker d would not receive information from some expert e.

There thus exist a decay threshold $\bar{\delta} = \max\{\bar{\delta}_1, \bar{\delta}_2\}$ and an intermediate link cost range $(c_-(\delta), c^+(\delta))$, with $\lim_{\delta \to 1} c_-(\delta) = 0$ and $\lim_{\delta \to 1} c^+(\delta) = \bar{c}$, such that for all $\delta \in (\bar{\delta}, 1]$ and $c \in (c_-(\delta), c^+(\delta))$, the ordered line is the unique optimal network N.

Part b. Because $\mathbb{E}[\text{Var}(x|\omega_d(h^T), \mu)] \to 0$ as $\delta \to 1$ for all e and d with the ordered line, for δ sufficiently close to 1, no coalition of players finds it beneficial to form extra links unless c is too low, nor to delete any links unless c is too high. It follows that the ordered line is immune to coalitional deviations for δ close to 1, unless c is too small or too large. Under the same conditions, it is also optimal, by Part a. Hence, no other network N can be immune to coalitional deviations, as the grand coalition would benefit from deviating to form the ordered line and possibly redistributing payoffs. This concludes that there exist a decay threshold $\bar{\delta} < 1$ and an intermediate cost range $(\hat{c}_-(\delta), \hat{c}^+(\delta))$, with $\lim_{\delta \to 1} \hat{c}_-(\delta) = 0$ and $\lim_{\delta \to 1} \hat{c}^+(\delta) = \hat{c}$, such that for all $\delta \in (\bar{\delta}, 1]$ and $c \in (\hat{c}_-(\delta), \hat{c}^+(\delta))$, the ordered line is the unique network immune to coalitional deviations. ■

Proposition 7 *Consider the disclosure game on a path p from the expert e to the decision maker d defined in Section 4.2. The expert's signal s satisfies the monotone likelihood ratio property, and each player i's loss function L_i is concave and supermodular.*

a. if the path p has no bias reversals, then the decision maker d learns s for every s in equilibrium, and plays $\hat{y} = y_d(s)$;

b. if the path p has bias reversals, then there exists a set $\tilde{S}$ of non-zero measure such that d cannot learn s for any $s \in \tilde{S}$ in equilibrium.

Proof. Part a. Say the path p has no bias reversals, e.g. $i > d$ for all $i \neq d$ on p. Then, proceeding as in the Proof of Proposition 1, I will show that there is a Perfect Bayesian Equilibrium such that every $i \neq d$ plays $m_{ij}(h^t) = \omega_j(h^t)$ at any history h^t she is active, and player d holds beliefs that $s = \min \omega_d(h^t)$ with probability one at every terminal history h^T, thus choosing $y_d(h^T) = y_d(\min \omega_d(h^T))$. As a result, the message $\hat{m}_{ij} = \{s\}$ travels along the path p from e to d on the equilibrium path, and d chooses $y_d(h^T) = y_d(s)$.

To prove this, first note that, as in the Proof of Proposition 1, the beliefs are admissible, and d's strategy y_d is optimal given her beliefs. Now consider the decision of any player $i \neq d$ at any history h^t in which she knows s and she is active. I show that player i does not gain by deviating from the equilibrium strategy $m_{ij}(h^t) = \omega_i(h^t) = \{s\}$. Player i anticipates that all her successors $j \neq d$ on p will play $\hat{m}^{\tau}_{jk} = \omega_j(h^{\tau})$ at any history h^{τ} that extends h^t, by the imputation that they adhere to μ. As in the Proof of Proposition 1, it follows that i anticipates that $\omega_d(h^T) = \hat{m}^t_{ij}$ at any terminal history h^T that extends h^t. If i sends $\hat{m}^t_{ij} = S$, then d plays $y_d(h^T) = y_d(\underline{s})$. If instead i sends $\hat{m}^t_{ij} = \{s\}$, then d chooses $y_d(h^T) = y_d(s)$.

By supermodularity, i's expected utility $\mathbb{E}[L_i(\hat{y},x)|s]$ given information $\omega_i(h^t) = \{s\}$ increases in $\hat{y}$ for $\hat{y} < y_i(s)$, because $y_i(s)$ is the bliss point of i's expected utility. Further, by supermodularity, $y_i(s) > y_d(s) \geq y_d(\underline{s})$, where the first inequality follows from $i > d$, and the second because $\underline{s} \leq s$. Hence, player i prefers to send $\hat{m}^t_{ij} = \{s\}$ over $\hat{m}^t_{ij} = X$ for all $s \in X$, thus concluding the verification that the stated strategy profile is a Perfect Bayesian Equilibrium.

For brevity, I omit the proof that in every equilibrium (μ, y_d), the message $\hat{m}_{ij} = \{s\}$ travels along the path p from e to d on the equilibrium path, and the decision maker chooses $\hat{y} = y_d(s)$ for all terminal histories h^T. This proof is a generalization of the proof the same results in Proposition 1.

Part b. Suppose now that the path p has bias reversals.

Pick any arbitrary equilibrium (μ, y_d). For any history h^t, the information $\omega_i(h^t)$ that player i has on s may either be that $\omega_i(h^t) = \{s\}$ or that $\omega_i(h^t) = S$. If $\omega_i(h^t) = S$, then by verifiability $\hat{m}^t_{ij} = S$ and $\omega_d(h^T) = S$ for every terminal history h^T that extends h^t.

Consider any player $i > d$ called to play at a history h^t such that $\omega_i(h^t) = \{s\}$. Consider the set $\tilde{S}_i(\mu) = \{s : y_d(s) < y_d(S;\mu) < y_i(s)\}$, where $y_d(S,\mu)$ denotes the decision of player d in case $\omega_d(h^T) = S$ at equilibrium (μ, y_d). Because $i > d$ and supermodularity, for all $s \in \tilde{S}_i(\mu)$, player i would rather that d plays $\hat{y} = y_d(S;\mu)$ than $\hat{y} = y_d(s)$. Suppose that under μ, all the successors j of i relay signal s along p when they are disclosed it. Then, player i strictly prefers sending message $\tilde{m}^t_{ij} = S$ over $\tilde{m}^t_{ij} = \{s\}$ as the former blocks the transmission of s to d. Of course if some successor j of i blocks signal s by playing $\tilde{m}^t_{jk} = S$ at the history h^τ such that $\omega_j(h^\tau) = \{s\}$, then player i is indifferent between $\tilde{m}^t_{ij} = S$ and $\tilde{m}^t_{ij} = \{s\}$ at h^t. In any case, there cannot exist any equilibrium (μ, y_d) in which $\omega_d(h^T) = \{s\}$ on the equilibrium path, for any $s \in \tilde{S}_i(\mu)$. An analogous, symmetric, argument implies that there cannot exist any equilibrium in which $\omega_d(h^T) = \{s\}$ on the equilibrium path, for any $s \in \tilde{S}_i(\mu) = \{s : y_i(s) < y_d(S;\mu) < y_d(s)\}$ and any $i < d$.

As in the Proposition 1, this concludes that for every equilibrium (μ, y_d) and corresponding decision $y_d(S;\mu) \in [y_d(\underline{s}), y_d(\overline{s})]$, there exists a set $\tilde{S}(\mu) = \cup_{i\in\mathcal{N}}\tilde{S}_i(\mu)$ of strictly positive measure such that d cannot learn s for any $s \in \tilde{S}(\mu)$. ■

Proposition 8 *Consider the model of strategic disclosure on networks defined in Section*

4.2. The expert e's signal s satisfies the monotone likelihood ratio property, and each player's loss function L_i *is concave and supermodular. Then, for every utility weights* $\boldsymbol{\alpha} > \mathbf{0}$*, and full support selection probability* P*, the ordered line is the unique optimal network, in the sense that it guarantees full disclosure of signal* s *from any expert* e *to any decision maker* d*, with minimal aggregate link cost* $c(N)$*.*

Proof. The proof is omitted as it is the same as the proof of Proposition 2, using Proposition 7 in lieu of Proposition 1. ■