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The Choice of Political Advisors*

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Abstract

We study the choice of multiple advisors, balancing political alignment, competence, and diverse perspectives. An imperfectly informed leader can consult one or two advisors. One has views closely aligned with the leader's, but his information is imprecise or correlated with the leader's own. The other is more biased but has independent or more precise information. We identify a trade-off between consulting the more aligned or the better informed expert, even when this entails small costs. Subtle comparative statics emerge: When the leader consults both advisors, increasing the bias of the more biased expert may result in the dismissal of the other advisor. The leader may opt to delegate consulting and decision-making, but only to the advisor who collects superior information in equilibrium. We then study the "uncertain trade-off" case where the most informed advisor is not necessarily also more biased. We find that reducing the probability that the better-informed expert is more biased may lead to hiring also the other advisor. The leader may delegate to the advisor with uncertain bias, although he is more biased in expectation, because he more easily aggregates information in equilibrium.

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1 Introduction

Political leaders use all sorts of advisors, associates, experts, and consultants. Access to good and reliable advice is widely recognized as making the difference between successful and poor decision-making, in the words of Niccoló Machiavelli (The Prince, Ch. 22):

> The first opinion that one forms of a prince, and of his understanding, is by observing the man he has around him.

The importance of advisors becomes apparent when we examine the frequent and abrupt changes in personnel during the Trump Presidency. By the conclusion of his term, there was a staggering 92% turnover in the most influential positions, with 45% of positions experiencing repeated turnovers (Tenpas, 2021). The institutional role of policy advisors has also come under scrutiny in British politics. Because of the predominant role in the Johnson Government granted to a (later dismissed) controversial Chief Advisor, tensions emerged between the tradition of independent civil servants and the adoption of politically-aligned consultants.

This paper studies the optimal choice of one or more advisors from a pool of available candidates who vary in terms of attributes such as political alignment, competence, and diversity of perspectives. A potential trade-off naturally arises between relying on closely aligned collaborators, and seeking the most competent advice. Broadening the set of advisors to include potentially less aligned experts with views different from the leader's own can provide access to a more diverse range of information. So, which advisors should a leader choose when faced with these trade-offs? How should a leader respond to changed circumstances? Under what circumstances should the leader delegate consulting and decision-making authority to an advisor, and which advisor should be entrusted with this responsibility?

The central role of advisors has long been recognized in economics and politics. Since the emergence of the modern state, acquiring technical knowledge has become a paramount attribute for effective leadership. Owing to the escalating intricacies of society, rulers can no longer solely depend on personal connections to govern a nation. Instead, they require the backing of a competent bureaucracy and the counsel of technical experts when making critical decisions.[1] However, identifying good advisors is not as straightforward as merely selecting those with competence. As Max Weber writes in Economy and Society Chapter XI, the leader is in a disadvantaged position due to informational asymmetry:

[1]These features persist in contemporary politics, exemplified by the tradition of parliamentary democracies regularly relying on the expertise of career civil servants. In the United States, the establishment of professionalized civil service careers resulted from 20th-century civil service reforms (Ash, Morelli, and Vannoni, 2022).

> Since the specialized knowledge of the expert became more and more the foundation for the power of the officeholder, an early concern of the ruler was how to exploit the special knowledge of experts without having to abdicate in their favor.

When advisors are chosen solely based on competence, the leader may unknowingly be influenced by advisors pursuing their own interests. Consequently, a trade-off between political alignment and competence naturally emerges. For instance, in the United States, presidential appointments have often been utilized to place aligned individuals in high-ranking positions, when presidents prioritize responsiveness to voters over impartial competence (Parsneau, 2013; Krause and O'Connell, 2019).

Beyond competence, the responsibility of a leader as a guardian of the public interest requires that she listens to the diverse voices within society.[2] Overreliance on advisors with similar views may lead to a "group-think" problem. Diverse information from different political camps provides a more comprehensive perspective in decision-making. Therefore, a political decision maker may consider enlisting advisors from ideologically distant groups, consistent with findings from studies on presidential appointments (Ingraham et al., 1995; Bertelli and Feldmann, 2007; Lewis, 2008). However, relying on advisors with divergent political views also risks receiving biased counsel.

The trade-offs between alignment and competence are significant in the UK and European countries. In line with the tradition of Weberian bureaucracy, senior civil servants have been appointed as advisors with a mandate for independence. This practice is based on the assumption that policy and administration can be separated, allowing bureaucrats to address issues objectively and impartially (Putnam, 1973). However, the roles of political advisors are inherently intertwined with politics, making it difficult to separate political and administrative aspects. Additionally, civil servants are required to withhold their ideological affiliations during their tenure. As a result, political leaders may suspect ideological discrepancies during the appointment process, potentially leading to an agenda that diverges from their programs. Furthermore, mitigating potential conflicts of interest poses a formidable challenge: Merely mandating that political advisors maintain political neutrality may hinder their ability to provide independent and impartial advice grounded in expertise.

Further to choosing one among advisors under these trade-offs, leaders often opt to enlist multiple advisors to augment their decision-making process. By choosing advisors with diverse viewpoints, leaders can cultivate a more comprehensive understanding of the state of the world. For example, the Bush administration appointed Colin Powell as Secretary of State despite his opposition to the Iraq war, while Donald Rumsfeld was chosen for his contrasting perspective (Saunders, 2018). Similarly, President Obama aimed for a diverse cabinet with varied perspectives, appointing Republican Ray LaHood as Secretary of Transportation to foster bipartisanship.

[2] E.g., James Madison, in his Federalist Paper No. 10, discusses the challenges of factions and advises that leaders broaden their sphere of influence to include multiple parties and interests.

This paper provides a formal theoretical analysis of these research questions using a cheap talk model in the tradition of Crawford and Sobel (1982). Unlike earlier work, we differentiate advisors not only by their alignment with the leader's preferences, but also in terms of competence, and access to diverse information. Further, we provide a formal model of multiple advisors choice, by positing that an imperfectly informed leader (she) may consult, at a small cost, either one or both of two advisors (both he), to improve decision-making. One advisor has views more closely aligned with the leader, and thus he is more likely to provide truthful advice in equilibrium. The other advisor has more valuable information due to greater competence or access to information less correlated with the leader's.

The significance of our formal analysis lies in uncovering the intricate and non-obvious predictions that arise from the strategic interaction between the leader and the advisors. Equilibrium truthtelling requires that an advisor is not too biased. Importantly, we find that this requirement becomes more stringent when the leader herself is better informed, i.e. if she also receives advice from the other advisor. This gives rise to an equilibrium trade-off between consulting the more aligned or the more informed expert, even when this entails small costs. Securing the more valuable information is the leader's priority. Hence, our model predicts that she hires the more biased but better-informed advisor whenever his equilibrium truthtelling conditions are met. She adds the more aligned expert only if the additional information received does not hinder truthtelling from the better-informed advisor.

Subtle comparative static results emerge. Beginning with a scenario where the leader relies on truthful advice from both experts, an increase in the misalignment of the better informed advisor leads to dismissing the other expert, who is less biased relative to the leader's views, but has less valuable information. Subsequently, if the better-informed expert's bias further increases, the leader switches advisors. The better-informed expert is dismissed, and the politically closer one is hired back. Indeed, there are multiple factors contributing to changes in preferences alignment between political officeholders and prospective advisors over time. Changes in the composition of an expert's team and political network, along with access to different information sources, can play a significant role. Major events such as terrorist attacks, the eruption of conflicts, or pandemics may also lead to divergences of opinion between leaders and advisors.

Most importantly, the retirement or electoral defeat of an incumbent policymaker may lead to a new leader with more distinct perspectives compared to long-term policy advisors. The key question is whether she should retain these experienced advisors or rely on new ideologically closer consultants. Our results suggest that, while her predecessors could seek advice from both experienced and politically aligned experts, the new leader should only listen to the most experienced consultants and avoid mixing them with inexperienced, politically-loyal advisors. As detailed later in the paper, newly elected policymakers often prioritize competence and continuity over political alignment when choosing collaborators, and they may face negative consequences when they do not.

While the pursuit of advice prioritizes information over political alignment, the opposite is true for delegating consulting and decision-making. Because the leader's information sources are not dominated by either advisor in our model, the leader may choose to delegate only to the more aligned advisor, and only if he is biased in the same direction as the other expert. Delegation occurs under these conditions: (i) the leader cannot obtain truthful information from the better-informed advisor, (ii) the political views of the closer advisor align sufficiently with the better-informed expert to access his information, and (iii) the bias of the closer advisor is not so large that it outweighs the superior information he gathers. In sum, the leader delegates only to an advisor positioned between her and the other advisor, who can gather information from both in equilibrium. Delegation to political agents with such "intermediate," moderate views is indeed common, as we report later in the paper.

The final part of the paper generalizes the analysis to account for the possibility that the political preferences of the better informed advisor are unknown. It is uncertain whether he is as aligned with the leader as the less informed expert, whose preferences are known. This framework is motivated by the observation that, while elected leaders need to make their political views manifest to gain electoral support, unelected advisors often keep their political leanings confidential. Indeed, refraining from disclosing one's political views is a crucial aspect of an expert's professional conduct aimed at establishing credibility of his advice.

Many of our earlier findings carry over to this model of "uncertain trade-off." However, the comparative static results are now richer. We demonstrate instances where, beginning with a situation in which the leader consults both experts, raising the bias of the more aligned advisor results in the dismissal of the better-informed expert. This cannot happen when the better-informed advisor is known to more biased: There, an increase in the bias of the more aligned advisor leads to his termination. When the more informed expert is possibly equally biased as the other advisor, the former becomes more attractive ex-ante, yet it is the latter who is consulted more.

Most importantly, significant comparative static results are no longer limited to changes in advisors' biases. We show that increasing the likelihood that the better-informed expert is not more biased than the other advisor can lead the leader to shift from consulting solely the better-informed expert to relying on both advisors. This finding complements our earlier results that a newly-elected leader should primarily rely on pre-existing experienced advisors. Over time, if the leader discovers that the views of experienced advisors are more aligned with hers than initially expected, she can improve decision-making by teaming them up with loyal consultants, even if they are less experienced. As we detail later in the paper, such dynamics are not uncommon in government appointments.

Another distinction compared to the case of "certain trade-off" pertains delegation. Now, the leader may optimally choose to delegate authority to the advisor who is more biased in expectation. This is not because of the better information he is endowed with, but, rather, because of the additional information gained in equilibrium from the other advisor, whose bias is certainly low. When communicating with one another, the former conditions his strategy on his type, while the latter does not. For some bias values, the expert with a known low bias communicates truthfully in equilibrium, whereas the more biased type of the other expert babbles. Consequently, the equilibrium information provided by the expert with a known low bias is of such inferior quality that the leader benefits more from delegating to the possibly more biased advisor.

As explained later, these findings may relate to the widespread norm in public administration of appointing experienced career civil servants who keep their political views private. Our findings suggest that, even if biased, they may provide better decision-making compared to elected politicians or political appointees. This is because career civil servants are required to maintain the confidentiality of their political views, unlike politicians who must disclose theirs to secure election support. As a result, they are better at gathering information from diverse sources and perspectives.

The paper is presented as follows. After describing related literature in Section 2, we formulate and solve a general model of multiple advisors' choice in Section 3. We specialize this model to address the trade-off between competence and political alignment in Section 4, and consider in Section 5 the case of uncertain trade-off in which the preferences of the better informed advisor are unknown. Section 6 concludes the paper, and all formal proofs are in Appendix.

2 Related Literature

The main focus of our paper is how a leader chooses multiple advisers who differ in terms of political alignment, competence and diversity of views. How the ability to collect information from advisors influences effective decision-making is a key question in political economy.[3] Several studies explore this research topic under various assumptions regarding the verifiability of advice and the motives of advisors. For instance, Battaglini (2002) shows how the information of perfectly informed experts can be extracted by a decision maker in a cheap talk model with multiple decisions. Dewatripont and Tirole (1999) analyze decision-making based on competition among advocates of special interests, who may conceal information but not manipulate it freely. Che and Kartik (2009)

[3]A separate, related strand of literature studies effective leadership as the ability to communicate information. Canes-Wrone, Herron, and Shotts (2001) see leadership as a counter to "pandering," which involves implementing policies that a leader considers valuable. Dewan and Myatt (2007, 2008, 2012) examine the qualities of a leader's judgment and communication skills in relation to effective leadership.

shows that an advisor whose priors differ from the leader's may exert more effort to acquire and disclose verifiable information. Morris (2001) studies communication by a single advisor concerned with enhancing his reputation as unbiased, whereas Ottaviani and Sörensen (2001) investigate the optimal order of speech by advisors with heterogeneous expertise, who all wish to appear well informed. Dewan and Squintani (2018) investigate the selection of political leaders who rely on the counsel of trustworthy associates. None of these papers consider the leader's trade-off between selecting aligned or competent advisors, which is the subject of our study.

In addition to consulting with advisors, we investigate delegating to advisors in order to improve decision-making. Delegation has been discussed in various contexts by political economists, including legislature delegation to special committees under the closed rule (Gilligan and Krehbiel, 1990), and legislature delegation to bureaucracies (Gailmard, 2002). An important result is the so-called "ally principle" (Bendor, Glazer, and Hammond, 2001): voters, legislators, or other principals should rationally delegate more authority to advisors who share their preferences. Numerous studies explore the trade-off between expertise and control when delegating tasks and obtain results that support the ally principle. Bendor and Meirowitz (2004) conduct a thorough analysis of delegation across various models and identify conditions under which the ally principle holds. They argue that the ally principle may be violated when advisors are of heterogeneous competence, as the principal may need to prioritize competence over preference similarity. Building upon the insights of Bendor and Meirowitz (2004), our study takes a further step. In our analysis, the primary determinants of delegation are political proximity and the information an advisor possesses in equilibrium, including the information he gathers from other experts.[4]

Our analysis contributes theoretical advancements to the literature on cheap talk, initiated by Crawford and Sobel (1982), wherein a perfectly informed expert communicates with an uninformed decision maker. Building upon the work of Galeotti, Ghiglino, and Squintani (2013), we formulate a rich yet tractable model for a leader's choice of advisors.[5] We introduce several features that have not been analyzed jointly before and some that are entirely novel. The decision maker is imperfectly informed (as in Moreno De Barreda, 2010), and she may choose to consult multiple imperfectly

[4]In the context of the organization design of firms, Alonso, Dessein and Matouschek (2008) discuss under which conditions it is optimal to centralize decisions or decentralize them to local divisions who may communicate with the center and among each other.

[5]The framework by Galeotti, Ghiglino, and Squintani (2013) has been the base of several formal investigations of matters related to multi-player communication in political economy. Patty and Penn (2014) study information transmission in small networks of decision makers; Dewan, Galeotti, Ghiglino, and Squintani (2015) investigate the optimal assignment of decision-making power in the executive of a parliamentary democracy; Penn (2016) studies the formation of stable aggregation of different units within an association; Dewan and Squintani (2016) analyze the formation of party factions; Schnakenberg and Turner (2021) study how campaign contributions affect elections and influence policy holders' choices; whereas Schnakenberg and Turner (2021) explore the influence of dark money on campaigns; and Patty (2024) determines the optimal exclusion and inclusion policies to maximize information sharing in meetings.

informed experts. We consider heterogeneous players' information and formalize notions of signal bias, precision, and correlation.[6] We allow for the possility that all players' preferences are private information (including the decision maker's, unlike earlier work),[7] and explore the scenario in which the leader delegates consulting and decision-making authority to one of the advisors, building on the ideas by Dessein (2002).

Our study is relevant to the literature on presidential appointments, where the trade-off between advisors' political alignment and competence is center-piece.[8] Appointing politicians can lead to "amateur government," as they may lack expertise and prioritize short-term success, advocating for career senior executives (e.g., Cohen, 1998). Conversely, Moe (1985) argues that presidents need "responsive competence" to meet voter expectations, and experienced officers may lack responsiveness. Studies suggest presidents make partisan appointments to enhance policy responsiveness. Presidents prioritize political alignment over experience when appointing subcabinet officers (Parsneau, 2013), with more pronounced trade-offs at top-level positions (Krause and O'Connell, 2019).[9]

The trade-off we investigate between selecting aligned or competent subordinates is not exclusive to democracies. Nevertheless, the forces at play in autocracies are likely considerably distinct from those we identify here. The leader is our model is not exposed to the risk of authority challenges. Instead, Egorov and Sonin (2011) examine how dictators choose advisors under the threat of treason. Appointing competent advisors enhances regime stability, but their competence may also heighten the risk of rebellion. Consequently, leaders in weak and fragile regimes opt for loyal but less capable subordinates.

3 The Choice of Advisors

The Model. This section presents a rich, yet tractable model of advice from many experts to a decision maker. Advisors may differ according to all sorts of characteristics, including their

[6] Levy (2004), Ottaviani and Sörensen (2006), and Denisenko, Hafer and Landa (2024) consider notions of signal precision (advisor competence) in a game where a single expert communicates to a decision maker. Aside from the fact that we consider multiple experts, there are further significant differences. The first two papers focus on the case where the expert's sole motivation is induce a high belief of competence, and the last one considers transmission of verifiable information.

[7] The literature has focused on the case in which there is a single expert, and only his bias is private information (e.g., Morgan and Stocken, 2003, and Li and Madarázs, 2008).

[8] Such trade-off is also related to studies on candidate selection in list proportional representation systems. Buisseret and Prato (2022) and Buisseret et al. (2022) consider the balance between ideological alignment with parties and the preferences of local voters.

[9] More distantly related are normative studies comparing between bureaucrats and politicians in decision-making can also be related to our work. Maskin and Tirole (2004) suggest that non-accountable bureaucrats suit technical decisions, while re-election incentives can address adverse selection and moral hazard for politicians but may lead to policies that neglect minority rights. Alesina and Tabellini (2007) find bureaucrats more effective when their technical capabilities outweigh moral hazard concerns.

ideological alignment with the decision maker, the precision of their information, the bias of their sources of information, and the likelihood that they gather information from the same sources as their decision maker. We characterize equilibrium and welfare, thus laying out the foundation for the specialized analyses of sections 4 and 5.

There is a set of players $N = \{0, 1, ..., n-1\}$, each observing a binary signal $s_i = \{0, 1\}$ informative of an unknown state x uniformly distributed on $[0, 1]$. In order to provide formal notions of signal bias, precision and correlation, we proceed as follows. We stipulate that there are n "real" signals $s'_j \in \{0, 1\}$, $j = 0, ..., n-1$, with $\Pr(s'_j = 1|x) = x$, i.i.d. The observable signals $s_j \in \{0, 1\}$, $j = 0, ..., n-1$, are an imprecise, possibly biased, representation of the signals s'_j: $\Pr(s_j = s|s'_j = s) = p_{js}$ for $s = \{0, 1\}$. Each player $i = 0, 1, ..., n-1$, observes signal each s_j, $j = 0, ..., n-1$, with probability $\rho_{ij} \in [0, 1]$, with $\sum_{j=0}^{n-1} \rho_{ij} = 1$.

The following game is played. First, player 0 chooses a set of advisors $A \subseteq N \backslash \{0\}$ to consult at a small cost each, and her choice A is public. Then, the consulted advisors $i \in A$ simultaneously send messages $\hat{m}_i \in \{0, 1\}$ to player 0, who also observes her signal s_0. Finally, player 0 makes a decision $\hat{y} \in \mathbb{R}$. The utility of each player $i \in N$ is $u_i(\hat{y}, x) = -(\hat{y} - x - b_i)^2$, where the bias b_i is private information, and may take a finite set of values B_i, with distribution q_i. We denote by the "average bias" of the decision maker by $\bar{b}_0 = \sum_{b_0 \in B_0} b_0 q_0(b_0)$. For expositional reasons, we assume that all bias types b_i of each advisor i are biased in the same direction relative to this average bias: for every $i > 0$, and every bias types b_i, b'_i, it is the case that $\left(b_i - \bar{b}_0\right)\left(b'_i - \bar{b}_0\right) > 0$. We say that i is biased rightward (leftward) if $b_i - \bar{b}_0 > 0$ (resp., if $b_i - \bar{b}_0 < 0$).

For any given choice of advisors A, we let the communication strategy of each $i \in A$ be denoted by $m_i : s_i \to \hat{m}_i$, and the leader's decision strategy be $y : (s_0, \hat{\mathbf{m}}_A) \mapsto \hat{y}$. For clarity, the analysis focuses on pure strategy Bayesian equilibrium $(A, \mathbf{m}, y)$. As we shall see, there are multiple equilibria. As is customary when studying cheap talk games, we will select the equilibrium with the highest welfare calculated ex-ante (see, e.g., Crawford and Sobel, 1982). We will later show that the ranking of equilibria of every player i in terms of ex-ante payoff is perfectly aligned (omitting the small cost of hiring advisors by player 0).

Analysis: Equilibrium and Welfare. In equilibrium, each consulted advisor i may be truthful or babbling, possibly depending on her type b_i. We denote a pure strategy m_{i,b_i} as truthful if $m_{i,b_i}(s_i) = s_i$, and as babbling if $m_{i,b_i}(0) = m_{i,b_i}(1)$. Fix any choice of consulted advisors A. Up to interchanging messages $\hat{m}_i$, every equilibrium of the ensuing communication game is uniquely identified by a collection of sets T_i of truthtelling bias types, for each each advisor $i \in A$. Each babbling bias type $b_i \notin T_i$ pools on message $\hat{m}_i = 1$ if i is biased rightward, and on $\hat{m}_i = 0$ if i

is biased leftward.[10] Further, in equilibrium, the leader will never hire an advisor she expects to always babble.

There always exists a (babbling) equilibrium in which A is empty. We prove in the appendix that any equilibrium such that A is non-empty is characterized by the following truthtelling conditions for every advisor $i \in A$.

Theorem 1 *Every equilibrium $(A, \mathbf{m}, y)$ is such that the set T_i of truthtelling bias types b_i is non-empty for each consulted advisor $i \in A$. The equilibrium truthtelling conditions are that for each $i \in A$, $b_i \in T_i$, $s_i = 0, 1$, and $\hat{m}_i = s_i$,*

$$\left|b_i - \bar{b}_0\right| \leq \left| \frac{\sum_{s_0 \in \{0,1\}} \sum_{\hat{\mathbf{m}}_{A-i} \in \{0,1\}^{N-2}} \Delta_i \left(s_0, \hat{m}_i, \hat{\mathbf{m}}_{A-i}\right)^2 \Pr(s_0, s_i, \hat{\mathbf{m}}_{A-i})}{2 \sum_{s_0 \in \{0,1\}} \sum_{\hat{\mathbf{m}}_{A-i} \in \{0,1\}^{N-2}} \Delta_i \left(s_0, \hat{m}_i, \hat{\mathbf{m}}_{A-i}\right) \Pr(s_0, s_i, \hat{\mathbf{m}}_{A-i})} \right| \tag{1}$$

where $\Delta_i \left(s_0, \hat{m}_i, \hat{\mathbf{m}}_{-i}\right) = E[x|s_0, 1 - \hat{m}_i, \hat{m}_{A-i}] - E[x|s_0, \hat{m}_i,\ \hat{m}_{A-i}]$. The equilibrium decision of a leader of type b_0 is $y\left(s_0, \hat{\mathbf{m}}_A\right) = E[x|s_0, \hat{\mathbf{m}}_A] + b_0$, for every $\hat{\mathbf{m}}_A \in \{0,1\}^A$. The equilibrium ex-ante payoff of the leader is:

$$Eu_0\left(A, \mathbf{m}, y\right) = -E_{s_0, \hat{\mathbf{m}}_A}[Var(x|s_0, \hat{\mathbf{m}}_A)], \tag{2}$$

for each player i of type b_i, ex-ante payoff is $Eu_{i,b_i}(A, \mathbf{m}, y) = Eu_0\left(A, \mathbf{m}, y\right) - \sum_{b_0 \in B_0} q_0\left(b_0\right)\left(b_0 - b_i\right)^2$.

An immediate consequence of the final result in the above Theorem is that the ranking of equilibria of every player i in terms of ex-ante welfare is perfectly aligned (omitting the small cost of hiring advisors). The expression $E_{s_0, \hat{\mathbf{m}}_A}[Var(x|s_0, \hat{\mathbf{m}}_A)]$ in (2) denotes the residual variance of x given the optimal choice $y(s_0, \hat{\mathbf{m}}) = E[x|s_0, \hat{\mathbf{m}}]$ based on the information $(s_0, \hat{\mathbf{m}})$ player 0 receives in equilibrium.

Consider now the equilibrium characterization in expression (1). $\Pr(s_0, s_i, \hat{m}_j)$ denotes the total probability that advisor i receives signal s_i, the leader observes signal s_0 and receives message $\hat{\mathbf{m}}_{A-i}$ from the other advisors $j \in A\backslash\{i\}$. Because the equilibrium decision of the leader is $y\left(s_0, \hat{\mathbf{m}}_A\right) = E[x|s_0, \hat{\mathbf{m}}_A] + b_0$, for every $\hat{\mathbf{m}}_A \in \{0,1\}^A$, the expression $\Delta_i\left(s_0, \hat{m}_i, \hat{\mathbf{m}}_{A-i}\right) = E[x|s_0, \hat{m}_i, \hat{\mathbf{m}}_{A-i}] - E[x|s_0, 1 - \hat{m}_i, \hat{\mathbf{m}}_{A-i}]$ denotes by how much advisor i of type b_i would move the leader's decision if lying, i.e., if sending message $1 - \hat{m}_i$ instead of the truthful message $\hat{m}_i = s_i$, in an equilibrium where b_i is supposed to tell the truth.

To gain some intuition about the truthtelling condition (1), we provide the following heuristic argument. Write the equilibrium condition *as if* it held "term-by-term" for every $(s_0, \hat{\mathbf{m}}_{-i})$, i.e., $2|b_i - \bar{b}_0| \leq |\Delta(s_0, \hat{m}_i, \hat{\mathbf{m}}_{A-i})|$. Consider Figure 1, in which we assume that the leader's bias type is

[10]To represent the possibility that all bias types of an advisor i pool, we adopt the convention that they all pool on the same message. Because off path beliefs are free, this is without loss of generality. For example, it is always possible to assign the same belief to off path and on path messages.

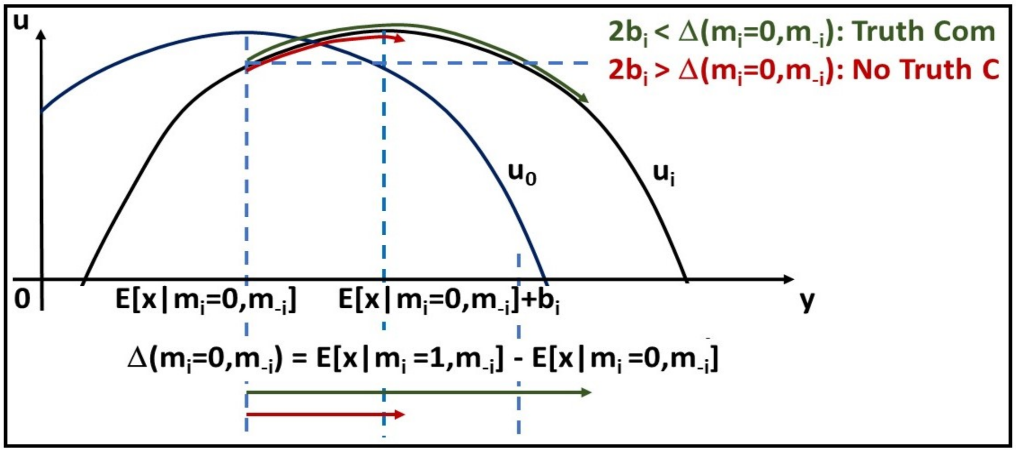

Figure 1: Heuristic argument for the truthtelling condition (1).

known to $b_0 = 0$, for simplicity, consider a rightward bias type $b_i > b_0 = 0$, and simplify notation in the expressions $E[x|s_0, \hat{m}_i, \hat{\mathbf{m}}_{A-i}]$ and $\Delta_i(s_0, \hat{m}_i, \hat{\mathbf{m}}_{A-i})$. Fix any realizations of the leader's signal s_0 and of the messages $\hat{\mathbf{m}}_{A-i}$ sent by the other consulted advisor. For any signal $s_i = 0, 1$, player i may deviate from equilibrium, by sending the false message $1 - \hat{m}_i$ instead of the truthful message $\hat{m}_i = s_i$. By doing this, i moves the leader's decision $\hat{y}$ from $E[x|s_0, \hat{m}_i, \hat{\mathbf{m}}_{A-i}]$ to $E[x|s_0, 1-\hat{m}_i, \hat{\mathbf{m}}_{A-i}]$ by $\Delta_i(s_0, \hat{m}_i, \hat{\mathbf{m}}_{A-i})$. Of course, player i has no reason to lie when $s_i = 1$, and this can only move $\hat{y}$ to the left and he is biased rightward. When $s_i = 0$, player i gains by lying if and only if he moves $\hat{y}$ to the right by not too much. Specifically, suppose that the leader's response $\Delta_i(s_0, \hat{m}_i = 0, \hat{\mathbf{m}}_{A-i})$ to i's lie $1 - \hat{m}_i = 1$ is such that $\Delta_i(s_0, \hat{m}_i = 0, \hat{\mathbf{m}}_{A-i}) > 2b_i$. Then, the leader's response to i's lie "overshoots" i's bliss point $E[x|s_0, \hat{m}_i = 0, \hat{\mathbf{m}}_{A-i}]+b_i$ so much that it makes player i worse off relative sending the truthful message $m_i = s_i = 0$. Conversely, when instead $\Delta_i(s_0, \hat{m}_i = 0, \hat{\mathbf{m}}_{A-i}) < 2b_i$, player i of type b_i is better off deviating from equilibrium and lying.

The following considerations are important about condition (1) and the above explanation. First, when expert i is less biased relative to the leader, $\left|b_i - \bar{b}_0\right|$ is smaller, condition (1) is more likely satisfied and equilibrium communication is more informative. This fact is a manifestation of a general feature of cheap talk games. In a model with a continuum of signals s_i and messages m_i, it is known since Crawford and Sobel (1982), that equilibrium communication must be "partitional." That is, the signal space S_i is partitioned into a finite set of intervals, and i sends the same message $\hat{m}_i$ and induces the same decision $\hat{y}$ for every s_i in the same interval.

Second, the fact that the truthtelling conditions (1) is more easily satisfied if $|\Delta(s_0, \hat{m}_i, \hat{\mathbf{m}}_{A-i})|$ is larger for all $s_0, \hat{m}_i, \hat{\mathbf{m}}_{A-i}$ is also not a special feature of the simple model with binary signals s_i considered here. The quantity $|\Delta(s_0, \hat{m}_i, \hat{\mathbf{m}}_{A-i})|$ measures the influence that i exerts on the leader's decision $\hat{y}$ by lying. It is larger if i's signals becomes more informative relative to the information the leader gathers from other sources, here her own signal s_0 and the messages $\hat{\mathbf{m}}_{A-i}$ of the other consulted advisors. As shown by Moreno de Barreda (2010) in a general continuous signal model with a unique sender, the equilibrium partition becomes more informative as the decision maker's own signal informativeness is reduced, holding the sender's information constant.

Indeed, even if player i's signal and message spaces are continuous, i cannot induce arbitrarily small changes in the leader's decision by lying. In every equilibrium, the set of possible decisions is finite, and consequently, so is the set of decisions that player i can induce by lying. Each equilibrium is characterized by conditions ensuring that when s_i lies on the boundary between two intervals in the signal space partition, player i is indifferent about inducing either decision associated with those intervals. These conditions share similar qualitative properties with the truthtelling conditions (1): they become less stringent as player i's signal becomes more informative relative to the leader's own information.

Returning to the description of this paper's formalization, we single out the main implications of the equilibrium characterizations in Proposition 1, as they will be driving most subsequent results on the trade-off between alignment and competence.

Corollary 2 *Each expert's equilibrium truthtelling condition is more stringent if the expert is less aligned with the leader, and if the leader receives information from more advisors in equilibrium.*

We conclude the section by noting that these results stem from analyzing advice as a communication game. In such games, the players' messages do not directly affect their payoffs, and are relevant only in how they influence the leader's decision. Here, advisors care solely about the receiver's response to their advice. Each advisor i aims to bias the leader's decision but is concerned about the leader overshooting his response if he lies. When the leader has better information independent of i's advice, there is less risk of overshooting if i lies. Consequently, it becomes more challenging to prevent advisors from lying, and equilibrium communication becomes less informative. As a result, each expert's equilibrium truthtelling condition becomes more stringent if the leader consults more advisors in equilibrium. As discussed in the concluding Section 6, this may change if advisors also care about their reputation for truthtelling, as the leader can cross-check their reports. We postpone to future research the investigation of this and other extensions of our model discussed in Section 6.

The next section specializes the model and results of this section, focusing on the trade-off faced by a leader when choosing advisors: whether to prioritize competence, diversity of views, or political alignment.

4 Political Alignment vs. Competence and Independence

Model of Advice and Advisor Choice. A leader (player 0) makes a decision $\hat{y} \in \mathbb{R}$ to maximize her expected utility $u_0(\hat{y}, x) = -(\hat{y} - x)^2$, based on an unknown state x uniformly distributed on $[0, 1]$. The leader receives a binary signal $s_0 \in \{0, 1\}$ such that $\Pr(s_0 = 1|x) = x$. Before observing s_0 and choosing $\hat{y}$, the leader may consult one or two advisors $i = 1, 2$, who each receive a binary signal $s_i \in \{0, 1\}$ informative of x. If consulted, each advisor sends a binary message $\hat{m}_i \in \{0, 1\}$ to the leader, simultaneously if both are consulted.

Each advisor i's utility is $u_i(\hat{y}, x) = -(\hat{y} - x - b_i)^2$, where the advisor's bias b_i is common knowledge. Advisor 1 is more closely aligned with the leader than Advisor 2, $|b_2| > |b_1| > 0$, but has information less valuable to the leader. Advisor 2's signal s_2 is independent of s_0 with $\Pr(s_2 = 1|x) = x$, while Advisor 1's signal s_1 may either be less precise than s_2 or correlated with s_0. Specifically, there is an unobserved signal $s_1' \in \{0, 1\}$, independent of s_0 and s_2, such that $\Pr(s_1' = 1|x) = x$. To model that s_1 is less precise than s_2, and hence expert 1 less competent than 2, we stipulate that $s_1 = s_1'$ with probability $p \in (1/2, 1)$, and otherwise $s_1 = 1 - s_1'$. To represent that advisor 1's signal s_1 is correlated with the leader's signal, we say that $s_1 = s_0$ with probability $\rho \in (0, 1)$, and otherwise $s_1 = s_1'$.

The leader's decision strategy is $y : (s_0, \hat{\mathbf{m}}_A) \mapsto \mathbb{R}$, and each advisor's message choice is $m_i : s_i \to \hat{m}_i$. The analysis focuses on pure strategy Bayesian equilibrium $(A, \mathbf{m}, y)$. Multiple equilibria exist, and the equilibrium with the highest welfare calculated ex-ante is selected, as is customary in communication games (see, e.g., Crawford and Sobel, 1982).

Communication Equilibrium Characterization. There are four possible equilibria to consider: (1) both advisors tell the truth, (2) only one advisor (either 1 or 2) is truthful, and (3) the babbling equilibrium. We consider them in order. Before proceeding, we briefly report results by Galeotti et al. (2013) to simplify the exposition. They cover the case where both advisors' signals $s_i \in \{0, 1\}$ are i.i.d. Bernoulli trials, i.e., $\Pr(s_i = 1|x) = x$. They show that in any communication equilibrium $(\mathbf{m}, y)$ where the leader's information consists of k signals, the bias b_i of each truthful advisor i must be such that $b_i \leq \frac{1}{2(k+2)}$, and the leader's equilibrium welfare is $W(\mathbf{m}, y) = -\frac{1}{6(k+2)}$.

Consider the equilibrium $\mathcal{E}_{12}$, where both advisors are truthful and consulted. The thresholds for advisor $i = 1, 2$, $\eta_{12.i}$, are calculated in the appendix. There is a fully revealing equilibrium $\mathcal{E}_{12}$ if $b_1 \leq \eta_{12.1}$ and $b_2 \leq \eta_{12.2}$. In Figure 2, we graph the thresholds $\eta_{12.1}$ and $\eta_{12.2}$ and the leader's ex-ante welfare W_{12} for both the case when signal s_1 is less precise than s_2, and when s_1 is correlated with s_0. The threshold $\eta_{12.1}$ lies below $\eta_{12.2}$ for all values of p and ρ. As the informativeness of signal s_1 increases (p increases or ρ decreases), $\eta_{12.1}$ becomes less stringent, and $\eta_{12.2}$ more demanding. Equilibrium welfare W_{12} increases and is concave in p (decreases and is convex in ρ). As s_1 becomes

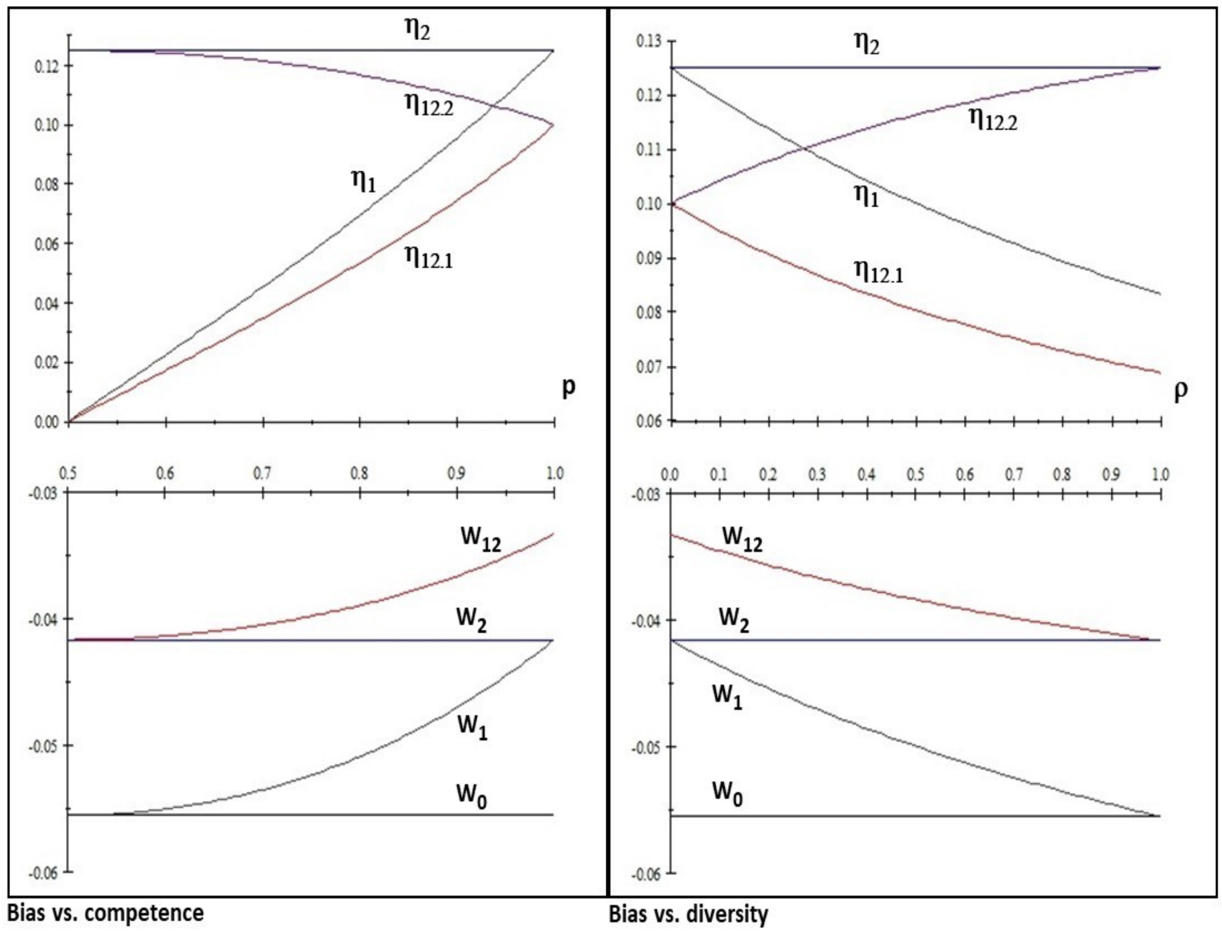

Figure 2: Bias vs. Competence/Diversity – Equilibrium and Welfare Thresholds

more informative, W_{12} increases with diminishing returns.[11]

Next, we consider the equilibrium $\mathcal{E}_1$ where only advisor 1 is consulted. Figure 2 illustrates the threshold η_1, which decreases in p and increases in ρ, always lies above $\eta_{12.1}$, but crosses $\eta_{12.2}$. Advisor 1's equilibrium truthtelling condition is more stringent if advisor 2 is also truthful. The welfare $W_1(p)$ increases in p and is concave, whereas $W_1(\rho)$ decreases in ρ and is convex. $W_1 < W_{12}$ for all p and ρ. Further, $\eta_1 = 1/8$ and $W_1 = -1/24$, when $p = 1$ or $\rho = 0$, as the leader holds 2 i.i.d. Bernoulli signals when choosing $\hat{y}$, whereas $\eta_1 = 1$ and $W_1 = -1/18$, when $p = 1/2$ or $\rho = 1$, because the leader only observes her own signal.

The equilibrium $\mathcal{E}_2$, where only advisor 2 is truthful, is simpler. Since s_2 is independent of s_0 and $\Pr(s_2 = 1|x) = x$, the leader is informed of 2 i.i.d. Bernoulli signals. This equilibrium exists if and only if $b_2 \le 1/8 \equiv \eta_2$, and the leader's welfare is $W_2 = -1/24$. Thus, η_2 and W_2 are independent of

[11]Further, when $p = 1$, or $\rho = 0$, the thresholds $\eta_{12.1}$ and $\eta_{12.2}$ equal 1/10 and the welfare is $W_{12} = -1/30$, as the leader's information consists of 3 i.i.d. Bernoulli signals. Instead, the information is 2 i.i.d. signals when $p = 0$ or $\rho = 1$, so that $\eta_{12.1} = 0$, $\eta_{12.2} = 1$, and $W_{12} = -1/24$.

p and ρ, and $W_2 < W_{12}$ for all p and ρ.

Finally, when both advisors' biases are too large ($b_1 > \eta_1$ and $b_2 > \eta_2$), neither advisor is consulted in the unique equilibrium $\mathcal{E}_0$, as they both babble. The leader's welfare is $W_0 = -1/18$, as she holds only one Bernoulli signal (her own) when choosing $\hat{y}$. W_0 is smaller than both W_1 and W_2: the equilibrium welfare is lowest if the leader decides on her own.

The Optimal Equilibrium. The results shown in Figure 2 that $\eta_{12.1} < \{\eta_{12.2}, \eta_1\} < \eta_2$ imply that not only there exist bias pairs (b_1, b_2) such that only the least biased advisor 1 tells the truth in equilibrium, but also (b_1, b_2) such that only expert 2 is truthful.[12] The former is the case when $b_1 \leq \eta_1$ and $b_2 > \eta_2$, and the latter when $\eta_1 < b_1 < b_2 \leq \eta_2$. Indeed, when $b_1 \leq \eta_1$ and $b_2 \leq \eta_2$, but either $b_1 > \eta_{12.1}$ or $b_2 > \eta_{12.2}$, the leader can get truthful information from either advisor 1 or 2, but not from both. This result identifies an "equilibrium" trade-off between advisors which exists even when the cost of advice is small, and that emerges due to the strategic interaction between advisors: The truthtelling requirements are more demanding when both advisors are consulted.

When the leader can get truthful information from either advisor 1 or 2, but not from both, she only consults advisor 2, who is better informed, as we proved that $W_2 > W_1$. This finding leads to the following takeaway: the leader hires the more biased but better-informed advisor 2 when his truthtelling conditions are met, and adds the more aligned expert 1 only if it does not hinder truthtelling from the better-informed advisor.

We summarise these results into the following characterization of the optimal equilibrium.

Proposition 1 *If $b_1 \leq \eta_{12.1}$ and $b_2 \leq \eta_{12.2}$, then the leader optimally consults both experts. If $\eta_{12.2} < b_2 \leq \eta_2$, then she hires only 2, the better informed advisor. If $b_2 > \eta_2$ and $b_1 \leq \eta_1$, then the leader consults only 1, the least biased advisor. If $b_1 > \eta_1$ and $b_2 > \eta_2$, then the leader does not consult any expert.*

The optimal equilibrium as a function of the biases b_1 and b_2 is depicted in Figure 3. Darker shades denote higher welfare equilibrium. In the darkest area, the leader consults both advisors. In the intermediate grey area, she consults expert 2, whose signal is most informative. In the lightest grey area, she resorts to the advice of advisor 1. In the white area, neither expert is consulted. Inspection of Figure 3 reveals some of the most interesting results of this section, particularly in terms of comparative statics.[13]

[12] We use the notation $S > S'$ to say that all elements of a set S dominate all elements of another S', and identify singleton sets S by their element for brevity.

[13] Proposition 2 reports only the most unexpected comparative statics results. There also exist instances such that an increase of bias b_1 leads to dismissing advisor 1 (this is the case when $b_1 \leq h_{12.1}$ and $b_2 \leq h_{12.2}$, and when $b_1 \leq h_1$ and $b_2 > h_2$), and that increasing b_2 causes the discharge of 2, when $b_1 > h_{12.1}$ and $b_2 \leq h_2$.

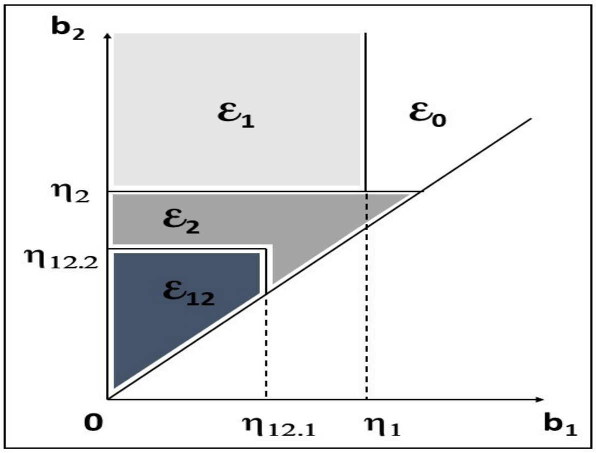

Figure 3: Bias vs. Competence/Diversity – Optimal Equilibrium

Proposition 2 *Suppose that $b_1 \leq \eta_{12.1}$ and $b_2 \leq \eta_{12.2}$, so that the leader consults both experts. Increasing the bias b_2 of the most biased advisor 2 leads to dismissing the other advisor 1, and to retaining expert 2, when $\eta_{12.2} < b_2 \leq \eta_2$. As the bias b_2 further increases, the leader dismisses advisor 2 and hires back expert 1, when $b_2 > \eta_2$.*

While it is unexpected that, as the bias of expert 2 grows, the other advisor is dismissed, this result is easy to understand with the aid of our analysis. Starting from a situation where $b_1 \leq \eta_{12.1}$ and $b_2 \leq \eta_{12.2}$ so that the leader extracts truthful information from both advisors, increasing the bias b_2 of advisor 2 leads to a situation where it is impossible for both experts to be truthful in equilibrium: $\eta_{12.2} < b_2 \leq \eta_2$. The leader then needs to choose an advisor, and she will secure the more valuable information of expert 2. As the bias b_2 grows further, however, expert 2 will not be truthful in equilibrium anymore. The leader will need to resort to the inferior information of the more aligned advisor 1.

Proposition 2 bears the following implications. Suppose the views of a newly-elected policymaker mark a significant change from those of previous officeholders. This results in a sharp increase in the distance between the views of the current leader and the most experienced advisors, who have been nominated and gained expertise under previous administrations. In the context of Proposition 2, this corresponds to a sudden increase in the bias b_2 of the competent expert 2 relative to the new leader. The question then arises: should the new leader retain the most experienced consultants or rely on ideologically closer advisors, represented by expert 1 in our model? Proposition 2 suggests that, while previous leaders could afford to seek advice from both the most experienced experts (advisor 2) and politically aligned consultants (advisor 1), the new leader should only listen to the most experienced consultants and avoid mixing them with inexperienced politically-loyal advisors.

Indeed, newly elected policymakers often prioritize competence and continuity over political alignment when selecting collaborators. For insteance, President Obama appointed Robert Gates in 2008, a prominent member of George W. Bush's cabinet, as Secretary of Defense. This decision was significant because of the contrasting positions of the Bush and Obama administrations on the Iraq War. While the Bush administration initiated the conflict, the Obama administration aimed to end it responsibly. To facilitate a successful withdrawal from Iraq, President Obama chose competence over political loyalty in selecting a defense secretary. Consequently, Obama appointed Gates for his pragmatism and effective management of the Pentagon during the Iraq surge and operations in Afghanistan, valuing continuity across administrations (Gates, 2010; Suri, 2018).

It is also not difficult to find instances where newly elected politicians prioritized political alignment over competence and continuity, sometimes leading to catastrophic consequences. When populist, pro-Brexit Boris Johnson became UK Prime Minister in December 2018, his extreme views marked a significant shift from his moderate predecessors. Unlike in the US, the UK does not have a spoil system; the permanent secretary and all top civil servants maintain their positions regardless of the Prime Minister. Traditionally, such career advisors are relied upon for top advising roles, with private consultants added only for specific policy campaigns.

However, Johnson broke with tradition by appointing Dominic Cummings, a political strategist with no policy experience and the mastermind behind the "Leave" campaign, to the most prominent advising role in government. Instead of depending on competent and experienced, but possibly misaligned, career civil servants, Johnson chose to rely mainly on the advice of an inexperienced but ideologically aligned private consultant. This departure from the norm can be attributed to their shared Brexit objectives.

Cummings' tenure in the Johnson government was disastrous and led to his eventual dismissal. His lack of experience and unrestrained conduct caused conflicts with other government members and advisors. As our analysis suggests, he prevented them from providing honest and effective advice (Seldon and Newell, 2023). One notable conflict occurred when Cummings dismissed Sonia Khan, a special advisor to the Chancellor of the Exchequer, Sajid Javid, resulting in Javid's resignation. In accordance with Proposition 2, Johnson should not have granted Cummings a prominent role in government.

While our analysis focuses on pure strategy equilibrium, we conclude this part of the section by showing the robustness of our results to the consideration of mixed strategy equilibrium.

Remark 1 We show in a supplementary appendix that the results in Propositions 1 and 2 generalize if considering also mixed strategy equilibrium, but the analysis becomes significantly more involved and requires non-negligible consulting costs. Take again Figure 3, and suppose that $b_1 < \eta_{12.1}$. The mixed strategy equilibrium $\mathcal{M}_{1.2}$ in which 1 tells the truth and 2 randomizes exists only for $b_2 < \eta_{12.2}$, where also E_{12} exists and dominates. For $\eta_{12.2} < b_2 < \eta_2$, there exist 3 equilibria, ordered in welfare as follows: $\mathcal{M}_{2.1}$, where 1 randomizes and 2 tells the truth, E_2, in which 1 babbles and 2 tells the truth, and E_1, where 2 babbles and 1 is truthful. As b_2 grows, the probability that 1 tells the truth in $\mathcal{M}_{2.1}$ decreases and shrinks to zero as $b_2 = \eta_2$. For $b_2 > \eta_2$, the only equilibrium is E_1. Hence, for any (not too large) consulting cost $c > 0$, there exists a threshold $\bar{\eta}_2(c) > \eta_{12.2}$ decreasing in c, such that for $b_2 < \bar{\eta}_2(c)$, one should hire both experts 1 and 2 (possibly relying on equilibrium $\mathcal{M}_{2.1}$), whereas as b_2 crosses $\bar{\eta}_2(c)$ it becomes optimal to consult only 2. Such a result generalizes Proposition 2, but the region in which consulting only 2 is optimal shrinks as c becomes small, because $\bar{\eta}_2(c) \to \eta_2$ for $c \to 0$. ◇

Delegation of Consulting and Decision-Making. The final part of this section determines whether the leader opts to maintain authority or delegate to one of the two advisors, who will subsequently consult the other two players before making a decision. In the appendix, we demonstrate that the leader delegates exclusively to advisor 1, and only if she anticipates that he will be more informed than she is in equilibrium. This requires that 1 and 2 are biased in the same direction, so that 1's bliss point b_1 is in an intermediate position relative to players 0 and 2. Further, Proposition 5 reveals a distinction in the trade-offs between political alignment and either competence or independence. The precise conditions under which delegation takes place are in appendix.

Proposition 3 *The leader delegates authority only to advisor* 1 *if and only if his bias* $|b_1|$ *is not too large and has the same sign as* b_2*. When expert 1's signal* s_1 *is less precise than* s_2*, the leader delegates if 1 is fully informed in equilibrium, and she would not be. When* s_1 *is correlated with the leader's signal* s_0*, she delegates if either 1 is fully informed in equilibrium and she would only receive either* s_1 *or* s_2*, or if expert 1 receives* s_2 *and she would.*

The leader never delegates to the more informed, more biased advisor 2. This is because signal s_2, while more valuable than s_1, is never more valuable than the leader's own signal s_0 in our model. However, this does not mean that the leader will never delegate decisions. Paradoxically, she may delegate to advisor 1, who is less informed than advisor 2. This happens when advisors 1 and 2 are biased in the same direction, and advisor 1 is not too biased relative to the leader. In this case, advisor 1 is not only ideologically closer to the leader than advisor 2, but is also ideally placed to gather information from both the leader and advisor 2, as his position lies between theirs. In sum, the leader delegates to advisor 1 not because his personal information s_1 is superior to hers, but because he obtains better information in equilibrium than she does.

Delegation to political agents with such "intermediate," moderate views is indeed commonplace. Consider, for instance, the appointment of William Ruckelshaus as the first administrator of the U.S. Environmental Protection Agency (EPA) in the Nixon administration. Known for his moderate Republican stance, Ruckelshaus skillfully managed the challenges of a divided government and was well remembered for his integrity and trust. His moderate position sometimes clashed with the Republican Party's more antagonistic stance on environmental regulation. This was evident in his decisive actions, such as the banning of DDT and the enforcement of pollution control technologies (Dobel, 1995). As another example, consider the appointment of Joe Lieberman, a moderate Democrat, as Chairman of the Senate Committee on Homeland Security and Governmental Affairs in 2001. During his tenure, Lieberman played a pivotal role in establishing the Department of Homeland Security (DHS). He worked closely with Republican Senator Susan Collins, fostering a bipartisan relationship to pass significant legislation.

5 Uncertain trade off

Model. We generalize the model of Section 4 by making advisor 2's bias private information. Specifically, 2's bias is $b_1 > 0$ with probability q, and $b_2 > b_1$ with probability $1 - q$. Expert 2 may or may not be less aligned than advisor 1, so that the trade off faced by the leader when choosing between 1 and 2 is "uncertain." Again, advisor 1's signal s_1 is less informative to the leader's than expert 2's. For brevity, we posit the analysis for the case in which s_1 is less precise. There is an unobserved signal $s_1' \in \{0, 1\}$, independent of s_0 and s_2, such that $\Pr(s_1' = 1|x) = x$, and $\Pr(s_1 = s_1'|s_1') = p \in (1/2, 1)$.

Equilibrium Characterization. The only difference with the analysis in Section 4 concerns the equilibria of the communication game in which the strategy of advisor 2 differs across his two types b_1 and b_2. Consequently, all the results previously derived for the equilibria $\mathcal{E}_{12}$, $\mathcal{E}_1$, $\mathcal{E}_2$ and $\mathcal{E}_0$ still hold, because, in all these equilibria, the strategy of expert 2 is the same across his types. Additionally, we must now consider two more equilibria where the two bias types of advisor 2 employ different strategies.

The equilibrium $\mathcal{E}_{12A}$ is such that expert 1 and the aligned bias type b_1 of advisor 2 are truthful, whereas the bias type b_2 babbles. It exists when the two truthtelling conditions $b_1 \leq \eta_{12A.1}$ and $b_1 \leq \eta_{12A.2}$ are met, one for advisor 1 and one for type b_1 of expert 2. Therefore, equilibrium $\mathcal{E}_{12A}$ exists if $b_1 \leq \eta_{12A}(p, q) \equiv \min\{\eta_{12A.1}(p, q), \eta_{12A.2}(p, q)\}$. The threshold functions $\eta_{12A.1}$ and $\eta_{12A.2}$ are calculated in the appendix. We show that $\eta_{12A.1}$ is concave, increases in p, and decreases in q, with $\eta_{12A.1}(1/2, q) = 0$ for all q, $\eta_{12A.1}(1, 0) = 1/8$, $\eta_{12A.1}(1, 1) = 1/10$, whereas $\eta_{12A.2}$ is convex, decreases in p, and increases in q, and such that $\eta_{12A.2}(1, 1) = 1/10$, $\eta_{12A.2}(1, 1/2) = 1/8$, $\eta_{12A.2}(0, 1/2) = 1/16$, and $\eta_{12A.2}(0, 1) = 1/20$. The two functions cross on an increasing line in the (p, q) space. The welfare W_{12A} of equilibrium $\mathcal{E}_{12A}$ increases and is convex in p and q, and such that $W_{12A}(1, 1) = -1/30$ as in the case where the leader holds 3 i.i.d. Bernoulli signals; $W_{12A}(1/2, 1) = W_{12A}(1, 0) = -1/24$, as for 2 i.i.d. signals; and $W_{12A}(1/2, 0) = -1/18$, as with 1 signal.

Turning to the equilibrium $\mathcal{E}_{2A}$ in which the aligned type b_1 of expert 2 is truthful, whereas advisor 1 and the bias type b_2 of expert 2 babble, the formula (1) yields η_{2A} such that the truthtelling condition of the aligned type of player 2 holds if $b_1 \leq \eta_{2A}$. The threshold η_{2A} increases and is concave in q and is independent of p, with $\eta_{2A}(0) = 1/16$ and $\eta_{2A}(1) = 1/8$. The welfare W_{2A} of equilibrium $\mathcal{E}_{2A}$ increases and is convex in q, with $W_{2A}(1) = -1/24$, as with 2 i.i.d. signals, and $W_{2A}(0) = -1/18$, as with 1 signal.

Optimal Equilibrium. We prove in the appendix that the equilibria are ranked in terms of welfare.

Lemma 1 *The welfare of communication game equilibria is ranked as follows:* $W_{12} > \{W_2, W_{12A}\} > \{W_{2A}, W_1\}$. *There exist functions* $g_1, g_2 : q \mapsto p$ *such that* $W_2 > W_{12A}(p,q)$ *if and only if* $p < g_2(q)$ *and* $W_{2A}(q) > W_1(p)$ *if and only if* $p < g_1(q)$. *The function* g_1 *strictly increases in* q *with* $g_1(0) = 0$ *and* $g_1(1) = 1$, *whereas* g_2 *strictly decreases in* q *with* $g_2(0) = 1$ *and* $g_2(1) = 0$.

In words, the equilibrium $\mathcal{E}_{12}$ in which all types of both advisors are truthful is top welfare ranked for all values of p and q. Then come both the equilibrium $\mathcal{E}_2$, in which both types of expert 2 tell the truth, and the equilibrium $\mathcal{E}_{12A}$, where expert 1 and type b_1 of advisor 2 are truthful. The ranking of these two equilibria among each other depends on the parameters (p,q). The equilibrium $\mathcal{E}_2$ is welfare superior when p and q are low, i.e., formally $p < g_2(q)$. Intuitively, the leader does not consult advisor 1 to secure truthful advice from both types of advisor 2, when 1's signal precision p is low, and when the probability q that expert 2 is aligned is low. Both $\mathcal{E}_2$ and $\mathcal{E}_{12A}$ uniformly dominate both equilibria $\mathcal{E}_1$, where only advisor 1 is truthful, and $\mathcal{E}_{2A}$, where only the aligned type of expert 2 is truthful. Again, these two equilibria are not welfare ranked for all (p,q). The equilibrium $\mathcal{E}_1$ dominates when p is high and q is low, that is, $p > g_1(q)$. As is intuitive, the leader prefers the advice of expert 1 to that of the aligned type of 2 when 1's signal precision p is high, and when the probability q that expert 2 is aligned is low.

Lemma 1 partitions the (p,q)-parameter space into four regions delimited by the functions g_1 and g_2, which we depict in Figure 4. In the region above g_1 and g_2, characterized by high p, the optimal equilibria involve consulting expert 1: equilibria $\mathcal{E}_{12A}$ and $\mathcal{E}_1$ dominate $\mathcal{E}_2$ and $\mathcal{E}_{2A}$, respectively. In the region to the right of g_1 and g_2, where q is high, the equilibria $\mathcal{E}_{12A}$ and $\mathcal{E}_2$ are better than $\mathcal{E}_{2A}$ and $\mathcal{E}_1$, because the risk that expert 2 is of type b_2 is sufficiently low. In the region below g_1 and g_2, where p is low, the leader prefers to consult expert 2: the equilibria $\mathcal{E}_2$ and $\mathcal{E}_{2A}$ outrank $\mathcal{E}_{12A}$ and $\mathcal{E}_1$. Finally, in the region to the left of g_1 and g_2, the equilibria $\mathcal{E}_2$ and $\mathcal{E}_1$ dominate $\mathcal{E}_{12A}$ and $\mathcal{E}_{2A}$: the leader is not interested in consulting advisor 2 unless his type b_2 tells the truth.

The optimal communication equilibrium characterization does not end with Lemma 1, however, because it is not always the case that the optimal equilibrium among $\mathcal{E}_{12A}$, $\mathcal{E}_1$, $\mathcal{E}_2$, and $\mathcal{E}_{2A}$ exists for

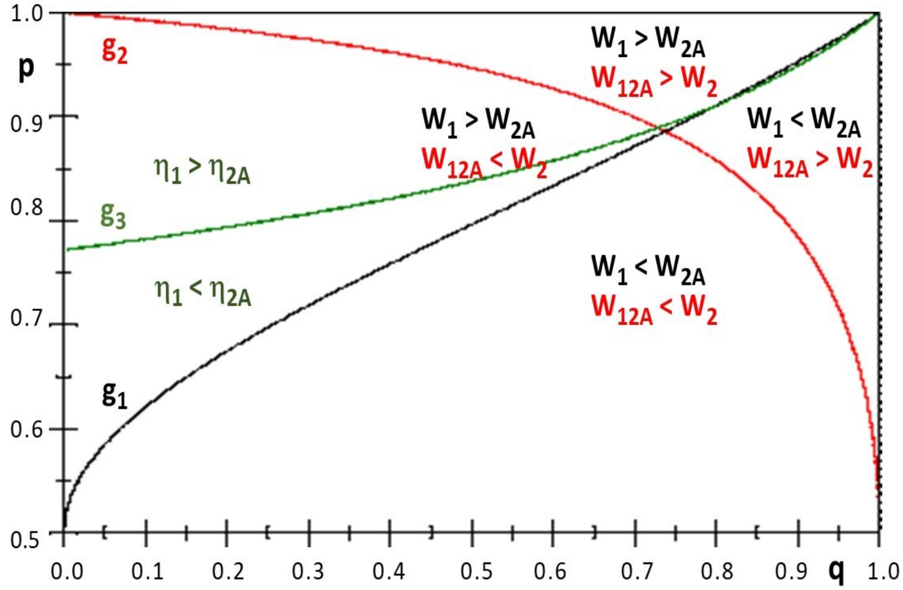

Figure 4: Uncertain Trade-off – Optimal Equilibrium Regions

all bias values (b_1, b_2). We prove in the appendix that the equilibrium existence threshold functions are such that: $\eta_{12.1} < \eta_{12A} < \{\eta_1, \eta_{2A}, \eta_{12.2}\} < \eta_2$. So, the existence conditions $b_1 < \eta_{12.1}(p)$ and $b_2 < \eta_{12.2}(p)$ of equilibrium $\mathcal{E}_{12}$ are satisfied only for smaller values of b_1 than equilibrium $\mathcal{E}_{12A}$'s condition $b_1 < \eta_{12A}(p, q)$, and for smaller b_2 than $\mathcal{E}_2$'s condition $b_2 < \eta_2$. Further, equilibrium $\mathcal{E}_{12A}$ only exists for smaller b_1 than both $\mathcal{E}_{12A}$ and $\mathcal{E}_1$. Instead, the existence ranges of $\mathcal{E}_{12A}$ and $\mathcal{E}_1$ are not ordered in terms of inclusion. Lemma 2 identifies a third threshold function g_3, determined by $\eta_1(p) = \eta_{2A}(q)$. This function indicates whether $\mathcal{E}_1$'s existence condition $b_1 \leq \eta_1$ is tighter or looser than $\mathcal{E}_{2A}$'s condition $b_1 \leq \eta_{2A}$. When $\eta_1(p) > \eta_{2A}(q)$, $\mathcal{E}_1$'s existence region in the bias parameter space (b_1, b_2) strictly contains that of $\mathcal{E}_{2A}$, and vice versa.

Lemma 2 *There exists a strictly increasing function $g_3 : q \mapsto p$ such that $\eta_1(p) > \eta_{2A}(q)$ if and only if $p > g_3(q)$. The function g_3 is such that $g_1(q) < g_3(q) < g_2(q)$ for low q. As q grows, g_3 first crosses g_2 and then g_1 to finally join g_1 again at $q = 1$, with $g_3(1) = 1$.*

While function g_1 determines whether equilibrium $\mathcal{E}_1$ is better or worse than $\mathcal{E}_{2A}$ in terms of welfare, g_3 indicates whether equilibrium $\mathcal{E}_1$ exists for lower bias values b_1 than equilibrium $\mathcal{E}_{2A}$, or vice versa. Consider Figure 4. In the region above both curves g_1 and g_3, $\mathcal{E}_1$ dominates $\mathcal{E}_{2A}$ and exists for a larger set of biases b_1. If the bias values (b_1, b_2) are such that none of the superior equilibria $\mathcal{E}_{12}$, $\mathcal{E}_2$, or $\mathcal{E}_{12A}$ are available, the leader hires only advisor 1, making $\mathcal{E}_1$ the optimal communication equilibrium. In the region above g_1 and below g_3, while $\mathcal{E}_1$ again dominates $\mathcal{E}_{2A}$, $\eta_1(p) < \eta_{2A}(q)$. When the equilibria $\mathcal{E}_{12}$, $\mathcal{E}_2$, and $\mathcal{E}_{12A}$ do not exist, the optimal equilibrium available to the leader will be either $\mathcal{E}_1$ or $\mathcal{E}_{2A}$, depending on whether $b_1 \leq \eta_1(p)$, or $\eta_1(p) < b_1 \leq \eta_{2A}(q)$. The

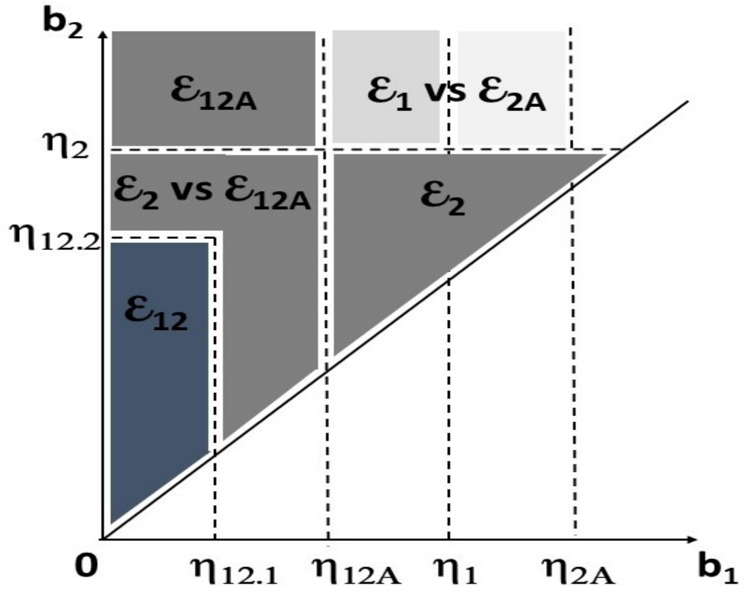

Figure 5: Uncertain Trade-off - Optimal Equilibrium

same arguments hold symmetrically in the region below g_1.

The results in Lemmas 1 and 2 lead to the following characterization of the optimal communication equilibrium, denoted as $\mathcal{E}^*$.

Proposition 4 *The optimal equilibrium $\mathcal{E}^*$ of the game of uncertain trade-off is as follows:*

1. If $b_1 \leq \eta_{12.1}(p)$ and $b_1 \leq \eta_{12.1}(p)$ then $\mathcal{E}^* = \mathcal{E}_{12}$; if $b_1 > \eta_{12A}(p,q)$ and $b_2 \leq \eta_2$ then $\mathcal{E}^* = \mathcal{E}_2$; and if $b_1 \leq \eta_{12A}(p,q)$ and $b_2 > \eta_2$ then $\mathcal{E}^* = \mathcal{E}_{12A}$;

2. If $\eta_{12.1}(p) < b_1 \leq \eta_{12A}(p,q)$ and $\eta_{12.2}(p) < b_2 \leq \eta_2$, then $\mathcal{E}^* = \mathcal{E}_2$ when $p < g_2(q)$, and $\mathcal{E}^* = \mathcal{E}_{12A}$ when $p > g_2(q)$;

3. If $\eta_{12A}(p,q) < b_1 \leq \max\{\eta_{2A}(q), \eta_1(p)\}$ and $b_2 > \eta_2$, then $\mathcal{E}^* = \mathcal{E}_{2A}$ when $p < g_1(q)$ or $\eta_1(p) < b_1 \leq \eta_{2A}(q)$, and $\mathcal{E}^* = \mathcal{E}_1$ when $p > g_1(q)$ or $\eta_{2A}(q) < b_1 \leq \eta_1(p)$.

The optimal communication equilibrium $\mathcal{E}^*$ is illustrated in Figure 5. The darker shaded area identifies the region where the optimal equilibrium is $\mathcal{E}_{12}$: the leader hires both advisors because neither is significantly biased. The leader also consults both experts in the top-left region where $b_1 \leq \eta_{12A}(p,q)$ and $b_2 > \eta_2$, although advisor 2 babbles if his bias is b_2, because the optimal equilibrium is $\mathcal{E}_{12A}$. In the bottom-right region close to the 45-degree line, only expert 2 is consulted. Here, $b_1 > \eta_{12A}(p,q)$ and $b_2 \leq \eta_2$, making $\mathcal{E}_2$ the optimal equilibrium. In the "inverse L-shaped" region where $\eta_{12.1}(p) < b_1 \leq \eta_{12A}(p,q)$ and $\eta_{12.2}(p) < b_2 \leq \eta_2$, both equilibria $\mathcal{E}_2$ and $\mathcal{E}_{12A}$ are available to the leader. Whether she hires both experts or only expert 2 depends on whether the information of expert 1 is sufficiently valuable and expert 2 is sufficiently aligned, i.e., on whether the point (p, q) is above or below curve g_2 in Figure 4. Somewhat unexpectedly, as expert 2 becomes more likely aligned, the leader may switch from consulting only him to hiring both advisors.

The description of the optimal equilibrium $\mathcal{E}^*$ in the top-right region of Figure 5, where $\eta_{12A}(p, q) < b_1 \leq \max\{\eta_{2A}(q), \eta_1(p)\}$ and $b_2 > \eta_2$, is more intricate. When $b_1 \leq \min\{\eta_{2A}(q), \eta_1(p)\}$, both equilibria $\mathcal{E}_{2A}$ and $\mathcal{E}_1$ exist. The leader consults either advisor 1 or 2 depending on whether 1 is sufficiently informed or 2 is sufficiently likely aligned, i.e., depending on whether (p, q) is above or below curve g_1 in Figure 4. When $\min\{\eta_{2A}(q), \eta_1(p)\} < b_1 \leq \max\{\eta_{2A}(q), \eta_1(p)\}$, only one of the equilibria $\mathcal{E}_{2A}$ or $\mathcal{E}_1$ exists: $\mathcal{E}_1$ if (p, q) is above g_3, and $\mathcal{E}_{2A}$ if (p, q) is below g_3. Thus, if (p, q) is above g_1 and below g_3, $\mathcal{E}_1$ would yield higher welfare than $\mathcal{E}_{2A}$, but the leader must resort to hiring only expert 2 because the equilibrium $\mathcal{E}_1$ does not exist. The opposite happens when (p, q) is below g_1 and above g_3.

The comparison with the analysis of Section 4 is intuitive. Now, the trade-off between advisors 1 and 2 is uncertain because 2's bias may be the same as 1's. So, the leader consults 2 for a broader set of bias parameters (b_1, b_2). It is no longer the case that expert 2 is dismissed for high values of b_2, as long as $b_1 \leq \eta_{12A}$. In this case, truthful communication from the aligned type of expert 2 is enough to seek 2's advice. Further, it may even be the case that advisor 2 is the only one consulted for large b_2. This happens for $\eta_{12A} < b_1 \leq \eta_{2A}$ if the likelihood q that 2 is not more biased than 1 is sufficiently high relative to 1's precision p, i.e., when (p, q) is below g_1 or g_3 in Figure 4.

Comparative Statics. As is the case for Section 4, also the current analysis uncovers non-trivial comparative statics, which we report below.

Proposition 5 *The main comparative statics results for the case of uncertain trade-off are:*

1. Suppose $b_1 \leq \eta_{12.1}(p)$ and $b_2 \leq \eta_{12.2}(p)$ so that both advisors are consulted, and bias b_2 increases. When $p > g_2(q)$, neither advisor is dismissed. If $p < g_2(q)$, then expert 1 is fired when $\eta_{12.2}(p) < b_2 \leq \eta_2$, and hired back as $b_2 > \eta_2$.

2. Suppose that $b_1 \leq \eta_{12A}(p, q)$ and $b_2 > \eta_2$, so that the leader hires both 1 and 2, and b_1 increases. If $p > g_1(q)$, then expert 2 is dismissed when $b_1 > \eta_{12A}(p, q)$. If also $p < g_3(p)$, then 2 is hired back, and 1 discharged, when $\eta_{12A}(p, q) < b_1 \leq \eta_1 (p)$. If $g_3(q) < p < g_1(p)$, expert 1 is fired when $\eta_{12A}(p, q) < b_1 \leq \eta_{2A}(q)$ and hired back as $b_1 > \eta_{2A}(q)$, when 2 is dismissed.

3. Suppose that $\eta_{12.1}(p) < b_1 \leq \eta_{12A}(p, q)$, $\eta_{12.2}(p) < b_2 \leq \eta_2$ and $p < g_2(q)$: the leader only consults expert 2. An increase in the probability q that 2 is aligned leads to hiring also advisor 1, when $p > g_2(q)$.

These results are richer than in the certain trade-off case seen in Section 4. There, the bias of advisor 2's bias is b_2 and its increase when $b_1 \leq \eta_{12.1}(p)$ and $b_2 \leq \eta_{12.2}(p)$ leads to first dismissing expert 1, and then hiring him back to fire advisor 2. Here, the bias of advisor 2 may be either b_1 or b_2. Increasing b_2 may only lead to dismissing advisor 1, when $p < g_2(q)$, and does not lead to dismissing either expert 1 or 2 when $p > g_2(q)$. Interestingly, the most biased in expectation advisor 2 is never fired, and making his bias more likely equal to 1's bias, leads to consulting expert 1 more often.

Further, increasing bias b_1 may now lead to dismissing advisor 2, instead of only expert 1. This occurs in the region above g_1 in Figure 4, when $b_2 > \eta_2$ and b_1 increases to cross $\eta_{12A}(p, q)$ (cf. Figure 5). In this scenario, because $b_2 > \eta_2$, the equilibrium $\mathcal{E}_2$ does not exist. When $b_1 < \eta_{12A}(p, q)$, the optimal equilibrium is $\mathcal{E}_{12A}$, where both expert 1 and the aligned type of 2 are consulted. However, as b_1 increases and crosses $\eta_{12A}(p, q)$, the equilibrium $\mathcal{E}_{12A}$ ceases to exist. Since $p > g_1(q)$, the optimal equilibrium becomes $\mathcal{E}_1$, and the leader dismisses expert 2 to gather information only from expert 1. As in the case of a certain trade-off, increasing one expert's bias may lead to dismissing the other advisor due to their strategic interaction. Here, however, this occurs not only when advisor 2's bias increases, but also when 1's bias is raised.[14]

Finally, non-trivial comparative statics are not limited to changes in the biases b_1 and b_2. In the "inverse-L" shaped region of Figure 5, where $\eta_{12.1}(p) < b_1 \leq \eta_{12A}(p, q)$ and $\eta_{12.2}(p) < b_2 \leq \eta_2$, as expert 2 becomes more likely aligned (i.e., q increases), the leader may switch from consulting only him to hiring both advisors. This occurs because g_2 decreases in q (cf. Figure 4). Hence, an increase in q while holding p constant may lead to crossing g_2, switching from the region where $W_2 > W_{12A}(p, q)$ to the region where this inequality is reversed. As the equilibrium $\mathcal{E}_{12A}$ now dominates $\mathcal{E}_2$, the leader hires expert 1, even though type b_2 of expert 2 will no longer be truthful. Intuitively, if the expert 2's type is more likely b_1, the leader is willing to forgo truthful advice from type b_2 of expert 2 to gain the benefit of hiring advisor 1 and receiving his truthful report.

These findings are related to our results in Proposition 2. There, we found that a newly-elected leader should primarily rely on pre-existing experienced advisors, rather than bringing in less competent, politically loyal consultants. Here, we complement this by considering the implications of learning the views of the pre-existing advisors over time. The final part of Proposition 5 suggests that if the leader learns that the views of experienced advisors are not as different from hers as expected, she can improve decision-making by teaming them up with other loyal consultants, even if they are less experienced.

Such dynamics are fairly common in the appointment of ministers and government advisors. A recent instance is found in South Korean politics. In 2017, newly elected liberal President

[14] In the region above both g_1 and g_3 in Figure 4, further increasing b_1 leads to dismissing expert 1 as well when b_1 crosses h_1. In the region above g_1 and below g_3, increasing b_1 above h_1 leads to dismissing advisor 1 and rehiring expert 2, who is then dismissed when b_1 crosses h_{2L}. The opposite pattern of dismissals occurs as b_1 increases when $g_3(q) < p < g_1(q)$. First, expert 1 is dismissed as b_1 crosses h_{12L}, then advisor 2 is dismissed and 1 rehired when b_1 crosses h_{2L}, and finally 1 is dismissed again when b_1 crosses h_1.

Moon Jae-in appointed Hong Nam-ki, a career civil servant who previously served in previous conservative governments, as Minister of Economy and Finance. There were concerns that he might not align with the current government's policy stance (e.g., Choi, 2018). However, Hong demonstrated loyalty to the current administration after his appointment. Several months later, President Moon appointed Kim Sang-jo, a professor with a background in economics known for his association with progressive and left-wing policy circles but lacking civil service experience, as Chief Presidential Secretary for Policy. Together, Hong Nam-ki and Kim Sang-jo played crucial roles in formulating and implementing economic policies and broader initiatives.

Delegation of Consulting and Decision-Making. As in Section 4, we conclude by considering delegation, as characterized in Proposition 6 below. Unlike in the case where advisor 2 is known to be more biased than 1, the leader delegates to advisor 2, in some cases. The precise conditions under which delegation takes place are in the appendix.

Proposition 6 *The leader delegates to advisor* 1 *if he is not too biased and either would access all information in equilibrium, whereas she would not, or he would receive information from the leader and the aligned type of advisor* 2*, whereas she would not receive any information in equilibrium. The leader delegates to advisor* 2 *if he is not too biased and would access all information in equilibrium, whereas neither she nor expert* 1 *would.*

It is interesting that the leader may prefer to delegate to advisor 2, despite him being more biased than 1 in expectation. This is not because 2 is a more competent advisor. But rather because it may be that advisor 2 obtains the leader's and expert 1's information in equilibrium, and not vice versa. For both types of 2 to truthfully inform a player $i = 0, 1$, the bias b_2 of 2's misaligned type must not be too different from b_i, player i's bias. Instead, truthtelling by (the unique type of) player i to advisor 2 requires only that b_i is not too different from $qb_1 + (1 - q)\, b_2$, the average bias of 2. In practice, it is easy for the leader and expert 1 to be truthful to expert 2 than vice versa exactly because players $i = 0, 1$ do not know 2's preferences. The possibility that expert 2 is no more misaligned than 1 softens their truthtelling conditions. But if expert 2 is of the misaligned type, then he will not be equally willing to share his information.

Delegation to highly competent political agents who do not reveal their political views is a common practice in public administration across several states. Our findings suggest that career civil servants, despite any biases, may outperform elected politicians and political appointees in decision-making because they are mandated to keep their political views private, unlike politicians who need to disclose them for election purposes. As a result, they are better at collecting information from all possible sources and perspectives than elected politicians or political appointees. For instance, the appointment of Cynthia L. Attwood to the Occupational Safety and Health Review

Commission (OSHRC) by both President Barack Obama and President Donald Trump exemplifies this practice. She possesses extensive legal experience and has never publicly disclosed her political affiliation (Hobbs and Jenkins, 2019). Another example of such delegation policies is observed during legislative elections in Korea, where parties often delegate decision-making authority to unaffiliated, competent senior politicians. The delegated authority includes candidate selection, allowing the party to appeal to swing voters beyond its ideological platform.[15]

6 Conclusion

Selecting advisors represents a pivotal yet unresolved aspect of evaluating effective leadership. Our investigation studies the optimal choice of advisors for a leader, considering the delicate balance between political alignment, competence, and diversity of perspectives. The leader can enlist either one or both of the two advisors. One is politically closer to the leader, and the other has more valuable information due to higher competence or access to less correlated data sources. She adds the aligned advisor only if his additional information does not disrupt equilibrium truthtelling by the better-informed advisor. Hence, information trumps political alignment in our analysis.

We uncover intriguing comparative statics. If the leader initially consults both advisors, increasing the bias of the better-informed advisor causes the dismissal of the other advisor, although he is politically more aligned with the leader. Further increasing the better-informerd advisor's bias eventually leads to his removal, and to rehiring the other expert.

Next, we analyze whether the leader may benefit from delegating decision-making to one of the advisors. In contrast to seeking advice, we find that political alignment is the dominant concern. The leader may delegate only to the more aligned advisor, and only when he is able to collect information that she would not have access to in equilibrium.

We subsequently examine the scenario of an 'uncertain trade-off' between a definitely aligned advisor and a more informed expert who may or may not have greater bias. The analysis reveals non-obvious implications. We identify situations where a lower probability of the better-informed expert being more biased results in increased reliance on the less competent but definitely aligned advisor. Delegating to the expert with uncertain bias, who may potentially be misaligned, might become the preferred choice. However, this preference does not stem from the better information the expert is endowed with, but rather from the possibility that he may access superior information in equilibrium.

[15] For example, to secure more seats in the 2016 legislative election, the Democratic Party of Korea entrusted Kim Chong-in with decision-making authority. A scholar and politician, Kim previously served as Minister of Health and Welfare and participated in amending the 1988 Constitution of the Sixth Republic. Despite causing conflict among voters and politicians with strong party loyalty, his leadership positively impacted the election results (Jun, 2016).

Our model makes a valuable contribution to the leadership literature by offering a framework for selecting a diverse pool of advisors based on multiple characteristics. While prior research has primarily concentrated on institutional constraints related to advisor appointments, such as the separation of powers or regime type, it has often overlooked the inherent trade-offs between political alignment and informativeness among advisors. Our framework has the potential for further extensions in multiple directions, and it leaves numerous questions open for exploration in future studies.

Beyond the specifications presented in Sections 4 and 5, for example, the model of Section 3 may be used to represent instances in which advisors favor information sources biased in the same direction as their preferences. This is achieved by stipulating that $p_{j1} > p_{j0}$ for rightward biased advisors j, and vice versa. Advisor j is more likely to mistakenly observe $s_i = 1$ when the "true" signal is $s_i = 0$ than to make the opposite mistake. Further, it is possible to represent a leader who likes to be told what he already independently presumes, simply by assuming that she overestimates the precision of her signal s_0 relative to the advisors' signals.

While our model focuses on advisors caring solely about the decision maker's choice, expanding the framework to incorporate a broader range of motivations for experts' advice selection would be valuable. Political advisors care about their reputation for competence and integrity. Previous studies centered on reputational concerns with one expert and one decision maker, as detailed in the literature review. Exploring leaders' choices among advisors with diverse political alignment and expertise, who also prioritize their reputation, is an intriguing avenue. Repeated interactions with a single advisor enable the leader to retrospectively assess advice, creating a scenario where misaligned advisors may offer valuable counsel to safeguard their reputation. In cases where multiple experts provide simultaneous advice, the leader's ability to cross-check reports may encourage more truthful messages.

Further, advisors highly value being consulted, creating a dual incentive structure that promotes both the costly acquisition of valuable information (see Argenziano, Severinov and Squintani, 2016) and strategically shaping their team and political network to align with the leader's perspectives. These strategic choices not only strengthen advisors influence but also enhance the perceived credibility of their advice in the eyes of the decision maker. Relatedly, the leader may adopt a 'tournament-like' mechanism to screen potential leaders and determine whose advice is more valuable.

It would also be interesting to study the dynamic relationship between a leader and her pool of allies/advisors. History has repeatedly shown the transformation of former adversaries into future allies and vice versa, underscoring the ever-evolving nature of political alliances. When advice lacks informative value or conflicts with the leader's objectives, the timing of an advisor's dismissal may need to be considered in light of its potential impact on the leader's effectiveness.

Other research questions arise from the insights of Hermann and Preston (1994), who observed that the composition of an advisory team significantly hinges on a president's leadership style. According to Johnson (1974), presidents' leadership styles can be categorized into three types. The formalistic style aims to minimize human error through hierarchical structures. The collegial approach places a premium on teamwork, shared responsibilities, consensus-building, and receptiveness to diverse information sources. In contrast, the competitive style encourages overlapping spheres of authority, promoting information gathering and diverse opinions. At the same time, different advisors in a team may be called to take upon different roles. The complementarities and synergies between "idea persons" and advisors focused on implementation are a major determinant for the well functioning of an optimal mix of advisors (Leonard Wantchekon, private conversation).

References

[1] Alesina, A., and Tabellini, G. (2007): Bureaucrats or Politicians? Part I: A Single Policy Task. *American Economic Review*, 97(1), 169–179.

[2] Alonso, R., W. Dessein, and N. Matouschek (2008): When Does Coordination Require Centralization? *American Economic Review*, 98(1): 145-79.

[3] Argenziano, R., S. Severinov and F. Squintani (2016): Strategic Information Acquisition and Transmission. *American Economic Journal: Microeconomics*, 8(3): 119-55.

[4] Ash, E., Morelli, M., and Vannoni, M. (2022): Divided Government, Delegation, and Civil Service Reform. *Political Science Research and Methods*, 10(1), 82–96.

[5] Battaglini, M. (2002): Multiple Referrals and Multidimensional Cheap Talk. *Econometrica*, 70(4), 1379–1401.

[6] Bendor, J., Glazer, A., and Hammond, T. (2001): Theories of Delegation. *Annual Review of Political Science*, 4(1), 235–269.

[7] Bendor, J., and Meirowitz, A. (2004): Spatial Models of Delegation. *American Political Science Review*, 98(2), 293–310.

[8] Bertelli, A., and Feldmann, S. E. (2007): Strategic Appointments. *Journal of Public Administration Research and Theory*, 17(1), 19–38.

[9] Buisseret, P., Folke, O., Prato, C., and Rickne, J. (2022): Party Nomination Strategies in List Proportional Representation Systems. *American Journal of Political Science*, 66(3), 714-729.

[10] Buisseret, P., and Prato, C. (2022): Competing Principals? Legislative Representation in List Proportional Representation Systems. *American Journal of Political Science*, 66(1), 156-170.

[11] Canes-Wrone, B., Herron, M. C., and Shotts, K.W. (2001): Leadership and Pandering: A Theory of Executive Policymaking. *American Journal of Political Science*, 532–550.

[12] Che, Y. K., and Kartik, N. (2009): Opinions as Incentives. *Journal of Political Economy*, 117(5), 815–860.

[13] Choi, B. (2018, 12, 24): [Editorial] Economic Policy Reversal Caused by Incompetence [English Transition of the Title]. *Hankyoreh.* Retrieved from `https://www.hani.co.kr/arti/opinion/column/875638.html`

[14] Cohen, D. M. (1998): Amateur Government. *Journal of Public Administration Research and Theory*, 8(4), 450–497.

[15] Crawford, V. P., and Sobel, J. (1982): Strategic Information Transmission. *Econometrica*, 50(6), 1431–1451.

[16] Denisenko, A., Hafer, C., and Landa D. (2024): Competence and Advice. *American Journal of Political Science,* forthcoming.

[17] Dessein W. (2002): Authority and Communication in Organizations. *Review of Economic Studies*, 69, 811-838.

[18] Dewan, T., A. Galeotti, C. Ghiglino, and Francesco Squintani (2015): Information Aggregation and Optimal Executive Structure. *American Journal of Political Science*, 2, 475–494.

[19] Dewan, T., and Myatt, D. P. (2007): Leading the Party: Coordination, Direction, and Communication. *American Political Science Review*, 101(4), 827–845.

[20] Dewan, T., and Myatt, D. P. (2008): The Qualities of Leadership: Direction, Communication, and Obfuscation. *American Political Science Review*, 102(3), 351–368.

[21] Dewan, T., and Myatt, D. P. (2012): On the Rhetorical Strategies of Leaders: Speaking Clearly, Standing Back, and Stepping Down. *Journal of Theoretical Politics*, 24(4), 431–460.

[22] Dewan, T., and F. Squintani (2016): In Defence of Factions. *American Journal of Political Science*, 60(4), 860–881.

[23] Dewan, T., and Squintani, F. (2018): Leadership with Trustworthy Associates. *American Political Science Review*, 112(4), 844–859.

[24] Dewatripont, M., and Tirole, J. (1999): Advocates. *Journal of Political Economy*, 107(1), 1–39.

[25] Dobel, J. P. (1995): Managerial Leadership in Divided Times: William Ruckelshaus and the Paradoxes of Independence. *Administration & Society*, 26(4), 488-514.

[26] Egorov, G., and Sonin, K. (2011): Dictators and Their Viziers: Endogenizing the Political Alignment–Competence Trade-Off. *Journal of the European Economic Association*, 9(5), 903–930.

[27] Gailmard, S. (2002): Expertise, Subversion, and Bureaucratic Discretion. *Journal of Law, Economics, and Organization*, 18(2), 536–555.

[28] Galeotti, A., Ghiglino, C., and Squintani, F. (2013): Strategic Information Transmission Networks. *Journal of Economic Theory*, 148(5), 1751–1769.

[29] Gates, R. M. (2010). Helping Others Defend Themselves: The Future of U.S. Security Assistance. *Foreign Affairs,* 89(3), 2-6.

[30] Gilligan, T. W., and Krehbiel, K. (1990): Organization of Informative Committees by a Rational Legislature. *American Journal of Political Science*, 531–564.

[31] Hobbs, E. E., and Jenkins, D. (2019, October 17): Three's Company: OSHRC's Third Panel Seat to Be Filled Soon. *National Law Review*. Retrieved from `https://natlawreview.com/article/three-s-company-oshrc-s-third-panel-seat-to-be-filled-soon/`

[32] Ingraham, P. W., Thompson, J. R., and Eisenberg, E. F. (1995): Political Management Strategies and Polical/Career Relationships: Where Are We Now in the Federal Government? *Public Administration Review*, 263–272.

[33] Johnson, R. T. (1974): Managing the White House: An Intimate Study of the Presidency. *Harper Collins Publishers*.

[34] Jun, J. (2016, April 14): Kim Chong-in Solidifies Leadership with Election Victory. *The Korea Times*. Retrieved from `https://www.koreatimes.co.kr/www/news/nation/2016/05/113_202625.html`

[35] Krause, G. A., and O'Connell, A. J. (2019): Loyalty–Competence Trade-Offs for Top US Federal Bureaucratic Leaders in the Administrative Presidency Era. *Presidential Studies Quarterly*, 49(3), 527–550.

[36] Levy, G. (2004): Anti-herding and strategic consultation. *European Economic Review*, 48(2), 503-525.

[37] Lewis, D. E. (2008): *The Politics of Presidential Appointments: Political Control and Bureaucratic Performance*. Princeton University Press.

[38] Li, M., and Madarász, K. (2008): When Mandatory Disclosure Hurts: Expert Advice and Conflicting Interests. *Journal of Economic Theory*, 139(1), 47-74.

[39] Moe, T. M. (1985): The Politicized Presidency. *The New Direction in American Politics*, 235(238), 244–63.

[40] Moreno De Barreda, I. (2010): Cheap Talk with Two-Sided Private Information. Mimeo, Oxford University.

[41] Morgan, J., and Stocken, P. C. (2003): An Analysis of Stock Recommendations. *RAND Journal of Economics*, 34(1), 183–203.

[42] Morris, S. (2001): Political Correctness. *Journal of Political Economy*, 109(2), 231–265.

[43] Ottaviani, M., and Sörensen P. N. (2001): Information Aggregation in Debate: Who Should Speak First? *Journal of Public Economics*, 81(3): 393–421.

[44] Ottaviani, M., and Sörensen, P. N. (2006): Reputational Cheap Talk. *RAND Journal of Economics*, 37(1): 155–175.

[45] Parsneau, K. (2013): Politicizing Priority Departments: Presidential Priorities and Subcabinet Experience and Political Alignment. *American Politics Research*, 41(3), 443–470.

[46] Patty, J. W. (2024): Designing Deliberation for Decentralized Decisions. *American Journal of Political Science*, forthcoming.

[47] Patty, J. W., and Penn, E. M. (2014): Sequential Decision Making and Information Aggregation in Small Networks. *Political Science Research and Methods*, 2, 1–29.

[48] Penn, E. M. (2016): Engagement, Disengagement, or Exit: A Theory of Equilibrium Associations. *American Journal of Political Science*, 60, 322–336.

[49] Putnam, R. D. (1973): The Political Attitudes of Senior Civil Servants in Western Europe: A Preliminary Report. *British Journal of Political Science*, 3(3), 257-290.

[50] Saunders, E. N. (2018): Leaders, Advisors, and the Political Origins of Elite Support for War. *Journal of Conflict Resolution*, 62(10), 2118–2149.

[51] Schnakenberg, K. E., and I. R. Turner (2021): Helping Friends or Influencing Foes: Electoral and Policy Effects of Campaign Finance, *American Journal of Political Science*, 65(1): 88-100.

[52] Schnakenberg, K. E., and I. R. Turner (2023): Political Influence and Dark Money, mimeo, Yale University.

[53] Seldon, A., and Newell, R. (2023): *Johnson at 10: The Inside Story: The Bestselling Political Biography of 2023.* Atlantic Books.

[54] Suri, J. (2018). Liberal Internationalism, Law, and the First African American President. In J. Zelizer (Ed.), *The Presidency of Barack Obama: A First Historical Assessment* (pp. 195-212). Princeton University Press.

[55] Tenpas, Kathryn D. (2021): Tracking Turnover in the Trump Administration, *Brookings Institution Reports,* January 2021.

Appendix

Proof of Theorem 1. In equilibrium, the decision maker of type b_0 chooses y to maximizes $Eu_0(y, x|s_0, \hat{\mathbf{m}}) = -E[(y - x - b_0)^2 |s_0, \hat{\mathbf{m}}]$, by taking a first-order and second-order condition, we see that the optimal strategy is $y(s_0, \hat{\mathbf{m}}; b_0) = E(x|s_0, \hat{\mathbf{m}}) + b_0$.

Hence, regardless of her type b_0, the leader's ex-ante payoff is

$$\begin{aligned} W(A, \mathbf{m}, y) = Eu_0(y, \mathbf{m}; b_0) &= E_{s_0,\hat{\mathbf{m}}} E_x \left[(y(s_0, \hat{\mathbf{m}}; b_0) - x - b_0)^2 |s_0, \hat{\mathbf{m}}\right] \\ &= -E_{s_0,\hat{\mathbf{m}}}[E_x \left[(E(x|s_0, \hat{\mathbf{m}}) + b_0 - x - b_0)^2 |s_0, \hat{\mathbf{m}}\right]] \\ &= -E_{s_0,\hat{\mathbf{m}}}[E_x \left[(E(x|s_0, \hat{\mathbf{m}}) - x)^2 |s_0, \hat{\mathbf{m}}\right]] = -E_{s_0,\hat{\mathbf{m}}} Var(x|s_0, \hat{\mathbf{m}}). \end{aligned} \tag{3}$$

The ex-ante payoff of each advisor i of type b_i is:

$$\begin{aligned} Eu_i(y, \mathbf{m}; b_i) &= -E_{b_0} E_{s_0,\hat{\mathbf{m}}} E_x \left[(y(s_0, \hat{\mathbf{m}}; b_0) - x - b_i)^2 |s_0, \hat{\mathbf{m}}\right] \\ &= -E_{s_0,\hat{\mathbf{m}}} E_x \left[E_{b_0}(E(x|s_0, \hat{\mathbf{m}}) + b_0 - x - b_i)^2 |s_0, \hat{\mathbf{m}}\right] \\ &= -E_{s_0,\hat{\mathbf{m}}} \left[E_x (E(x|s_0, \hat{\mathbf{m}}) - x)^2 + E_{b_0}(b_0 - b_i)^2 + 2E_{b_0}(b_0 - b_i) E_x (E(x|s_0, \hat{\mathbf{m}}) - x)) |s_0, \hat{\mathbf{m}}\right] \\ &= -E_{s_0,\hat{\mathbf{m}}} E_x \left[(E(x|s_0, \hat{\mathbf{m}}) - x)^2 |s_0, \hat{\mathbf{m}}\right] - \sum_{b_0 \in B_0} q_0(b_0)(b_0 - b_i)^2 \\ &= Eu_0(y, \mathbf{m}; b_0) - \sum_{b_0 \in B_0} q_0(b_0)(b_0 - b_i)^2. \end{aligned}$$

Consider a communication strategy profile $\mathbf{m}$ and suppose that it is an equilibrium together with the strategy y. Consider a rightward biased advisor i of bias type $b_i \in T_i$. Given signal s_i, the advisor chooses $\hat{m}_i$ to maximize her equilibrium utility

$$\begin{aligned} Eu_i(y, \mathbf{m}; b_i|s_i) &= -E_{b_0} E_{s_0,\hat{\mathbf{m}}|s_i} E_x \left[(y(s_0, \hat{\mathbf{m}}; b_0) - x - b_i)^2 |s_0, \hat{\mathbf{m}}, s_i\right] \\ &= -E_{b_0} E_{s_0,\hat{\mathbf{m}}|s_i} E_x \left[(E(x|s_0, \hat{m}_i, \hat{\mathbf{m}}_{-i}) + b_0 - x - b_i)^2 |s_0, s_i, \hat{\mathbf{m}}_{-i}\right]. \end{aligned}$$

When $s_i = 1$, the advisor's dominant strategy is to truthfully report $\hat{m}_i = s_i$. Because $b_0 - b_i < 0$, the advisor would lose by lowering $E(x|s_0, \hat{\mathbf{m}})$ by sending $\hat{m}_i = 0$.

When $s_i = 0$, the advisor does not deviate from reporting truthfully $\hat{m}_i = s_i$ to the leader of uncertain bias $b_0 \in B_0$ if and only if

$$\begin{aligned} -\int_0^1 \sum_{b_0 \in B_0} \sum_{s_0 \in \{0,1\}} \sum_{\hat{\mathbf{m}}_{-i} \in \{0,1\}^{N-2}} &\left[(y(s_0, \hat{m}_i, \hat{\mathbf{m}}_{-i}; b_0) - x - b_i)^2 - (y(s_0, 1 - \hat{m}_i, \hat{\mathbf{m}}_{-i}; b_0) - x - b_i)^2\right] \cdot \\ &f(x, s_0, \hat{\mathbf{m}}_{-i}|s_i) dx q_0(b_0) \geq 0. \end{aligned}$$

Simplifying, we obtain:

$$\begin{aligned} -\sum_{b_0 \in B_0} \int_0^1 \sum_{s_0 \in \{0,1\}} \sum_{\hat{\mathbf{m}}_{-i} \in \{0,1\}^{N-2}} &(y(s_0, \hat{m}_i, \hat{\mathbf{m}}_{-i}; b_0) - y(s_0, 1 - \hat{m}_i, \hat{\mathbf{m}}_{-i}; b_0)) \cdot \\ &\left[\frac{y(s_0, \hat{m}_i, \hat{\mathbf{m}}_{-i}; b_0) + y(s_0, 1 - \hat{m}_i, \hat{\mathbf{m}}_{-i}; b_0)}{2} - (x + b_i)\right] f(x, s_0, \hat{\mathbf{m}}_{-i}|s_i) dx q_0(b_0) \geq 0. \end{aligned}$$

Next, observing that

$$y(s_0, \hat{m}_i, \hat{\mathbf{m}}_{-i}; b_0) = E\left[x|s_0, \hat{m}_i, \hat{\mathbf{m}}_{-i}\right] + b_0,$$

we obtain

$$-\sum_{b_0\in B_0}\int_0^1 \sum_{s_0\in\{0,1\}}\sum_{\hat{\mathbf{m}}_{-i}\in\{0,1\}^{N-2}} \left(E\left[x|s_0, \hat{m}_i, \hat{\mathbf{m}}_{-i}\right] - E\left[x|s_0, 1-\hat{m}_i, \hat{\mathbf{m}}_{-i}\right]\right)\cdot$$
$$\cdot\left[\frac{E\left[x|s_0, \hat{m}_i, \hat{\mathbf{m}}_{-i}\right] + E\left[x|s_0, 1-\hat{m}_i, \hat{\mathbf{m}}_{-i}\right]}{2} + b_0 - x - b_i\right] f(x, s_0, \hat{\mathbf{m}}_{-i}|s_i) dx q_0\left(b_0\right) \geq 0.$$

Denoting

$$\Delta_i\left(s_0, \hat{m}_i, \hat{\mathbf{m}}_{-i}\right) = E\left[x|s_0, 1-\hat{m}_i, \hat{\mathbf{m}}_{-i}\right] - E\left[x|s_0, \hat{m}_i, \hat{\mathbf{m}}_{-i}\right],$$

observing that:

$$f(x, s_0, \hat{\mathbf{m}}_{-i}|s_i) = f(x|s_0, \hat{\mathbf{m}}_{-i}, s_i)\Pr(s_0, \hat{\mathbf{m}}_{-i}|s_i),$$

and simplifying, we get:

$$\sum_{b_0\in B_0}\sum_{s_0\in\{0,1\}}\sum_{\hat{\mathbf{m}}_{-i}\in\{0,1\}^{N-2}}\int_0^1 \Delta_i\left(s_0, \hat{m}_i, \hat{\mathbf{m}}_{-i}\right)\left(\frac{E\left[x|s_0, \hat{m}_i, \hat{\mathbf{m}}_{-i}\right] + E\left[x|s_0, 1-\hat{m}_i, \hat{\mathbf{m}}_{-i}\right]}{2} + b_0 - x - b_i\right)$$
$$\cdot f(x|s_0, \hat{\mathbf{m}}_{-i}, s_i)\Pr(s_0, \hat{\mathbf{m}}_{-i}|s_i) q_0\left(b_0\right) \geq 0.$$

Furthermore, note that

$$\int_0^1 x f(x|s_0, \hat{\mathbf{m}}_{-i}, s_i) dx = E\left[x|s_0, s_i, \hat{\mathbf{m}}_{-i}\right],$$

and that, because advisor i is rightward biased, $\hat{m}_i = 0$ only if $s_i = 0$,

$$E\left[x|s_0, s_i, \hat{\mathbf{m}}_{-i}\right] = E\left[x|s_0, \hat{m}_i, \hat{\mathbf{m}}_{-i}\right]$$

because the information $b_i \in T_i$ obtained from $m_i = 0$ is irrelevant for the inference on x.

Hence, we obtain:

$$\sum_{b_0\in B_0}\sum_{s_0\in\{0,1\}}\sum_{\hat{\mathbf{m}}_{-i}\in\{0,1\}^{N-2}} \Delta_i\left(s_0, \hat{m}_i, \hat{\mathbf{m}}_{-i}\right)\left(\frac{\Delta_i\left(s_0, \hat{m}_i, \hat{\mathbf{m}}_{-i}\right)}{2} + b_0 - b_i\right)\Pr(s_0, \hat{\mathbf{m}}_{-i}|s_i) q_0\left(b_0\right) \geq 0,$$

and, rearranging:

$$b_i - \sum_{b_0\in B_0}\left(b_0\right) \leq \frac{\sum_{s_0\in\{0,1\}}\sum_{\hat{\mathbf{m}}_{-i}\in\{0,1\}^{N-2}} \Delta_i\left(s_0, \hat{m}_i, \hat{\mathbf{m}}_{-i}\right)\frac{\Delta_i\left(s_0, \hat{m}_i, \hat{\mathbf{m}}_{-i}\right)}{2}\Pr(s_0, \hat{\mathbf{m}}_{-i}|s_i)}{\sum_{s_0\in\{0,1\}}\sum_{\hat{\mathbf{m}}_{-i}\in\{0,1\}^{N-2}} \Delta_i\left(s_0, \hat{m}_i, \hat{\mathbf{m}}_{-i}\right)\Pr(s_0, \hat{\mathbf{m}}_{-i}|s_i)}$$

The argument for a leftward biased advisor i is symmetric, leading to the condition:

$$b_i - \sum_{b_0 \in B_0} b_0 q_0(b_0) \geq \frac{\sum_{s_0 \in \{0,1\}} \sum_{\hat{\mathbf{m}}_{-i} \in \{0,1\}^{N-2}} \Delta_i(s_0, \hat{m}_i, \hat{\mathbf{m}}_{-i}) \left(\frac{\Delta_i(s_0, \hat{m}_i, \hat{\mathbf{m}}_{-i})}{2} \right) \Pr(s_0, \hat{\mathbf{m}}_{-i} | s_i)}{\sum_{s_0 \in \{0,1\}} \sum_{\hat{\mathbf{m}}_{-i} \in \{0,1\}^{N-2}} \Delta_i(s_0, \hat{m}_i, \hat{\mathbf{m}}_{-i}) \Pr(s_0, \hat{\mathbf{m}}_{-i} | s_i)},$$

where both the left-hand side and the right-hand side are negative.

Wrapping them up together, we obtain condition (1). ■

We continue with derivations that will be useful to calculate the welfare expression (3), as well as the terms $\Delta_i(s_0, \hat{m}_i, \hat{\mathbf{m}}_{-i})$ and $\Pr(s_0, s_i, \hat{\mathbf{m}}_{-i})$ in expression condition (1), so as to obtain the equilibrium and welfare calculations in Sections 4 and 5.

For any player i and profile of leader's information $(s_0, \hat{m}_i, \hat{\mathbf{m}}_{-i})$, we let the n-th moment of x be:

$$P_n(s_0, \hat{m}_i, \hat{\mathbf{m}}_{-i}) = \int_0^1 x^n \Pr(s_0, \hat{m}_i, \hat{\mathbf{m}}_{-i} | x)\, dx.$$

Expression (3) can be written as:

$$\begin{aligned} W(A, \mathbf{m}, y) &= -E_{s_0, \hat{\mathbf{m}}}\left[E_x\left[(y(s_0, \hat{\mathbf{m}}) - x)^2 | s_0, \hat{\mathbf{m}}\right]\right] = -\sum_{s_0 \in \{0,1\}} \sum_{\hat{\mathbf{m}} \in \{0,1\}^N} E_x\left[\left(\frac{P_1(s_0, \hat{\mathbf{m}})}{P_0(s_0, \hat{\mathbf{m}})} - x\right)^2 \middle| s_0, \hat{\mathbf{m}}\right] \Pr(s_0, \hat{\mathbf{m}}) \\ &= -\sum_{s_0 \in \{0,1\}} \sum_{\hat{\mathbf{m}} \in \{0,1\}^N} \left[\frac{P_1(s_0, \hat{\mathbf{m}})^2}{P_0(s_0, \hat{\mathbf{m}})^2} + \frac{P_2(s_0, \hat{\mathbf{m}})}{P_0(s_0, \hat{\mathbf{m}})} - 2\frac{P_1(s_0, \hat{\mathbf{m}})^2}{P_0(s_0, \hat{\mathbf{m}})^2}\right] \Pr(s_0, \hat{\mathbf{m}}) \\ &= -\sum_{s_0 \in \{0,1\}} \sum_{\hat{\mathbf{m}} \in \{0,1\}^N} w(s_0, \hat{\mathbf{m}}),\ w(s_0, \hat{\mathbf{m}}) \equiv \left[P_2(s_0, \hat{\mathbf{m}}) - \frac{P_1(s_0, \hat{\mathbf{m}})^2}{P_0(s_0, \hat{\mathbf{m}})}\right]. \end{aligned}$$

Exploiting the fact that the prior of x is uniform, we obtain:

$$\begin{aligned} E[x | s_0, \hat{m}_i, \hat{\mathbf{m}}_{-i}] &= \frac{\int_0^1 x \Pr(s_0, \hat{m}_i, \hat{\mathbf{m}}_{-i} | x)\, dx}{\int_0^1 \Pr(s_0, \hat{m}_i, \hat{\mathbf{m}}_{-i} | x)\, dx} = \frac{P_1(s_0, \hat{m}_i, \hat{\mathbf{m}}_{-i})}{P_0(s_0, \hat{m}_i, \hat{\mathbf{m}}_{-i})}, \\ \Delta(s_0, \hat{m}_i, \hat{\mathbf{m}}_{-i}) &= \frac{P_1(s_0, 1 - \hat{m}_i, \hat{\mathbf{m}}_{-i})}{P_0(s_0, 1 - \hat{m}_i, \hat{\mathbf{m}}_{-i})} - \frac{P_1(s_0, \hat{m}_i, \hat{\mathbf{m}}_{-i})}{P_0(s_0, \hat{m}_i, \hat{\mathbf{m}}_{-i})}. \end{aligned}$$

Recalling that each player $j = 0, ..., N$ observes signal s_ℓ, $\ell = 0, ..., N$, with probability $\rho_{j\ell}$, and that signal $s_\ell = s$ is correct with probability $\tau_{\ell s}$, we derive explicit formulas for $\Pr(s_0, s_i, \hat{\mathbf{m}}_{-i})$ and $P_n(s_0, \hat{m}_i, \hat{\mathbf{m}}_{-i})$.

First, we note that:

$$\Pr(s_0, s_i, \hat{\mathbf{m}}_{-i}) = \sum_{\mathbf{s} \in \{0,1\}^{N+1}} \prod_{i=1,...,N} \int_0^1 \Pr(s_i | \mathbf{s}') \Pr(s_0 | \mathbf{s}') \Pr(\mathbf{s}' | x)\, dx \tag{4}$$

$$\begin{aligned} P_n(s_0, \hat{m}_i, \hat{\mathbf{m}}_{-i}) &= \int_0^1 x^n \Pr(s_0, \hat{m}_i, \hat{\mathbf{m}}_{-i} | x)\, dx = \int_0^1 \sum_{\mathbf{s} \in \{0,1\}^{N+1}} x^n \Pr(\hat{m}_i, \hat{\mathbf{m}}_{-i} | \mathbf{s}') \Pr(s_0 | \mathbf{s}') \Pr(\mathbf{s}' | x)\, dx \\ &= \sum_{\mathbf{s} \in \{0,1\}^{N+1}} \prod_{i=1,...,N} \int_0^1 x^n \Pr(\hat{m}_i | \mathbf{s}') \Pr(s_0 | \mathbf{s}') \Pr(\mathbf{s}' | x)\, dx, \end{aligned} \tag{5}$$

Then, for any signal realization profile $\mathbf{s}'$, letting $Z(\mathbf{s}')$ be the set of real signals s'_ℓ equal to 0, and $z(\mathbf{s}')$ be the cardinality of $Z(\mathbf{s}')$, we write:

$$\Pr\left(\mathbf{s}'|x\right) = x^{N+1-z(\mathbf{s}')}(1-x)^{z(\mathbf{s}')}. \tag{6}$$

The probability of any player i's observed signal s_i is:

$$\Pr\left(s_i|\mathbf{s}'\right) = \sum_{\ell:s_i=s_\ell} \rho_{i\ell}\tau_{\ell s_\ell} + \sum_{\ell:s_i\neq s_\ell} \rho_{i\ell}\left(1-\tau_{\ell s_\ell}\right), \tag{7}$$

The first term denotes the probability that i observes signal s_ℓ "correctly," the second term captures incorrect signal observations.

The probability of any advisor i's message $\hat{m}_i$ is:

$$\begin{aligned}\Pr\left(\hat{m}_i|\mathbf{s}\right) &= \sum_{b_i\in B_i(\hat{m}_i)} q_i\left(b_i\right) + \Pr(\hat{m}_i|\mathbf{s},T_i)\sum_{b_i\in T_i} q_i\left(b_i\right) \\ &= \tilde{q}_i\left(\hat{m}_i\right) + \tilde{q}_i\left(T_i\right)\left[\sum_{\ell:s_\ell=\hat{m}_i}\rho_{i\ell}\tau_{\ell\hat{m}_i} + \sum_{\ell:s_\ell\neq\hat{m}_i}\rho_{i\ell}(1-\tau_{\ell\hat{m}_i})\right],\end{aligned} \tag{8}$$

using the notation: $\tilde{q}_i\left(\hat{m}_i\right) = \sum_{b_i\in B_i(\hat{m}_i)} q_i\left(b_i\right)$ and $\tilde{q}_i\left(T_i\right) = \sum_{b_i\in T_i} q_i\left(b_i\right)$. Expression $\tilde{q}_i\left(\hat{m}_i\right)$ denotes the probability that advisor i pools on $\hat{m}_i$, and $\tilde{q}_i\left(T_i\right)$ the probability that i is telling the truth.

Equilibrium $\mathcal{E}_{12}$. We first consider the case in which signal s_1 is imprecise. The existence conditions for equilibrium $\mathcal{E}_{12}$ in which both expert 1 and expert 2 are truthful to the leader are $b_1 \leq \eta_{12.1}(p)$, and $b_2 \leq \eta_{12.2}(p)$, respectively. Simplifying condition (1), we obtain:

$$\eta_{12.1} = \frac{\sum_{s_0\in\{0,1\}}\sum_{s_2\in\{0,1\}}\Delta_1\left(s_0,s_2\right)^2 P_0(s_0,s_1=0,s_2)}{2\sum_{s_0\in\{0,1\}}\sum_{s_2\in\{0,1\}}\Delta_1\left(s_0,s_2\right)P_0(s_0,s_1=0,s_2)},\ \Delta_1\left(s_0,s_2\right) = \frac{P_1(s_0,s_1=1,s_2)}{P_0(s_0,s_1=1,s_2)} - \frac{P_1(s_0,s_1=0,s_2)}{P_0(s_0,s_1=0,s_2)}. \tag{9}$$

Using (5) together with (6)-(8), we obtain:

$$P_n\left(s_0,s_1,s_2\right) = p\frac{(s_0+s_1+s_2+n)!\,(3-s_0-s_1-s_2)!}{(4+n)!} + (1-p)\,\frac{(s_0-s_1+s_2+n+1)!\,(2+s_1-s_0-s_2)!}{(4+n)!}.$$

Hence, the calculations of $P_n\left(s_0,s_1,s_2\right)$ for $n=0,1$, $s_1=0$ and $(s_0,s_2)\in\{0,1\}^2$ are as follows:

$$P_0(0,0,0) = p\frac{0!3!}{4!} + (1-p)\,\frac{1!2!}{4!} = \frac{2p+1}{12},\ P_1(0,0,0) = p\frac{1!3!}{5!} + (1-p)\,\frac{2!2!}{5!} = \frac{p+2}{60},$$
$$P_0(0,0,1) = P_0(1,0,0) = p\frac{1!2!}{4!} + (1-p)\,\frac{2!1!}{4!} = \frac{1}{12},\ P_1(0,0,1) = P_1(1,0,0) = p\frac{2!2!}{5!} + (1-p)\,\frac{3!1!}{5!} = \frac{3-p}{60},$$
$$P_0(1,0,1) = p\frac{2!1!}{4!} + (1-p)\,\frac{3!0!}{4!} = \frac{3-2p}{12},\ P_1(1,0,1) = p\frac{3!1!}{5!} + (1-p)\,\frac{4!0!}{5!} = \frac{4-3p}{20}.$$

Noting that the formula of $P_n(s_0,1,s_2)$ is obtained from $P_n(s_0,0,s_2)$ by interchanging p with $1-p$:

$$P_0(0,1,0) = \frac{3-2p}{12},\ P_1(0,1,0) = \frac{3-p}{60},\ P_0(1,1,1) = \frac{2p+1}{12},\ P_1(1,1,1) = \frac{3p+1}{20},$$
$$P_0(0,1,1) = P_0(1,1,0) = \frac{1}{12},\ P_1(0,1,1) = P_1(1,1,0) = \frac{2+p}{60}.$$

Plugging in $P_0(s_0, s_1, s_2)$, and $P_1(s_0, s_1, s_2)$ into $\Delta_1(s_0, s_2)$, for $(s_0, s_2) \in \{0,1\}^2$, and simplifying:

$$\Delta_1(0,0) = \frac{3}{5}\frac{2p-1}{(2p+1)(3-2p)},\ \Delta_1(0,1) = \Delta_1(1,0) = \frac{1}{5}(2p-1),\ \Delta_1(1,1) = \frac{3}{5}\frac{2p-1}{(2p+1)(3-2p)}.$$

Substituting $\Delta_1(s_0, s_2)$ and $P_0(s_0, 0, s_2)$ into the expression (9), and simplifying, we get:

$$\eta_{12.1}(p) = \frac{1}{10}\frac{\left(27+24p-8p^2-32p^3+16p^4\right)(2p-1)}{(2p+1)(3-2p)(9+4p-4p^2)}.$$

The truthtelling constraint of advisor 2 in equilibrium $\mathcal{E}_{12}$ is $b_2 \leq \eta_{12.2}$, with, again simplifying (1),

$$\eta_{12.2} \equiv \frac{\sum_{s_0\in\{0,1\}}\sum_{s_1\in\{0,1\}}\Delta_2(s_0,\hat{m}_1)^2 P_0(s_0,s_1,s_2=0)}{2\sum_{s_0\in\{0,1\}}\sum_{s_1\in\{0,1\}}\Delta_2(s_0,\hat{m}_1)P_0(s_0,s_1,s_2=0)},\ \Delta_2(s_0,s_1) = \frac{P_1(s_0,s_1,s_2=1)}{P_0(s_0,s_1,s_2=1)} - \frac{P_1(s_0,s_1,s_2=0)}{P_0(s_0,s_1,s_2=0)}. \tag{10}$$

Plugging in $P_0(s_0, s_1, s_2)$ and $P_1(s_0, s_1, s_2)$ derived above into $\Delta_2(s_0, s_1)$, and simplifying,

$$\Delta_2(0,0) = \frac{1}{5}\frac{4p-2p^2+1}{2p+1},\ \Delta_2(0,1) = \frac{1}{5}\frac{3-2p^2}{3-2p},\ \Delta_2(1,0) = \frac{1}{5}\frac{3-2p^2}{3-2p},\ \Delta_2(1,1) = \frac{1}{5}\frac{4p-2p^2+1}{2p+1}.$$

Substituting $\Delta_2(s_0, s_1)$ and $P_0(s_0, s_1, s_2 = 0)$ into (10), and simplifying, we obtain:

$$\eta_{12.2}(p) = \frac{1}{10}\frac{27+132p+88p^2-360p^3-20p^4+240p^5-80p^6}{(2p+1)(3-2p)(9+4p-4p^2)(1+2p-2p^2)}.$$

To calculate the welfare $W_{12}(p)$, we simplify formula (3) and use the symmetry of the prior $\Pr(s_0, s_1, s_2)$, $\Pr(s_0, s_1, s_2) = \Pr(1-s_0, 1-s_1, 1-s_2)$ for all $(s_0, s_1, s_2) \in \{0,1\}^3$, to obtain:

$$W_{12}(p) = -2\sum_{(s_1,s_2)\in\{0,1\}^2} w(s_0=0,s_1,s_2),\ w(0,s_1,s_2) = P_2(0,s_1,s_2) - \frac{P_1(0,s_1,s_2)^2}{P_0(0,s_1,s_2)}\ \text{for}\ (s_1,s_2)\in\{0,1\}^2. \tag{11}$$

Using the above expressions for $P_0(0, s_1, s_2)$ and $P_1(0, s_1, s_2)$, together with: $P_2(0,0,0) = p\frac{2!3!}{6!} + (1-p)\frac{3!2!}{6!} = \frac{1}{60}$, $P_2(0,0,1) = p\frac{3!2!}{6!} + (1-p)\frac{4!1!}{6!} = \frac{2-p}{60}$, $P_2(0,1,0) = \frac{1}{60}$, $P_2(0,1,1) = \frac{1+p}{60}$, we obtain, after simplification:

$$w(0,0,0) = \frac{1+6p-p^2}{300(2p+1)},\ w(0,0,1) = w(0,1,0) = \frac{1+p-p^2}{300},\ w(0,1,1) = \frac{6-4p-p^2}{300(3-2p)}.$$

Plugging these expressions into (11), we obtain, after simplification:

$$W_{12}(p) = -\frac{1}{150}\frac{15+38p-30p^2-16p^3+8p^4}{(2p+1)(3-2p)}.$$

We now turn to consider the case in which the expert 1's signal s_1 is correlated with the leader's signal s_0. Again using (5) together with (6)-(8), we see that, here, $P_n(s_0, s_1, s_2)$ take the form, for $(s_0, s_1, s_2) \in \{0,1\}^3$

$$P_n(s_0,s_1,s_2) = \rho\frac{(s_0+s_1+s_2+n)!\,(3-s_0-s_1-s_2)!}{(4+n)!} + (1-\rho)\frac{(s_0+s_2+n)!\,(2-s_0-s_2)!}{(3+n)!}.$$

Once calculated $P_n(s_0, s_1, s_2)$ for $n = 0, 1, 2$ and $(s_0, s_1, s_2) \in \{0, 1\}^3$, the thresholds $\eta_{12.1}(\rho)$ and $\eta_{12.2}(\rho)$, and the welfare expression $W_{12}(\rho)$ are obtained using the same procedure as in the case of imprecise signals s_1 seen above. Omitting the repetition of such algebric manipulations we obtain:

$$\eta_{12.1}(\rho) = \frac{\left(42\rho + 19\rho^2 + 27\right)}{10(\rho+3)(\rho+1)(7\rho+9)}, \ \eta_{12.2}(\rho) = \frac{96\rho + 118\rho^2 + 52\rho^3 + 7\rho^4 + 27}{10(\rho+3)(\rho+1)(16\rho+5\rho^2+9)},$$

$$W_{12}(\rho) = -\frac{1}{150}\frac{26\rho + 9\rho^2 + 15}{(\rho+3)(\rho+1)}.$$

Detailed calculations are available in a supplementary appendix.

Equilibrium $\mathcal{E}_1$. Simplifying condition (1), we obtain the truthtelling constraint:

$$b_1 \leq \eta_1 \equiv \frac{\sum_{s_0\in\{0,1\}} \Delta_1(s_0)^2 P_0(s_0, s_1 = 0)}{2\sum_{s_0\in\{0,1\}} \Delta_1(s_0) P_0(s_0, s_1 = 0)}, \ \Delta_1(s_0) = \frac{P_1(s_0, s_1 = 1)}{P_0(s_0, s_1 = 1)} - \frac{P_1(s_0, s_1 = 0)}{P_0(s_0, s_1 = 0)}, \text{ for } s_0 = 0, 1. \tag{12}$$

Simplifying condition (2) and using symmetry of prior $\Pr(s_0, s_1)$, we obtain:

$$W_1 = -2\sum_{s_1\in\{0,1\}} w(0, s_1), \text{ where } w(0, s_1) = P_2(0, s_1) - \frac{P_1(0, s_1)^2}{P_0(0, s_1)}, \text{ for all } s_1 \in \{0, 1\}.$$

When signal s_1 is imprecise, using (5)-(8), $P_n(s_0, s_1)$ take the form, for $(s_0, s_1) \in \{0, 1\}^2$,

$$P_n(s_0, s_1) = p\frac{(s_0 + s_1 + n)!(2 - s_0 - s_1)!}{(3+n)!} + (1-p)\frac{(s_0 - s_1 + 1 + n)!(3 - s_0 + s_1)!}{(3+n)!}.$$

Instead, when signal s_1 is correlated with the leader's signal s_0:

$$P_n(s_0, s_1) = \rho\frac{(s_0 + s_1 + n)!(1 - s_0 - s_1)!}{(3+n)!} + (1-\rho)\frac{(s_0 + n)!(1 - s_0)!}{(2+n)!}.$$

Once calculated $P_n(s_0, s_1)$ for $n = 0, 1$ and $(s_0, s_1) \in \{0, 1\}^2$, we plug them into expression (12) and obtain the threshold η_1 and welfare formula W_1. (Detailed calculations are in a supplementary appendix.)

When signal s_1 is imprecise:

$$\eta_1(p) = \frac{1}{4}\frac{2p - 1}{(p+1)(2-p)} \text{ and } W_1(p) = -\frac{1}{12}\frac{1 + 2p - 2p^2}{(p+1)(2-p)}.$$

When s_1 is correlated with signal s_0:

$$\eta_1(\rho) = \frac{1}{4(2+\rho)} \text{ and } W_1(\rho) = -\frac{1}{12}\frac{1+\rho}{2+\rho}.$$

It is easy to show that $\eta_{12.1} < \eta_{12.2} < \eta_2$ and $\eta_{12.1} < \eta_1 < \eta_2$, and that $W_0 < W_1 < W_2 < W_{12}$ for all $p \in (1/2, 1)$ and all $\rho \in (0, 1)$.

The equilibria when either 1 or 2 makes decisions. We begin with the case in which signal s_1 is imprecise. Here, the signals s_0 and s_2 of players 0 and 2 are exchangeable. So, the equilibrium truthtelling thresholds when 2 is the decision maker are the same as when the leader retains authority. The equilibrium $\mathcal{E}_{01}^2$ in which both 0 and 1 are truthful to 2 exists if and only if $b_2 \leq \eta_{12.2}(p)$ and $b_2 - b_1 \leq \eta_{12.1}(p)$, the equilibrium $\mathcal{E}_0^2$ such that only the leader is truthful to 2 exists whenever $b_2 \leq \eta_2$, and the equilibrium $\mathcal{E}_1^2$ where only advisor 1 is truthful if and only if $b_2 - b_1 \leq \eta_1(p)$. The associated welfares are $W_{01}^2 = W_{12} - b_2^2$, $W_0^2 = W_2 - b_2^2$ and $W_1^2 = W_1 - b_2^2$ respectively.

Suppose now that expert 1 is the decision maker. The equilibrium $\mathcal{E}_{02}^1$ in which both 0 and 2 are truthful to 1 exists if and only if $b_1 \leq \eta_{12.2}(p)$ and $b_2 - b_1 \leq \eta_{12.2}(p)$, and its welfare is $W_{02}^1 = W_{12} - b_1^2$. This is because the decision maker, player 1, holds the same equilibrium information as the leader in the equilibrium $\mathcal{E}_{12}$, and the signals s_0 and s_2 of the advising players 0 and 2 are identically distributed. Hence, the thruthtelling threshold of both 0 and 2 in $\mathcal{E}_{02}^1$ is the same as that of player 2 in $\mathcal{E}_{12}$. The characterization of the equilibria $\mathcal{E}_0^1$ and $\mathcal{E}_2^1$, in which only player 0 (respectively, player 2) is truthful to player 1 is more involved. Because s_0 and s_2 are exchangeable, the same threshold, η^1, applies, so that the existence conditions are $b_1 \leq \eta^1(p)$ and $b_2 - b_1 \leq \eta^1(p)$, respectively, and the equilibrium welfares are $W_0^1(p) = W_2^1(p) = W_1(p) - b_1^2$. But the threshold η^1 is different from any of the thresholds calculated above for the case in which the leader decides. Using formula (1),

$$\eta^1 \equiv \frac{\sum_{s_1 \in \{0,1\}} \Delta_2(s_1)^2 P_0(s_1, s_2 = 0)}{2 \sum_{s_1 \in \{0,1\}} \Delta_2(s_1) P_0(s_1, s_2 = 0)}, \text{ with } \Delta_2(s_1) = \frac{P_1(s_1, s_2 = 1)}{P_0(s_1, s_2 = 1)} - \frac{P_1(s_1, s_2 = 0)}{P_0(s_1, s_2 = 0)}, \text{ for } s_1 = 0, 1.$$

Using the expressions for $P_n(s_1, s_2)$ for $n = 1, 2$ and $(s_1, s_2) \in \{0, 1\}^2$ calculated earlier, and simplifying,

$$\eta^1(p) \equiv \frac{1}{4} \frac{1 + 2p - 2p^2}{(1 + p)(2 - p)}.$$

It is easy to see that $\eta^1 > \eta_1$: the condition for truthtelling by the better informed players 0 or 2 to the less informed player 1 is less stringent than vice versa. Further, $\eta^1 > \eta_2$, because the leader's signal s_0 is more precise than player 1's.

Turning to the case in which signal s_1 is correlated with s_0, we now that, now, signals s_0 and s_1 are exchangeable. So, the equilibrium truthtelling thresholds when 1 decides are the same as when the leader decides. The existence condition for equilibrium $\mathcal{E}_{02}^1$ in which both 0 and 2 are truthful to 1 is $b_1 \leq \eta_{12.1}(\rho)$ and $b_2 - b_1 \leq \eta_{12.2}(\rho)$, equilibrium $\mathcal{E}_0^1$ such that only the leader is truthful to 1 exists whenever $b_1 \leq \eta_1(\rho)$, and the equilibrium $\mathcal{E}_1^2$ where only 2 is truthful if and only if $b_2 - b_1 \leq \eta_2$. The equilibrium welfares are $W_{02}^1(\rho) = W_{12}(\rho) - b_1^2$, $W_0^1 = W_1(\rho) - b_1^2$ and $W_2^1 = W_2 - b_1^2$.

Suppose now that 2 is the decision maker. The equilibrium $\mathcal{E}_{01}^2$ in which both 0 and 1 exists if and only if $b_2 \leq \eta_{12.1}(\rho)$ and $b_2 - b_1 \leq \eta_{12.1}(\rho)$. In fact, player 2 decides with the same information as the leader in equilibrium $\mathcal{E}_{12}$, and the advising players 0 and 1 have identically distributed signals. Hence, the thruthtelling threshold of both 0 and 1 in $\mathcal{E}_{01}^2$ is the same as player 1's in $\mathcal{E}_{12}$. Further, the existence conditions of equilibria $\mathcal{E}_0^2$ and $\mathcal{E}_1^2$ are $b_2 \leq \eta_2$ and $b_2 - b_1 \leq \eta_2$, respectively, because in both equilibria player 2 decides with 2 identically distributed signals, independent conditionally on x. The equilibrium welfares are $W_{01}^2(\rho) = W_{12}(\rho) - b_2^2$, $W_0^2 = W_2 - b_2^2$ and $W_1^2 = W_2 - b_2^2$.

The leader's delegation decision We first consider the case in which signal s_1 is imprecise. Letting $\delta_j(p) \equiv \sqrt{W_{12}(p) - W_j(p)}$, for $j = 0, 1, 2$, the complete characterization of leader's delegation is as follows.

Proposition 7 *Suppose that signal s_1 is imprecise. The leader delegates only to advisor 1 and only if he is fully informed in equilibrium, and specifically:*

1. if $b_1 < \delta_2(p)$, $\eta_{12.2}(p) < b_2 \leq \eta_2$ and $b_2 - b_1 \leq \eta_{12.2}(p)$, when only advisor 2 is truthful in equilibrium if the leader retains authority,

2. if $b_1 \leq \min\{\delta_1(p), \eta_1(p)\}$, $b_2 > \eta_2$, and $b_2 - b_1 \leq \eta_{12.2}(p)$, when only 1 is truthful in equilibrium if the leader decides,

3. if $\eta_1(p) < \eta_{12.2}(p)$, $\eta_1(p) < b_1 < \min\{\delta_0(p), \eta_{12.2}(p)\}$, $b_2 > \eta_2$ and $b_2 - b_1 \leq \eta_{12.2}(p)$, when the leader cannot collect any information in equilibrium.

Proof. Of course, when $b_1 \leq \eta_{12.1}(p)$ and $b_2 \leq \eta_{12.2}(p)$, the equilibrium $\mathcal{E}_{12}$ exists in which the leader is fully informed and she does not delegate.

When $b_1 > \eta_{12.1}(p)$ and $b_2 \leq \eta_2$: the optimal equilibrium without delegation is $\mathcal{E}_2$. The leader delegates to 1 if and ond only if equilibrium $\mathcal{E}_{02}^1$ exists and $W_{02}^1 > W_2$, i.e., $b_1 \leq \delta_2(p)$, but this is impossible because we verified that $\delta_2 < \eta_{12.1}$, and hence condition $b_1 > \eta_{12.1}(p)$ is contradicted. A fortiori, there is also no delegation to 2.

Say $\eta_{12.2}(p) < b_2 \leq \eta_2$: the optimal equilibrium without delegation is $\mathcal{E}_2$. Delegation to advisor 2 is pointless, because $\eta_2 > \eta_{12.2}(p)$ and so equilibrium $\mathcal{E}_{01}^2$ does not exist. The leader delegates to expert 1 if and only if $\mathcal{E}_{02}^1$ exists, i.e., $b_1 \leq \eta_{12.2}(p)$ and $b_2 - b_1 \leq \eta_{12.2}(p)$, and $b_1 \leq \delta_2(p)$. Because we verified that $\delta_2 < \eta_{12.2}$, the conditions for delegation are: $\eta_{12.2}(p) < b_2 \leq \eta_2$, $b_1 < \delta_2(p)$, $b_2 - b_1 \leq \eta_{12.2}(p)$.

When $b_1 \leq \eta_1(p)$ and $b_2 > \eta_2$, the optimal equilibrium without delegation is $\mathcal{E}_1$. Again, delegation to advisor 2 is pointless, because $b_2 > \eta_2 > \eta_{12.1}(p)$ and hence equilibria $\mathcal{E}_{01}^2$ and $\mathcal{E}_0^2$ do not exist. The leader delegates to expert 1 if and only if $\mathcal{E}_{02}^1$ exists, i.e., $b_1 \leq \eta_{12.2}(p)$ and $b_2 - b_1 \leq \eta_{12.2}(p)$, and $b_1 \leq \delta_1(p)$. We verified that $\delta_1 < \eta_{12.2}$, and $\eta_1(p) < \delta_1(p)$ if and only if p is low.

Suppose $b_1 > \eta_1(p)$ and $b_2 > \eta_2$: then the optimal equilibrium without delegation is the babbling equilibrium $\mathcal{E}_0$. Again, the leader delegates to advisor 1 if $\mathcal{E}_{02}^1$ exists, $b_1 \leq \eta_{12.2}(p)$ and $b_2 - b_1 \leq \eta_{12.2}(p)$, and if $b_1 \leq \delta_0(p)$. We verified that $\eta_1 < \bar{b}_0$, $\eta_1(p) < \eta_{12.2}(p)$ if p is not too high, and $\bar{b}_0(p) < \eta_{12.2}(p)$ for low values of p. Further, the leader delegates to advisor 1 if $\mathcal{E}_{02}^1$ does not exist, as long as either equilibrium $\mathcal{E}_0^1$ or $\mathcal{E}_2^1$ exists, and $W_1(p) - b_1^2 > W_0$, i.e., $b_1 < \delta_{0.1}(p) \equiv \sqrt{W_1(p) - W_0}$. But this contradicts $b_1 > \eta_1(p)$, because $\eta_1 > \delta_{0.1}$. Because $b_2 > \eta_2 > \eta_{12.2}(p)$, equilibrium $\mathcal{E}_{01}^2$ and $\mathcal{E}_0^2$ do not exist. Hence, the leader delegates to 2 if and only if equilibrium $\mathcal{E}_1^2$ exists, and $b_1 < \delta_{0.1}(p)$, but again this contradicts $b_1 > \eta_1(p)$. ■

Turning to the case in which signal s_1 is correlated with s_0, using $\delta_{j.2}(\rho) \equiv \sqrt{W_2 - W_j}$ for $j = 0, 1$,

Proposition 8 *Suppose signal s_1 is correlated with s_0. Leader delegates only to expert 1, and only if:*
1. $\eta_{12.2}(\rho) < b_2 \leq \eta_2$, $b_1 < \delta_2(\rho)$, and $b_2 - b_1 \leq \eta_{12.2}(\rho)$, because only advisor 2 is truthful in equilibrium to the leader if she decides, whereas 1 is fully informed,
2. $b_1 \leq \min\{\eta_{12.1}(\rho), \delta_1(\rho)\}$, $b_2 > \eta_2$, $b_2 - b_1 \leq \eta_{12.2}(\rho)$, since only 1 is truthful in equilibrium to the leader, whereas 1 is fully informed,
3. $b_1 < \min\{\eta_1(\rho), \delta_{1.2}(\rho)\}$, $b_2 > \eta_2$, $b_2 - b_1 \leq \eta_2$, and $b_1 > \eta_{12.1}(\rho)$ or $b_2 - b_1 > \eta_{12.2}(\rho)$, since only 1 is truthful in equilibrium to the leader, whereas 2 is truthful to 1,
4. $\eta_1(\rho) < b_1 < \delta_{0.2}(\rho)$, $b_2 > \eta_2$ and $b_2 - b_1 \leq \eta_2(\rho)$, as neither expert informs the leader in equilibrium, whereas 2 is truthful to 1.

Proof. Of course, when $b_1 \leq \eta_{12.1}(\rho)$ and $b_2 \leq \eta_{12.2}(\rho)$, the equilibrium $\mathcal{E}_{12}$ exists in which the leader is fully informed, and she does not delegate.

When $b_1 > \eta_{12.1}(\rho)$ and $b_2 \leq \eta_2$: the optimal equilibrium without delegation is $\mathcal{E}_2$. The leader delegates to 1 if and ond only if equilibrium $\mathcal{E}_{02}^1$ exists and $W_{02}^1 > W_2$, i.e., $b_1 \leq \delta_2(\rho)$, but this is impossible because we verified that $\delta_2 < \eta_{12.1}$, and hence condition $b_1 > \eta_{12.1}(\rho)$ is contradicted. A fortiori, there is also no delegation to 2.

Suppose that $\eta_{12.2}(\rho) < b_2 \leq \eta_2$: the optimal equilibrium without delegation is $\mathcal{E}_2$. The leader delegates to 1 if and ond only if equilibrium $\mathcal{E}_{02}^1$ exists, i.e., $b_2 - b_1 \leq \eta_{12.2}(\rho)$ and $b_1 \leq \eta_{12.1}(\rho)$, and if $W_{02}^1 > W_2$, i.e., $b_1 \leq \delta_2(\rho)$. Because $\delta_2 < \eta_{12.1}$, we can omit the constraint $b_1 \leq \eta_{12.1}(\rho)$. The leader delegates to 2 if and only if equilibrium $\mathcal{E}_{12}^2$ exists and $\mathcal{E}_{02}^1$ does not, i.e., $b_2 - b_1 \leq \eta_{12.1}(\rho)$ and $b_2 \leq \eta_{12.1}(\rho)$, but $b_2 - b_1 > \eta_{12.2}(\rho)$ or $b_1 > \eta_{12.1}(\rho)$. Because $b_2 > b_1$, condition $b_2 \leq \eta_{12.1}(\rho)$ contradicts the possibility $b_1 > \eta_{12.1}(\rho)$, and because $\eta_{12.1} < \eta_{12.2}$, condition $b_2 - b_1 \leq \eta_{12.1}(\rho)$ contradicts $b_2 - b_1 > \eta_{12.2}(\rho)$.

When $b_1 \leq \eta_1(\rho)$, and $b_2 > \eta_2$, the optimal equilibrium without delegation is $\mathcal{E}_1$. The leader delegates to 1 if equilibrium $\mathcal{E}_{02}^1$ exists and if $W_{02}^1 > W_1$, i.e., if $b_2 - b_1 \leq \eta_{12.2}(\rho)$ and $b_1 \leq \min\{\eta_{12.1}(\rho), \delta_1(\rho)\}$. Because $\eta_{12.1} < \eta_1$, we can omit constraint $b_1 \leq \eta_1(\rho)$. The leader would delegate to 2 if $\mathcal{E}_{12}^2$ existed and $\mathcal{E}_{02}^1$ did not, but this is impossible. The leader also delegates to 1 if equilibrium $\mathcal{E}_2^1$ exists and $\mathcal{E}_{02}^1$ does not (as the latter implies that $\mathcal{E}_{12}^2$ exist either) and $W_2^1 > W_1$, i.e., if $b_2 - b_1 \leq \eta_2$ and $b_1 < \delta_{1.2}(\rho)$, but $b_1 > \eta_{12.1}(\rho)$ or $b_2 - b_1 > \eta_{12.2}(\rho)$. Because $\eta_{12.1} < \eta_1$, $\eta_1(\rho) < \delta_{1.2}(\rho)$ if and only if ρ is large, and $\eta_2 - \min\{\eta_1(\rho), \delta_{1.2}(\rho)\} > \eta_{12.2}(\rho)$ if and only if ρ is small, none of the conditions can be omitted. Finally, the leader would delegate to 2 if equilibrium $\mathcal{E}_1^2$ existed, $b_2 - b_1 \leq \eta_2$, and $\mathcal{E}_2^1$ did not, $b_2 - b_1 > \eta_2$, which is of course impossible.

When $b_1 > \eta_1(\rho)$ and $b_2 > \eta_2$, the optimal equilibrium without delegation is $\mathcal{E}_0$. Equilibria $\mathcal{E}_0^1$ and $\mathcal{E}_{02}^1$ do not exist, because $b_1 > \eta_1(\rho) > \eta_{12.1}(\rho)$, nor do $\mathcal{E}_0^2$ and $\mathcal{E}_{01}^2$, because $b_2 > \eta_2$ and $b_2 > b_1 > \eta_{12.1}(\rho)$. The leader delegates to 1 if and only if equilibrium $\mathcal{E}_2^1$ exists and $W_2^1(\rho) > W_0$, i.e., $b_2 - b_1 \le \eta_2$ and $b_1 < \delta_{0.2}(\rho)$. The leader delegates to 2 if and only if equilibrium $\mathcal{E}_1^2$ exists, $b_2 - b_1 \le \eta_2$, and $\mathcal{E}_2^1$ does not, $b_2 - b_1 > \eta_2$, which is obviously impossible. ■

Equilibrium $\mathcal{E}_{12A}$. The equilibrium $\mathcal{E}_{12A}$ in which expert 1 and type b_1 of expert 2 are truthful exists if and only if $b_1 \le \eta_{12A.1}(p,q)$ and $b_1 \le \eta_{12A.2}(p,q)$, i.e., $b_1 \le \eta_{12A}(p,q) \equiv \min\{\eta_{12A.1}(p,q), \eta_{12A.2}(p,q)\}$. Simplifying condition (1), we obtain:

$$\eta_{12A.2} \equiv \frac{\sum_{(s_0,s_1)\in\{0,1\}^2} \Delta_{2A}(s_0,s_1)^2 \Pr(s_0,s_1,s_2=0)}{2\sum_{(s_0,s_1)\in\{0,1\}^2} \Delta_{2A}(s_0,s_1) \Pr(s_0,s_1,s_2=0)},\ \Delta_{2A}(s_0,s_1) = \frac{P_1(s_0,s_1,\hat{m}_2=1)}{P_0(s_0,s_1,\hat{m}_2=1)} - \frac{P_1(s_0,s_1,\hat{m}_2=0)}{P_0(s_0,s_1,\hat{m}_2=0)}. \tag{13}$$

Using (4)-(8), we calculate $P_n(s_0,s_1,\hat{m}_2)$ for $n=0,1,2$, and $(s_0,s_1,\hat{m}_2) \in \{0,1\}^3$:

$$P_n(s_0,s_1,\hat{m}_2=0) = qp\frac{(s_0+s_1+n)!\,(3-s_0-s_1)!}{(4+n)!} + q(1-p)\frac{(1+s_0-s_1+n)!\,(2-s_0+s_1)!}{(4+n)!}.$$

$$\begin{aligned}P_n(s_0,s_1,\hat{m}_2=1) &= (1-q)p\frac{(s_0+s_1+n)!\,(2-s_0-s_1)!}{(3+n)!} + (1-q)(1-p)\frac{(s_0+1-s_1+n)!\,(1-s_0+s_1)!}{(3+n)!}\\ &+ qp\frac{(s_0+s_1+1+n)!\,(2-s_0-s_1)!}{(4+n)!} + q(1-p)\frac{(s_0-s_1+2+n)!\,(1-s_0+s_1)!}{(4+n)!},\end{aligned}$$

as well as $\Pr(s_0,s_1,s_2)$ for $(s_0,s_1,s_2) \in \{0,1\}^3$,

$$\Pr(s_0,s_1,s_2) = p\frac{(s_0+s_1+s_2)!\,(3-s_0-s_1-s_2)!}{4!} + (1-p)\frac{(s_0-s_1+s_2+1)!\,(2-s_0+s_1-s_2)!}{4!}.$$

Proceeding as with equilibria $\mathcal{E}_{12}$ and $\mathcal{E}_1$, we then derive $\Delta_{2A}(s_0,s_1)$ and $\Pr(s_0,s_1,0)$ for $(s_0,s_1) \in \{0,1\}^2$, and substitute them into (13) to obtain:

$$\eta_{12A.2}(p,q) = \frac{\left(\frac{1+4p-2p^2}{2+2p-q-2qp}\right)^2 \frac{1}{2p+1} + \left(\frac{3-2p^2}{4-3q-2p+2qp}\right)^2 \frac{1}{3-2p} + \left(\frac{2p^2-3}{q+2p-4}\right)^2 + \left(\frac{1+4p-2p^2}{2+2p-q}\right)^2}{10\left(\frac{1+4p-2p^2}{2+2p-q-2qp} + \frac{3-2p^2}{4-3q-2p+2qp} + \frac{2p^2-3}{q+2p-4} + \frac{1+4p-2p^2}{2+2p-q}\right).}.$$

Proceeding in the same fashion as for $\eta_{12A.2}$,, we obtain:

$$\eta_{12A.1}(p,q) = (2p-1)\frac{\left(\frac{3}{3-2p}\right)^2 q\frac{1}{2p+1} + q + \left(\frac{10-12q+3q^2}{4-3q-2p+2qp}\right)^2 \frac{1}{2-q+2p-2qp} + \left(\frac{10-8q+q^2}{(2+2p-q)}\right)^2 \frac{1}{4-q-2p}}{10\left(\frac{3q}{3-2p} + q + \frac{10-12q+3q^2}{4-3q-2p+2qp} + \frac{10-8q+q^2}{2+2p-q}\right)}.$$

Simplifying formula (3), we obtain:

$$W_{12A}(p,q) = -\sum_{(s_0,s_1,\hat{m}_2)\in\{0,1\}^3} w(s_0,s_1,\hat{m}_2),\ w(s_0,s_1,\hat{m}_2) = P_2(s_0,s_1,\hat{m}_2) - \frac{P_1(s_0,s_1,\hat{m}_2)^2}{P_0(s_0,s_1,\hat{m}_2)}, \tag{14}$$

so that substituting in $P_n(s_0, s_1, \hat{m}_2)$ for $(s_0, s_1, \hat{m}_2) \in \{0,1\}^3$, and simplifying, we get:

$$\begin{aligned}W_{12A}(p,q) = &-\frac{1}{300}q\frac{1+6p-p^2}{2p+1} - \frac{1}{300}q\left(1+p-p^2\right) - \frac{1}{300}q\frac{6-4p-p^2}{3-2p} - \frac{1}{300}q\left(1+p-p^2\right)\\ &-\frac{1}{300}\frac{5-p^2q^2+10p^2q-10p^2+6pq^2-25pq+20p+q^2-5q}{2+2p-q-2pq}\\ &-\frac{1}{300}\frac{15-10q+pq^2+10p^2q-p^2q^2-5pq-10p^2+q^2+}{4-q-2p}\\ &-\frac{1}{300}\frac{15-p^2q^2+10p^2q-10p^2-4pq^2+5pq+6q^2-20q}{4-2p-3q+2pq}\\ &-\frac{1}{300}\frac{5+20p-5q+pq^2+10p^2q-p^2q^2-15pq-10p^2+q^2}{2+2p-q}.\end{aligned}$$

Equilibrium $\mathcal{E}_{2A}$. Only the aligned advisor 2 is trutful. Using (1), the equilibrium condition is

$$b_1 \leq \eta_{2A}(q) \equiv \frac{\sum_{s_0\in\{0,1\}} \Delta_{2A}(s_0)^2 \Pr(s_0, s_2 = 0)}{2\sum_{s_0\in\{0,1\}} \Delta_{2A}(s_0) \Pr(s_0, s_2 = 0)},\ \Delta_{2A}(s_0) = \frac{P_1(\hat{m}_2 = 1, s_0)}{P_0(\hat{m}_2 = 1, s_0)} - \frac{P_1(\hat{m}_2 = 0, s_0)}{P_0(\hat{m}_2 = 0, s_0)}. \quad (15)$$

Again, using (4)-(8), and simplifying, we obtain:

$$\Pr(s_0, s_2 = 0) = \frac{2-s_0}{6},\ P_n(s_0, \hat{m}_2 = 1) = (1-q)\frac{(s_0+n)!\,(1-s_0)!}{(2+n)!} + q\frac{(1+s_0+n)!\,(1-s_0)!}{(3+n)!},$$
$$P_n(s_0, \hat{m}_2 = 0) = q\frac{(s_0+n)!\,(2-s_0)!}{(3+n)!},\ \text{for } s_0 \in \{0,1\}.$$

This allows us to find $\Delta_{2A}(s_0)$ for $s_0 \in \{0,1\}$ so that, substituting into (15) and simplifying, we conclude:

$$\eta_{2A}(q) = \frac{1}{8}\frac{9-10q+3q^2}{(3-2q)(3-q)(2-q)}.$$

Simplifying formula (3), we obtain:

$$W_{2A}(q) = -\sum_{(s_0,\hat{m}_2)\in\{0,1\}^2} w_2(s_0,\hat{m}_2),\ w_2(s_0,\hat{m}_2) \equiv P_2(s_0,\hat{m}_2) - \frac{P_1(s_0,\hat{m}_2)^2}{P_0(s_0,\hat{m}_2)} \text{ for } (s_0,\hat{m}_2) \in \{0,1\}^2. \quad (16)$$

Substituting in $P_n(\hat{m}_2, s_0)$, $n = 1,2,3$ and simplifying:

$$W_{2A}(q) = -\frac{1}{48}\frac{24-27q+7q^2}{(3-2q)(3-q)}.$$

Proof of Proposition 4. We verified that some equilibria are Pareto ranked in terms of ex-ante welfare. The equilibrium $\mathcal{E}_{12}$ is top ranked. Both equilibria $\mathcal{E}_{12A}$ and $\mathcal{E}_2$ dominate $\mathcal{E}_{2A}$ and $\mathcal{E}_1$. However $\mathcal{E}_{12A}$ and $\mathcal{E}_2$ are not ranked among each other, and nor are $\mathcal{E}_{2A}$ and $\mathcal{E}_1$. Further, we determined that there exist functions $g_1, g_2 : q \mapsto p$ such that $W_2(p,q) > W_{12A}(p,q)$ if and only if $p < g_2(q)$ and $W_{2A}(p,q) > W_1(p,q)$ if and only if $p < g_1(q)$. The function g_1 strictly increases in q with $g_1(0) = 0$ and $g_1(1) = 1$, whereas g_2 strictly decreases in q with $g_2(0) = 1$ and $g_2(1) = 0$.

Turning to consider equilibrium existence, we first note that $\mathcal{E}_{12}$ is the only equilibrium whose existence requires joint conditions on b_1 and b_2. The existence of equilibria η_{12A}, $\mathcal{E}_{2A}$ and $\mathcal{E}_1$ requires only conditions on b_1, and the existence of $\mathcal{E}_2$ only conditions on b_2. Then, we verified that some equilibrium existence thresholds are ordered for all (p, q), and specifically: $\eta_{12.1} < \eta_{12A.1} < \eta_1 < \eta_2$, $\eta_{12.1} < \eta_{12.2} < \eta_2$, $\eta_{12A.2} < \eta_{12.2} < \eta_2$, $\eta_{12A.2} < \eta_{2A} < \eta_2$. Hence $\eta_{12A} = \min\{\eta_{12A.1}, \eta_{12A.2}\} < \{\eta_1, \eta_{2A}, \eta_{12.2}\}$, and we obtain the threshold ordering $\eta_{12.1} < \eta_{12A} < \{\eta_1, \eta_{2A}, \eta_{12.2}\} < \eta_2$.

This means that for any fixed b_1, the existence range $[0, \eta_2]$ of equilibrium $\mathcal{E}_2$ in the b_2 dimension strictly contains the existence range $[0, \eta_{12.2}]$ of equilibrium $\mathcal{E}_{12}$ (which is empty if $b_1 > \eta_{12.1}$). Most importantly, for any fixed b_2, the existence regions $[0, \eta_{2A}]$ and $[0, \eta_1]$ of equilibria $\mathcal{E}_{2A}$ and $\mathcal{E}_1$ in the b_1 dimension strictly contain the existence range $[0, \eta_{12A}]$ of equilibrium $\mathcal{E}_{12A}$, and the latter strictly contains the existence range $[0, \eta_{12.1}]$ of equilibrium $\mathcal{E}_{12}$ (which is empty when $b_2 > \eta_{12.2}$). Whether the existence range of equilibrium $\mathcal{E}_1$ contains the range of $\mathcal{E}_{2A}$ or viceversa depends on p and q. We determined that there is a strictly increasing function $g_3 : q \mapsto p$ such that $\eta_1(p) > \eta_{2A}(q)$ if and only if $p > g_3(q)$. The function g_3 is such that $g_1(q) < g_3(q) < g_2(q)$ for low q. As q grows, g_3 first crosses g_2 and then g_1 to finally join g_1 again at $q = 1$, with $g_3(1) = 1$.

Jointly taken, these results yield the complete characterization of the optimal equilibrium $\mathcal{E}^*$ as a function of (p, q) and (b_1, b_2) reported in Proposition 4. ■

Delegation. We begin the study of delegation by calculating the equilibria when agent 2 is delegated decision-making. Because the signals s_0 and s_2 are exchangeable, and both players 0 and 1 have only one bias types, the existence conditions and welfare formulas are analogous to the equilibria $\mathcal{E}_{12}$, $\mathcal{E}_1$ and $\mathcal{E}_2$ where the leader decides and player 2's communication strategy is the same across types. Specifically, equilibrium $\mathcal{E}^2_{01}$ where both players 0 and 1 are truthful exists if and only if $\bar{b}_2 = b_1 q + b_2 (1 - q) \leq \eta_{12.2}(p)$ and $\bar{b}_2 - b_1 = (b_2 - b_1)(1 - q) \leq \eta_{12.1}(p)$, and has welfare $W^2_{01}(p, q) = -qb_1^2 - (1 - q) b_2^2 + W_{12}(p)$; the equilibrium $\mathcal{E}^2_0$ where only 0 is truthful exists whenever $\bar{b}_2 \leq \eta_2$ and $W^2_0(p, q) = -qb_1^2 - (1 - q) b_2^2 + W_2$; and the equilibrium $\mathcal{E}^2_0$ where only 1 is truthful exists whenever $(b_2 - b_1)(1 - q) \leq \eta_1(p)$, with $W^2_1(p, q) = -qb_1^2 - (1 - q) b_2^2 + W_1$.

Let us turn to the case in which player 1 is delegated authority. First, we note that, in all equilibria, type $2A$ of player 2 is truthful as his bias is the same as 1's. The equilibrium $\mathcal{E}^1_{02}$, in which 0 and both types of 2 are truthful to 1, exists if $b_1 \leq \eta_{12.2}(p)$, and $b_2 - b_1 \leq \eta_{12.2}(p)$, and the associated welfare is $W^1_{02}(p) = W_{12}(p) - b_1^2$. The reason is that the decision maker, 1, acts fully informed as is the case for the leader in the $\mathcal{E}_{12}$ equilibrium, and both players 0 and 2 have the same information as player 2 in the $\mathcal{E}_{12}$ equilibrium. The equilibrium $\mathcal{E}^1_{02A}$, in which 0 and type $2A$ are truthful to 1, has welfare

$W_{02A}^1(p,q,b_1) = W_{12A}(p,q) - b_1^2$, and it exists if $b_1 \leq \eta_{02A}^1(p,q)$ such that, simplifying (1):

$$\eta_{02A}^1 \equiv \frac{\sum_{(s_1,\hat{m}_2)\in\{0,1\}^2} \Delta_0(s_1,\hat{m}_2)^2 P_0(s_0=1,s_1,\hat{m}_2)}{2\sum_{(s_1,\hat{m}_2)\in\{0,1\}^2} \Delta_0(s_1,\hat{m}_2) P_0(s_0=1,s_1,\hat{m}_2)},\ \Delta_0(s_1,\hat{m}_2) = \frac{P_1(s_0=0,s_1,\hat{m}_2)}{P_0(s_0=0,s_1,\hat{m}_2)} - \frac{P_1(s_0=1,s_1,\hat{m}_2)}{P_0(s_0=1,s_1,\hat{m}_2)}. \tag{17}$$

Using the formulas for $P_n(s_0,s_1,\hat{m}_2)$ calculated for equilibrium $\mathcal{E}_{12A}$ to find $\Delta_0(s_1,\hat{m}_2)$ for $(s_1,\hat{m}_2) \in \{0,1\}^2$, and simplifying, we obtain the expression of η_{02A}^1, which is very cumbersome and so relegated to the supplementary appendix.

The equilibrium $\mathcal{E}_2^1$, in which both types of player 2 are truthful to 1, exists if $b_2 - b_1 \leq \eta_2^1(p)$, and the associated welfare is $W_2^1 = W_1(p) - b_1^2$. In fact, expert 1 acts with one precise and one imprecise signal, as the leader in the E_1 equilibrium. The truthtelling condition is the same as for delegation without uncertain trade-off. Finally, there is always the equilibrium $\mathcal{E}_{2A}^1$ in which only type $2A$ is truthful to 1. The associated welfare $W_{2A}^1(p,q)$ is obtained using (2):

$$W_{2A}^1(p,q) = -\sum_{(s_1,\hat{m}_2)\in\{0,1\}^2} w_2(s_1,\hat{m}_2) - b_1^2,\ w_2(s_1,\hat{m}_2) \equiv P_2(s_1,\hat{m}_2) - \frac{P_1(s_1,\hat{m}_2)^2}{P_0(s_1,\hat{m}_2)} \text{ for } (s_1,\hat{m}_2) \in \{0,1\}^2. \tag{18}$$

Using (5) with (6)-(8), and simplifying, we obtain, for $n = 0,1,2$ and $s_1 \in \{0,1\}$,

$$\begin{aligned} P_n(s_1,\hat{m}_2=0) &= qp\frac{(s_1+n)!(2-s_1)!}{(3+n)!} + q(1-p)\frac{(1-s_1+n)!(1+s_1)!}{(3+n)!} \\ P_n(s_1,\hat{m}_2=1) &= (1-q)p\frac{(s_1+n)!(1-s_1)!}{(2+n)!} + (1-q)(1-p)\frac{(1-s_1+n)!s!}{(2+n)!} \\ &\quad + qp\frac{(s_1+1+n)!(1-s_1)!}{(3+n)!} + q(1-p)\frac{(2-s_1+n)!s_1!}{(3+n)!}, \end{aligned}$$

and substituting into (18), and simplifying,

$$W_{2A}^1(p,q) = -\frac{1}{3} + \frac{1}{24}\frac{(2+2p-q)^2}{3-2q+pq} + \frac{1}{24}\frac{q}{2-p} + \frac{1}{24}\frac{(4-q)^2}{3-q-pq} + \frac{1}{6}q\frac{(2+p)^2}{1+p}.$$

We wrap up the analysis by listing all the instances in which the leader optimally delegates. In the results below, we make use of the threshold functions $\delta_{j.k} \equiv \sqrt{W_j - W_k}$, for $j = 1,2,12A,2A$ and $k \neq j$, and of the notation $\hat{b}_2 = \sqrt{b_1^2 q + b_2^2(1-q)}$.

Proposition 9 *Delegation to advisor 1 is optimal if:*

1. $b_1 \leq \delta_2(p)$, $\eta_{12.2}(p) < b_2 \leq \eta_2$, $b_2 - b_1 \leq \eta_{12.2}(p)$ *and, either* $W_2 > W_{12A}(p,q)$ *and* $b_1 \leq \eta_{12.2}(p)$, *or* $W_2 < W_{12A}(p,q)$ *and* $b_1 > \eta_{12A}(p,q)$, *when it yields* $\mathcal{E}_{02}^1$ *instead of the equilibrium without delegation* $\mathcal{E}_2$;

2. $b_1 \leq \min\{\eta_{12.2}(p), \delta_{12A}(p,q)\}$, $b_2 - b_1 \leq \eta_{12.2}(p)$ *and, either* $W_{12A}(p,q) > W_2$, $b_2 > \eta_{12.2}(p)$, *and* $b_1 \leq \eta_{12A}$, *or* $W_{12A}(p,q) < W_2$ *and* $b_2 > \eta_2$, *when it yields* $\mathcal{E}_{02}^1$ *in lieu of* $\mathcal{E}_{12A}$;

3. $\eta_{12A}(p,q) < b_1 < \min\{\delta_1, \eta_{12.2}(p), \eta_1(p)\}$, $b_2 > \eta_2$, $b_2 - b_1 \leq \eta_{12.2}(p)$ *and* $W_1(p) > W_{2A}(q)$, *when it yields* $\mathcal{E}_{02}^1$ *in place of* $\mathcal{E}_1$;

4. $b_2 > \eta_2$, $\eta_{12A}(p,q) < b_1 \le \min\{\eta_{2A}(q), \eta_{12.2}(p), \delta_{2A}(p,q)\}$, $b_2 - b_1 \le \eta_{12.2}(p)$, *and, either* $W_{2A}(q) > W_1(p)$, *or* $W_{2A}(q) < W_1(p)$ *and* $b_1 > \eta_1(q)$, *when it yields* $\mathcal{E}_{02}^1$ *in place of* $\mathcal{E}_{2A}$;

5. $\max\{\eta_1(p), \eta_{2A}(q)\} < b_1 < \min\{\delta_0, \eta_{12.2}(p)\}$, *and* $b_2 - b_1 \le \eta_{12.2}(p)$, *when it yields* $\mathcal{E}_{02}^1$ *in place of* $\mathcal{E}_0$;

6. $\max\{\eta_1(p), \eta_{2A}(q)\} < b_1 < \min\{\delta_0, \eta_{02A}^1(p,q)\}$, *and, either* $b_1 > \eta_{12.2}(p)$ *or* $b_2 - b_1 > \eta_{12.2}(p)$ *or both, when it yields* $\mathcal{E}_{02A}^1$ *in place of* $\mathcal{E}_0$.

Delegation to advisor 2 is optimal if equilibrium $\mathcal{E}_{02}^1$ *does not exist and:*

1. $W_2 > W_{12A}(p,q)$, $\eta_{12.2}(p) < b_2 \le \eta_2$, $\eta_{12.2}(p) < b_2 - b_1 \le \frac{\eta_{12.1}(p)}{1-q}$, $b_1 q + b_2(1-q) \le \eta_{12.2}(p)$, $\hat{b}_2 < \delta_2(p)$, *when it yields equilibrium* $\mathcal{E}_{01}^2$ *and the top equilibrium without delegation is* $\mathcal{E}_2$;

2. $b_1 \le \eta_{12A}(p,q)$, $\hat{b}_2 < \delta_{12A}(p,q)$, $b_1 q + b_2(1-q) \le \eta_{12.2}(p)$, $\eta_{12.2}(p) < b_2 - b_1 \le \frac{\eta_{12.1}(p)}{1-q}$ *and, either* $W_{12A}(p,q) > W_2$ *and* $b_2 > \eta_{12.2}(p)$, *or* $W_{12A}(p,q) < W_2$ *and* $b_2 > \eta_2$, *when it yields* $\mathcal{E}_{01}^2$ *in place of* $\mathcal{E}_{12A}$;

3. $W_{12A}(p,q) < W_2$, $b_1 \le \eta_{12A}(p,q)$, $b_2 > \eta_2$, $b_1 q + b_2(1-q) \le \eta_2$, $\hat{b}_2 < \delta_{2.12A}(p,q)$, $b_2 - b_1 > \max\{\eta_{12.2}(p), \frac{\eta_{12.1}(p)}{1-q}\}$, *when it yields* $\mathcal{E}_0^2$ *in place of* $\mathcal{E}_{12A}$;

4. $W_1(p) > W_{2A}(q)$, $\eta_{12A}(p,q) < b_1 \le \eta_1(p)$, $b_2 > \eta_2$, $\eta_{12.2}(p) < b_2 - b_1 \le \eta_{12.1}(p)/(1-q)$, $b_1 q + b_2(1-q) \le \eta_{12.2}(p)$, $\hat{b}_2 < \delta_1(p,q)$, *when it yields* $\mathcal{E}_{02}^1$ *in place of* $\mathcal{E}_1$.

Proof. The proof is lengthy and for reasons of space it is relegated to a supplementary appendix. ■

Supplementary Appendix not submitted for publication

Equilibrium $\mathcal{E}_{12}$, correlated signal s_1. The expressions $P_n(s_0, s_1, s_2)$ for $(s_0, s_1, s_2) \in \{0,1\}^3$ are:

$$P_n(s_0, s_1, s_2) = \rho \frac{(s_0 + s_1 + s_2 + n)!\,(3 - s_0 - s_1 - s_2)!}{(4+n)!} + (1-\rho)\frac{(s_0 + s_2 + n)!\,(2 - s_0 - s_2)!}{(3+n)!}.$$

Hence, we obtain:

$$\begin{aligned}
P_0(0,0,0) &= \rho\frac{0!2!}{3!} + (1-\rho)\frac{0!3!}{4!} = \frac{\rho+3}{12},\ P_1(0,0,0) = \rho\frac{1!2!}{4!} + (1-\rho)\frac{1!3!}{5!} = \frac{2\rho+3}{60},\\
P_0(0,0,1) &= \rho\frac{1!1!}{3!} + (1-\rho)\frac{1!2!}{4!} = \frac{\rho+1}{12},\ P_1(0,0,1) = \rho\frac{2!1!}{4!} + (1-\rho)\frac{2!2!}{5!} = \frac{3\rho+2}{60},\\
P_0(0,1,0) &= (1-\rho)\frac{1!2!}{4!} = \frac{1-\rho}{12},\ P_1(0,1,0) = (1-\rho)\frac{2!2!}{5!} = \frac{1-\rho}{30},\\
P_0(0,1,1) &= (1-\rho)\frac{2!1!}{4!} = \frac{1-\rho}{12},\ P_1(0,1,1) = (1-\rho)\frac{3!1!}{5!} = \frac{1-\rho}{20},\\
P_0(1,1,1) &= \rho\frac{2!0!}{3!} + (1-\rho)\frac{3!0!}{4!} = \frac{\rho+3}{12},\ P_1(1,1,1) = \rho\frac{3!0!}{4!} + (1-\rho)\frac{4!0!}{5!} = \frac{\rho+4}{20},\\
P_0(1,1,0) &= \rho\frac{1!1!}{3!} + (1-\rho)\frac{2!1!}{4!} = \frac{1+\rho}{12},\ P_1(1,1,0) = \rho\frac{2!1!}{4!} + (1-\rho)\frac{3!1!}{5!} = \frac{3+2\rho}{60},\\
P_0(1,0,1) &= (1-\rho)\frac{2!1!}{4!} = \frac{1-\rho}{12},\ P_1(1,0,1) = (1-\rho)\frac{3!1!}{5!} = \frac{1-\rho}{20},\\
P_0(1,0,0) &= (1-\rho)\frac{1!2!}{4!} = \frac{1-\rho}{12},\ P_1(1,0,0) = (1-\rho)\frac{2!2!}{5!} = \frac{1-\rho}{30}.
\end{aligned}$$

By plugging the expressions for $P_0(s_0, s_1, m_2)$, and $P_1(s_0, s_1, m_2)$ into $\Delta_1(s_0, m_1)$, for $s_0 = 0,1$ and $\hat{m}_2 = 0,1$, we get:

$$\begin{aligned}
\Delta_1(0,0) &= \frac{P_1(0,1,0)}{P_0(0,1,0)} - \frac{P_1(0,0,0)}{P_0(0,0,0)} = \frac{\frac{1-\rho}{30}}{\frac{1-\rho}{12}} - \frac{\frac{2\rho+3}{60}}{\frac{\rho+3}{12}} = \frac{3}{5(3+\rho)}\\
\Delta_1(0,1) &= \frac{P_1(0,1,1)}{P_0(0,1,1)} - \frac{P_1(0,0,1)}{P_0(0,0,1)} = \frac{\frac{1-\rho}{20}}{\frac{1-\rho}{12}} - \frac{\frac{3\rho+2}{60}}{\frac{\rho+1}{12}} = \frac{1}{5(\rho+1)}\\
\Delta_1(1,0) &= \frac{P_1(1,1,0)}{P_0(1,1,0)} - \frac{P_1(1,0,0)}{P_0(1,0,0)} = \frac{\frac{3+2\rho}{60}}{\frac{1+\rho}{12}} - \frac{\frac{1-\rho}{30}}{\frac{1-\rho}{12}} = \frac{1}{5(1+\rho)}\\
\Delta_1(1,1) &= \frac{P_1(1,1,1)}{P_0(1,1,1)} - \frac{P_1(1,0,1)}{P_0(1,0,1)} = \frac{\frac{\rho+4}{20}}{\frac{\rho+3}{12}} - \frac{\frac{1-\rho}{20}}{\frac{1-\rho}{12}} = \frac{3}{5(\rho+3)}
\end{aligned}$$

Plugging $\Delta_1(s_0, \hat{m}_2)$ and $P_0(s_0, s_1 = 0, \hat{m}_2)$ for $s_0 = 0,1$ and $\hat{m}_2 = 0,1$, into into the formula (9), we obtain:

$$\begin{aligned}
\eta_{12.1}(\rho) &= \frac{1}{2}\frac{(\frac{3}{5(3+\rho)})^2\left(\frac{\rho+3}{12} + \frac{1-\rho}{12}\right) + (\frac{1}{5(\rho+1)})^2\left(\frac{\rho+1}{12} + \frac{1-\rho}{12}\right)}{(\frac{3}{5(3+\rho)})\left(\frac{\rho+3}{12} + \frac{1-\rho}{12}\right) + (\frac{1}{5(\rho+1)})\left(\frac{\rho+1}{12} + \frac{1-\rho}{12}\right)}\\
&= \frac{1}{2}\frac{\left(42\rho + 19\rho^2 + 27\right)}{5(\rho+3)(\rho+1)(7\rho+9)}.
\end{aligned}$$

Turning to consider the truthtelling constraint of advisor 2, by plugging in the expressions for $P_0(s_0, \hat{m}_1, m_2)$, and $P_1(s_0, \hat{m}_1, m_2)$ into $\Delta_2(s_0, \hat{m}_1)$, we obtain:

$$\Delta_2(0,0) = \frac{P_1(0,0,1)}{P_0(0,0,1)} - \frac{P_1(0,0,0)}{P_0(0,0,0)} = \frac{\frac{3\rho+2}{60}}{\frac{\rho+1}{12}} - \frac{\frac{2\rho+3}{60}}{\frac{\rho+3}{12}} = \frac{1}{5}\frac{6\rho+\rho^2+3}{(\rho+3)(\rho+1)}$$
$$\Delta_2(0,1) = \frac{P_1(0,1,1)}{P_0(0,1,1)} - \frac{P_1(0,1,0)}{P_0(0,1,0)} = \frac{\frac{1-\rho}{20}}{\frac{1-\rho}{12}} - \frac{\frac{1-\rho}{30}}{\frac{1-\rho}{12}} = \frac{1}{5}$$
$$\Delta_2(1,0) = \frac{P_1(1,0,1)}{P_0(1,0,1)} - \frac{P_1(1,0,0)}{P_0(1,0,0)} = \frac{\frac{1-\rho}{20}}{\frac{1-\rho}{12}} - \frac{\frac{1-\rho}{30}}{\frac{1-\rho}{12}} = \frac{1}{5}$$
$$\Delta_2(1,1) = \frac{P_1(1,1,1)}{P_0(1,1,1)} - \frac{P_1(1,1,0)}{P_0(1,1,0)} = \frac{\frac{\rho+4}{20}}{\frac{\rho+3}{12}} - \frac{\frac{3+2\rho}{60}}{\frac{1+\rho}{12}} = \frac{1}{5}\frac{6\rho+\rho^2+3}{(\rho+3)(\rho+1)}$$

By plugging these expressions into the formula (10), we get:

$$\eta_{12.2}(\rho) = \frac{1}{2}\frac{\left(\frac{1}{5}\frac{6\rho+\rho^2+3}{(\rho+3)(\rho+1)}\right)^2\left(\frac{\rho+3}{12}+\frac{1+\rho}{12}\right)+2\frac{1-\rho}{12}\left(\frac{1}{5}\right)^2}{\left(\frac{1}{5}\frac{6\rho+\rho^2+3}{(\rho+3)(\rho+1)}\right)\left(\frac{\rho+3}{12}+\frac{1+\rho}{12}\right)+2\frac{1-\rho}{12}\left(\frac{1}{5}\right)}$$
$$= \frac{1}{10}\frac{96\rho+118\rho^2+52\rho^3+7\rho^4+27}{(\rho+3)(\rho+1)(16\rho+5\rho^2+9)}.$$

To calculate welfare in the full information equilibrium, we use the already calculated $P_0(0, s_1, s_2)$, $P_2(0, s_1, s_2)$ and:

$$P_2(0,0,0) = \rho\frac{2!2!}{5!} + (1-\rho)\frac{2!3!}{6!} = \frac{\rho+1}{60},\ P_2(0,0,1) = \rho\frac{3!1!}{5!} + (1-\rho)\frac{3!2!}{6!} = \frac{2\rho+1}{60},$$
$$P_2(0,1,0) = (1-\rho)\frac{3!2!}{6!} = \frac{1-\rho}{60},\ P_2(0,1,1) = (1-\rho)\frac{4!1!}{6!} = \frac{1-\rho}{30},$$

to obtain:

$$w(0,1,1) = \frac{1-\rho}{30} - \frac{\left(\frac{1-\rho}{20}\right)^2}{\frac{1-\rho}{12}} = \frac{1-\rho}{300},\ w(0,1,0) = \frac{1-\rho}{60} - \frac{\left(\frac{1-\rho}{30}\right)^2}{\frac{1-\rho}{12}} = \frac{1-\rho}{300},$$
$$w(0,0,1) = \frac{2\rho+1}{60} - \frac{\left(\frac{3\rho+2}{60}\right)^2}{\frac{\rho+1}{12}} = \frac{1}{300}\frac{3\rho+\rho^2+1}{\rho+1},\ w(0,0,0) = \frac{\rho+1}{60} - \frac{\left(\frac{2\rho+3}{60}\right)^2}{\frac{\rho+3}{12}} = \frac{1}{300}\frac{8\rho+\rho^2+6}{\rho+3}.$$

We plug these expressions into (11) and get:

$$W_{12}(\rho) = -2\left(\frac{1-\rho}{300} + \frac{1-\rho}{300} + \frac{1}{300}\frac{3\rho+\rho^2+1}{\rho+1} + \frac{1}{300}\frac{8\rho+\rho^2+6}{\rho+3}\right)$$
$$= -\frac{1}{150}\frac{26\rho+9\rho^2+15}{(\rho+3)(\rho+1)}.$$

Equilibrium $\mathcal{E}_1$, imprecise signal s_1. The expressions $P_n(s_0,s_1)$ for $(s_0,s_1)\in\{0,1\}^2$ are:

$$\begin{aligned}
P_0(0,0)&=p\frac{0!2!}{3!}+(1-p)\frac{1!1!}{3!}=\frac{1+p}{6},\ P_1(0,0)=p\frac{1!2!}{4!}+(1-p)\frac{2!1!}{4!}=\frac{1}{12},\\
P_0(1,0)&=p\frac{1!1!}{3!}+(1-p)\frac{2!0!}{3!}=\frac{2-p}{6},\ P_1(1,0)=p\frac{2!1!}{4!}+(1-p)\frac{3!0!}{4!}=\frac{3-2p}{12},\\
P_0(0,1)&=p\frac{1!1!}{3!}+(1-p)\frac{0!2!}{3!}=\frac{2-p}{6},\ P_1(0,1)=p\frac{2!1!}{4!}+(1-p)\frac{1!2!}{4!}=\frac{1}{12},\\
P_0(1,1)&=p\frac{2!0!}{3!}+(1-p)\frac{1!1!}{3!}=\frac{p+1}{6},\ P_1(1,1)=p\frac{3!0!}{4!}+(1-p)\frac{2!1!}{4!}=\frac{2p+1}{12}.
\end{aligned}$$

Plugging these expressions into (12), we obtain:

$$\eta_1(p)=\frac{1}{2}\frac{\left(\frac{\frac{1}{12}}{\frac{2-p}{6}}-\frac{\frac{1}{12}}{\frac{1+p}{6}}\right)^2\frac{1+p}{6}+\left(\frac{\frac{2p+1}{12}}{\frac{p+1}{6}}-\frac{\frac{3-2p}{12}}{\frac{2-p}{6}}\right)^2\frac{2-p}{6}}{\left(\frac{\frac{1}{12}}{\frac{2-p}{6}}-\frac{\frac{1}{12}}{\frac{1+p}{6}}\right)\frac{1+p}{6}+\left(\frac{\frac{2p+1}{12}}{\frac{p+1}{6}}-\frac{\frac{3-2p}{12}}{\frac{2-p}{6}}\right)\frac{2-p}{6}}=\frac{1}{4}\frac{2p-1}{(p+1)(2-p)}.$$

Then, we calculate the welfare:

$$\begin{aligned}
W_1(p)&=-2\sum_{s_1\in\{0,1\}}w(s_1)=-2\left(\frac{1}{120}\frac{1+4p-2p^2}{p+1}+\frac{1}{120}\frac{3-2p^2}{2-p}\right)\\
&=-\frac{1}{12}\frac{1+2p-2p^2}{(p+1)(2-p)},
\end{aligned}$$

using

$$P_2(0,0)=p\frac{2!2!}{5!}+(1-p)\frac{3!1!}{5!}=\frac{3-p}{60},P_2(0,1)=p\frac{3!1!}{5!}+(1-p)\frac{2!2!}{5!}=\frac{p+2}{60},$$

to calculate:

$$w(0)=\frac{3-p}{60}-\frac{\left(\frac{1}{12}\right)^2}{\frac{1+p}{6}}=\frac{1}{120}\frac{1+4p-2p^2}{p+1},\ w(1)=\frac{p+2}{60}-\frac{\left(\frac{1}{12}\right)^2}{\frac{2-p}{6}}=\frac{1}{120}\frac{3-2p^2}{2-p}.$$

Equilibrium $\mathcal{E}_1$, correlated signal s_1. Using (5) together with (6)-(8), the expressions $P_n(s_0,s_1)$ are:

$$\begin{aligned}
P_0(0,0)&=\rho\frac{0!1!}{2!}+(1-\rho)\frac{0!2!}{3!}=\frac{2+\rho}{6},\quad P_1(0,0)=\rho\frac{1!1!}{3!}+(1-\rho)\frac{1!2!}{4!}=\frac{1+\rho}{12},\\
P_0(0,1)&=P_0(1,0)=(1-\rho)\frac{1!1!}{3!}=\frac{1-\rho}{6},\quad P_1(0,1)=P_1(1,0)=(1-\rho)\frac{2!1!}{4!}=\frac{1-\rho}{12},\\
P_0(1,1)&=\rho\frac{1!0!}{2!}+(1-\rho)\frac{2!0!}{3!}=\frac{2+\rho}{6},\quad P_1(1,1)=\rho\frac{2!0!}{3!}+(1-\rho)\frac{3!0!}{4!}=\frac{3+\rho}{12}.
\end{aligned}$$

Plugging these expressions in the formulas for $\Delta_1(s_0)$, we obtain:

$$\begin{aligned}
\Delta_1(0)&=\frac{P_1(0,1)}{P_0(0,1)}-\frac{P_1(0,0)}{P_0(0,0)}=\frac{\frac{1-\rho}{12}}{\frac{1-\rho}{6}}-\frac{\frac{1+\rho}{12}}{\frac{2+\rho}{6}}=\frac{1}{2(2+\rho)}\\
\Delta_1(1)&=\frac{P_1(1,1)}{P_0(1,1)}-\frac{P_1(1,0)}{P_0(1,0)}=\frac{\frac{3+\rho}{12}}{\frac{2+\rho}{6}}-\frac{\frac{1-\rho}{12}}{\frac{1-\rho}{6}}=\frac{1}{2(2+\rho)}
\end{aligned}$$

Wrapping up, we obtain:

$$\eta_1(\rho) = \frac{1}{2}\frac{\left(\frac{1}{2(2+\rho)}\right)^2\left(\frac{2+\rho}{6}+\frac{1-\rho}{6}\right)}{\left(\frac{1}{2(2+\rho)}\right)\left(\frac{2+\rho}{6}+\frac{1-\rho}{6}\right)} = \frac{1}{4(2+\rho)}.$$

Turning to calculating welfare, we obtain

$$\begin{aligned} W_1(\rho) &= -2\sum_{s_1\in\{0,1\}} w(s_1) = -2\left(\frac{1}{120}\frac{3+6\rho+\rho^2}{2+\rho}+\frac{1-\rho}{120}\right) \\ &= -\frac{1}{12}\frac{1+\rho}{2+\rho}, \end{aligned}$$

using:

$$P_2(0,0) = \rho\frac{2!1!}{4!}+(1-\rho)\frac{2!2!}{5!} = \frac{2+3\rho}{60},\ P_2(0,1) = (1-\rho)\frac{3!1!}{5!} = \frac{1-\rho}{20},$$

to calculate:

$$w(0) = \frac{2+3\rho}{60} - \frac{\left(\frac{1+\rho}{12}\right)^2}{\frac{2+\rho}{6}} = \frac{1}{120}\frac{3+6\rho+\rho^2}{2+\rho},\ w(1) = \frac{1-\rho}{20} - \frac{\left(\frac{1-\rho}{12}\right)^2}{\frac{1-\rho}{6}} = \frac{1-\rho}{120}.$$

Mixed Strategy Equilibria $\mathcal{M}_{1.2}$ and $\mathcal{M}_{2.1}$. Suppose that $b_1 < \eta_{12.1}(p)$, $b_2 > \eta_{12.2}(p)$ but $b_2 - \eta_{12.2}(p) = \varepsilon > 0$ small. We focus on an equilibrium such that either 1 randomizes for $s_1 = 0$, or 2 randomizes for $s_2 = 0$. The other advisor tells the truth. Generically, there cannot exist an equilibrium in which both $i = 1, 2$ randomize for $s_i = 0$. And of course, when $s_i = 1$, and advisor i says $\hat{m}_i = 1$.

Consider the equilibrium in which 1 randomizes. Let $\Pr(\hat{m}_1 = 1|s_1 = 0) = 1 - \sigma_1 \in (0,1)$. Note that, because $y(s_0, \hat{m}_1, \hat{m}_2) = E[x|s_0, \hat{m}_1, s_2]$, the equilibrium indifference condition for advisor 1 is:

$$b_1 = \eta_{12.1}(\sigma_1; p) \equiv \frac{\sum_{(s_0,s_2)\in\{0,1\}^2} \Delta_1(s_0,s_2)^2 P_0(s_0, s_1 = 0, s_2)}{2\sum_{(s_0,s_2)\in\{0,1\}^2} \Delta_1(s_0,s_2) P_0(s_0, s_1 = 0, s_2)} \tag{19}$$

where:

$$\Delta_1(s_0,s_2) = \frac{P_1(s_0, \hat{m}_1 = 1, s_2)}{P_0(s_0, \hat{m}_1 = 1, s_2)} - \frac{P_1(s_0, \hat{m}_1 = 0, s_2)}{P_0(s_0, \hat{m}_1 = 0, s_2)}.$$

When $s_1 = 0$, the message $\hat{m}_1 = 0$ is sent with probability σ_1. This message reveals that $s_1 = 0$. Hence,

$$\begin{aligned} P_n(s_0, \hat{m}_1 = 0, s_2) =& \sigma_1 P_n(s_0, s_1 = 0, s_2) \\ =& \sigma_1 p\frac{(s_0+s_2+n)!\,(3-s_0-s_2)!}{(4+n)!} + \sigma_1(1-p)\frac{(s_0+1+s_2+n)!\,(2-s_0-s_2)!}{(4+n)!}. \end{aligned}$$

On the other hand, the message $\hat{m}_1 = 1$ could come from $s_1 = 1$ or from $s_1 = 0$. By definition,

$$\begin{aligned} P_n(s_0, \hat{m}_1 = 1, s_2) &= \int_0^1 x^n \Pr(s_0, \hat{m}_1 = 1, s_2|x)\,dx \\ &= \int_0^1 \sum_{s_1'=0,1}\ \sum_{s_1\in\{s_1', 1-s_1'\}} x^n \Pr(\hat{m}_1 = 1|s_1)\Pr(s_1|s_1')\Pr\left(s_0, s_1', s_2|x\right)dx, \end{aligned} \tag{20}$$

where

$$\Pr\left(s_0, s_1', s_2|x\right) = x^{s_0+s_1'+s_2}\left(1-x\right)^{3-s_0-s_1'-s_2}$$

$$\Pr(s_1|s_1 = s_1') = p \text{ and } \Pr(s_1|s_1 = 1-s_1') = 1-p$$

$$\Pr\left(\hat{m}_1 = 1|s_1\right) = 1 \text{ if } s_1 = 1 \text{ and } \Pr\left(\hat{m}_1 = 1|s_1\right) = 1-\sigma_1 \text{ if } s_1 = 0.$$

Hence,

$$\begin{aligned} P_n\left(s_0, \hat{m}_1 = 1, s_2\right) = {} & p\left(1-\sigma_1\right)\int_0^1 x^n x^{s_0+s_2}\left(1-x\right)^{3-s_0-s_2}dx + \left(1-p\right)\int_0^1 x^n x^{s_0+s_2}\left(1-x\right)^{3-s_0-s_2}dx \\ & + p\int_0^1 x^n x^{s_0+1+s_2}\left(1-x\right)^{2-s_0-s_2}dx + \left(1-p\right)\left(1-\sigma_1\right)\int_0^1 x^n x^{s_0+s_2}\left(1-x\right)^{3-s_0-s_2}dx \end{aligned}$$

We conclude that

$$\begin{aligned} P_n\left(s_0, \hat{m}_1 = 1, s_2\right) = {} & \left[p\left(1-\sigma_1\right) + \left(1-p\right)\right]\frac{\left(s_0+s_2+n\right)!\left(3-s_0-s_2\right)!}{\left(4+n\right)!} \\ & + \left[p + \left(1-p\right)\left(1-\sigma_1\right)\right]\frac{\left(s_0+1+s_2+n\right)!\left(2-s_0-s_2\right)!}{\left(4+n\right)!}. \end{aligned}$$

Using $P_n(s_0, \hat{m}_1 = 0, s_2)$ and $P_n(s_0, \hat{m}_1 = 1, s_2)$, we get $P_0(s_0, \hat{m}_1, s_2)$:

$$\begin{aligned} & P_0(0,0,0) = \frac{\sigma_1 + 2p\sigma_1}{12}, && P_0(1,0,0) = P_0(0,0,1) = \frac{\sigma_1}{12}, && P_0(1,0,1) = \frac{3\sigma_1 - 2p\sigma_1}{12}, \\ & P_0(0,1,0) = \frac{4-\sigma_1-2p\sigma_1}{12}, && P_0(1,1,0) = P_0(0,1,1) = \frac{2-\sigma_1}{12}, && P_0(1,1,1) = \frac{4-3\sigma_1+2p\sigma_1}{12}. \end{aligned}$$

We also derive $P_1(s_0, \hat{m}_1, s_2)$:

$$\begin{aligned} & P_1(0,0,0) = \frac{2\sigma_1 + p\sigma_1}{60}, && P_1(1,0,0) = P_1(0,0,1) = \frac{-3\sigma_1 + p\sigma_1}{60}, && P_1(1,0,1) = \frac{4\sigma_1 - 3p\sigma_1}{20}, \\ & P_1(0,1,0) = \frac{5-2\sigma_1-p\sigma_1}{60}, && P_1(1,1,0) = P_1(0,1,1) = \frac{5-3\sigma_1+p\sigma_1}{60}, && P_1(1,1,1) = \frac{5-4\sigma_1+3p\sigma_1}{20} \end{aligned}$$

Using $P_0(s_0, \hat{m}_1, s_2)$ and $P_1(s_0, \hat{m}_1, s_2)$, $\Delta_1(s_0, s_2)$ is derived as

$$\Delta_1(0,0) = \frac{3-6p}{5(1+2p)(\sigma_1+2p\sigma_1-4)}, \Delta_1(0,1) = \Delta_1(1,0) = \frac{1-2p}{5(\sigma_1-2)}, \Delta_1(1,1) = \frac{3-6p}{5(2p-3)(4+(2p-3)\sigma_1)}.$$

Substituting $\Delta_1(s_0, s_2)$ and $P_0(s_0, s_1, s_2)$ to the expression (19), we can derive $\eta_{12.1}(\sigma_1; p)$. The truthtelling condition for advisor 1 as a pure strategy $\eta_{12.1}(p)$ is

$$\eta_{12.1}(p) = \frac{(2p-1)(27+24p-8p^2-32p^3+16p^4)}{10(-3+2p)(1+2p)(-9-4p+4p^2)}$$

We see that $\eta_{12.1}(\sigma_1; p) < \eta_{12.1}(p)$ for all $p \in (0.5, 1]$ and $\sigma_1 \in (0,1)$ and $\eta_{12.1}(\sigma_1; p) \to \eta_{12.1}(p)$ if $\sigma_1 \to 1$.

The equilibrium condition for advisor 2 is that

$$b_2 < \eta_{12.2}(\sigma_1; p) \equiv \frac{\sum_{(s_0,\hat{m}_1)\in\{0,1\}^2} \Delta_2(s_0,\hat{m}_1)^2 \Pr(s_0,\hat{m}_1,s_2=0)}{2\sum_{(s_0,\hat{m}_1)\in\{0,1\}^2} \Delta_2(s_0,\hat{m}_1) \Pr(s_0,\hat{m}_1,s_2=0)},$$

where

$$\Delta_2(s_0,\hat{m}_1) = E[x|s_0,\hat{m}_1,s_2=1] - E[x|s_0,\hat{m}_1,s_2=0] = \frac{P_1(s_0,\hat{m}_1,s_2=1)}{P_0(s_0,\hat{m}_1,s_2=1)} - \frac{P_1(s_0,\hat{m}_1,s_2=0)}{P_0(s_0,\hat{m}_1,s_2=0)}.$$

We verify that this condition holds with a sufficiently small $\varepsilon > 0$. Using $P_0(s_0,\hat{m}_1,s_2)$ and $P_0(s_0,\hat{m}_1,s_2)$, we derive $\Delta_2(s_0,\hat{m}_1)$ as

$$\Delta_2(0,0) = \begin{cases} \frac{1+4p-2p^2}{5+10p} & \text{if} \quad \sigma_1 > 0 \\ 0 & \text{if} \quad \sigma_1 = 0 \end{cases} \qquad \Delta_2(0,1) = \frac{10-4(2+p)\sigma_1 + (1+4p-2p^2)\sigma_1^2}{5(2-\sigma_1)(4-\sigma_1-2p\sigma_1)},$$

$$\Delta_2(1,0) = \begin{cases} \frac{3-2p^2}{15-10p} & \text{if} \quad \sigma_1 > 0 \\ 0 & \text{if} \quad \sigma_1 = 0 \end{cases} \qquad \Delta_2(1,1) = \frac{10+4(-3+p)\sigma_1 + (-3+2p^2)\sigma_1^2}{5(2-\sigma_1)(4+(-3+2p)\sigma_1)}.$$

Substituting $\Delta_2(s_0,\hat{m}_1)$ and $P_0(s_0,s_1,s_2)$, we can derive $\eta_{12.2}(\sigma_1;p)$, which is strictly greater than the truthtelling condition for advisor 2, $\eta_{12.2}$, when advisor 1 is truthtelling for all $p \in (0.5,1)$ and $\sigma_1 \in (0,1)$. Therefore, for a sufficiently small $\varepsilon > 0$, advisor 2's truthtelling condition holds.

Finally, we calculate the welfare for the equilibrium in which 1 randomizes:

$$W_{12}(\sigma_1;p) = -\sum_{s_0\in\{0,1\}} \sum_{(\hat{m}_1,s_2)\in\{0,1\}^2} \left[P_2(s_0,\hat{m}_1,s_2) - \frac{P_1(s_0,\hat{m}_1,s_2)^2}{P_0(s_0,\hat{m}_1,s_2)} \right]. \tag{21}$$

We want to show that the welfare is higher in the equilibrium in which 1 randomizes and 2 tells the truth for $b_1 < \eta_{12.1}(p)$, $b_2 > \eta_{12.2}(p)$ but $b_2 - \eta_{12.2}(p) = \varepsilon > 0$ small. We derive $P_2(s_0,\hat{m}_1,s_2)$:

$$P_2(0,0,0) = \frac{\sigma_1}{60}, \qquad P_2(0,0,1) = P_2(1,0,0) = \frac{2\sigma_1 - p\sigma_1}{60}, \qquad P_2(1,0,1) = \frac{5\sigma_1 - 4p\sigma_1}{30}$$
$$P_2(0,1,0) = \frac{2-\sigma_1}{60}, \qquad P_2(0,1,1) = P_2(1,1,0) = \frac{3-2\sigma_1+p\sigma_1}{60}, \qquad P_2(1,1,1) = \frac{6+-5\sigma_1+4p\sigma_1}{30}$$

Let $w(s_0,\hat{m}_1,s_2)$ be

$$w(s_0,\hat{m}_1,s_2) = P_2(s_0,\hat{m}_1,s_2) - \frac{P_1(s_0,\hat{m}_1,s_2)^2}{P_0(s_0,\hat{m}_1,s_2)}.$$

which is derived as

$$w(0,0,0) = \frac{\sigma_1+6p\sigma_1-p^2\sigma_1}{300(1+2p)}, \quad w(0,0,1) = \frac{\sigma_1+p\sigma_1-p^2\sigma_1}{300}, \quad w(0,1,0) = \frac{15-10(1+p)\sigma_1+(1+6p-p^2)\sigma_1^2}{300(4-\sigma_1-2p\sigma_1)},$$
$$w(0,1,1) = \frac{5-5\sigma_1+(1+p-p^2)\sigma_1^2}{300(2-\sigma_1)}, \quad w(1,0,0) = \frac{\sigma_1+p\sigma_1-p^2\sigma_1}{300}, \quad w(1,0,1) = \frac{(6-4p-p^2)\sigma_1}{300(3-2p)},$$
$$w(1,1,0) = \frac{5-5\sigma_1+(1+p-p^2)\sigma_1^2}{300(2-\sigma_1)}, \quad w(1,1,1) = \frac{15+10(p-2)\sigma_1+(6-4p-p^2)\sigma_1^2}{300(4+(2p-3)\sigma_1)}.$$

By substituting into (21), we may derive $W_{12}(\sigma_1; p)$. For $\sigma_1 \in (0,1)$ and $p \in (1/2, 1)$, if $b_2 < \eta_{12.2} < \eta_{12.2}(\sigma_1; p)$, we see that $W_{12}(p) > W_{12}(\sigma_1; p)$. When $\eta_{12.2} < b_2 \leq \eta_{12.2}(\sigma_1; p)$, there exist 3 equilibria, ordered in welfare as follows: the equilibrium in which 1 randomizes and 2 tells the truth, the equilibrium in which 1 babbles and 2 tells the truth, the equilibrium in which 2 babbles and 1 tells the truth. Also, as b_2 grows, the probability that 1 tells the truth in the mixed strategy equilibrium shrinks and becomes zero as $b_2 = \eta_2$. For b_2 above η_2, there is only one equilibrium: 2 babbles and 1 tells the truth.

Next, consider the equilibrium in which 2 randomizes. Let $\Pr(\hat{m}_2 = 1 | s_2 = 0) = 1 - \sigma_2 \in (0,1)$. Since $y(s_0, \hat{m}_1, \hat{m}_2) = E[x | s_0, s_1, \hat{m}_2]$, the equilibrium indifference condition for advisor 2 is

$$b_2 = \eta_{12.2}(\sigma_2; p) \equiv \frac{\sum_{(s_0,s_1)\in\{0,1\}^2} \Delta_2(s_0,s_1)^2 P_0(s_0,s_1,s_2=0)}{2\sum_{(s_0,s_1)\in\{0,1\}^2} \Delta_2(s_0,s_1) P_0(s_0,s_1,s_2=0)} \tag{22}$$

where

$$\Delta_2(s_0,s_1) = \frac{P_1(s_0,s_1,\hat{m}_2=1)}{P_0(s_0,s_1,\hat{m}_2=1)} - \frac{P_1(s_0,s_1,\hat{m}_2=0)}{P_0(s_0,s_1,\hat{m}_2=0)}$$

When $s_2 = 0$, the message $\hat{m}_2 = 0$ is sent with probability σ_2, which reveals that $s_2 = 0$.

$$\begin{aligned} P_n(s_0,s_1,\hat{m}_2=0) =& \sigma_2 P_n(s_0,s_1,s_2=0) \\ =& \sigma_2 \left(p\frac{(s_0+s_1+n)!(3-s_0-s_1)!}{(4+n)!} + (1-p)\frac{(s_0+(1-s_1)+n)!(2+s_1-s_0)!}{(4+n)!} \right) \end{aligned}$$

The message $\hat{m}_2 = 1$ could come from $s_2 = 1$ or from $s_2 = 0$. Hence,

$$\begin{aligned} P_n(s_0,s_1,\hat{m}_2=1) =& p\frac{(s_0+s_1+1)!(2-s_0-s_1)!}{(4+n)!} + (1-p)\frac{(s_0-s_1+n+2)!(1+s_1-s_0)!}{(4+n)!} \\ &+ (1-\sigma_2)\left(p\frac{(s_0+s_1+n)!(3-s_0-s_1)!}{(4+n)!} + (1-p)\frac{(s_0-s_1+n+1)!(2+s_1-s_0)!}{(4+n)!} \right) \end{aligned}$$

The calculation of $P_n(s_0,s_1,\hat{m}_2)$ for $n = 0,1$ is

$$\begin{aligned}
&P_0(0,0,0) = \frac{\sigma_2 + 2p\sigma_2}{12}, && P_0(0,1,0) = \frac{3\sigma_2 - 2p\sigma_2}{12}, && P_0(1,0,0) = P_0(1,1,0) = \frac{\sigma_2}{12} \\
&P_0(0,0,1) = \frac{2+2p(1-\sigma_2)-\sigma_2}{12}, && P_0(0,1,1) = \frac{4-2p(1-\sigma_2)-3\sigma_2}{12}, && P_0(1,0,1) = \frac{4-2p-\sigma_2}{12}, \\
&P_0(1,1,1) = \frac{2+2p-\sigma_2}{12}, && P_1(0,0,0) = \frac{2\sigma_2 + p\sigma_2}{60}, && P_1(0,1,0) = P_1(1,0,0) = \frac{3\sigma_2 - p\sigma_2}{60}, \\
&P_1(1,1,0) = \frac{2\sigma_2 + p\sigma_2}{60}, && P_1(0,0,1) = \frac{5-2\sigma_2-p\sigma_2}{60}, && P_1(0,1,1) = \frac{5-3\sigma_2+p\sigma_2}{60}, \\
&P_1(1,0,1) = \frac{15-3\sigma_2-10p+p\sigma_2}{60}, && P_1(1,1,1) = \frac{5+10p-2\sigma_2-p\sigma_2}{60}.
\end{aligned}$$

Using $P_0(s_0,s_1,\hat{m}_2)$ and $P_1(s_0,s_1,\hat{m}_2)$, $\Delta_2(s_0,s_1)$ is derived as

$$\begin{aligned}
&\Delta_2(0,0) = \frac{1+4p-2p^2}{5(1+2p)(2+2p(1-\sigma_2)-\sigma_2)}, && \Delta_2(0,1) = \frac{3-2p^2}{5(3-2p)(4-2p(1-\sigma_2)-3\sigma_2)}, \\
&\Delta_2(1,0) = \frac{3-2p^2}{5(4-2p-\sigma_2)}, && \Delta_2(1,1) = \frac{1+4p-2p^2}{10(1+p)-5\sigma_2}
\end{aligned}$$

Substituting $\Delta_2(s_0, s_1)$ and $P_0(s_0, s_1, s_2)$, we can derive $\eta_{12.2}(\sigma_2; p)$. We see that $\eta_{12.2}\sigma_2; p < \eta_{12.2}(p)$ for all $p \in (0.5, 1]$ and $\sigma_2 \in (0, 1)$.

The equilibrium condition for advisor 1 is that

$$b_1 < \eta_{12.1}(\sigma_2; p) \equiv \frac{\sum_{(s_0,\hat{m}_2)\in\{0,1\}^2} \Delta_1(s_0, \hat{m}_2)^2 \Pr(s_0, s_1 = 0, \hat{m}_2)}{2\sum_{(s_0,\hat{m}_2)\in\{0,1\}^2} \Delta_1(s_0, \hat{m}_2) \Pr(s_0, s_1 = 0, \hat{m}_2)}$$

where

$$\Delta_1(s_0, \hat{m}_2) = E[x|s_0, s_1 = 1, \hat{m}_2] - E[x|s_0, s_1 = 0, \hat{m}_2] = \frac{P_1(s_0, s_1 = 1, \hat{m}_2)}{P_0(s_0, s_1 = 1, \hat{m}_2)} - \frac{P_1(s_0, s_1 = 0, \hat{m}_2)}{P_0(s_0, s_1 = 0, \hat{m}_2)}$$

$\Delta_1(s_0, \hat{m}_2)$ is derived as

$$\Delta_1(0,0) = \frac{-3+6p}{5(3+4p-4p^2)} \quad \Delta_1(0,1) = \frac{(-1+2p)(10-12\sigma_2+3\sigma_2^2)}{5(4-2p(1-\sigma_2)-3\sigma_2)(-2-2p(1-\sigma_2)+\sigma_2)}$$
$$\Delta_1(1,0) = \frac{-1+2p}{5} \quad \Delta_1(1,1) = \frac{(-1+2p)(10-8\sigma_2+\sigma_2^2)}{5(2+2p-\sigma_2)(-4+2p+\sigma_2)}$$

Then, by substituting $\Delta_1(s_0, \hat{m}_2)$ and $P_0(s_0, s_1 = 0, \hat{m}_2)$, we can derive $\eta_{12.1}(\sigma_2; p)$. For any $p \in (0.5, 1]$ and $\sigma_2 \in (0, 1)$, we can see that $\eta_{12.1}(\sigma_2; p) > \eta_{12.1}(p)$.

We calculate the welfare for the equilibrium in which 2 randomizes:

$$W_{12}(\sigma_2; p) = -\sum_{s_0\in\{0,1\}} \sum_{(s_1,\hat{m}_2)\in\{0,1\}^2} \left[P_2(s_0, s_1, \hat{m}_2) - \frac{P_1(s_0, s_1, \hat{m}_2)^2}{P_0(s_0, s_1, \hat{m}_2)}\right]. \tag{23}$$

$P_2(s_0, s_1, \hat{m}_2)$ is derived as

$$P_2(0,0,0) = P_2(0,1,0) = \frac{\sigma_2}{60}, \qquad P_2(1,0,0) = \frac{2\sigma_2 - p\sigma_2}{60}, \quad P_2(1,1,0) = \frac{(1+p)\sigma_2}{60},$$
$$P_2(0,0,1) = \frac{3-p-\sigma_2}{60}, \qquad P_2(0,1,1) = \frac{2+p-\sigma_2}{60}, \quad P_2(1,0,1) = \frac{2(6-\sigma_2)+p(-9+\sigma_2)}{60},$$
$$P_2(1,1,1) = \frac{3+p(9-\sigma_2)-\sigma_2}{60}.$$

Let

$$w(s_0, s_1, \hat{m}_2) = P_2(s_0, s_1, \hat{m}_2) - \frac{P_1(s_0, s_1, \hat{m}_2)^2}{P_0(s_0, s_1, \hat{m}_2)}.$$

By substituting $P_n(s_0, s_1, \hat{m}_2)$ for $n = 1, 2, 3$, $w(s_0, s_1, \hat{m}_2)$ is derived as

$$w(0,0,0) = \frac{\sigma_2 + 6p\sigma_2 - p^2\sigma_2}{300+600p}, \quad w(0,0,1) = \frac{-5+5\sigma_2-\sigma_2^2+p(-20+25\sigma_2-6\sigma_2^2)+p^2(10-10\sigma_2+\sigma_2^2)}{300(-2-2p(1-\sigma_2)+\sigma_2)},$$
$$w(0,1,0) = \frac{(-6+4p+p^2)\sigma_2}{300(-3+2p)}, \quad w(0,1,1) = \frac{15-20\sigma_2+p(5-4\sigma_2)\sigma_2+6\sigma_2^2-p^2(10-10\sigma_2+\sigma_2^2)}{300(4+2p(-1+\sigma_2)-3\sigma_2)},$$
$$w(1,0,0) = \frac{(1+p-p^2)\sigma_2}{300}, \quad w(1,0,1) = \frac{15-10\sigma_2+p(-5+\sigma_2)\sigma_2+\sigma_2^2-p^2(10-10\sigma_2+\sigma_2^2)}{300(4-2p-\sigma_2)},$$
$$w(1,1,0) = \frac{(1+p-p^2)\sigma_2}{300}, \quad w(1,1,1) = \frac{5-5\sigma_2+\sigma_2^2+p(20-15\sigma_2+\sigma_2)-p^2(10-10\sigma_2+\sigma_2^2)}{300(2+2p-\sigma_2)}.$$

Then, substituting into (23), we can derive $W_{12}(\sigma_2;p)$. Notice that because $\eta_{12.2}(\sigma_2;p) < \eta_{12.2}(p)$, when $b_1 < \eta_{12.1}(p)$, $W_{12}(\sigma_2;p)$ is lower than the welfare of the pure strategy equilibrium where both 1 and 2 tell the truth, $W_{12}(p)$. When $\eta_{12.1}(p) < b_1 < \eta_{12.1}(\sigma_2;p)$, there are three equilibria: the equilibrium in which 1 tells the truth and 2 randomizes, the equilibrium in which 1 babbles and 2 tells the truth, and the equilibrium in which 1 tells the truth and 2 babbles. And $W_{12}(\sigma_2;p) > W_1$ for all $p \in (1/2,1]$ and $\sigma_2 \in (0,1)$ and $W_{12}(\sigma_2;p) = W_1$ if $\sigma_2 = 0$. However, whether $W_{12}(\sigma_2;p)$ or W_2 is greater depends on σ_2 and p. Because 2 is more informative than 1, if σ_2 and p are low, hearing the truth only from 2 is better than hearing the truth from 1 and the mixed strategy report from 2. On the other hand, if both σ_2 and p are high enough, $W_{12}(\sigma_2;p)$ is greater than W_2.

Equilibrium $\mathcal{E}_{12A}$. Using the formulas for $P_n(s_0,s_1,\hat{m}_2)$, $n=0,1$, we calculate $\Delta_{2A}(s_0,s_1)$ and $\Pr(s_0,s_1,0)$ for $(s_0,s_1)\in\{0,1\}^2$:

$$\begin{aligned}\Delta_{2A}(0,0) &= \frac{p(1-q)\frac{1!2!}{4!}+pq\frac{2!2!}{5!}+(1-p)(1-q)\frac{2!1!}{4!}+(1-p)q\frac{3!1!}{5!}}{p(1-q)\frac{0!2!}{3!}+pq\frac{1!2!}{4!}+(1-p)(1-q)\frac{1!1!}{3!}+(1-p)q\frac{2!1!}{4!}} - \frac{qp\frac{1!3!}{5!}+q(1-p)\frac{2!2!}{5!}}{qp\frac{0!3!}{4!}+q(1-p)\frac{1!2!}{4!}} \\ &= \frac{1}{5}\frac{1-2p^2+4p}{(2p+1)(2+2p-q-2pq)} \\ \Delta_{2A}(0,1) &= \frac{p(1-q)\frac{2!1!}{4!}+pq\frac{3!1!}{5!}+(1-p)(1-q)\frac{1!2!}{4!}+(1-p)q\frac{2!2!}{5!}}{p(1-q)\frac{1!1!}{3!}+pq\frac{2!1!}{4!}+(1-p)(1-q)\frac{0!2!}{3!}+(1-p)q\frac{1!2!}{4!}} - \frac{qp\frac{2!2!}{5!}+q(1-p)\frac{1!3!}{5!}}{qp\frac{1!2!}{4!}+q(1-p)\frac{0!3!}{4!}} \\ &= \frac{1}{5}\frac{3-2p^2}{(3-2p)(4-2p-3q+2pq)},\end{aligned}$$

$$\Delta_{2A}(1,0) = \frac{p(1-q)\frac{2!1!}{4!}+pq\frac{3!1!}{5!}+(1-p)(1-q)\frac{3!0!}{4!}+(1-p)q\frac{4!0!}{5!}}{p(1-q)\frac{1!1!}{3!}+pq\frac{2!1!}{4!}+(1-p)(1-q)\frac{2!0!}{3!}+(1-p)q\frac{3!0!}{4!}} - \frac{qp\frac{2!2!}{5!}+q(1-p)\frac{3!1!}{5!}}{qp\frac{1!2!}{4!}+q(1-p)\frac{2!1!}{4!}} = \frac{1}{5}\frac{3-2p^2}{4-2p-q}$$

$$\Delta_{2A}(1,1) = \frac{p(1-q)\frac{3!0!}{4!}+pq\frac{4!0!}{5!}+(1-p)(1-q)\frac{2!1!}{4!}+(1-p)q\frac{3!1!}{5!}}{p(1-q)\frac{2!0!}{3!}+pq\frac{3!0!}{4!}+(1-p)(1-q)\frac{1!1!}{3!}+(1-p)q\frac{2!1!}{4!}} - \frac{qp\frac{3!1!}{5!}+q(1-p)\frac{2!2!}{5!}}{qp\frac{2!1!}{4!}+q(1-p)\frac{1!2!}{4!}} = \frac{1}{5}\frac{1+4p-2p^2}{2p-q+2}.$$

Substituting into expression (13), we obtain:

$$\begin{aligned}\eta_{12A.2}(p,q) &\equiv \frac{\left(\begin{array}{c}\left(\frac{1}{5}\frac{1+4p-2p^2}{(2p+1)(2+2p-q-2pq)}\right)^2\frac{1}{12}(2p+1)+\left(\frac{1}{5}\frac{3-2p^2}{(3-2p)(4-3q-2p+2pq)}\right)^2\frac{1}{12}(3-2p)\\ +\left(\frac{1}{5}\frac{3-2p^2}{4-2p-q}\right)^2\frac{1}{12}+\left(\frac{1}{5}\frac{1+4p-2p^2}{2+2p-q}\right)^2\frac{1}{12}\end{array}\right)}{2\left(\begin{array}{c}\frac{1}{5}\frac{1+4p-2p^2}{(2p+1)(2+2p-q-2qp)}\frac{1}{12}(2p+1)+\frac{1}{5}\frac{3-2p^2}{(3-2p)(4-2p-3q+2pq)}\frac{1}{12}(3-2p)\\ +\frac{1}{5}\frac{3-2p^2}{4-2p-q}\frac{1}{12}+\frac{1}{5}\frac{1+4p-2p^2}{2+2p-q}\frac{1}{12}\end{array}\right)} \\ &= \frac{\left(\frac{1+4p-2p^2}{2+2p-q-2qp}\right)^2\frac{1}{2p+1}+\left(\frac{3-2p^2}{(4-3q-2p+2qp)}\right)^2\frac{1}{3-2p}+\left(\frac{2p^2-3}{q+2p-4}\right)^2+\left(\frac{1+4p-2p^2}{2+2p-q}\right)^2}{10\left(\frac{1+4p-2p^2}{2+2p-q-2qp}+\frac{3-2p^2}{4-3q-2p+2qp}+\frac{2p^2-3}{q+2p-4}+\frac{1+4p-2p^2}{2+2p-q}\right)}.\end{aligned}$$

Further, using (1), we obtain:

$$\eta_{12A.1}(p,q) \equiv \frac{\sum_{s_0\in\{0,1\}}\sum_{\hat{m}_2\in\{0,1\}}\Delta_1(s_0,\hat{m}_2)^2 P_0(s_0,s_1,\hat{m}_2)}{2\sum_{s_0\in\{0,1\}}\sum_{\hat{m}_2\in\{0,1\}}\Delta_1(s_0,\hat{m}_2)P_0(s_0,s_1,\hat{m}_2)}, \tag{24}$$

$$\text{with } \Delta_1(s_0,\hat{m}_2) = \frac{P_1(s_0,s_1=1,\hat{m}_2)}{P_0(s_0,s_1=1,\hat{m}_2)} - \frac{P_1(s_0,s_1=0,\hat{m}_2)}{P_0(s_0,s_1=0,\hat{m}_2)},\ \text{for } (s_0,\hat{m}_2)\in\{0,1\}^2. \tag{25}$$

We calculate $\Delta_1(s_0,\hat{m}_2)$ for $(s_0,\hat{m}_2)\in\{0,1\}^2$,

$$\Delta_1(0,0)=\frac{qp\frac{2!2!}{5!}+q(1-p)\frac{1!3!}{5!}}{qp\frac{1!2!}{4!}+q(1-p)\frac{0!3!}{4!}}-\frac{qp\frac{1!3!}{5!}+q(1-p)\frac{2!2!}{5!}}{qp\frac{0!3!}{4!}+q(1-p)\frac{1!2!}{4!}}=\frac{3}{5}\frac{2p-1}{(2p+1)(3-2p)}$$

$$\Delta_1(1,0)=\frac{qp\frac{3!1!}{5!}+q(1-p)\frac{2!2!}{5!}}{qp\frac{2!1!}{4!}+q(1-p)\frac{1!2!}{4!}}-\frac{qp\frac{2!2!}{5!}+q(1-p)\frac{3!1!}{5!}}{qp\frac{1!2!}{4!}+q(1-p)\frac{2!1!}{4!}}=\frac{1}{5}(2p-1)$$

$$\begin{aligned}\Delta_1(0,1)&=\frac{pq\frac{3!1!}{5!}+p(1-q)\frac{2!1!}{4!}+(1-p)q\frac{2!2!}{5!}+(1-p)(1-q)\frac{1!2!}{4!}}{pq\frac{2!1!}{4!}+p(1-q)\frac{1!1!}{3!}+(1-p)q\frac{1!2!}{4!}+(1-p)(1-q)\frac{0!2!}{3!}}\\&\quad-\frac{pq\frac{2!2!}{5!}+p(1-q)\frac{1!2!}{4!}+(1-p)q\frac{3!1!}{5!}+(1-p)(1-q)\frac{2!1!}{4!}}{pq\frac{1!2!}{4!}+p(1-q)\frac{0!2!}{3!}+(1-p)q\frac{2!1!}{4!}+(1-p)(1-q)\frac{1!1!}{3!}}\\&=\frac{1}{5}\frac{(2p-1)\left(10-12q+3q^2\right)}{(4-2p-3q+2pq)(2+2p-q-2pq)}\end{aligned}$$

$$\begin{aligned}\Delta_1(1,1)&=\frac{pq\frac{4!0!}{5!}+p(1-q)\frac{3!0!}{4!}+(1-p)q\frac{3!1!}{5!}+(1-p)(1-q)\frac{2!1!}{4!}}{pq\frac{3!0!}{4!}+p(1-q)\frac{2!0!}{3!}+(1-p)q\frac{2!1!}{4!}+(1-p)(1-q)\frac{1!1!}{3!}}\\&\quad-\frac{pq\frac{3!1!}{5!}+p(1-q)\frac{2!1!}{4!}+(1-p)q\frac{4!0!}{5!}+(1-p)(1-q)\frac{3!0!}{4!}}{pq\frac{2!1!}{4!}+p(1-q)\frac{1!1!}{3!}+(1-p)q\frac{3!0!}{4!}+(1-p)(1-q)\frac{2!0!}{3!}}\\&=\frac{1}{5}\frac{\left(10-8q+q^2\right)(2p-1)}{(2+2p-q)(4-2p-q)}.\end{aligned}$$

Substituting into (24):

$$\begin{aligned}\eta_{12A.1}(p,q)&=\frac{\begin{pmatrix}\left(\frac{3}{5}\frac{2p-1}{(2p+1)(3-2p)}\right)^2\frac{1}{12}q(2p+1)+\left(\frac{1}{5}(2p-1)\right)^2\frac{1}{12}q\\+\left(\frac{1}{5}\frac{(2p-1)(10-12q+3q^2)}{(4-2p-3q+2pq)(2+2p-q-2pq)}\right)^2\frac{1}{12}(2-q+2p-2qp)+\left(\frac{1}{5}\frac{(10-8q+q^2)(2p-1)}{(2+2p-q)(4-2p-q)}\right)^2\frac{1}{12}(4-q-2p)\end{pmatrix}}{2\begin{pmatrix}\frac{3}{5}\frac{2p-1}{(2p+1)(3-2p)}\frac{1}{12}q(2p+1)+\frac{1}{5}(2p-1)\frac{1}{12}q\\+\frac{1}{5}\frac{(2p-1)(10-12q+3q^2)}{(4-2p-3q+2pq)(2+2p-q-2pq)}\frac{1}{12}(2-q+2p-2qp)+\frac{1}{5}\frac{(10-8q+q^2)(2p-1)}{(2+2p-q)(4-2p-q)}\frac{1}{12}(4-q-2p)\end{pmatrix}}\\&=(2p-1)\frac{\left(\frac{3}{3-2p}\right)^2q\frac{1}{2p+1}+q+\left(\frac{10-12q+3q^2}{4-3q-2p+2qp}\right)^2\frac{1}{2-q+2p-2qp}+\left(\frac{10-8q+q^2}{(2+2p-q)}\right)^2\frac{1}{4-q-2p}}{10\left(\frac{3q}{3-2p}+q+\frac{10-12q+3q^2}{4-3q-2p+2qp}+\frac{10-8q+q^2}{2+2p-q}\right)}.\end{aligned}$$

Using the formulas for $P_n(s_0,s_1,\hat{m}_2)$, $n=0,1,2,$ we obtain:

$$w(0,0,0)=qp\frac{2!3!}{6!}+q(1-p)\frac{3!2!}{6!}-\frac{\left(qp\frac{1!3!}{5!}+q(1-p)\frac{2!2!}{5!}\right)^2}{qp\frac{0!3!}{4!}+q(1-p)\frac{1!2!}{4!}}=-\frac{1}{300}q\frac{-6p+p^2-1}{2p+1}$$

$$w(1,0,0)=qp\frac{3!2!}{6!}+q(1-p)\frac{4!1!}{6!}-\frac{\left(qp\frac{2!2!}{5!}+q(1-p)\frac{3!1!}{5!}\right)^2}{qp\frac{1!2!}{4!}+q(1-p)\frac{2!1!}{4!}}=-\frac{1}{300}q\left(-p+p^2-1\right)$$

$$w(0,1,0)=qp\frac{3!2!}{6!}+q(1-p)\frac{2!3!}{6!}-\frac{\left(qp\frac{2!2!}{5!}+q(1-p)\frac{1!3!}{5!}\right)^2}{qp\frac{1!2!}{4!}+q(1-p)\frac{0!3!}{4!}}=\frac{1}{300}q\frac{4p+p^2-6}{2p-3}$$

$$w(1,1,0)=qp\frac{4!1!}{6!}+q(1-p)\frac{3!2!}{6!}-\frac{\left(qp\frac{3!1!}{5!}+q(1-p)\frac{2!2!}{5!}\right)^2}{qp\frac{2!1!}{4!}+q(1-p)\frac{1!2!}{4!}}=-\frac{1}{300}q\left(-p+p^2-1\right)$$

$$w\left(0,0,1\right)=p(1-q)\frac{2!2!}{5!}+pq\frac{3!2!}{6!}+\left(1-p\right)\left(1-q\right)\frac{3!1!}{5!}+\left(1-p\right)q\frac{4!1!}{6!}$$
$$-\frac{\left(p(1-q)\frac{1!2!}{4!}+pq\frac{2!2!}{5!}+\left(1-p\right)\left(1-q\right)\frac{2!1!}{4!}+\left(1-p\right)q\frac{3!1!}{5!}\right)^2}{p(1-q)\frac{0!2!}{3!}+pq\frac{1!2!}{4!}+\left(1-p\right)\left(1-q\right)\frac{1!1!}{3!}+\left(1-p\right)q\frac{2!1!}{4!}}$$
$$=\frac{1}{300}\frac{-20p+5q-6pq^2-10p^2q+p^2q^2+25pq+10p^2-q^2-5}{-2p+q+2pq-2}$$

$$w\left(1,0,1\right)=p(1-q)\frac{3!1!}{5!}+pq\frac{4!1!}{6!}+\left(1-p\right)\left(1-q\right)\frac{4!0!}{5!}+\left(1-p\right)q\frac{5!0!}{6!}$$
$$-\frac{\left(p(1-q)\frac{2!1!}{4!}+pq\frac{3!1!}{5!}+\left(1-p\right)\left(1-q\right)\frac{3!0!}{4!}+\left(1-p\right)q\frac{4!0!}{5!}\right)^2}{p(1-q)\frac{1!1!}{3!}+pq\frac{2!1!}{4!}+\left(1-p\right)\left(1-q\right)\frac{2!0!}{3!}+\left(1-p\right)q\frac{3!0!}{4!}}$$
$$=\frac{1}{300}\frac{10q-pq^2-10p^2q+p^2q^2+5pq+10p^2-q^2-15}{2p+q-4}$$

$$w\left(0,1,1\right)=p(1-q)\frac{3!1!}{5!}+pq\frac{4!1!}{6!}+\left(1-p\right)\left(1-q\right)\frac{2!2!}{5!}+\left(1-p\right)q\frac{3!2!}{6!}$$
$$-\frac{\left(p(1-q)\frac{2!1!}{4!}+pq\frac{3!1!}{5!}+\left(1-p\right)\left(1-q\right)\frac{1!2!}{4!}+\left(1-p\right)q\frac{2!2!}{5!}\right)^2}{p(1-q)\frac{1!1!}{3!}+pq\frac{2!1!}{4!}+\left(1-p\right)\left(1-q\right)\frac{0!2!}{3!}+\left(1-p\right)q\frac{1!2!}{4!}}$$
$$=-\frac{1}{300}\frac{20q+4pq^2-10p^2q+p^2q^2-5pq+10p^2-6q^2-15}{-2p-3q+2pq+4}$$

$$w\left(1,1,1\right)=p(1-q)\frac{4!0!}{5!}+pq\frac{5!0!}{6!}+\left(1-p\right)\left(1-q\right)\frac{3!1!}{5!}+\left(1-p\right)q\frac{4!1!}{6!}$$
$$-\frac{\left(p(1-q)\frac{3!0!}{4!}+pq\frac{4!0!}{5!}+\left(1-p\right)\left(1-q\right)\frac{2!1!}{4!}+\left(1-p\right)q\frac{3!1!}{5!}\right)^2}{p(1-q)\frac{2!0!}{3!}+pq\frac{3!0!}{4!}+\left(1-p\right)\left(1-q\right)\frac{1!1!}{3!}+\left(1-p\right)q\frac{2!1!}{4!}}$$
$$=-\frac{1}{300}\frac{-20p+5q-pq^2-10p^2q+p^2q^2+15pq+10p^2-q^2-5}{2p-q+2}.$$

Plugging into expression (14), we obtain:

$$W_{12A}(p,q)=-\frac{1}{300}q\frac{1+6p-p^2}{2p+1}-\frac{1}{300}q\left(1+p-p^2\right)-\frac{1}{300}q\frac{6-4p-p^2}{3-2p}-\frac{1}{300}q\left(1+p-p^2\right)$$
$$-\frac{1}{300}\frac{5-p^2q^2+10p^2q-10p^2+6pq^2-25pq+20p+q^2-5q}{2+2p-q-2pq}$$
$$-\frac{1}{300}\frac{15-10q+pq^2+10p^2q-p^2q^2-5pq-10p^2+q^2+}{4-q-2p}$$
$$-\frac{1}{300}\frac{15-p^2q^2+10p^2q-10p^2-4pq^2+5pq+6q^2-20q}{4-2p-3q+2pq}$$
$$-\frac{1}{300}\frac{5+20p-5q+pq^2+10p^2q-p^2q^2-15pq-10p^2+q^2}{2+2p-q}.$$

Equilibrium $\mathcal{E}_{2A}$. Using $P_n(s_0, \hat{m}_2)$, $n = 0, 1$, we find $\Delta_{2A}(s_0)$ for $s_0 = 0, 1$:

$$\Delta_2(0) = \frac{(1-q)\frac{1!1!}{3!} + q\frac{2!1!}{4!}}{(1-q)\frac{0!1!}{2!} + q\frac{1!1!}{3!}} - \frac{q\frac{1!2!}{4!}}{q\frac{0!2!}{3!}} = \frac{1}{4(3-2q)} \quad \Delta_2(1) = \frac{(1-q)\frac{2!0!}{3!} + q\frac{3!0!}{4!}}{(1-q)\frac{1!0!}{2!} + q\frac{2!0!}{3!}} - \frac{q\frac{2!1!}{4!}}{q\frac{1!1!}{3!}} = \frac{1}{2(3-q)},$$

so that, plugging into (15), we obtain:

$$\eta_{2A}(q) \equiv \frac{\left(\frac{1}{4(3-2q)}\right)^2 \frac{1}{3} + \left(\frac{1}{2(3-q)}\right)^2 \frac{1}{6}}{2\left(\frac{1}{4(3-2q)}\right)\frac{1}{3} + 2\left(\frac{1}{2(3-q)}\right)\frac{1}{6}} = \frac{1}{8}\frac{9-10q+3q^2}{(3-2q)(3-q)(2-q)}.$$

Substituting $P_n(\hat{m}_2, s_0)$, $n = 0, 1, 2$ in the expressions for $w_2(s_0, \hat{m}_2)$ and (16), we obtain:

$$\begin{aligned} W_{2A}(q) &= -q\frac{2!2!}{5!} + \frac{\left(q\frac{1!2!}{4!}\right)^2}{q\frac{0!2!}{3!}} - (1-q)\frac{2!1!}{4!} - q\frac{3!1!}{5!} + \frac{\left((1-q)\frac{1!1!}{3!} + q\frac{2!1!}{4!}\right)^2}{(1-q)\frac{0!1!}{2!} + q\frac{1!1!}{3!}} \\ &\quad - q\frac{3!1!}{5!} + \frac{\left(q\frac{2!1!}{4!}\right)^2}{q\frac{1!1!}{3!}} - (1-q)\frac{3!0!}{4!} - q\frac{4!0!}{5!} + \frac{\left((1-q)\frac{2!0!}{3!} + q\frac{3!0!}{4!}\right)^2}{(1-q)\frac{1!0!}{2!} + q\frac{2!0!}{3!}} \\ &= -\frac{1}{48}\frac{24-27q+7q^2}{(3-2q)(3-q)}. \end{aligned}$$

Threshold η^1_{02A}. Using the expressions $P_n(s_0, s_1, \hat{m}_2)$ calculated for equilibrium $\mathcal{E}_{12A}$, we find $\Delta_0(s_1, \hat{m}_2)$ for $(s_1, \hat{m}_2) \in \{0, 1\}^2$

$$\begin{aligned} \Delta_0(0,0) &= \frac{\frac{1}{60}q(p+2)}{\frac{1}{12}q(2p+1)} - \frac{\frac{1}{60}q(3-p)}{\frac{1}{12}q}, \ \Delta_0(1,0) = \frac{\frac{1}{60}q(3-p)}{\frac{1}{12}q(3-2p)} - \frac{\frac{1}{60}q(p+2)}{\frac{1}{12}q} \\ \Delta_0(0,1) &= \frac{-\frac{1}{60}(2q+qp-5)}{-\frac{1}{12}(q-2p+2qp-2)} - \frac{\frac{1}{60}(-3q-10p+qp+15)}{-\frac{1}{12}(q+2p-4)} \\ \Delta_0(1,1) &= \frac{\frac{1}{60}(-3q+qp+5)}{\frac{1}{12}(-3q-2p+2qp+4)} - \frac{-\frac{1}{60}(2q-10p+qp-5)}{-\frac{1}{12}(q-2p-2)}. \end{aligned}$$

Substituting into (17), we obtain:

$$\eta^1_{02A}(p,q) \equiv -\frac{\begin{pmatrix} \left(\frac{\frac{1}{60}q(p+2)}{\frac{1}{12}q(2p+1)} - \frac{-\frac{1}{60}q(p-3)}{\frac{1}{12}q}\right)^2 \frac{1}{12}q + \left(\frac{-\frac{1}{60}(2q+qp-5)}{-\frac{1}{12}(q-2p+2qp-2)} - \frac{\frac{1}{60}(-3q-10p+qp+15)}{-\frac{1}{12}(q+2p-4)}\right)^2 \frac{1}{12}(4-q-2p) \\ + \left(\frac{\frac{1}{60}q(3-p)}{\frac{1}{12}q(3-2p)} - \frac{\frac{1}{60}q(p+2)}{\frac{1}{12}q}\right)^2 \frac{1}{12}q + \left(\frac{\frac{1}{60}(-3q+qp+5)}{\frac{1}{12}(-3q-2p+2qp+4)} - \frac{-\frac{1}{60}(2q-10p+qp-5)}{-\frac{1}{12}(q-2p-2)}\right)^2 \frac{1}{12}(2-q+2p) \end{pmatrix}}{\begin{pmatrix} 2\left(\frac{\frac{1}{60}q(p+2)}{\frac{1}{12}q(2p+1)} - \frac{-\frac{1}{60}q(p-3)}{\frac{1}{12}q}\right) \frac{1}{12}q + 2\left(\frac{-\frac{1}{60}(2q+qp-5)}{-\frac{1}{12}(q-2p+2qp-2)} - \frac{\frac{1}{60}(-3q-10p+qp+15)}{-\frac{1}{12}(q+2p-4)}\right) \frac{1}{12}(4-q-2p) \\ +2\left(\frac{\frac{1}{60}q(3-p)}{\frac{1}{12}q(3-2p)} - \frac{\frac{1}{60}q(p+2)}{\frac{1}{12}q}\right) \frac{1}{12}q + 2\left(\frac{\frac{1}{60}(-3q+qp+5)}{\frac{1}{12}(-3q-2p+2qp+4)} - \frac{-\frac{1}{60}(2q-10p+qp-5)}{-\frac{1}{12}(q-2p-2)}\right) \frac{1}{12}(2-q+2p) \end{pmatrix}}$$

Welfare function W^1_{2A}. The expressions $P_n(s_1, \hat{m}_2)$, for $n = 0, 1, 2$, are

$$P_0(0,0) = qp\frac{0!2!}{3!} + q(1-p)\frac{1!1!}{3!} = \frac{1}{6}q(p+1) \quad P_1(0,0) = qp\frac{3!2!}{4!} + q(1-p)\frac{2!1!}{3!} = \frac{1}{6}q(p+2)$$
$$P_2(0,0) = qp\frac{2!2!}{5!} + q(1-p)\frac{3!1!}{5!} = -\frac{1}{60}q(p-3)$$
$$P_0(0,1) = p(1-q)\frac{0!2!}{3!} + p\frac{1!1!}{3!} + (1-p)(1-q)\frac{1!1!}{3!} + (1-p)\frac{2!0!}{3!} = -\frac{1}{6}(q+qp-3)$$
$$P_1(0,1) = p(1-q)\frac{1!2!}{4!} + p\frac{3!1!}{4!} + (1-p)(1-q)\frac{2!1!}{4!} + (1-p)\frac{3!0!}{4!} = -\frac{1}{12}(q-4)$$
$$P_2(0,1) = p(1-q)\frac{2!2!}{5!} + p\frac{3!1!}{5!} + (1-p)(1-q)\frac{3!1!}{5!} + (1-p)\frac{4!0!}{5!} = \frac{1}{60}(-3q-10p+qp+15)$$
$$P_0(1,0) = qp\frac{1!1!}{3!} + q(1-p)\frac{0!2!}{3!} = -\frac{1}{6}q(p-2) \quad P_1(1,0) = qp\frac{2!1!}{4!} + q(1-p)\frac{1!2!}{4!} = \frac{1}{12}q$$
$$P_2(1,0) = qp\frac{3!1!}{5!} + q(1-p)\frac{2!2!}{5!} = \frac{1}{60}q(p+2)$$
$$P_0(1,1) = p(1-q)\frac{1!1!}{3!} + p\frac{2!0!}{3!} + (1-p)(1-q)\frac{0!2!}{3!} + (1-p)\frac{1!1!}{3!} = \frac{1}{6}(-2q+qp+3)$$
$$P_1(1,1) = p(1-q)\frac{2!1!}{4!} + p\frac{3!0!}{4!} + (1-p)(1-q)\frac{1!2!}{4!} + (1-p)\frac{2!1!}{4!} = -\frac{1}{12}(q-2p-2)$$
$$P_2(1,1) = p(1-q)\frac{3!1!}{5!} + p\frac{4!0!}{5!} + (1-p)(1-q)\frac{2!2!}{5!} + (1-p)\frac{3!1!}{5!} = -\frac{1}{60}(2q-10p+qp-5).$$

Substituting into (18),

$$W^1_{2A}(p,q) = -\left(\begin{array}{c} -\frac{1}{60}q(p-3) - \frac{\left(\frac{1}{6}q(p+2)\right)^2}{\frac{1}{6}q(p+1)} + \frac{1}{60}(-3q-10p+qp+15) - \frac{\left(-\frac{1}{12}(q-4)\right)^2}{-\frac{1}{6}(q+qp-3)} \\ +\frac{1}{60}q(p+2) - \frac{\left(\frac{1}{12}q\right)^2}{-\frac{1}{6}q(p-2)} - \frac{1}{60}(2q-10p+qp-5) - \frac{\left(-\frac{1}{12}(q-2p-2)\right)^2}{\frac{1}{6}(-2q+qp+3)} \end{array} \right)$$
$$= -\frac{1}{3} + \frac{1}{24}\frac{(2+2p-q)^2}{3-2q+pq} + \frac{1}{24}\frac{q}{2-p} + \frac{1}{24}\frac{(4-q)^2}{3-q-pq} + \frac{1}{6}q\frac{(2+p)^2}{1+p}.$$

Proof of Proposition 9. There are several cases to consider, depending on which one is the top equilibrium without delegation.

1. If $b_1 \leq \eta_{12.1}(p)$, $b_2 \leq \eta_{12.2}(p)$, equilibrium $\mathcal{E}_{12}$ exists and there is no delegation.

2a. The top equilibrium without delegation is $\mathcal{E}_2$ and $W_2 > W_{12A}(p,q)$. Then $b_2 \leq \eta_2$ and either $b_1 > \eta_{12.1}(p)$ or $b_2 > \eta_{12.2}(p)$, or both.

2aa. $W_2 > W_{12A}(p,q)$, $b_2 \leq \eta_2$ and $b_1 > \eta_{12.1}(p)$.

Because $W_2 > W_{12A}(p,q)$, the leader delegates to 1 if and only if $\mathcal{E}^1_{02}$ exists, i.e., $b_1 \leq \eta_{12.2}(p)$ and $b_2 - b_1 \leq \eta_{12.2}(p)$, and if $\mathcal{E}^1_{02}$ dominates $\mathcal{E}_2$, i.e., $b_1 < \delta_2(p)$. But the latter is incompatible with $b_1 > \eta_{12.1}(p)$ because we verify that $\delta_2(p) < \eta_{12.1}(p)$. A fortiori, the leader does not delegate to 2 either, as this would require that $\mathcal{E}^2_{12}$ exists and that $\hat{b}_2(q) < \delta_2(p)$, and this is tighter than $b_1 < \delta_2(p)$ because $b_2 > b_1$.

2ab. $W_2 > W_{12A}(p,q)$, and $\eta_{12.2}(p) < b_2 \leq \eta_2$.

The leader delegates to 1 if and only if $\mathcal{E}^1_{02}$ exists, i.e., $b_1 \leq \eta_{12.2}(p)$ and $b_2 - b_1 \leq \eta_{12.2}(p)$, and $b_1 < \delta_2(p,q)$. These conditions do not contradict $W_2 > W_{12A}(p,q)$, and $\eta_{12.2}(p) < b_2 \leq \eta_2$. The leader delegates to 2 if and only if $\mathcal{E}_{01}^{2}$ exists, i.e., $(b_2 - b_1)(1-q) \leq \eta_{12.1}(p)$ and $b_1 q + b_2(1-q) \leq \eta_{12.2}(p)$, it dominates $\mathcal{E}_2$, i.e., $\hat{b}_2(q) < \delta_2(p)$, and $\mathcal{E}_{02}^{1}$ does not exist, i.e. $b_1 > \eta_{12.2}(p)$ or $b_2 - b_1 > \eta_{12.2}(p)$. Because $b_2 > b_1$, it cannot be that $b_1 > \eta_{12.2}(p)$, as it would imply that $\bar{b}_2(q) > \eta_{12.2}(p)$. Hence, it must be that $b_2 - b_1 > \eta_{12.2}(p)$. We verify numerically that these conditions do not contradict the conditions characterizing case 2ab.

2b. The top equilibrium without delegation is $\mathcal{E}_2$ and $W_2 < W_{12A}(p,q)$. Then $b_2 \leq \eta_2$, $b_1 > \eta_{12A}(p,q)$, and either $b_1 > \eta_{12.1}(p)$ or $b_2 > \eta_{12.2}(p)$, or both.

2ba. $W_2 < W_{12A}(p,q)$, $b_2 \leq \eta_2$, $b_1 > \min\{\eta_{12.1}(p), \eta_{12A}(p,q)\}$.

The leader does not delegate to either 1 or 2 because, as in 2aa, equilibria $\mathcal{E}_{02}^{1}$ and $\mathcal{E}_{01}^{2}$ cannot dominate $\mathcal{E}_2$, and a fortiori $\mathcal{E}_{02A}^{1}$ cannot dominate $\mathcal{E}_2$ either.

2bba. $W_2 < W_{12A}(p,q)$, $\eta_{12.2}(p) < b_2 \leq \eta_2$ and $b_1 > \eta_{12A}(p,q) = \eta_{12A.1}(p,q)$, i.e., $\eta_{12A.1}(p,q) \leq \eta_{12A.2}(p,q)$.

Because $\eta_{12.1}(p) < \eta_{12A.1}(p,q)$, the same arguments of cases 2aa and 2ba imply that the leader does not delegate to either 1 or 2.

2bbb. $W_2 < W_{12A}(p,q)$, $\eta_{12.2}(p) < b_2 \leq \eta_2$ and $b_1 > \eta_{12A}(p,q) = \eta_{12A.2}(p,q)$,.i.e., $\eta_{12A.2}(p,q) < \eta_{12A.1}(p,q)$.

Let's consider delegation to advisor 1. Equilibrium $\mathcal{E}_{02}^{1}$ exists when $b_1 \leq \eta_{12.2}(p)$ and $b_2 - b_1 \leq \eta_{12.2}(p)$, and dominates $\mathcal{E}_2$ if $b_1 < \delta_2(p)$. Condition $b_1 \leq \eta_{12.2}(p)$ is redundant, because $\delta_2(p) < \eta_{12.1}(p) < \eta_{12.2}(p)$. We verify that the remaining conditions do not contradict those characterizing 2bbb. Equilibrium $\mathcal{E}_{02A}^{1}$ exists when $b_1 \leq \eta_{02A}^{1}(p,q)$ and dominates $\mathcal{E}_2$ if $b_1 < \delta_{2.12A}(p,q)$, but the latter contradicts $b_1 > \eta_{12A.2}(p,q)$ because $\delta_{2.12A}(p,q) > \eta_{12A.2}(p,q)$. The leader delegates to 2 if and only if $\mathcal{E}_{01}^{2}$ exists, i.e., $(b_2 - b_1)(1-q) \leq \eta_{12.1}(p)$, $\bar{b}_2(q) \leq \eta_{12.2}(p)$, $\mathcal{E}_{01}^{2}$ dominates $\mathcal{E}_2$, i.e., $\hat{b}_2(q) \leq \delta_2(p)$, and $\mathcal{E}_{02}^{1}$ does not exist, i.e., $b_2 - b_1 > \eta_{12.2}(p)$ or $b_1 > \eta_{12.2}(p)$. Because $b_2 > b_1$, the latter contradicts $\bar{b}_2(q) \leq \eta_{12.2}(p)$, hence it must be that $b_2 - b_1 > \eta_{12.2}(p)$. We verify that these conditions contradict those of 2bbb.

3a. The top equilibrium without delegation is $\mathcal{E}_{12A}$ and $W_{12A}(p,q) > W_2$. Then $b_1 \leq \eta_{12A}$, and either $b_1 > \eta_{12.1}(p)$, or $b_2 > \eta_{12.2}(p)$, or both.

3aa. $W_{12A}(p,q) > W_2(p)$ and $\eta_{12.1}(p) < b_1 \leq \eta_{12A}$. The leader delegates to advisor 1 if and only if $\mathcal{E}_{02}^{1}$ exists, i.e., $b_1 \leq \eta_{12.2}(p)$ and $b_2 - b_1 \leq \eta_{12.2}(p)$, and $\mathcal{E}_{02}^{1}$ dominates $\mathcal{E}_{12A}$, i.e., $b_1 < \delta_{12A}(p,q)$. We verify that all these conditions are incompatible. Delegation to 2 requires that $\mathcal{E}_{12}^{2}$ exists and dominates $\mathcal{E}_{12A}$, i.e., $\bar{b}_2(q) - b_1 \leq \eta_{12.1}(p)$, $\bar{b}_2(q) \leq \eta_{12.2}(p)$, and $\hat{b}_2(q) \leq \delta_{12A}(p,q)$. Because $b_2 > b_1$, the latter 2 conditions are tighther than $b_1 \leq \eta_{12.2}(p)$ and $b_1 < \delta_{12A}(p,q)$, so also delegation to 2 is ruled out

3ab. $W_{12A}(p,q) > W_2$, $b_1 \leq \eta_{12A}(p,q)$ and $b_2 > \eta_{12.2}(p)$. As in 3aa, the leader delegates to advisor 1 if and only if $b_1 \leq \eta_{12.2}(p)$, $b_2 - b_1 \leq \eta_{12.2}(p)$, and $b_1 < \delta_{12A}(p,q)$. The resulting conditions do not contradict with those of 3ab. As in 3aa, delegation to 2 requires that $\bar{b}_2(q) - b_1 \leq \eta_{12.1}(p)$, $\bar{b}_2(q) \leq \eta_{12.2}(p)$, $\hat{b}_2(q) \leq \delta_{12A}(p,q)$, and $b_1 > \eta_{12.2}(p)$ or $b_2 - b_1 > \eta_{12.2}(p)$. Because $b_2 > b_1$, $\bar{b}_2(q) \leq \eta_{12.2}(p)$ contradicts $b_1 > \eta_{12.2}(p)$, so it is required that $b_2 - b_1 > \eta_{12.2}(p)$. We verify that the resulting conditions are compatible with those of 3ab.

3b. The top equilibrium without delegation is $\mathcal{E}_{12A}$ and $W_{12A}(p,q) < W_2$. Then $b_1 \leq \eta_{12A}(p,q)$, $b_2 > \eta_2$

and either $b_1 > \eta_{12.1}(p)$ or $b_2 > \eta_{12.2}(p)$, or both. Because $\eta_2 > \eta_{12.2}(p)$, the latter is always satisfied, and so also $b_1 > \eta_{12.1}(p)$ drops out.

As in 3aa, the leader delegates to advisor 1 if and only if $b_1 \leq \eta_{12.2}(p)$, $b_2 - b_1 \leq \eta_{12.2}(p)$, and $b_1 < \delta_{12A}(p,q)$. We verify that the resulting conditions are compatible. The leader delegates to advisor 2 if the same conditions as in 3aa hold: $\bar{b}_2(q) - b_1 \leq \eta_{12.1}(p)$, $\bar{b}_2(q) \leq \eta_{12.2}(p)$, $\hat{b}_2(q) \leq \delta_{12A}(p,q)$, and $b_2 - b_1 > \eta_{12.2}(p)$. We verify that these conditions are compatible with those of 3b. There is delegation to 2 also if equilibrium $\mathcal{E}_0^2$ exists and dominates $\mathcal{E}_{12A}$, $\bar{b}_2(q) \leq \eta_2$ and $\hat{b}_2(q) < \delta_{2.12A}(p,q)$, and if neither $\mathcal{E}_{02}^1$ nor $\mathcal{E}_{10}^2$ exist, i.e., $b_1 > \eta_{12.2}(p)$ or $b_2 - b_1 > \eta_{12.2}(p)$, and $\bar{b}_2(q) > \eta_{12.2}(p)$ or $\bar{b}_2(q) - b_1 > \eta_{12.2}(p)$. Because $b_2 > b_1$ and $\delta_{2.12A}(p,q) < \delta_{12A}(p,q) < \eta_{12.2}(p,q)$, condition $\hat{b}_2(q) < \delta_{2.12A}(p,q)$ contradicts $b_1 > \eta_{12.2}(p)$, and because $\hat{b}_2(q) < \bar{b}_2(q)$ it also contradicts $\bar{b}_2(q) > \eta_{12.2}(p)$. We verify that the resulting conditions are compatible with those of 3b.

4a. The top equilibrium without delegation is $\mathcal{E}_1$ and $W_1(p) > W_{2A}(q)$. Then $b_1 \leq \eta_1(p)$, $b_2 > \eta_2$, and $b_1 > \eta_{12A}(p,q)$. The leader delegates to advisor 1 if $\mathcal{E}_{02}^1$ exists and dominates $\mathcal{E}_1$: $b_1 \leq \eta_{12.2}(p)$, $b_2 - b_1 \leq \eta_{12.2}(p)$ and $b_1 < \delta_1(p)$. We verify that these conditions are compatible with those of 4a. There is delegation to 1 also if $\mathcal{E}_{02A}^1$ (exists and) dominates $\mathcal{E}_1$, i.e., $b_1 < \delta_{1.12A}(p,q)$ but this contradicts $W_1(p) > W_{2A}(q)$ and $b_1 > \eta_{12A}(p,q)$. The leader delegates to 2 if $\mathcal{E}_{01}^2$ exists and dominates $\mathcal{E}_1$, i.e., $\bar{b}_2(q) - b_1 \leq \eta_{12.1}(p)$, $\bar{b}_2(q) \leq \eta_{12.2}(p)$ and $\hat{b}_2(q) \leq \delta_1$, and $\mathcal{E}_{02}^1$ does not exist, i.e. $b_2 - b_1 > \eta_{12.2}(p)$, because as in 3ab, $b_1 > \eta_{12.2}(p)$ contradicts $\bar{b}_2(q) \leq \eta_{12.2}(p)$. We verify that the resulting conditions are compatible with those of 4a. There is delegation to 2 also if $\mathcal{E}_0^2$ exists and dominates $\mathcal{E}_1$, i.e., $\bar{b}_2(q) \leq \eta_2$ and $\hat{b}_2(q) < \delta_{2.1}(p)$, and neither $\mathcal{E}_{02}^1$ nor $\mathcal{E}_{01}^2$ exist, i.e., $b_1 > \eta_{12.2}(p)$ or $b_2 - b_1 > \eta_{12.2}(p)$, and $\bar{b}_2(q) - b_1 > \eta_{12.1}(p)$ or $\bar{b}_2(q) > \eta_{12.2}(p)$, and finally $b_1 > \eta_{02A}^1(p,q)$ or $W_2 > W_{12A}(p,q)$, or else equilibrium $\mathcal{E}_{02A}^1$ exists and dominates $\mathcal{E}_0^2$. We verify that $b_1 > \eta_{12.2}(p)$, $\bar{b}_2(q) > \eta_{12.2}(p)$ and $b_1 > \eta_{02A}^1(p,q)$ are all incompatible with $\hat{b}_2(q) \leq \delta_1$. Further, we verify that the remaining conditions are incompatible with those of 4a.

4b. The top equilibrium without delegation is $\mathcal{E}_1$ and $W_1(p) < W_{2A}(q)$. Then $\eta_{2A}(q) < b_1 \leq \eta_1(p)$, $b_2 > \eta_2$, $b_1 > \eta_{12A}(p,q)$. As in 4a, there is delegation to 1 if $b_1 \leq \eta_{12.2}(p)$, $b_2 - b_1 \leq \eta_{12.2}(p)$ and $b_1 < \delta_1(p)$, or if $b_1 \leq \eta_{02A}^1(p,q)$ and $b_1 < \delta_{1.12A}(p,q)$. We verify that neither of the resulting two sets of conditions are compatible with those of 4b. Likewise, there is delegation to 2 under the same two sets of conditions found in 4a, which we both verify are incompatible with those of 4b.

5a. The top equilibrium without delegation is $\mathcal{E}_{2A}$, and $W_{2A}(q) > W_1(p)$. Then $b_1 \leq \eta_{2A}(q)$ and $b_2 > \eta_2$ and $b_1 > \eta_{12A}(p,q)$.

There is delegation to 1 to induce $\mathcal{E}_{02}^1$ if $b_1 \leq \eta_{12.2}(p)$, $b_2 - b_1 \leq \eta_{12.2}(p)$ and $b_1 < \delta_{2A}(p,q)$, and these conditions are compatible with those of 5a. There is also delegaton to 1 if $\mathcal{E}_{02A}^1$ exists and dominates $\mathcal{E}_{2A}$, i.e., $b_1 \leq \eta_{02A}^1(p,q)$ and $b_1 < \delta_{2A.12A}$, and $\mathcal{E}_{02}^1$ does not exist, i.e., $b_1 > \eta_{12.2}(p)$ or $b_2 - b_1 > \eta_{12.2}(p)$, but we verify that these conditions are incompatible with those of 5a. There is delegation to 2 to induce $\mathcal{E}_{01}^2$ if $\bar{b}_2(q) - b_1 \leq \eta_{12.1}(p)$, $\bar{b}_2(q) \leq \eta_{12.2}(p)$, $\hat{b}_2(q) < \delta_{2A}(p,q)$ and $b_1 > \eta_{12.2}(p)$ or $b_2 - b_1 > \eta_{12.2}(p)$, but we verify that these conditions are incompatible with those of 5a. There is delegation to 2 to induce $\mathcal{E}_0^2$ if

$\bar{b}_2(q) \leq \eta_2$, $\hat{b}_2(q) < \delta_{2A.2}(p,q)$, and $b_1 > \eta_{12.2}(p)$ or $b_2 - b_1 > \eta_{12.2}(p)$, but we verify that these conditions are incompatible with those of 5a.

5b. The top equilibrium without delegation is $\mathcal{E}_{2A}$, and $W_{2A}(q) < W_1(p)$. Then $b_1 > \eta_1(q)$, $b_1 \leq \eta_{2A}(q)$, $b_2 > \eta_2$ and $b_1 > \eta_{12A}(p,q)$.

As in 5a, there is delegation to 1 to induce $\mathcal{E}^1_{02}$, as the conditions $b_1 \leq \eta_{12.2}(p)$, $b_2 - b_1 \leq \eta_{12.2}(p)$ and $b_1 < \delta_{2A}(p,q)$ are compatible with those of 5b, whereas there is no delegation to 1 to obtain $\mathcal{E}^1_{02A}$, because $b_1 < \eta^1_{02A}(p,q)$, $b_1 < \delta_{12A.2}(p,q)$, and $b_1 > \eta_{12.2}(q)$ or $b_2 - b_1 > \eta_{12.2}(q)$ are incompatible with case 5b. There is also no delegation to 1 to induce $\mathcal{E}^1_2$, because $b_2 - b_1 \leq \eta^1_2(p)$, $b_1 < \delta_{1.2A}(p,q)$ and $b_1 > \eta_{12.2}(q)$ or $b_2 - b_1 > \eta_{12.2}(q)$ contradict those of 5b. As in 5a, there is no delegation to 2 to achieve $\mathcal{E}^2_{01}$ nor $\mathcal{E}^2_0$, because neither $\bar{b}_2(q) - b_1 \leq \eta_{12.1}(p)$, $\bar{b}_2(q) \leq \eta_{12.2}(p)$, $\hat{b}_2(q) < \delta_{2A}(p,q)$ and ($b_1 > \eta_{12.2}(p)$ or $b_2 - b_1 > \eta_{12.2}(p)$), nor $\hat{b}_2(q) < \delta_{2.2A}(p,q)$, are compatible with case 5b. Finally, there is no delegation to 2 to obtain $\mathcal{E}^2_1$ because condition $\hat{b}_2(q) < \delta_{1.2A}(p,q)$ is tighter than $\hat{b}_2(q) < \delta_{2.2A}(p,q)$.

6. The top equilibrium without delegation is $\mathcal{E}_0$, so that $b_1 > \eta_1(p)$ and $b_1 > \eta_{2A}(q)$. There is delegation to 1 to induce $\mathcal{E}^1_{02}$ and also to obtain $\mathcal{E}^1_{02A}$, as both the conditions $b_1 \leq \eta_{12.2}(p)$, $b_2 - b_1 \leq \eta_{12.2}(p)$ and $b_1 < \delta_0(p)$, and the conditions $b_1 \leq \eta^1_{02A}(p,q)$, $b_1 < \delta_{12A.0}$, and ($b_1 > \eta_{12.2}(p)$ or $b_2 - b_1 > \eta_{12.2}(p)$) are compatible with case 6. There is no delegation to achieve $\mathcal{E}^1_2$ or $\mathcal{E}^1_{2A}$, because condition $b_1 < \delta_{2.0}$ and hence, a fortiori, $b_1 < \sqrt{W^1_{2A}(p,q) - W_0(q)}$ are incompatible with case 6. Finally, the leader never delegates to 2. Getting $\mathcal{E}^2_{01}$ would be optimal if and only if $\bar{b}_2(q) - b_1 \leq \eta_{12.1}(p)$, $\bar{b}_2(q) \leq \eta_{12.2}(p)$, $\hat{b}_2(q) \leq \delta_0(p)$ and $b_1 > \eta_{12.2}(p)$ or $b_2 - b_1 > \eta_{12.2}(p)$; whereas inducing $\mathcal{E}^2_0$ only if $\hat{b}_2(q) \leq \delta_{0.2}(p)$, and obtaining $\mathcal{E}^2_1$ only if $\delta_{0.1}(p)$. Neither of these 3 sets of conditions is compatible with case 6. ■