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Joint estimation of time and risk preferences using a representative sample of UK households' subjective perceptions of time

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Jeremy Smith (Head of the Department of Economics, University of Warwick) and Michael Ward (Head of the Department of Economics, Monash University)

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Aidan Aungles*

Joint estimation of time and risk preferences using a representative sample of UK households' subjective perceptions of time

Abstract

I use real money choices from the Innovation Panel of the UK Household Longitudinal Survey to jointly estimate time and risk preferences via Maximum Likelihood Estimation using different specifications of subjective time. First, a survey-elicited individual estimation of subjective time is utilised. Second, I use two sample-level estimations of subjective time based upon psychophysical laws which have been found to hold for the perception of stimuli such as light and heat, and apply them to the perception of time (To clarify, here, the sample's average curvature of subjective time is estimated). These specifications are examined closely and compared to that of objective time. Lastly, I also add to the literature on the heterogeneity of time and risk preferences utilising the wide range of variables available.

JEL Classification: D81 (Criteria for Decision-Making under Risk and Uncertainty)
Keywords: Discount rate, Risk aversion, Subjective time, Heterogeneity

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1 Introduction

This project centres around the estimation of a representative sample of the UK population's time and risk preferences- the extent to which individuals prefer present payoffs and are risk averse (though risk aversion is not assumed). These questions are of fundamental significance to any consideration of choices made by individuals which involve uncertainty or have non-immediate implications (the vast majority of those considered in Economics).

Subjective time denotes the idea that individuals' perception of time does not increase linearly i.e. the perception of 12 months may not equal 12x the perception of 1 month. Consideration of subjective time perception could provide significant insight into how individuals discount future outcomes, as discussed below.

Additionally, a brief investigation of heterogeneity in preferences seeks to further illuminate the differences between individuals in those regards. This consideration has policy implications; it also provides a basis for further research into causal relationships.

2 Literature review

The estimation of future discounting by incentive-compatible methods, largely following Coller and Williams (1999), assumed risk neutrality until Andersen et al. (2008); they relaxed this assumption, and found evidence for a constant rate of discounting. This result goes against the literature on hyperbolic discounting, based upon Loewenstein and Prelec (1992), in which discount rates are higher for more immediate periods than more distant ones. In a later paper, Andersen et al. (2014) note that evidence for such models largely comes from university student subjects, with little evidence from field experiments using adult respondents and real money choices. Likewise some have found no evidence of this declining discounting both by traditional methods, such as Andreoni and Spenger (2012), and replicated by alternative ones such as the 'direct method' of Attema et al. (2018).

This method developed by Attema et al. (2016) measures discounting without measuring utility by finding indifference points between weekly cashflows across two different periods so that utility drops out of the equations. The analysis, however, relies upon the assumption of 'additivity'[1] which has long been called into question (Read, 2001). That respondents' subjective perceptions of time are generally non-linear and concave (Zauberman et al. 2009) further questions central assumptions of previous methods of time preference elicitation, with the aforementioned finding of declining impatience possibly attributable to subjective time perceptions rather than hyperbolic discounting in objective time.

Subsequently, Bradford et al. (2019) contribute by measuring subjective time as well as considering time intervals as short as 1 day to investigate the 'first period effect' (FPE). They find evidence for subjective time concavity and constant discounting after the FPE.

[1] It relies upon the cumulative discount weight of a subset of the overall period (finishing at the end of the period) being equal to that of the whole period, minus that of the period up until the beginning of the subset. However, Read (2001) found that discounting over a delay is greater when subdivided.

The principal contributions to the literature of this project are therefore to use a relatively larger and more representative sample to jointly estimate time and risk preference via Maximum Likelihood Estimation based upon specifications of subjective time. (Further contributions, such as individual estimates of background consumption are discussed below)

3 Data and Methodology

3.1 Experimental Design

The methods used to elicit time and risk preferences are real money Multiple Price List (MPL) methods (outlined below). Within the experiment, one choice was selected at random with each respondent having a one-in-ten chance of receiving the payout that they opted for in that choice. The reason for the lottery payout design is one of budgetary constraint- it allows for a far larger sample than would otherwise be possible. Crucially, it is also believed that this elicitation method provides unbiased estimates of the elicited preferences (See, for example, Starmer and Sugden (1991); Beattie and Loomes (1997)).

Beginning with time preferences, the method developed by Coller and Williams (1999) is used in which respondents are asked to choose between a smaller payoff in one period 'smaller sooner' (SS) and a larger payoff in a later period 'larger later' (LL). The repetition of these choices provides an estimate of the respondent's discount rate across the time interval by virtue of which payoff they preferred being compared with the choice's discount rate (for example, opting for SS in choice 25 implies an annual discount rate in excess of 5%). The choices proceed towards higher discount rates within each set of twelve.

Table 1 shows the first 36 choices faced by respondents within the module. These are the choices with a 'front end delay' (FED) of one month, i.e. the SS payoff is £100 to be paid one month from the date the survey was taken, with the LL payoffs being paid 1, 3, and 12 months after that.

Table 2 displays the time preference choices presented to the respondents without a FED, i.e. the SS payoff is £100 to be paid immediately upon completion of the survey, with the LL payoffs being paid 1, 3, and 12 months after that.

The second MPL method elicits risk preferences following Holt and Laury (2002). Here, respondents are asked for their preference between two lotteries. Lottery A is relatively safer- lower variation between payoffs as well as the two payoffs both lying between those of Lottery B. Lottery B is relatively riskier- higher variation of payoffs with one being the highest of the four, and the other the lowest. The choices begin with a small probability of the higher payoff in each lottery and a higher probability of the lower payoff. In each subsequent choice the probability of the higher payoff is increased by 10% and the lower is decreased accordingly.

This process is illustrated in Table 3 with the expected difference being positive when Lottery A provides a higher expected payoff. A risk neutral individual would therefore choose Lottery A in the first four choices, and Lottery B subsequently. The risk preference of the individual can therefore be estimated based upon these choices, i.e. whether the individual

Choice	LL (2 months)	Choice	LL (4 months)	Choice	LL (13 months)
1	£100.42	13	£101.25	25	£105.00
2	£100.83	14	£102.50	26	£110.00
3	£101.25	15	£103.75	27	£115.00
4	£101.67	16	£105.00	28	£120.00
5	£102.08	17	£106.25	29	£125.00
6	£102.50	18	£107.50	30	£130.00
7	£103.33	19	£110.00	31	£140.00
8	£104.17	20	£112.50	32	£150.00
9	£105.00	21	£115.00	33	£160.00
10	£106.66	22	£120.00	34	£180.00
11	£108.33	23	£125.00	35	£200.00
12	£112.50	24	£137.50	36	£250.00

Table 1: SS payoff £100.00 in 1 month

Choice	LL (1 month)	Choice	LL (3 months)	Choice	LL (12 months)
37	£100.42	49	£101.25	61	£105.00
38	£100.83	50	£102.50	62	£110.00
39	£101.25	51	£103.75	63	£115.00
40	£101.67	52	£105.00	64	£120.00
41	£102.08	53	£106.25	65	£125.00
42	£102.50	54	£107.50	66	£130.00
43	£103.33	55	£110.00	67	£140.00
44	£104.17	56	£112.50	68	£150.00
45	£105.00	57	£115.00	69	£160.00
46	£106.66	58	£120.00	70	£180.00
47	£108.33	59	£125.00	71	£200.00
48	£112.50	60	£137.50	72	£250.00

Table 2: SS payoff £100.00 today

follows the aforementioned pattern or deviates in either direction from risk neutrality.

Choice	Lottery A	Lottery B	Expected difference
73	1/10 of £40, 9/10 of £32	1/10 of £77, 9/10 of £2	+£23.30
74	2/10 of £40, 8/10 of £32	2/10 of £77, 8/10 of £2	+£16.60
75	3/10 of £40, 7/10 of £32	3/10 of £77, 7/10 of £2	+£9.90
76	4/10 of £40, 6/10 of £32	4/10 of £77, 6/10 of £2	+£3.20
77	5/10 of £40, 5/10 of £32	5/10 of £77, 5/10 of £2	-£3.50
78	6/10 of £40, 4/10 of £32	6/10 of £77, 4/10 of £2	-£10.20
79	7/10 of £40, 3/10 of £32	7/10 of £77, 3/10 of £2	-£16.90
80	8/10 of £40, 2/10 of £32	8/10 of £77, 2/10 of £2	-£23.60
81	9/10 of £40, 1/10 of £32	9/10 of £77, 1/10 of £2	-£30.30

Table 3: Risk preference section, lower payoffs

The final set of choices outlined in Table 4 principally differ from Table 3 in the magnitude of their payoffs (the risk neutral switching point also differs). This allows the investigation of whether risk preference differs due to the size of payoffs. The higher risk aversion estimate upper bound here relative to the previous section is not salient- I estimate risk aversion to be lower for the higher payoff choices (Section 4.2.2).

Choice	Lottery A	Lottery B	Expected difference
82	1/10 of £100, 9/10 of £40	1/10 of £180, 9/10 of £2	+£26.20
83	2/10 of £100, 8/10 of £40	2/10 of £180, 8/10 of £2	+£14.40
84	3/10 of £100, 7/10 of £40	3/10 of £180, 7/10 of £2	+£2.60
85	4/10 of £100, 6/10 of £40	4/10 of £180, 6/10 of £2	-£9.20
86	5/10 of £100, 5/10 of £40	5/10 of £180, 5/10 of £2	-£21.00
87	6/10 of £100, 4/10 of £40	6/10 of £180, 4/10 of £2	-£32.80
88	7/10 of £100, 3/10 of £40	7/10 of £180, 3/10 of £2	-£44.60
89	8/10 of £100, 2/10 of £40	8/10 of £180, 2/10 of £2	-£56.40
90	9/10 of £100, 1/10 of £40	9/10 of £180, 1/10 of £2	-£68.20

Table 4: Risk preference section, higher payoffs

3.2 Sub-sample Selection

The data for this project come from an ESRC project linking experimental and survey data from a representative sample of the UK population using the Innovation Panel of the UK Household Longitudinal Survey, also known as Understanding Society (ESRC Future Research Leader Grant ES/K001965/1: Linking Experimental and Survey Data: Behavioural Experiments in Health; PI: M.M. Galizzi: https://reshare.ukdataservice.ac.uk/852514/)[2].

[2]This is the author's requested citation format

Specifically, a nationally representative, randomly selected sample of 707 individuals responded to a module with real money rewards in wave 6 of the Innovation Panel. Of whom, 592 also answered the accompanying module which included questions on subjective time perception; respondents were asked on a sliding scale how long 1, 3, and 12 months feel. While 661 of the 707 responded to both the time and risk preference modules.

The method of estimation of a respondent's subjective time perception is to divide the subjective duration of 3 and 12 months by that of 1 month, to provide two calculated estimates and normalise the perception of 1 month to 1 for all respondents. This process left 571 respondents for whom both estimations were calculable. While all 571 of these respondents answered the time preference module, only 537 of them also completed the risk preference module.

Returning to the 571 respondents, upon calculation 114 of them returned subjective time perceptions of at least one of the longer periods of less than one. The interpretation of this would be that one months feels longer than 3 and/or 12 months. The fact that this is hard to rationalise motivates the separate consideration of the 457 respondents with increasing perceptions of time, of whom 432 answered both the risk and time modules.

I refer to the 368 full respondents that, within each set, never switched from their initial option, or only did so once as 'Consistent'. The reason for this is that Option B (LL for time preferences and Lottery B for risk preferences) becomes strictly better with each new choice. This means that if Option B is initially preferred, it ought to be preferred for all the choices within each set, and switching back from B to A, having earlier switched from A to B, is inconsistent.

Thus, I have five samples: The full sample of 707, the 'With RP' sample of 661, the 'Unrestricted' sample of 537, the 'Restricted' sample of 432, and the 'Consistent' sample of 368. (N.B. the first, second and fifth samples do not make restrictions based upon respondents participation in the subjective time module)

3.3 Statistical Specification

The following outlines the log likelihood specification for joint estimation of the risk parameter r and the discount rate δ.

3.3.1 Discounting Functions

Since Samuelson (1947) the standard way to discount future utility has been 'exponential discounting' where the discount rate is constant for each period, leading to exponential decay as time, t, increases.

$$D^E(t) = \left(\frac{1}{1+\delta}\right)^t \tag{1}$$

More recently, the idea of the discount rate being constant for all periods has been questioned. 'Hyperbolic discounting' has become the most widely used alternative, typically following the Loewenstein and Prelec (1992) specification:

$$D^{WF}(t) = \left(\frac{1}{1+\alpha t}\right)^{\frac{\beta}{\alpha}} \tag{2}$$

I follow Bradford et al.(2019) in using the two subjective time specifications they outline, which are grounded in psychophysical theory and experiment. Firstly the Weber-Fechner law:

$$f(t) = \left(\frac{1}{\alpha}\right) ln(1+\alpha t) \tag{3}$$

where $f(t)$ is the subjective perception of time which contracts more as α becomes more positive, and expands more as it becomes more negative- so long as $(1+\alpha t)$ remains positive. t denotes objective time. Bradford et al.(2019) show that substituting $f(t)$ into the exponential discounting function gives the Loewenstein-Prelec hyperbolic discounting function where $\beta = ln(1+\delta)$. Therefore, in all that follows I reframe hyperbolic discounting as a subjective time specification and refer to it as such.

The second specification is Stevens' Power Law:

$$f(t) = t^{\alpha} \tag{4}$$

Which gives the following when substituted into the exponential discounting function:

$$D^{SPL}(t) = \left(\frac{1}{1+\delta}\right)^{t^{\alpha}} \tag{5}$$

Where time is contracting as α decreases below one, and expanding as it increases above one. To clarify, these specifications including the parameter α are overall subjective time specifications, as opposed to the individual surveyed subjective time specification based upon the calculation outlined in Section 3.2. In the former case t is objective time (with α allowing for deviation from objective time), in the latter, survey-elicited subjective time.

3.3.2 Log Likelihood

First, the contribution to the overall likelihood from the risk preference responses. Here, $p(M_j)$ denotes the probability of payoff M_j as specified in the experiment making the expected utility the probability weighted utility of each outcome in each lottery. The expected utility of lottery i is therefore

$$EU_i = \sum_{j=1,2} (p(M_j) \times U(\omega_r + M_j)) \tag{6}$$

Where ω_r is the background consumption into which the respondent integrates the potential prize. Rather than assume a value of ω_r as does the literature, here it is calculated separately for each individual using their responses on monthly consumption of non-essential items. These responses are scaled to a week for ω_r and half a month for ω_δ. These are arbitrarily feasible durations for integration given the nature of the tasks, but a lack of research into prize integration necessitates robustness checks where they will be altered (Section 5). The utility function is specified as

$$U(M) = \frac{(\omega_r + M)^{(1-r)}}{1-r} \tag{7}$$

with r being the coefficient of risk attitude, r=0 denotes risk neutrality, while positive and negative values denote risk aversion and seeking respectively.

∇EU specifies the likelihood of selecting option A, it is the excess of the expected utility of choosing option A above that of option B, scaled by the range of between the highest and lowest possible utilities.

$$\nabla EU = \frac{EU_A - EU_B}{U_{MAX} - U_{MIN}} \tag{8}$$

U_{MAX} is the utility of receiving the higher payoff in option B for sure, and U_{MIN}, that of the lower payoff in option B for sure. This process means that only the relative sizes of the utilities and payoffs effect the estimated parameters and not their absolute magnitude. This has particular implications for investigating different values of ω_r. Once scaled, the change in the estimated parameters comes only from the effect that this has on relative utilities and not, for example, from the fact that a higher value of ω_r will increase all of their absolute magnitudes.

This specification is then transformed using an inverse logit function to bound the value asymptotically between 0 and 1 for maximisation. The noise term μ is added such that respondents need not adhere exactly to Expected Utlity Theory (EUT).

$$\nabla EU^* = \frac{e^{\frac{\nabla EU}{\mu}}}{1 + e^{\frac{\nabla EU}{\mu}}} \tag{9}$$

Here, as $\mu \rightarrow 1$ this specification collapses to EUT, the choice becomes noisier and more random the further away from one it is.

Moving on to the contribution of the time preference responses to the overall likelihood. The present value of choosing the smaller sooner payoff, PV_A, is considered to be the discounted sum of all future payoffs from choosing that option, however only the periods in which these payoffs differ between option A and option B need be considered here.

$$PV_A = D(FED)(\omega_\delta + M_A)^{(1-r)} + D(t + FED)\omega_\delta^{(1-r)} \tag{10}$$

As before, payoffs are denoted by M, background consumption by ω, and the risk parameter by r. D() one of the three specifications from Section 3.3.1, FED is the Front End Delay and t is the time stated time in months between the sooner payoff and the later one. As previously mentioned, the surveyed subjective perception of 1 month is normalised to one such that here FED does not differ from objective time. This specification differs from the literature in that t can be the individual's surveyed subjective perception of time, and that the different discounting functions are used to consider overall subjective time.

Similarly the present value of choosing the larger later payoff is

$$PV_B = D(FED)\omega_\delta^{(1-r)} + D(t + FED)(\omega_\delta + M_B)^{(1-r)} \tag{11}$$

The likelihood of selecting option A is as follows (it is due to this that only the periods in which PV_A and PV_B differ are considered, the rest cancel)

$$\nabla PV = \frac{PV_A - PV_B}{B_{Sooner} - A_{Sooner}} \tag{12}$$

The use of scaling in this specification is less straightforward given the lack of an unequivocal best and worst payoff as in (8). However, using B_{Sooner}, the present value of receiving the larger payoff in the sooner time period (therefore encompassing both those with no discounting of future payoffs and infinite), gives a reasonable upper bound. Correspondingly, A_{Sooner}, the present value of the smaller sooner payoff, allows for suitable scaling.

This is then transformed with precisely the same rationale and interpretation as (9), with the noise parameter in this case being ν

$$\nabla PV^* = \frac{e^{\frac{\nabla PV}{\nu}}}{1 + e^{\frac{\nabla PV}{\nu}}} \tag{13}$$

Finally we have the overall log likelihood specification.

$$\begin{aligned} lnL(r, \delta, \mu, \nu; y, \omega, \lambda, t, \mathbf{X}) = &\sum_i \left((ln(\nabla EU^*) | y_i^{RP} = 1) + (ln(-\nabla EU^*) | y_i^{RP} = -1) \right) \\ &+ \sum_i \left((ln(\nabla PV^*) | y_i^{TP} = 1) + (ln(-\nabla PV^*) | y_i^{TP} = -1) \right) \end{aligned} \tag{14}$$

$y_i^{RP} = 1(-1)$ corresponds to choosing option A (B) in choice i of the risk preference task. $y_i^{TP} = 1(-1)$ corresponds to choosing option A (B) in choice i of the time preference task.

4 Results

N.B. All coefficients in sections 4.1, 4.2 and 5 have p-values below 0.001 unless stated otherwise. Section 4.3 and Table 16 follow the usual convention on asterisks. It is also worth noting that discussion of ν and μ will be limited due principally to the inability to know the true extent of deviation from EUT in individuals' mental processes, such that the 'correct' extent to which the parameters differ from one is unknown. What's more the difference in specifications across the literature mean that these estimations are not directly comparable e.g. the same absolute change in noise parameter estimates implies far less deviation from EUT in my specification than that of Andersen et al. (2008).

4.1 Initial Estimates

The first results to be considered are the parameters estimated using each respondent's surveyed subjective time estimate, compared with those solely using objective time. Table 5 shows that, for every sample, using surveyed subjective time leads to greatly higher estimates of individuals' discount rate (although they are not precisely comparable rates given that they use different measures of time), and slightly, but significantly, higher levels of risk aversion.

Sample	δ_s	r_s	μ_s	ν_s	δ_o	r_o	μ_o	ν_o
Full Sample (707)	0.438	0.662	0.378	23.462	0.138	0.565	0.376	9.449
	(0.0315)	(0.0383)	(0.0136)	(1.0999)	(0.0076)	(0.0332)	(0.0135)	(0.4408)
With RP (661)	0.457	0.687	0.379	24.894	0.133	0.607	0.377	9.456
	(0.0346)	(0.0393)	(0.0137)	(1.2128)	(0.0077)	(0.0337)	(0.0135)	(0.4689)
'Unrestricted' (537)	0.330	0.880	0.356	21.716	0.111	0.812	0.352	9.404
	(0.0267)	(0.0459)	(0.0138)	(1.2250)	(0.0085)	(0.0400)	(0.0133)	(0.6380)
'Restricted' (432)	0.271	1.027	0.311	20.184	0.103	0.952	0.305	9.305
	(0.0241)	(0.0472)	(0.0121)	(1.2911)	(0.0103)	(0.0414)	(0.0115)	(0.8224)
'Consistent' (368)	0.463	0.977	0.275	22.920	0.165	0.971	0.274	10.843
	(0.0449)	(0.0518)	(0.0107)	(1.4068)	(0.0122)	(0.0456)	(0.0105)	(0.6526)

Table 5: Parameter estimates for surveyed subjective time, and objective time

Considering goodness of fit, the Akaike Information Criterion supported the objective time specification across all the samples, suggesting that it is a better fit for the data than is the survey elicited subjective time specification (Table 6). Notwithstanding the complications of determining the 'correct' level of ν, the higher level for surveyed subjective time *may* add credence to this suggestion of worse model fit.

Sample	Surveyed Subjective Time	Objective Time
Full Sample (707)	80421.81	78278.93
With RP (661)	76386.96	74340.14
'Unrestricted' (537)	61591.46	60671.22
'Restricted' (432)	49038.41	48605.83

Table 6: Comparing Akaike Information Criteria for surveyed subjective time with objective time

From the above results, it appears that individual survey-elicited estimates of subjective time do not offer greater insight into these respondents' time and risk preferences, nor the way in which they make their choices. The other method of considering subjective time, as outlined above, is to consider the phenomenon on a sample-wide level, forming one overarching estimate of subjective time's curvature. Of the two potential forms discussed, the Weber-Fechner specification is more computationally demanding. This is not a detrimental issue when the sample is altered between individuals, i.e. removing some individuals from the sample. However, when it is altered within individuals, i.e. some of each individual's choices are ignored, the reduction in the range of different values t can take can cause parameter estimation to fail.

Intuitively, the model is asked: for different values of objective time what values of subjective time make the respondents choices most likely? Reducing the range of objective time can therefore lead to flat or discontinuous regions of the log likelihood function's derivative where maximisation can fail. This provides a lens through which to compare the specifications: does the Weber-Fechner significantly outperform?

Sample	δ_{wf}	r_{wf}	μ_{wf}	ν_{wf}	α_{wf}	Log Likelihood
Full Sample (707)	0.146	0.559	0.376	11.144	-0.065	-39123.436
	(0.0061)	(0.0335)	(0.0135)	(0.4570)	(0.0096)	
With RP (661)	0.143	0.601	0.377	11.291	-0.063	-37155.838
	(0.0064)	(0.0340)	(0.0135)	(0.5179)	(0.0106)	
'Consistent' (368)	0.175	0.968	0.274	12.768	-0.075	-20003.229
	(0.0091)	(0.0457)	(0.0105)	(0.5533)	(0.0100)	

Table 7: Weber-Fechner

Table 7 displays the estimated parameters and the log likelihood for the relevant samples using the Weber-Fechner subjective time specification. The estimates of risk and time preference are broadly in line with that of objective time (subject to the comparability caveathere, more comparable as $\alpha_{wf} \to 0$). The fact that α_{wf} is slightly negative and significant implies a small level of time expansion, in contrast to the average surveyed estimate of subjective time contraction.

Only the final row in Table 8 provides an estimate of α_{spl} statistically different from one, in which time expansion is found. The estimates of δ and r are broadly similar to those from Weber-Fechner, with the 'Consistent' sample again providing higher estimates of risk aversion and discounting. Mechanically, increasing r would decrease δ, which implies that these differences come from genuine underlying differences between the samples, not coincidental statistical deviation. Those that made consistent choices may have answered the questions more carefully or been less likely to switch from their initial option. It seems reasonable that individuals that behaved in either of those ways were more risk averse. The potential implications of more considered respondents' estimates of r and δ being higher could be wide ranging and require further study- discussion here would be overly speculative.

When Likelihood Ratio (LR) tests are used to compare the subjective time specifications with that of objective time, Weber-Fechner provides statistically significant improvement to the model fit in all samples, Stevens' Power Law only for 'Consistent' (Appendix Table

Sample	δ_{spl}	r_{spl}	μ_{spl}	ν_{spl}	α_{spl}	Log Likelihood
Full Sample (707)	0.137	0.564	0.376	9.524	1.011	-39135.36
	(0.0076)	(0.0332)	(0.0135)	(0.4691)	(0.0228)	
With RP (661)	0.133	0.606	0.377	9.534	1.010	-37165.979
	(0.0077)	(0.0337)	(0.0135)	(0.5033)	(0.0237)	
'Consistent' (368)	0.160	0.971	0.274	11.790	1.117	-20005.922
	(0.0107)	(0.0457)	(0.0105)	(0.6706)	(0.0367)	

Table 8: Stevens' Power Law

16) Furthermore, according to the Akaike Information Criterion Weber-Fechner outperforms Stevens' Power Law across the board (Table 9)

Sample	Weber-Fechner	Stevens' Power Law
Full Sample (707)	78256.87	78280.72
With RP (661)	74321.86	74342.13
'Consistent' (368)	40016.46	40021.84

Table 9: Comparing Akaike Information Criteria for the sample-level subjective time specifications

The results are more supportive of the overarching subjective time models than the individually estimated one, particularly Weber-Fechner. They provide intriguing results on the curvature of subjective time, pointing towards time expansion. However, the analysis so far has not considered two important parts of the experimental design, the FED, and the scale of payoff in the risk preference task. In Section 4.2 these will be considered, seeking to shed further light on the both the values of δ and r as well as the curvature of subjective time.

4.2 Restricted Choice Samples

4.2.1 Front End Delay

In order to investigate the effect of having a FED as outlined in Section 3.1, estimations are run for each specification first ignoring the choices 1-36 (keeping only those without the FED), and then ignoring choices 37-72 (keeping only those with it). Here, the aforementioned computational difficulties mean that the estimation of the Weber-Fechner specification fails, though samples which keep only one of the choices which should be dropped can be estimated successfully (Appendix Table 17) though they are purely illustrative and are not rigorously comparable with Table 10.

In Table 10 we see that the estimate of δ is much lower in the sample with FED. This supports the prevailing idea of a FPE (for example, Bradford et al. 2019) whereby the discount rate is higher when comparing a payoff immediately with one some interval into the future than it is when comparing two future payoffs with the same interval between them.

Sample	δ	r	μ	ν	α	Log Likelihood
Objective Time no FED	0.182	0.621	0.377	11.713	-	-22411.79
	(0.0149)	(0.0399)	(0.0135)	(0.7561)	(-)	
Objective Time FED only	0.103	0.588	0.377	7.757	-	-22528.088
	(0.0079)	(0.0392)	(0.0135)	(0.5450)	(-)	
SPL no FED	0.195	0.622	0.377	13.342	1.179	-22405.775
	(0.0137)	(0.0400)	(0.0136)	(0.7784)	(0.0529)	
SPL FED only	0.061	0.583	0.377	9.730	1.493	-22505.897
	(0.0064)	(0.0394)	(0.0135)	(0.5512)	(0.0852)	

Table 10: FED effect amongst With RP sample

Encouragingly in Table 10 the values of risk aversion appear to only differ due to the mechanical influence of the lower discount rates rather than a large difference in underlying aversion between the samples. The values of μ are equal, as before, given that the contribution to the overall likelihood function of the risk preference task is unchanged. Here, the Stevens' Power Law provides statistically significant improvement to model fit for both samples (Appendix Table 16)

The higher value of α within the FED only sample implies an increased rate of time expansion without the FPE. This growing impatience- though, beginning from a lower level than in the no FED sample- when the FPE is absent is an intriguing result. Next, I further investigate whether this is driven by choices across the range of time periods or, potentially, by the shorter intervals. In order to have sufficient data for reliable estimation, I will no longer segregate by FED. This is a limitation as regards the investigation of growing impatience in the absence of the FPE specifically, which merits future study.

The estimates of α in Table 11 appear surprising in light of the previous result, as time contraction is observed when only the three and twelve month intervals are considered. This, however, need not be inconsistent, instead it could be indicative of an inflection point, whereby the large initial time expansion slows and reverses to time contraction over a longer horizon. In simpler terms this would imply that shorter periods like one month feel 'manageable' with three months feeling far more distant into the future; the difference, however, between this seemingly distant point and another, such as twelve months, may feel relatively shorter than their objective separation. Further and more rigorous investigation of this unfortunately falls beyond the scope of this data and project.

Specification	δ	r	μ	ν	α	Log Likelihood
Objective Time	0.092	0.613	0.377	6.608	-	-27444.557
	(0.0101)	(0.0369)	(0.0135)	(0.6056)	(-)	
Stevens' Power Law	0.089	0.622	0.377	5.060	0.860	-27437.197
	(0.0087)	(0.0364)	(0.0136)	(0.4823)	(0.0304)	
Weber-Fechner	0.078	0.623	0.377	4.851	0.061	-27437.382
	(0.0069)	(0.0363)	(0.0136)	(0.4485)	(0.0176)	

Table 11: Estimation for 3 & 12 month intervals only

4.2.2 Scale Effects

The estimations of the parameters firstly ignoring the larger payoff choices 82-90, and secondly ignoring the smaller payoff choices 73-81 for each specification are displayed in Table 12. The effect of an increase of the size of payoffs amongst this sample is to decrease the estimated level of risk aversion in the sample. This runs contrary to the prevailing view that risk aversion increases with increased stakes (for example, Baltussen et al., 2008; Burks et al., 2009; and, for a wider discussion, Fehr-Duda et al. 2010). Such a contradiction may partially be explained by the relatively small range of stakes being offered within the survey, richer investigation would require a wider range and/or more granular data.

Sample	δ	r	μ	ν	α	Log Likelihood
Objective Time Smaller Payoffs	0.132	0.650	0.440	9.576	-	-33286.059
	(0.0077)	(0.0395)	(0.0228)	(0.4817)	(-)	
Objective Time Larger Payoffs	0.133	0.583	0.301	9.395	-	-33243.123
	(0.0077)	(0.0341)	(0.0145)	(0.4645)	(-)	
SPL Smaller Payoffs	0.132	0.649	0.440	9.651	1.010	-33285.98
	(0.0077)	(0.0395)	(0.0228)	(0.5168)	(0.0237)	
SPL Larger Payoffs	0.133	0.583	0.301	9.476	1.010	-33243.025
	(0.0077)	(0.0342)	(0.0145)	(0.4982)	(0.0237)	
Weber-Fechner Smaller Payoffs	0.143	0.642	0.440	11.441	-0.063	-33276.037
	(0.0064)	(0.0401)	(0.0228)	(0.5384)	(0.0108)	
Weber-Fechner Larger Payoffs	0.143	0.577	0.301	11.210	-0.064	-33232.771
	(0.0064)	(0.0344)	(0.0145)	(0.5101)	(0.0106)	

Table 12: Payoff scale effect amongst With RP sample

4.3 Heterogeneity

A few common themes run through the literature on heterogeneity of time and risk preferences. Typically discount rate has been found to fall with income, age, education, and cognitive ability; and be lower for males, majority ethnic groups, and non-smokers (for example; Lawrance, 1991; Samwick, 1998; Warner and Pleeter, 2001; Dohmen et al. 2010).

These findings have, however, not been universal. Bozio et al. (2017) find discount rate increasing with education and numeracy amongst the elderly. While Chesson and Viscusi (2000) find discount rates to be lower for smokers and those with low income, and higher for those above average age and those with a postgraduate qualification (they also find sex to be insignificant). These examples are far from being exhaustive, but are outlined to motivate the investigation of heterogeneity given the lack of consensus.

The most frequent finding pertaining to risk preference is that risk aversion decreases with cognitive ability, though this dataset does not allow such investigation. The findings of Dohmen et al. (2011) highlight other common results, that men have lower estimates of risk aversion, and that risk aversion increases with age. In addition they find that height is significantly negatively correlated with risk aversion. A final prevailing result is that there is heterogeneity in risk preference by income, with it usually being found that risk aversion is lower for higher income individuals (for example, Barsky et al., 1997; Hopland et al., 2016)

I will be considering further attributes to those above given the wide range available in this dataset. Below, 'male', 'higher education', 'non-white', 'smoker', 'above average income', and 'above average height' are dummy variables; the rest are ordinal variables with 'impatience', and 'impulsivity' being the self-rated level of those characterstics; while 'general risks', 'health risks', and 'financial risks' are self-rated willingness to take on those risks. (N.B. the wording of the survey question was 'risks' only, the word 'general' is added here for clarity). Both overall subjective time specifications are considered to determine whether specification has a large effect on estimations of heterogeneity.

This sample of UK households accords with the prevailing results in terms of discounting being significantly lower for males and the more educated; significantly higher for non-white individuals and smokers; and falling with age. Here however, income is not significant for δ, going against the expectation, with this result being robust to specification changes. The significant positive coefficient on impatience is encouraging insofar as those that consider themselves to be more impatient have higher estimated discount rates (a measure of impatience).

Considering risk aversion these results are broadly surprising, with gender and age being insignificant, and both those with above average height and above average income being more risk averse. The scale of the income effect is also very large, remaining comparably so if an ordinal variable is used in its place. The fact that this result comes from a more representative sample than is typical in the literature may imply that further investigation into the effect of income on risk aversion across a wide range of incomes is required. The significant negative coefficients on 'general risk' and 'impulsivity' provide credence to the estimates, as calculated risk aversion is lower for those that consider themselves more willing to take risk or more impulsive. The contrast of the coefficients on 'health risks' and 'financial risks' with each other and that of 'general risks' offers further evidence for the distinct risk preferences for different risks. Such domain dependence has been found repeatedly such as between gains and losses (Kahneman and Tversky, 1979; Harrison and Rutström, 2008), different risks such as health and finances (Zaleskiewicz, 2001), and even between different financial decisions (Einav et al., 2012).

δ	Weber Fechner	Stevens' Power Law
male	-0.0240**	-0.0245**
	(0.0043)	(0.0044)
age	-0.0005**	-0.0005**
	(0.0001)	(0.0001)
impatience	0.0011**	0.0011**
	(0.0003)	(0.0003)
higher education	-0.0632**	-0.0646**
	(0.0052)	(0.0053)
non-white	0.0198**	0.0200**
	(0.0070)	(0.0072)
smoker	0.0232**	0.0236**
	(0.0057)	(0.0058)
above average income	-0.0002	-0.0004
	(0.0046)	(0.0047)
constant	0.1575**	0.1616**
	(0.0121)	(0.0125)
r		
male	-0.0165	-0.0202
	(0.0705)	(0.0705)
age	-0.0004	-0.0003
	(0.0020)	(0.0020)
above average height	0.3849**	0.3909**
	(0.0638)	(0.0638)
above average income	1.1251**	1.1291**
	(0.0760)	(0.0757)
general risks	-0.1706**	-0.1709**
	(0.0228)	(0.0228)
health risks	0.1912**	0.1920**
	(0.0269)	(0.0269)
financial risks	-0.0063	-0.0065
	(0.0175)	(0.0175)
impulsivity	-0.0384**	-0.0383**
	(0.0125)	(0.0125)
constant	-0.0109	-0.0208
	(0.1411)	(0.1411)

Table 13: Heterogeneity in time and risk preference for With RP sample

5 Robustness

The above consideration of alternative samples and specifications has already gone some way towards a consideration of robustness. However, given the lack of concrete grounding for the calculation of background consumption used so far, analysis of the robustness of the model to alternative values is required.

'OIE' denotes Original Individual Estimations of background consumption, with the other rows rescaling this value for each individual. The rescaled values can be considered in terms of changing the time frame within which individuals aggregate their winnings, for OIE this is a week for ω_r and half a month for ω_δ. While the changes in the estimates of discounting and α appear very slight, that of risk aversion is relatively large. This higher sensitivity of r appears intuitively reasonable since its estimation relies principally on fewer data points (see Section 3.1). Coupling this with the somewhat surprising findings of Section 4.3 suggests that the method of elicitation of risk aversion could be improved upon with further questions. The Log Likelihoods (and therefore information criteria, in this case) suggest that OIE is the best fit for the data. From this, the results appear to be broadly robust to rescaling of calculated background consumption, with the slight exception of r.

Background Consumption	δ	r	ν	μ	α	Log Likelihood
2 * OIE	0.145	0.669	0.382	10.840	-0.065	-37171.214
	(0.0063)	(0.0398)	(0.0138)	(0.4665)	(0.0101)	
$\frac{3}{2}$ * OIE	0.144	0.641	0.379	11.019	-0.064	-37164.026
	(0.0063)	(0.0371)	(0.0137)	(0.4867)	(0.0103)	
OIE	0.143	0.601	0.377	11.291	-0.062	-37144.838
	(0.0064)	(0.0340)	(0.0135)	(0.5179)	(0.0106)	
$\frac{2}{3}$ * OIE	0.142	0.561	0.375	11.572	-0.063	-37150.358
	(0.0065)	(0.0315)	(0.0135)	(0.5517)	(0.0110)	
$\frac{1}{2}$ * OIE	0.142	0.533	0.378	11.770	-0.062	-37148.22
	(0.0066)	(0.0301)	(0.0135)	(0.5761)	(0.0112)	

Table 14: Estimations using different individual values of ω for Weber-Fechner specification, With RP sample

The literature has generally not had access to personal levels of consumption, so now I consider assumed values which do not vary between individuals in Table 15. As background consumption is lowered, the slight changes in δ and α become more ambiguous, while the positive relationship between background consumption and risk aversion is even more stark. Along with the evidence from the Log Likelihoods (and therefore information criteria), this higher sensitivity of r implies a sizeable improvement to the estimation of the overall model as a result of using individual estimates of background consumption, i.e. using OIE. (N.B. the 3rd row uses the mean values of ω_r and ω_δ from OIE but fixed for all individuals)

Background Consumption	δ	r	ν	μ	α	Log Likelihood
ω_r=150 ω_δ=300	0.147	1.230	0.369	11.445	-0.063	-37247.653
	(0.0065)	(0.1003)	(0.0133)	(0.5239)	(0.0108)	
OIE	0.143	0.601	0.377	11.291	-0.062	-37144.838
	(0.0064)	(0.0340)	(0.0135)	(0.5179)	(0.0106)	
ω_r=88 ω_δ=194	0.146	0.872	0.369	11.439	-0.063	-37245.624
	(0.0065)	(0.0698)	(0.0133)	(0.5213)	(0.0107)	
ω_r=50 ω_δ=100	0.145	0.649	0.370	11.771	-0.063	-37241.307
	(0.0066)	(0.0503)	(0.0133)	(0.5599)	(0.0111)	
ω_r=0 ω_δ=50	0.147	0.267	0.376	10.881	-0.065	-37241.857
	(0.0063)	(0.0202)	(0.0135)	(0.4535)	(0.0099)	
ω_r=0 ω_δ=0	0.140	0.290	0.378	13.128	-0.061	-37226.224
	(0.0068)	(0.0201)	(0.0139)	(0.7328)	(0.0118)	

Table 15: Estimations using different assumed values of ω for Weber-Fechner specification, With RP sample

6 Concluding Remarks

I found that survey-elicited subjective time was outperformed both by using objective time, and by overall subjective time specifications. Of these, the Weber-Fechner specification was a better fit for the choices made than was the Stevens' Power Law, though the latter was also often found to be a better fit than objective time. Evidence was found for time expansion, though this appeared to be driven by more proximate choices, suggesting that this data is best described by a subjective time function with an inflection point, from convex to concave. In line with the literature, a significant 'first period effect' was found. The heterogeneity in discounting was in line with expectations as regards gender, education, ethnicity, smoking status and age, but income was not found to be significant. That of risk aversion was more controversial, with gender and age being found insignificant, while income was found to be positively related to risk aversion. The latter result provides motivation for further study given the wider range of incomes considered in this sample than is typical.

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Appendix

Sample	LR test statistic WF	LR test statistic SPL
Full Sample (707)	24.06**	0.21
With RP (661)	20.46**	0.18
'Consistent' (368)	16.96**	11.57**
no FED	-	12.03**
FED only	-	44.38**
3&12 Months Only	14.35**	14.72**

Table 16: Likelihood ratio tests for WF and SPL specifications compared with that of objective time

Sample	δ	r	μ	ν	α
WF choices 1 & 37-90	0.140	0.617	0.377	10.661	0.022
	(0.0121)	(0.0397)	(0.0135)	(0.9840)	(0.0340)
					[p=0.509]
WF choices 1-36 & 72-90	0.112	0.575	0.377	10.337	-0.076
	(0.0056)	(0.0396)	(0.0135)	(0.4311)	(0.0039)

Table 17: Illustrative nearby FED Weber-Fechner estimate