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Intergenerational human capital, risk aversion, and the poverty trap

Chau Pham*

Abstract

This paper addresses two issues: the underinvestment in education and the poverty trap that ensues for poor households. In a setting where the end outcome is binary, an investing agent faces two levels of risk, one in the intermediate outcome - how much human capital she obtains for a given amount of investment, and one inherent in the end outcome - whether she gets the high-paid job. We show that when human capital is inheritable, risk-averse agents are deterred from investing because their parents are not sufficiently educated. Moreover, the U-shaped expected utility means the optimal investment occurs at either corners. If this investment or underinvestment is sustained through generations, a separating equilibrium such that poor households do not invest while wealthier ones do emerges. The divergence in educational attainment translates into a divergence in wealth between those who invest and those who do not. A simple calibration employing data from the NLSY97 demonstrates the existence of these equilibria at different levels of risk-aversion.

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Keywords: intergenerational human capital, poverty trap, risk-aversion, underinvestment

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Appendix is available at https://github.com/ChauPPham/MSc-Dissertation-appendix.git
1 Introduction

It might be an exaggeration to say “there can be as many causes of poverty as there are poor people,” but the idea that every individual trapped in poverty might face different poverty-perpetuating mechanisms or the combinations thereof deserves some recognition. From a theoretical standpoint, poverty trap arises when there is a mechanism that gives rise to the discontinuity of equilibrium wealth as, for example, capital falls below the cutoff threshold.\(^1\) In earlier works, external frictions accompanied by nonconvex technology have been utilized to generate an S-shaped production function which in turn perpetuates poverty (e.g. capital market imperfections with education in Galor and Zeira (1993); occupational choice in Banerjee and Newman (1993); and human capital in Mookherjee and Ray (2003)). More recent works adopted a behavioural approach, for instance, non-homothetic preference in regards to temptation goods in Banerjee and Mullainathan (2010), and self-control problem in Bernheim et al. (2015).

The ubiquity of capital market frictions as one of the defining features of poverty-inducing mechanisms in earlier works lends itself to the fact that the impoverished, unlike wealthier individuals, often face binding constraints due to the lack of collateral to take out loans (Banerjee and Duflo, 2007). With the advent of microcredit, the expectation is that the poor acquire funds and invest in high-yielding projects to lift themselves out of poverty. Unfortunately, recent research has shown that successful stories are few and far between as we witness low take-up rates and negligible effects on the living conditions of those who did apply for loans (Crépon et al., 2011; Augsburg et al., 2015; Banerjee et al., 2015). Furthermore, there is little difference in human development outcomes encompassing health, education, and women’s empowerment to name a few.

Focusing on education as one such high-yielding project and a ticket out of poverty, this paper addresses the question of why poor individuals do not invest or invest very little in education even when the credit constraint is nonbinding. A natural follow-up question “can underinvestment in education engender a poverty trap?” is also examined. This paper builds up on Khazanov et al. (2018) by synthesizing risk aversion and the intergenerational transmission of human capital to show that underinvestment in education can indeed lead to a poverty trap as a result of a sequence of utility optimization under uncertainty. In a setting where the probability of getting a good job is increasing in the level of human capital, the endowment of human capital from one’s predecessor influences the investment decision of the agent as well as the stock of human capital that will be passed down to the next generation.

\(^1\)Note that the threshold does not need to be fixed. See Matsuyama (2011) for a comprehensive review on the literature of poverty trap with fixed and variable thresholds.
An extension over Khazanov et al. (2018), the intergenerational human capital, shifts the focus to human capital externalities as a determinant of investment decision rather than wealth. As will be shown, different initial endowments of human capital compel agents to invest different amounts and through the intergenerational linkage, the investment decision of one generation affects generations to come. Under CARA utility, initial wealth has little effect on the investment decision. Under CRRA utility, however, parents’ education levels complement wealth in determining one’s level of education and wealth.

This paper is closely related to two strands of literature: underinvestment on account of risk aversion, and intergenerational human capital. Investment under uncertainty is an area laden with studies whose origin goes back to personal and corporate finance (see Arrow, 1964; Hirshleifer, 1966; Stiglitz, 1969 among many others). In relation to the poor, Banerjee (2004) showed that underinvestment stemming from risk aversion with respect to total wealth can trap an individual in poverty. There is an abundance of evidence on the underinvestment in risky high-yielding projects such as profitable but risky technologies in agriculture in Morduch (1990), fertilizer use in Duflo et al. (2011), and inventories in Kremer et al. (2013). On the contrary, there is little support for risk aversion as an explanation to why poor people leave these low-risk-high-return opportunities unexploited, especially so in the microcredit literature. Rather, there is a copious amount of study supporting loss aversion and the prospect theory as an explanation to this underinvestment (see, for instance, Tanaka et al., 2010; Kremer et al., 2013; Liu, 2013). To the best of our knowledge, Khazanov et al. (2018) is the first to offer an explanation based solely on risk aversion divorced from any of the aforementioned frictions. When the probability of success is endogenous and the outcome is binary, the investment decision of an agent is discontinuous in her level of risk aversion. Assuming risk aversion decreases with wealth, it follows that wealthier agents invest while poorer ones do not. Similar to most other studies, the uncertainty in their model manifests in the end outcomes whereas, in this paper, the uncertainty manifests both in the intermediate outcome, i.e. the level of human capital, as well as the end outcome. As an analogy to fertilizer use in Duflo et al. (2011), farmers in our model know fertilizer increases the chance of a fruitful harvest but wonder how much fertilizer they would get with a fixed amount of investment. *Prima facie*, it does not sound very realistic for tangible investment, but in the context of health and education, how much nutrition one can get or how much knowledge she obtains with a fixed amount of investment is at best an expectation.

The transmission of human capital through generations is also an issue much studied in the social mobility literature. The explicit question addressed in this area is “how does parental education affect a child’s education?” Using an education reform in Norway in the 1960s, Black et al. (2005) found little causal relationship between parental education and child
education. Chevalier (2004) and Oreopoulos et al. (2006), on the other hand, suggested that there is a positive and significant effect of parental education on grade retention of children using minimum school leaving age law in the UK and US respectively. A survey of the literature conducted by Holmlund et al. (2011) acknowledged the small yet causal effects while also noting the systemic difference across identification strategies. Another survey conducted by Black and Devereux (2010) also yields a similar conclusion. Having established the intergenerational persistence in human capital, it is helpful to look at how this is best modelled from a theoretical viewpoint. The pioneering work of Cunha and Heckman (2007) on skill formation links the level of human capital of a child to that of her parents, the investment made as well as other genetic or environmental factors. Becker et al. (2018) further took this to prove the imperfect mobility while assuming the complementarity of human capital and investment. In this dissertation, we adopt an additive functional form rather than the Cobb-Douglas form as in Becker et al. (2018).\(^2\) Unlike their formulation, the assumption of disproportionate return to education, which is used to generate nonconvexity and hence imperfect mobility, is relaxed by construction. As will be shown, disproportionate return to education, or more generally, nonconvexity is not a necessary condition for the existence and persistence of poverty in our model.

To corroborate our findings, we calibrate the model using parameters estimated from NLSY97 data. If utility is CARA, we show that among agents who are similarly risk-averse, agents whose parents are less educated are barred from investing in education altogether. For CRRA utility, however, investment is continuous in the amount of inherited human capital such that above a certain threshold, investment is increasing in the amount of inherited human capital. On the contrary to the single-period setting, the underinvestment in both cases translates into the divergence in long-run wealth between those who invest in education and those who do not at the infinite horizon. The implication of this is straightforward, on average, households that house non-investing agents remain poor.

The rest of this paper is organised as follows. The second section presents the basic setting of our model. The third section incorporates the bequest motives to allow for the analysis of wealth over generations. The fourth section presents the results of a simple calibration exercise using data from NLSY97. The last section concludes the paper.

\(^2\)Irrespective of the functional form of the production function, the results of our model hold under certain conditions. More details are available in appendix A.1.
2 Underinvestment in education

2.1 The basic model

Consider first the single-dynasty static case. An agent chooses the amount of investment in education and enters employment. There are two types of jobs in the economy, the good job, which pays a high wage, $H$, and the bad job, which pays a low wage, $L$, where $H > L \geq 0$. With probability $p$, an agent receives an offer and take the good job, and with probability $1 - p$, she does not receive one and, in which case, takes the bad job. The main results of our model rest on the following key identifying assumptions, prefixed A.

Assumption 1 (A1). Let $h$ be the level of human capital. The probability of receiving an offer to take the good job is linear in the stock of human capital, $p \equiv p(h) = \gamma h$ ($\gamma > 0$).

This assumption incentivises agents to increase the level of human capital in order to improve their chances of getting the offer. If an agent chooses not to invest, she still enjoys a low probability of getting the offer to work the high-paid job.

Assumption 2 (A2). Let $\hat{h}$ be the stock of human capital of the agent’s predecessor and $I$ the amount of investment in education. The level of human capital of an agent, $h$, is uniformly distributed from $\hat{h}^\alpha$ to $\hat{h}^\alpha + I$. That is, $h \sim U(\hat{h}^\alpha, \hat{h}^\alpha + I)$ with $\alpha$ satisfying $1 > \alpha > 0$.

Assumption 2 predisposes an agent to a starting stock of human capital from parent. She then invests in education an amount $I$ and her resulting level of human capital is a random draw from the continuous interval regulated by the level of human capital of parent and the amount of investment. This assumption implies that with a fixed amount of investment, agents whose parents are more educated benefit more on average from investing. Combining A1 and A2, an agent who invests faces two levels of uncertainty, one in the determination of the returns to education, and another one inherent in the job selection process. An agent who does not invest knows for certain her level of human capital yet also wonders whether she can get the high-paid job.

On top of the two identifying assumptions, we further introduce two simplifying assumptions, prefixed B. For assumption A2 to be valid, it is required that the level of human capital and the amount of investment are scaled to the same unit of measurement.

Assumption 3 (B1). The level of human capital, $h$ and $\hat{h}$ are bounded within the interval $[1, 2]$. The cost of investment, $I$ is bounded within the interval $[0, \bar{I}]$ where $0 < \bar{I} < 1$. 

4
Combining A2 and B1, it follows that if $\hat{h}^\alpha + \bar{I} \geq 2$, an agent would prefer to choose the amount of investment such that $h \sim U(\hat{h}^\alpha, 2)$ rather than the maximum amount of investment. Additionally, A1 and B1 imply that the minimum probability of getting the offer is $\gamma$ while the maximum is $2\gamma$. In reality, educated people stand a much higher chance of getting a high-paid job than do the uneducated. Depending on the realised probability, this assumption can be modified accordingly to accommodate empirical analysis.

Lastly, for tractability of analysis, agents are assumed to be risk-averse with respect to total wealth and their utility function is CARA,

\textbf{Assumption 4 (B2).} $u(y) = -e^{-\lambda y}$ where $\lambda > 0$ denotes Arrow-Pratt coefficient of absolute risk-aversion.

\section{2.2 Standard results}

Within this setting, an agent chooses an amount of investment that maximises her expected utility from working. Her objective function can be written as,

\[ \max_I U \equiv p(h)u(H - I) + (1 - p(h))u(L - I) \quad (2.1) \]

This objective function corresponds to the objective function of Khazanov et al. (2018, p.9) with the choice variable replaced by $I$, and the probability of success is now a function of human capital. The first and second proposition follows from their results.

\textbf{Proposition 1.} The agent’s expected utility $U$ is U-shaped in $I$. Specifically, there exists $\hat{I}$ such that $U$ is non-increasing on the interval $[0, \hat{I}]$ and increasing on the interval $[\hat{I}, \bar{I}]$ where $\hat{I} \in [0, \bar{I}]$.

\textbf{Proof}

First, rewrite the expected utility as,

\[ U = p(h)u(H - I) + (1 - p(h))u(L - I) \]

\[ = u(L - I) + \left[ u(H - I) - u(L - I) \right] \int_{\hat{h}^\alpha}^{\hat{h}^\alpha + I} I^{-1} \gamma h \, dh \]

\[ = -e^{-\lambda(L-I)} + \left[ -e^{-\lambda(H-I)} + e^{-\lambda(L-I)} \right] \frac{\gamma}{2} (2\hat{h}^\alpha + \bar{I}) \quad (2.2) \]
The first and second derivative with respect to $I$ are then,

\[ U' = -\lambda e^{\lambda I} e^{-\lambda L} + \frac{\gamma}{2} e^{\lambda I} (e^{-\lambda L} - e^{-\lambda H})(2\lambda \hat{h}^\alpha + \lambda I + 1) \]  
\[ U'' = -\lambda^2 e^{\lambda I} e^{-\lambda L} + \frac{\gamma}{2} \lambda e^{\lambda I} (e^{-\lambda L} - e^{-\lambda H})(2\lambda \hat{h}^\alpha + \lambda I + 1) + \frac{\gamma}{2} \lambda e^{\lambda I} (e^{-\lambda L} - e^{-\lambda H}) \]

\[ = \lambda U' + \frac{\gamma}{2} \lambda e^{\lambda I} (e^{-\lambda L} - e^{-\lambda H}) \]  

(2.3) 

(2.4)

For $I$ such that the first derivative is positive, the second derivative is also positive, in other words, if $U$ is increasing at an arbitrary value of $I$, then it is also increasing for all $I' > I$. \[ \square \]

A simple manipulation of algebraic terms shows that the local minimum occurs at the point $\hat{I} \equiv \frac{2e^{-\lambda L}}{\gamma(e^{-\lambda L} - e^{-\lambda H})} - 2\hat{h}^\alpha - \frac{1}{\lambda}$. Subject to parameter values, $\hat{I}$ might be negative, and in which case, the agent’s expected utility is increasing over the entire interval $[0, \hat{I}]$. By contrast, the agent’s expected utility is decreasing over the entire interval $[0, \hat{I}]$ if $\hat{I} > \bar{I}$. If the solution is interior, however, due to the U-shaped expected utility, an agent chooses an amount of investment that is either $\bar{I}$ or 0, whichever yields the higher expected utility. For an agent who chooses not to invest, her utility is $u(H)$ with probability $\gamma \hat{h}^\alpha$ and $u(L)$ with probability $1 - \gamma \hat{h}^\alpha$, that is she is uncertain whether she can get the offer but faces no risk otherwise. For an agent that chooses to invest, her utility is $u(H - I)$ with probability $p(h)$ and $u(L - I)$ with probability $1 - p(h)$, that is she is uncertain how much human capital she obtains from the investment as well as whether she can get the offer. For two agents with the same level of risk-aversion, investment decisions may vary owing to the difference in human capital endowment. This leads to our second proposition.

**Proposition 2.** For agents with the same degree of risk-aversion $\lambda$, the agents’ investment decision is discontinuous in the amount of inherited human capital, $\hat{h}$. There exists a threshold level for $\hat{h}$, denoted $\bar{h} > 0$, above which, agents invest the maximum amount $\bar{I}$ and below which, $I = 0$. Note that the threshold level $\bar{h}$ is not necessarily fixed but varies with $\lambda$.

**Proof**

i) Following from Proposition 1, the U-shaped expected utility compels an agent to strictly prefer corner solutions to interior solutions. Namely, she either invests the maximum amount $\bar{I}$ or does not invest. The investment decision then rests on whether $U(\bar{I}) > U(0)$ holds.
ii) From (2.3), $U$ is continuous in $\hat{h}$. It follows that for a given $\lambda$, agents whose $\hat{h}$ is higher enjoys higher expected utility when investing the maximum amount.

The discontinuity of $I$ in (i) and the continuity of $U$ in (ii) imply that there exists a threshold level $\tilde{h} > 0$ such that given $\lambda$, agents with $\hat{h} > \tilde{h}$ invest $\bar{I}$; this holds true for $\hat{h}$ satisfying $\hat{h}^\alpha \leq 2 - \bar{I}$, for $\hat{h}^\alpha > 2 - \bar{I}$, agents invest an amount $2 - \hat{h}^\alpha$.

For the maximum amount of investment to be made, it must also be the case that $\tilde{h}^\alpha + \bar{I} < 2$. On the other hand, agents with $\hat{h} < \tilde{h}$ do not invest, i.e. $I = 0$. Since $U$ varies with $\lambda$, the threshold level $\tilde{h}$ also varies with $\lambda$. \hfill $\Box$

In the single-period case, Proposition 1 shows the emergence of corner solutions as a result of the U-shaped expected utility; Proposition 2 establishes the existence of a threshold level of parental human capital that influences the corner solutions. Both propositions also apply to the degree of risk-aversion such that sufficiently risk-averse agents do not invest while sufficiently risk-neutral agents invest the maximum amount. However, here and throughout the rest of the paper, we abstract from the discussion regarding the level of risk aversion and assume it is exogenous as its interaction with education is unclear. We instead bring to attention the effect of having higher inherited human capital to explain the underinvestment that ensues for poor agents. This shift in focus neither refutes the risk-aversion explanation nor offers an exclusive explanation to the poverty trap. It simply introduces a mechanism solely based on human capital that coexists with other mechanisms to reinforce poverty.

3 The poverty trap

3.1 Underinvestment and the intergenerational linkage

The previous section establishes the discontinuity of investment in the stock of inherited human capital. An immediate implication of this discontinuity is the diverging end-of-period expected income as agents who do not invest have decreasing expected income from working. On the contrary, children of agents who invest in education are more likely to invest and enjoy higher expected income from working. It is straightforward to see that an agent whose parents did not invest also chooses not to invest in education as her human capital remains below the threshold. This result is obtained in an environment absent capital market. In this section, we show that when the bequest motive is operational and the capital market is perfect, the result from the last section still holds. Furthermore, underinvestment in human capital constitutes a poverty trap in the longer run under certain conditions.

Since the variables are now linked to their past values, a slight change of notation and clarity on the specific interaction between two immediate generations are necessary. An agent
lives for one period, in which she invests, works a certain job, consumes a fixed portion of her total wealth and bequeaths the rest to her immediate descendant. With the introduction of the capital market, an agent can now invest in education, which increases her stock of human capital, or in the capital market, which increases her total wealth directly, or in both. Notice that in this scenario, we allow for lending and borrowing so that agents can invest in education the maximum amount even if they do not have enough bequest to do so.

As agents only live for one period, let \( h_{i,t} \) denote the level of human capital of an agent from family \( i \) in period \( t \). Similarly, let \( b_{i,t} \) be the amount of bequest said agent sets aside for her child, i.e. the agent from family \( i \) living in period \( t+1 \) will receive this bequest. An important issue that arises is how an agent bequeaths and how her risk-averse attitudes are reflected in the current setting. To this end, we follow Banerjee (2004, p.2).

**Assumption 5 (A3).** An agent first decides what to invest in, and subsequently allocates the resulting wealth between consumption and bequest. For the purpose of ex-post resource allocation, her utility can be represented by,

\[
U_{i,t} = -e^{-\lambda W_{i,t}}
\]

\[
W_{i,t} \equiv c_{i,t}^{1-\beta} b_{i,t}^\beta = b_{i,t} + c_{i,t}
\]

From A3, standard optimisation under constraint delivers \( b_{i,t} = \beta W_{i,t} \) and \( c_{i,t} = (1-\beta)W_{i,t} \) for \( \beta \in (0,1) \). Note that this allocation of wealth is made ex-post while the investment decision is made ex-ante. Consequently, the investment decision determines the total wealth and homothetic preference determines the allocation of wealth. As the allocation of wealth does not interact with the determination of wealth, we show that the standard result from the previous section still holds, that is, the investment decision in education is discontinuous in the level of inherited human capital.

**Proof.** The modified objective function of an agent is,

\[
\max_{I_{i,t}} U_{i,t} = p(h)u\big((b_{i,t-1} - I_{i,t})R + H\big) + (1 - p(h))u\big((b_{i,t-1} - I_{i,t})R + L\big)
\]

(3.1)

where \( R > 1 \) is the gross interest rate in the capital market. First, rewrite the objective function,

\[
U_{i,t} = -e^{-\lambda L}e^{-\lambda(b_{i,t-1}-I_{i,t})R} + \frac{\gamma}{2} (2h_{i,t-1}^\alpha + I_{i,t})e^{-\lambda(b_{i,t-1}-I_{i,t})R}(e^{-\lambda L} - e^{-\lambda H})
\]

(3.2)
The corresponding first and second derivative with respect to $I_{i,t}$ are

$$U' = e^{-\lambda(b_{i,t} - I_{i,t})}R \left[ -\lambda R e^{\lambda L} + \frac{\gamma}{2}(e^{-\lambda L} - e^{-\lambda H})(2\lambda R h_{i,t-1}^\alpha + \lambda RI_{i,t} + 1) \right]$$  \hfill (3.3)

$$U'' = \lambda RU' + \lambda R e^{-\lambda(b_{i,t} - I_{i,t})}R(e^{-\lambda L} - e^{-\lambda H})$$  \hfill (3.4)

If $U$ is increasing at an arbitrary value of $I_{i,t}$, it is also increasing for $I_{i,t} > I_{i,t}$ subject to $I_{i,t} + h_{i,t-1}^\alpha \leq 2$. The U-shaped utility means whether an agent invests the maximum amount she can or none at all in education hinges on the initial human capital endowment.

3.2 Multiple equilibria and persistent poverty

Unlike the single-period case, the evolution of bequests and human capital over generations of a household is more complicated. Let $M$ be the limit to which $E[h_{i,k}]$ converges as $k$ approaches infinity.\(^3\) It is trivial to see that if $h_{i,0} < \tilde{h}$, generations of this household will never invest in education and only invest in the capital market. Given $h_{i,0} < \tilde{h}$ and $b_{i,0}$, the level of human capital and expected bequest in the next period are,

$$h_{i,1} = h_{i,0}^\alpha$$  \hfill (3.5)

$$E[b_{i,1}] = \beta(b_{i,0}R + L + \gamma h_{i,0}^\alpha(H - L))$$  \hfill (3.6)

Through repeated substitutions, the corresponding level of human capital and expected bequest at time $k$ are

$$h_{i,k} = h_{i,0}^{\alpha^k}$$  \hfill (3.7)

$$E[b_{i,k}] = (\beta R)^k b_0 + \beta L \sum_{j=0}^{k-1} (\beta R)^j + \beta \gamma(H - L) \sum_{j=0}^{k-1} (\beta R)^{k-j} h_{i,0}^{\alpha^j}$$  \hfill (3.8)

As $k$ approaches infinity, the above yields,

$$\lim_{k \to \infty} h_{i,k} = h_{i,0}^{\lim_{k \to \infty} \alpha^k} = h_{i,0}^0 = 1$$  \hfill (3.9)

$$\lim_{k \to \infty} E[b_{i,k}] = \frac{\beta L}{1 - \beta R} + \beta \gamma(H - L) \lim_{k \to \infty} \sum_{j=0}^{k-1} (\beta R)^{k-j} h_{i,0}^{\alpha^j} \quad (\beta R < 1)$$  \hfill (3.10)

Consider now the case when $h_{i,0} > \tilde{h}$, there are four possible scenarios: (i) increasing $E[h_{i,k}]$ and $M > 2$; (ii) increasing $E[h_{i,k}]$ and $\tilde{h} < M < 2$; (iii) decreasing $E[h_{i,k}]$ and

\(^3\)See Appendix A.3 for proof of convergence.
\( \tilde{h} < M < 2; \) and (iv) decreasing \( E[h_{i,k}] \) and \( M < \tilde{h} \). Let \( \bar{k} \) be the largest \( k \) such that \( E[h_{i,k}]^{\alpha} + \bar{I} \leq 2 \) and \( E[h_{i,k}] > \tilde{h} \), we have

\[
E[h_{i,k}] = E[h_{i,k-1}]^{\alpha} + \frac{1}{2} \bar{I}
\]

(3.11)

\[
E[b_{i,k}] = (\beta R)^{\bar{k}} b_{i,0} + \left[ \frac{\beta \gamma}{2} \bar{I}(H - L) + \beta L - \beta R \bar{I} \right] \sum_{j=0}^{k-1} (\beta R)^{j} + \beta \gamma (H - L) \sum_{j=0}^{k-1} (\beta R)^{k-j} E[h_{i,j}]^{\alpha}
\]

(3.12)

It is possible to show that, with the exception of (iv), generations of household \( i \) still invest in education at the infinite horizon.\(^4\) The divergence in investment decision, thus, results in a divergence in long-run wealth provided that the returns to education are sufficiently high. In the following section, we conduct a simple calibration using data from NLSY97 to corroborate our findings.

4 A simple calibration

4.1 Methodology

Using data from NLSY97, we illustrate, first the underinvestment in education in a one-period setting, and second the poverty trap that emerges in a multi-period setting. Considering the number of parameters to be configured in the model, a number of steps need to be taken prior to the calibration.

First, to estimate \( H \) and \( L \), we take the respective averages of the top and bottom 25% wages for the single-period calibration, whereas only 15% top and bottom wages are used for the multi-period calibration. A brief summary and description of the data are available in Appendix B.1. Next, we calculate the probability of getting the offer by creating a dummy \( p \) and assign its value to 1 for individuals who hold the high-paid job and 0 otherwise. The restricted samples are then divided into 8 groups sorted by the highest qualifications of parents, biological and residential inclusive. Specifically, those belonging to the 1\(^{st}\) group are children of parents who hold no qualifications, 2\(^{nd}\) are children whose most educated parent has a General Educational Development (GED) qualification, and continuing on until the 8\(^{th}\) group which houses individuals who have at least one parent holding a Professional Degree. For each group, the probability of getting the high-paid job is calculated by taking the average of \( p \).

Second, data on the level of education for parents as well as children are re-scaled to fit

\(^4\)A more detailed treatment of these cases and their comparison are available in appendix C.
within the range \([1, a]\), where \(a\) is the ratio between the highest and lowest probability. In so doing, the level of education is treated in a manner synonymous with \textit{years of schooling} such that the amount of investment made and knowledge obtained are assumed to be homogeneous across different qualifications. To the extent that the expected wage elasticity of investment remains the same, this assumption simplifies our analysis. Moreover, by re-scaling the level of human capital, \(\gamma\) corresponds to the minimum probability while the maximum probability is \(a\gamma\).

Third, we create a variable to proxy for the amount of investment made in education by summing the tuition fees over the years and the total amount of educational loans, including both federal loans and personal loans. The annual investment is thus obtained by dividing the previously generated variable by the highest grade achieved as of 2017. Unsurprisingly, the group that invests the most in education is the 8th group. To exclude outliers, among those who invested, we associate the highest amount of investment made, or \(\bar{I}\) in our model, with the 95th percentile rather than the maximum. Furthermore, we assume that \(\bar{p}\), the highest probability of getting the offer, is obtained at the highest level of parental education, i.e. \(\bar{h} = a\), in conjunction with the maximum investment \(\bar{I}\). Through this association, \(H\), \(L\), and \(\bar{I}\) can be scaled to suit our needs for calibration. If \(\bar{I} > a - 1\), however, we impose another restriction, \(\bar{I} = a - 1\), so that absent uncertainty, an agent can only invest to reach the upper limit of \(h\).

Fourth, \(\alpha\) is estimated using the following regression equation,

\[
\log(h_{i,j}) = \alpha \log(\hat{h}_i) + \beta + \theta FE_j + \epsilon_i
\]

where \(FE_j\) is a 2 \(\times\) 1 vector of fixed-effects encompassing birth year and ethnicity factors. This regression equation is derived from our formulation in A1, where the human capital of parents and oneself are log-linear while the educational investment is excluded due to endogeneity by assumption. Results of the regression are available in Appendix B.2.

For the multi-period analysis, on top of the above procedure, \(R\) is estimated as an average of the gross interest rate of the three types of educational loans: direct subsidized loans, direct unsubsidized loans, and direct plus loans.\(^5\)

\subsection{4.2 Discontinuity of investment decision}

We restrict the sample to contain only observations with wages either below the 25th percentile or above the 75th percentile.

\(^5\)Data obtained from The Federal Student Aid, available at https://studentaid.gov/understand-aid/types/loans/interest-rates
Figure 4.1: Expected Utility and Expected Wealth for different \( \hat{h} \) and \( I \)

**Panel A: Expected Utility**

**Panel B: Expected Wealth**

Calibrated parameters: \( H = 18.94, L = 2.82, \lambda = 0.23, \alpha = 0.41, \gamma = \bar{p} = 0.25, \bar{p} = 0.82, \bar{I} = 2.27, \hat{h} \in [1, 3.27] \). The corresponding \( \hat{h} = 2.42 \). In Panel A, levels are adjusted by adding constants for ease of presentation. In Panel B, real values of \( H \) and \( L \) are used.

Figure 1 presents the expected utility and expected wealth for different levels of parental human capital. In Panel A, the expected utility is plotted against the amount of investment \( I \), and the corresponding expected probability of getting the offer. In Panel B, the expected wealth is calculated in accordance with the rules described in section 2, that is, agents whose inherited human capital is below the threshold do not invest whereas agents above the threshold invest the maximum amount. For the given parameters and estimated values, any investing agents invest an amount equal to \( 3.27 - \hat{h}_i^{0.41} \) rather than \( \bar{I} \). The calculated threshold is the actual threshold that investing agents face. Additionally, a higher inherited human capital predisposes an agent to a higher probability of getting the offer for a given level of investment. This translates into an increasing expected wealth in \( \hat{h} \), a pattern that persists throughout the entire range of \( \hat{h} \). Further, a jump in the expected wealth results from the change in investment decision as \( \hat{h} \) crosses the threshold level. It is also apparent that the slope of expected wealth becomes flatter as agents start investing. This stems from the uncertainty in the “schooling” process, lowering the expected returns to educational investment. We next examine the effects of this discontinuity of investment decision in the multi-period setting.

4.3 The evolution of bequest

For the multi-period analysis, we further restrict the sample to contain only the top and bottom 15% wage. This restriction is imposed as the introduction of the capital market allows agents who do not invest in education to invest in capital, thereby raising the threshold level \( \hat{h} \). By shrinking the sample, we effectively increase the gap between \( H \) and \( L \), making the good job rewarding enough for agents to invest in education. We present two scenarios, a
Figure 4.2: Expected bequest, human capital and educational investment over time

Panel A: Separating equilibrium

Calibrated parameters: \( H = 16.73, \ L = 1.35, \ \lambda = 0.24, \ \alpha = 0.43, \ \gamma = p = 0.28, \ \bar{p} = 0.79, \ \beta = 0.3, \ R = 1.05, \ \bar{I} = 1.86, \ h \in [1, 2.86], \ h_{1,0} = 2, \ h_{2,0} = 1.9, \ b_{1,0} = b_{2,0} = 1. \) The corresponding threshold \( \tilde{h} = 1.97. \)

Panel B: Pooling equilibrium

Modified parameters: \( \lambda = 0.246, \ h_{1,0} = 2.5. \) The corresponding threshold \( \tilde{h} = 2.165. \)

separating equilibrium and a pooling equilibrium.

The amount of investment made in this case is similar to the previous sub-section, that is an agent invests an amount \( 2.86 - h_{1,0}^{0.43} \) rather than \( \bar{I}. \) In Panel A of Figure 4.2, the expected level of human capital of agents from household 1 increases until the second period and remains stable thereafter. The amount of investment made increases sharply in the first period then stabilises at a slightly lower level in subsequent periods. We readily observe the divergence in the evolution path of expected bequest between household 1 and 2. In Panel B, household 1 invests in the first and second periods when the expected inherited human capital is above the threshold. From the third period onward, future members of household 1 shy away human capital investment in favor of capital investment. Consequently, we observe a large gap in the amount of expected bequest between the two households in the beginning, which gradually decays over time to reach almost zero by the 9th period.
5 Conclusion

In the microcredit literature, it has been documented that the poor often avoid investing in risky projects with high payoffs. This puzzling phenomenon has sparked heated debates on the cause thereof. To this end, Khazanov et al. (2018) proved that risk-aversion could lead to said underinvestment in a setting where the probability of success is endogenous. However, in the context of education and health wherein the intermediate outcomes, i.e. human capital and calories, are both uncertain and inheritable from parents, an agent faces two level of uncertainty if she invests while her investing capability is constrained by her inheritance. In this paper, we explore how an agent invests in the current setting and demonstrate the discontinuity of investment in the level of inherited human capital. Further, we show that through the intergenerational linkage of human capital, poverty persists for households with sufficiently low level of human capital. Our results are obtained in an environment absent any fixed-cost, credit constraints, behavioural factors or non-homotheticity.

What does it mean for persistent poverty and poverty alleviation? Our schematic approach to analysis provides a simple yet straightforward message “if educational investment can be sustained, the poor can escape poverty.” If governments subsidise education to effectively elevate the human capital of poor households, future generations will benefit immensely as they can sustain their own educational investment. Note that our suggestion is to improve “human capital,” and not years of schooling. If the poor are entitled to, say 10 years of schooling of poor quality, they are much better off spending their time working. While our analysis using CARA ignores wealth, the results using CRRA links poverty to both inputs, namely human and non-human capital. As we acknowledge the importance of capital, using data from NLSY97, we also show that it is crucial to examine the elasticity of substitution between the two inputs to design and implement the most effective policy. Further, different steady states carry different implications for policy design. If no household invests in perpetuity, periodic, as opposed to lump-sum, transfer must be put in place.

Thus far, we have assumed constant the degree of risk-aversion with varying levels of human capital. However, to some degree, we expect children of more educated parents to be less risk-averse, which further bolsters our results. There has been mixed views on this issue, however. A literature review conducted by Outreville (2015) showed that indeed there is support for the view that education reduces risk-aversion while also noting the limited external validity. Studies supporting the opposing view mainly use education as proxy for risk-aversion when studying the demand for insurance. More directly, Jung (2015) used the British educational reform to prove the positive relationship between education and risk-aversion, albeit only for the cohort most affected by this reform. More extensive research
is necessary to reconcile those opposing views, figure out their intricate interactions and explore their implications for poverty reduction. Further expansion to our study may also consider modelling risk-aversion as a function of human capital.

References


Morduch, J. (1990). Risk, production and saving: Theory and evidence from indian house-
holds. *Harvard University, Manuscript.*


