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Mann's imperial march: Modelling the role of marcher lords in ancient state development and expansion

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Mann's Imperial March: Modelling the role of marcher lords in ancient state development and expansion

Andrew Moore*

Abstract

This article uses economic analysis of the 'marcher lord' actor theorised by sociology literature, particularly the work of Michael Mann (1986), to explain the shifting nature of power in the ancient world. We model the marcher lord to argue that a first-mover advantage from developing state institutions first does not bring a lasting military advantage as lesser developed lords located closer to the periphery of a region are better able to diversify their armies. We develop a second model to analyse Mann's strategies of control and a rational calculus of technology adoption to consider whether new empires will emerge to dominate existing empires. We find that advancement does not emanate from the seat of power, rather marcher lords on the periphery of civilisation development have the opportunity to expand leading to them being more innovative and able to shift the centre of power away from established empires in later time periods.

Keywords: Marcher Lord, Michael Mann, Military Power, Ancient States

JEL Classifications: H10, N40, O33

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1 Introduction

Comparing the modern world to maps of ancient empires raises the question, if Mesopotamia developed civilisation first why did it not become the ruler of the Mediterranean like Rome did later? The answer presented by Michael Mann (1986) and sociologists like Wallerstein (1974) and Chase-Dunn (1988) is that the driver of state development and military power were peripheral states. Specifically, they identify an actor known as the marcher lord, named due to their home region being the march between developed regions and other states, who were given a uniquely advantageous location to dominate their neighbours. This paper raises two questions. First, does a model of Michal Mann's (1986) marcher lord explain ancient empire expansion and military development? Second, do peripheral lords have a relative advantage over their more developed counterparts?

A pattern in the ancient historical record is the constant movement of the dominant state or city, especially in western history, and the lack of advantage granted to those cities that first developed complex states. Economic literature so far on ancient history broadly falls into one of two categories: the creation of states (Schönholzer, 2020, Mayshar et al. 2018, Sierra, 2013), or the analysis of external forces which affected state development and history (Alesina et al. 1995, Bockstette et al., 2002, Levine & Modica, 2013, Fernández-Villaverde et al., 2020, Yu Ko et al., 2018). Such work by economists brings a unique contribution to a field dominated by historians and sociologists for two reasons. First, identification of recurring themes and variables in this competitive process of development can explain the success and failure of states in the historical record to explain the makeup of the modern world. Second, given that the ancient record is inherently incomplete due to erosion of records over time, economic models provide mathematical insight to supplement and extend existing historical analysis and intuition.

Sociologists have considered the arc of history and attempt to identify common forces and trends that have dictated the flow of history and civilisation. In particular, Carneiro (1970) and Mann (1986) have theorised on the conditions that laid the foundation for early states and how states expanded from there. Wallerstein (1974 and 1979) created a theory to capture the interaction between developed, developing, and semi-developed states to understand how each of those actors develop. While the theory is primarily applied in the modern world some authors (Chase-Dunn, 1988, Chase-Dunn & Hall, 1991, Hall 1991) have applied the theory to ancient history and marcher lord conquerors.

We contribute to the literature by creating a model based off Mann's (1986) analysis of this unique actor who's power arises from being at the periphery of other states and regions. Such an actor is counter intuitive given the line of literature on the importance and advantage of developing institutions (for example Bocketette et al., 2002) and the general principle in economics of the first mover advantage. The marcher lord does not follow this logic, they inherently develop later than their neighbours as they exist in an area less conducive to state development and they are located at the periphery of a region. We model how this actor uses its peripheral location to consolidate a more diverse army than states located in the core of a particular region.

Furthermore, we model the strategies of control outlined by Mann (1986), thus providing an endogenous analysis on the size of empires based on the revenue and costs of a particular style of empire management. Fernández-Villaverde et al. (2020) and Yu Ko et al. (2018) argue that the size of empires is due to geography and external threats, suggesting a strong exogenous component to where a state will draw its borders. Our contribution is more in line with analysis by Alesina & Spolaore (2005), Alesina & Enrico (1995) and Alesina et al. (2002) who consider the limits to the size of an empire separate from their total military power. We find that states may have the capacity to expand more but it would be unprofitable to do so, thus leading to a smaller empire than expected.

Finally, we adopt a rational calculus for why an individual state chooses to adopt technology, that is the benefit outweighs the cost. While simpler than the forms of technology innovation discussed by McNeill (1982), for example, we integrate it with our consideration of the size of empire to find that there is an advantage to being in the peripheral region. The advantage is the ability to grow and expand as military power is the limiting factor of expansion. Thus, peripheral states are more likely to adopt technologies than established empires.

2 Marcher Lord Model

2.1 One March Model

2.1.1 Model Setup

The first model illustrates the relative military power available to states in ancient history. We consider the location of the strongest lord in a single period game where geography defines the distribution of different types of soldiers and the 'cost' to integrate those soldiers into an army. Particularly in ancient history

where the costs of travel were so high, the characteristics of a region influenced the nature of warfare and lead to cities and lords specialising in a certain type of soldier. For example, lords and cities located in 'civilised' regions, around the Nile or Tigris and Euphrates Rivers, were agriculturalists who lived in more developed cities. This societal makeup encourages the development of infantry forces as agriculturalists have sufficient surpluses to produce the heavy armour required by infantry and a level of cohesion necessary to fight in formation. Conflict between agriculturalists involves besieging and garrisoning other cities, the winner determined by which infantry are relatively better at fighting. In the highland regions that surrounded Mesopotamia. These areas could not rely on agriculture to the same extent and thus generated food surpluses from hunting with bows and herding. Where the agriculturalists turned farming tools into weapons, the people of the highlands turned the bow from a hunting tool into a weapon. The steppe people's in the grassland regions developed a style of warfare completely different to their agricultural or highland counterparts. Their pastoralist heritage meant they had plenty of horses and were expert at riding them. Therefore, they developed cavalry warfare which was highly mobile and effective at raiding in the areas of the grasslands where garrisoned cities were virtually no existent.

There are two actors: individual states, representing military lords and rulers who are seeking to expand their realm, and supplier cities, representing communities that provide soldiers to any given state or lord's army. States are represented by x where $x \in [0, 1]$, cities are represented by y where $y \in [0, 1]$.

We assume these actors are distributed at all points across a Hotelling line of one unit length[1]. The line represents an area of civilisation, for example Mesopotamia. Each state draws upon all supplier cities to create their army. Creating an army is affected by a linear discount rate of $i(y)[1 - |x - y|]$, where cities further away from any given state provide less valuable soldiers than cities neighbouring the state.

The supplier cities are split into two categories, divided by the march which is represented by m. The type of soldiers supplied by by the city depends on which side of m they are located on (see Equations 1 and 2) as cities are specialised in providing only one type of soldier.

In this particular case suppliers to the left of/less than m are located in an 'agricultural core' region and therefore specialise in supplying infantry at rate i. Suppliers to the right of/greater than m are located in a 'grasslands steppe' region and therefore specialise in supplying cavalry at rate k.

[1] In some ways this is a modern application of Von Thünen's "isolated state" model as any given location along the line represents a trade off between access to one type of solider over another. I thank Lionel Frost for taking the time to introduce the concept to me.

$$i(y) = \begin{cases} i & y < m \\ 0 & y \geq m \end{cases} \tag{1}$$

$$k(y) = \begin{cases} 0 & y < m \\ k & y \geq m \end{cases} \tag{2}$$

The final aspect of this game is that we use an integral to calculate the military power of a lord at any given point x. Specifically a state's military power is a function of $M(x) = I(x) + K(x)$ where $I(x) = \int_0^1 i(y)[1 - |x - y|]dy$ and $K(x) = \int_0^1 k(y)[1 - |x - y|]dy$. That is, a state's military power is the aggregation of every city along the line providing a soldier with a specific value discounted by the distance required to reach the state.

2.1.2 Model Analysis

Mann (1986) argues that no individual style of warfare is dominant and therefore the strongest army was one that could combine multiple soldier variants to utilise all their strengths together. Due to geography dictating the accessibility of various types of soldiers, the most powerful lords are the ones located at the march, the point separating two different areas. In other words, the marcher lord.

We assume that lords will always amass the biggest/strongest army they can and cities will always supply the lords. Furthermore we assume that $i > 0$ and $k > 0$ and their value is exogenous. Military power is the only metric of power, therefore a lord with a greater M will always beat lords with a lesser M.

We draw inferences from the model by solving with integral calculus the following Equation $x^* = \arg\max_x M(x) = \arg\max_x [I(x) + K(x)]$. Thus we find the optimal location of x or the marcher lord[2] in this model. Due to the split in the type of soldier provided along the line we find two sets of solutions as shown in Equations 3 and 4, the twin solutions reflect the power for any given lord on either side of point m.

$$M(x) = \begin{cases} \overbrace{i\{m - \frac{m^2}{2} + mx - x^2\}}^{I(x)} + \overbrace{k[\frac{m^2}{2} - m + \frac{1}{2} + (1-m)x]}^{K(x)} & \text{for } x \in [0, m) \\ \overbrace{i[(1-x)m + \frac{m^2}{2}]}^{I(x)} + \overbrace{k[\frac{1}{2} - \frac{m^2}{2} - m + (m+1)x - x^2]}^{K(x)} & \text{for } x \in [m, 1] \end{cases} \tag{3}$$

[2]For the step by step process see Appendix 1.

$$\frac{dM(x)}{dx} = \begin{cases} i(m-2x) + k(1-m) & \text{for } x \in [0, m) \\ k[(m+1) - 2x] - im & \text{for } x \in [m, 1] \end{cases} \tag{4}$$

IMPLICATION 1 (SUPPLY AND VALUE OF SOLDIERS) - *Equation 5 represents the ratio determining if moving towards the march results in an increase in military power of a lord*[3] .

$$\frac{\text{importance of calvary in fight}}{\text{importance of infantry in fight}} \geq \frac{\text{size of agricultural land}}{\text{size of grassland}} \tag{5}$$

For this ratio to hold true, then $\frac{dM(x)}{dx} \geq \frac{dM(x \to m)}{dx}$ must hold true, that is the military power of an agricultural lord increases as they shift their location towards the march. Therefore, this means $k(1-m) - im \geq 0$, or else military power actually decreases as the lord moves closer to the march. The ratio highlights that the value of infantry compared to cavalry matters but so does the number of suppliers of each type of soldier. For a shift to be a net positive the extra value gained from being closer, and thus able to access a soldier for 'cheaper', must outweigh the increase in travel costs/discounting rate from being further away from the majority of suppliers of the other type of soldier. Logically this makes sense, in an extreme case being close to a highly powerful but rare type of soldier would means a lord is more powerful than another lord who is very distant and only able to access a relatively less powerful but common type of soldier. We argue this is also an expansion on Mann's (1986) analysis of the armies of marcher lords, even if the types of soldiers have the same value to an army's military power they may have different amounts of suppliers thus changing the relationship of how close the marcher lord is to the march.

IMPLICATION 2 (MARCHER LORD'S LOCATION) - *The marcher lord will sit within a range between the strongest infantry and cavalry lords instead of always sitting at the march.*

We identify the location of the strongest infantry and cavalry lords, lords who specialise in one type of soldier. The strongest infantry lord will always sit at $\frac{m}{2}$ and the strongest cavalry lord will sit at $\frac{m+1}{2}$[4]. The first result from these two values is that a lord specialising in one type of infantry will always be located at the exact middle of the region that produces that type of soldier as that is the 'cheapest' location to amass the most powerful single soldier army. As a corollary to that, we find that the location of the march, m, is the determining

[3] both sides of the inequality are inverted in the case of a grassland lord.

[4] We find these values by using differential calculus. We derive $I(x)$ for when $x < m$, as the strongest infantry lord will reside in the region that produces infantry, and we derive $K(x)$ for when $x > m$.

factor of their location on the line as they will be an equal distance from the march and the other limit of the line.

Equation four give the following results for the location of x; for values less than m (agricultural lords) $x^* = \frac{m+1}{2} - \frac{im}{2k}$ and for values greater than m (grassland lords) $x^* = \frac{m}{2} + \frac{k(1-m)}{2i}$. Each Equation has two parts. The first part is the location of the strongest infantry or cavalry lord. The second part modifies the location relative to that lord based on ratio identified above, the value of each type of soldier and their relative supply. Therefore, the marcher lord will always sit at a point between these two lords as even in very extreme cases there is still some military power to be gained by moving slightly to the right of the infantry lord or slightly to the left of the cavalry lord and therefore lowering the 'cost' to access a relatively stronger army.

Mann (1986) argues that there is something unique and special about the locations of the march which facilitates the creation of the marcher lord, concluding that that marcher lord arises in the march. We make two key findings that expand on Mann's (1986) conclusion. First, that marcher lord will only appear at the march in one special case (depicted in Figure 1) - when the march is located at $\frac{1}{2}$ and i and k have equivalent value. Otherwise, the marcher lord will exist within a range of locations between the strongest infantry and cavalry lords. Thus, highlighting that relative location plays an important role in a lord's military power. Equally, developing or specialising in one type of warfare does not create a first mover advantage as doing so does not contribute to having a better mixed army. Second, the location of the march heavily influences the position of the strongest infantry and cavalry lords. This is because it sets the boundary of how large a region is and thus the number of suppliers of a type of soldier. In a case where there is no mixing of soldier types and lords must speciailise in one type of soldier, there is a clear advantage to being at the center of a region and away from the periphery.

2.1.3 Depiction of the One March Model

Assume that the march (m) is at $\frac{1}{2}$, therefore the size of agricultural land and grassland is equal. We also assume that i and k both have equal value (i.e. one infantry solider is just as valuable as one cavalry soldier).

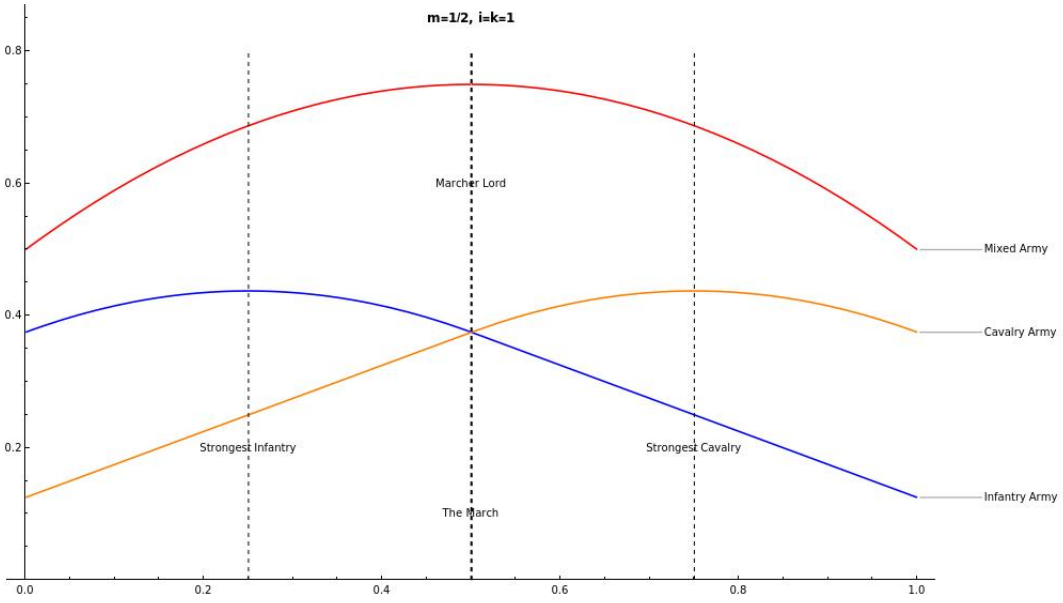

Figure 1. *One Marcher Lord - Mann's case*

Figure 1 reinforces Mann's (1986) original theory that the marcher lord appears exactly at the march. Due to Infantry and Cavalry having equal contribution to military strength the strongest lord resides at exactly the middle point of the line. The march of the agricultural and grassland regions becomes the strongest position because it is the 'cheapest' location to access the troops supplied by each region.

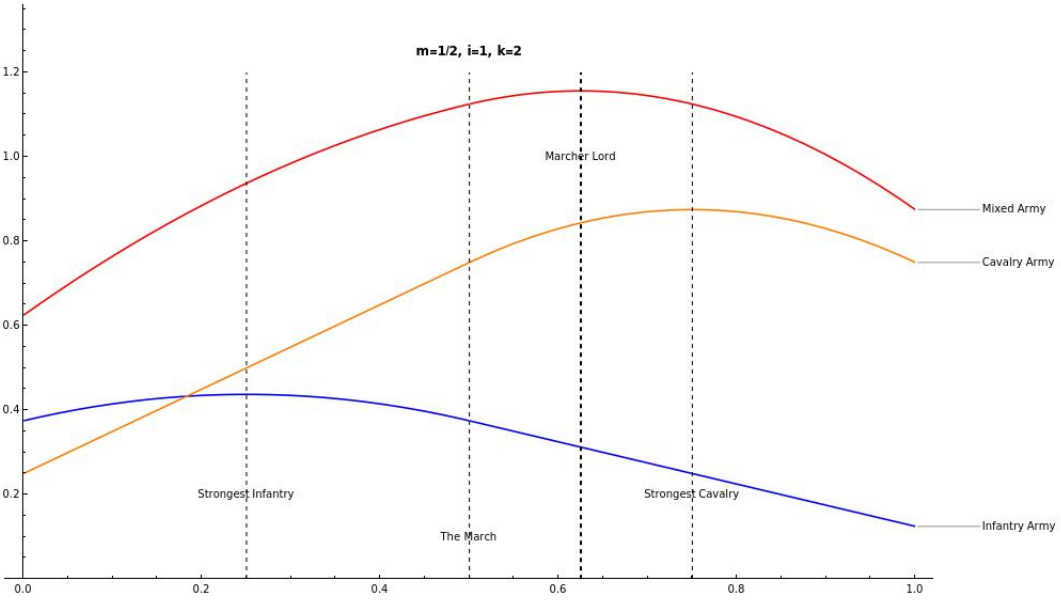

Figure 2. *One Marcher Lord - Cavalry Variation*

We analyse a variation where the march is still the middle of the line (i.e. m is $\frac{1}{2}$) but i and k have different values. k is now two while i is still one, that is one cavalry soldier is worth two infantry soldiers (or alternatively each supplier in the grasslands can provide twice as many cavalry soldiers as a agricultural supplier). Figure 2 highlights our findings from implication 1 and 2; the marcher lord no longer sits at the intersection of agricultural and grassland, rather the lord sits to the right as access to cavalry is relatively more important. However, the marcher lord is to the left of the strongest cavalry lord as accessing infantry is important for the overall strongest army. Similar to Figure 1, being closer to the periphery means greater military power but the march, or the exact peripheral location, is actually weaker than being slightly within the grasslands. The strongest infantry and cavalry lords still occupy the same positions but there is a large disparity in military power due to the increased power of cavalry.

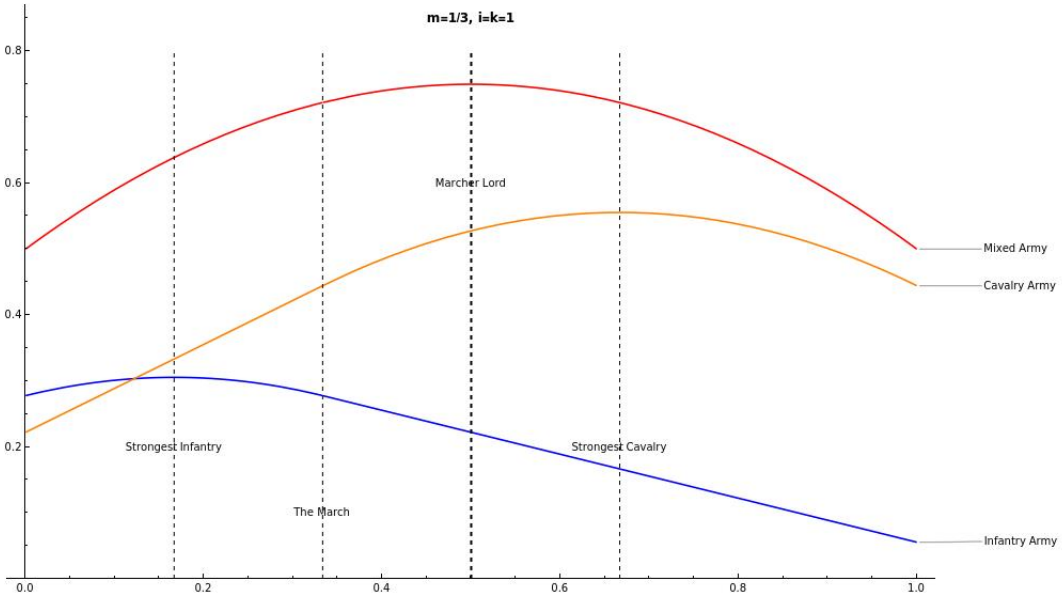

Figure 3. *One Marcher Lord - March Location Variation*

Finally, we consider a variation where i and k have the same value but the march is located at $\frac{1}{3}$, thus there are approximately twice as many cavalry suppliers as there are infantry suppliers. As argued in implication 2, the location of the march does not affect the location of the marcher lord if the value of the soldiers are identical and in line with Figures 1 and 2, the marcher lord is closer to the periphery than the strongest cavalry and infantry lords. Conversely, the locations of the strongest infantry and cavalry lords shift slightly to maintain their position in the middle of the agricultural and grassland areas.

2.2 Two March Model

2.2.1 Model Setup

To consider a more complex case, for example an agricultural area is surrounded by grasslands or highlands, we use two marches splitting the area into three sections. There are two marches that separate an 'agricultural core' from the 'grassland steppe'. The march m is split into f, representing the 'first march', and s, representing the 'second march'. Lords from $0 \to f$ and $s \to 1$ are 'grassland' lords and have direct access to cavalry. Lords from $f \to s$ are 'agricultural' lords and have direct access to infantry. Due to this change, the

density functions are modified slightly from the one marcher lord case (see appendix for further mathematical detail). Otherwise, all other parts of the game are the same as in the one march model.

2.2.2 Model Analysis

We test whether the two implications identified in the previous section also apply when there are two marches. First, the ratio of supply and value of soldiers. We consider the case where $\frac{dM(x)}{dx} \geq \frac{dM(x \to s)}{dx}$ military power increases as the lord shifts closer to the 'first march' or f. Therefore, $i(s-f) - kf \geq 0$ which can also be written as $\frac{i}{k} \geq \frac{f}{s-f}$, which represents the ratio in Equation 6. This ratio is an inverse of the ratio covered above and as such supports the implication - that moving to the periphery/march makes sense if the increase in 'cost' from moving away from the majority of suppliers is outweighed by the increase in military power due to greater access to more valuable soldiers.

$$\frac{\text{importance of infantry in fight}}{\text{importance of cavalry in fight}} \geq \frac{\text{size of grassland}}{\text{size of agricultural land}} \tag{6}$$

The second implication is that the marcher lord will be located within a range of values rather than always being at the march. To test this we consider a case for the optimal location (i.e. $\frac{dM(x)}{dx}$) for $x \in [0, f)$. We find the following first order condition ($i(s-f)+k(f-2x^*) = 0$) and the following location of the marcher lord $x^* = \frac{f}{2} + \frac{i(s-f)}{2k}$. Similar to above, the Equation has two parts, the first part reflects the strongest cavalry lord in the first grasslands region while the second part is the increase in military power gained by a lord being closer to infantry suppliers. Therefore, even if cavalry is the single most important factor for military power, a lord slightly to the right of the strongest cavalry lord will be more powerful due to their access to at least some infantry to supplement their forces.

2.2.3 Depiction of the Two March Model

Assume that the first march (f) is at $\frac{1}{3}$ and the second march (s) is at $\frac{2}{3}$, therefore the size of agricultural land and grassland is equal. We also assume that i and k both have equal weight.

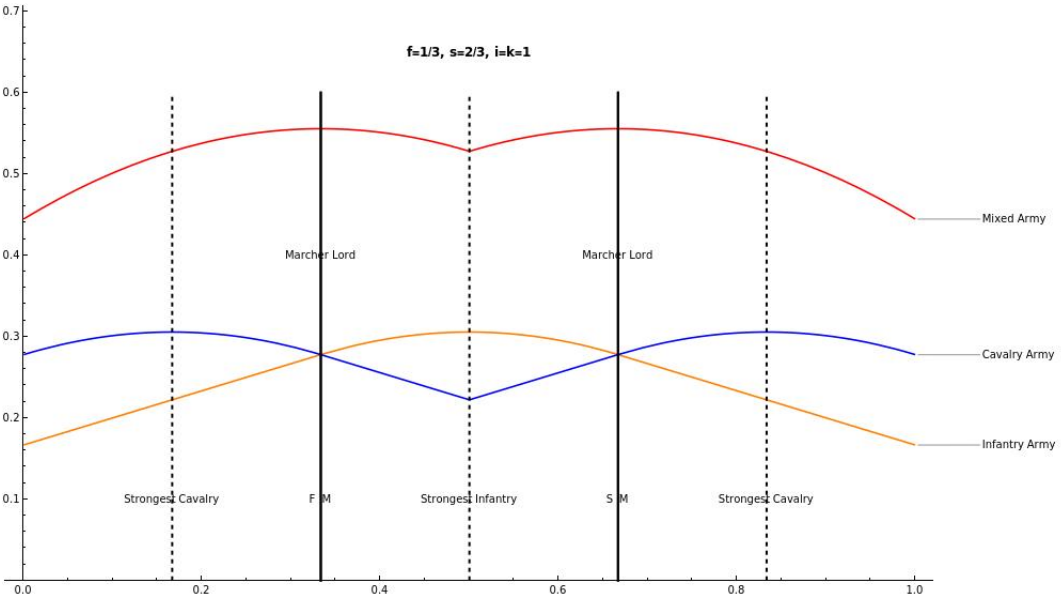

Figure 4. *Two Marcher Lords - Mann's Case*

We find that in the case there is no difference between the soldier varieties and the number of suppliers, Mann's (1986) intuition is right as the marcher lord will reside exactly at the march due that position having the best access to cavalry and infantry.

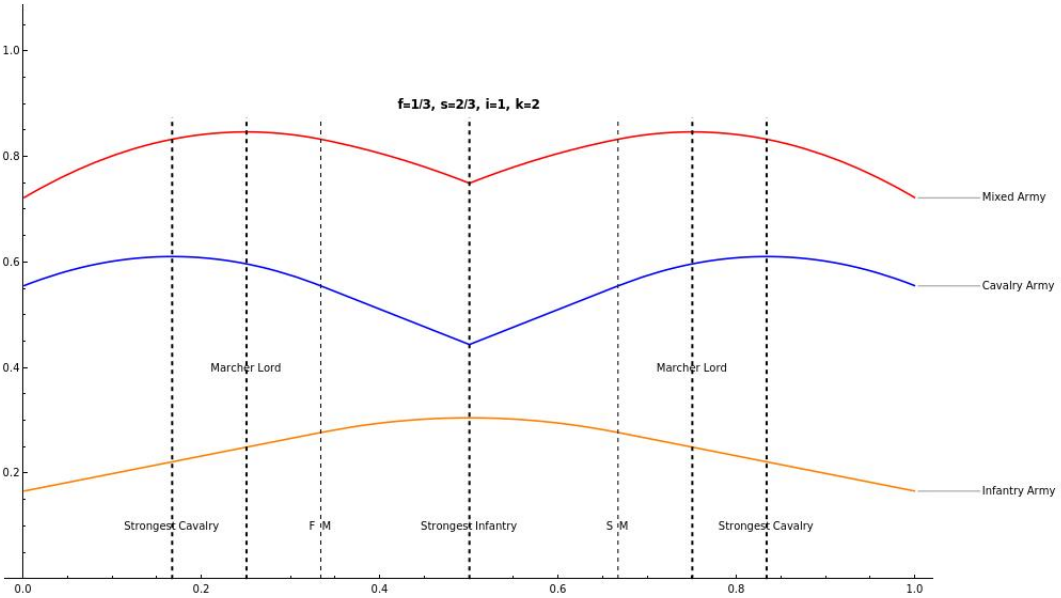

Figure 5. *Two Marcher Lords - Cavalry Variation*

We consider the more powerful cavalry variation again by doubling the importance of cavalry to military power and leaving the marches in the same location. In line with implications 1 and 2, the marcher lords shift away from the march slightly as greater access to cavalry is a major factor in military power. However, they are still located slightly closer to the periphery than the strongest cavalry lords.

2.2.4 Limitations of the Marcher Model

We must note that this model has two key limitations. First, it is a one period game where the optimal location for a marcher lord identified by the model will always be the optimal location. A new marcher lord can only arise if a second period is added and the relative value of soldiers or the location of the march changes for exogenous reasons. Second, this model is predicated on the assumption that each region only specialises in one type of solider. This assumption makes sense in the ancient world as there was insufficient surpluses, technologies, and population to enable a single area to produce a diverse army by itself.

3 Methods of Control

The above models of the optimal location of the marcher lord indicate how dominant states arise across a region. We consider a second illustrative model that captures the trade off between revenue/tribute, cost, and empire size that states are subjected to when they forge their empire. These strategies reflect how the logistics of empire bottle necked the size of empires as opposed to how strong their military was (Mann, 1986, McNeill, 1982).

We analyse three strategies of ancient states identified by Mann (1986). First, rule through clients or rule through army both rely on a ruler 'ruling' by controlling cities through an intermediary either a vassal or an appointed governor. In the case of ruling through clients, tribute extraction is inherently limited and the chance of rebellion is ever present due to the arms-length nature of the over lordship. Similarly, ruling through army means greater tribute extraction but increased costs from garrisoning the city and the chance of rebellion remains as local governors may choose to become independent.

Second, there is compulsory cooperation. The state pays an upfront cost to establish centralised infrastructure, for examples markets and basic quality standards or military protection to facilitate trade, to create a carrot and stick approach to ruling their lands. Subjugation now brings an economic benefit to cities under an overlord which when paired with fear of military force reduces the chance of rebellion and separately increases the revenue of the overlord.

Finally, there is shared ruling class or shared ruling identity. The key flaw with the two previous strategies is the regionalisation of culture and effect of distance making it increasingly difficult to extract the same value from those cities while maintaining control overly an increasingly diverse empire. Creating a common culture or language touch point among the elite of an empire removes this cost by ensuring that the difference in ruling and revenue extraction in the capital or a city on the border is effectively insignificant. However, establishing such a culture requires investment greater than the amount required for compulsory cooperation to develop a language that can be easily taught to others and construction of common points of culture such as theaters or temples.

3.1 Model Setup

We model these strategies in a single period game where a state has full information regarding how much land they can conquer with their military power. We use a hotelling line with cities and empires distributed at all points along the line. In this game the state is the only actor which are represented by x where $x \in [0, 1]$. Each state chooses an amount R for their empire to control. This decision is constrained by either a lack of military power to defeat other states or it being more profitable to control a smaller empire due to the exponential costs of empire. Similar to the calculation of military power of states in the previous model, we use an integral to calculate the total revenue and cost for ruling over R cities.

We also introduce the same linear discount rate that applied to a state's military power. In this case the discount rate reflects the increased logistics cost of managing cities and the local elites that are further away from the capital.

States collect t, where $t > 0$, which represents tax income. For compulsory cooperation and common ruling class or identity, states also collect an trade income or economic benefit represented by e, where $e > 0$. Given the relative importance of tribute to state treasuries and the indirect nature of the state's role in their empire's commercial dealings, we argue that often $t > e$ but a more trade focused empires have an inverse relationship.

We use α represents the cost required per city to establish a ruling apparatus. This could take the form of bureaucrats to oversee the city or garrison forces

needing to be in the area to control the city. β on the other hand represents the efficiency of running a given empire through this strategy. Compulsory cooperation uses γ instead. Both β and γ are greater than zero. They can have the same or different values. It must be noted that because this model takes place on a hotelling line, the vast majority of empires will have a value less than one, as such this efficiency number is counter-intuitive as a higher value means the empire is more efficiently run, thus lowering the cost for each city that it manages.

3.2 Rule Through Clients or Rule Through Army

The following function considers the revenue and cost of this strategy. We use Θ to represent the probability of rebellion by clients or subdued cities and it has the following nature, $1>\Theta>0$. The revenue function is represented by Equations 7 and 8.

$$\Pi_A(R) = (1 - \Theta) \int_{x-R}^{x+R} t(1 - |x - R|)dR \tag{7}$$

$$\Pi_A(R) = (1 - \Theta)(2R - R^2)t \tag{8}$$

For the cost function there are two key variables.

$$c_A(R) = \alpha R^\beta, \beta > 0 \tag{9}$$

3.3 Compulsory Cooperation

The revenue Equation is the same except the profit function is slightly different. We remove Θ as this strategy makes conquered cities more compliant due to conquered cities receiving an economic benefit and the lack direct military oppression. We also introduce e to represent the economic benefit that the ruler receives from being involved in facilitating trade through this strategy.

$$\Pi_{CC}(R) = t \int_{x-R}^{x+R} 1 - |x - R|dR + e \int_{x-R}^{x+R} 1 - |x - R|dR \tag{10}$$

$$\Pi_{CC}(R) = (2R - R^2)t + (2R - R^2)e \tag{11}$$

The cost function is very similar to ruling through clients/army. Ψ represents a fixed cost required to establish the set of infrastructure needed for this strategy, i.e. markets. In this case the fixed cost is not as substantial as in the final strategy as the degree of infrastructure needed is much less.

$$c_{CC}(R) = \alpha R^{\gamma} + \frac{\Psi}{2}, \gamma > 0 \tag{12}$$

3.4 Shared Ruling Class or Shared Ruling Identity

Mann (1986) argues that the best strategy is shared ruling class or ruling identity. Where the ruling elites of all the cities of the empire have a common culture point to interact with and thus feel a much closer bond with each other. The importance of this is to fully realise the maximum revenue that can be extracted from every city in the empire as distance is considerably less of a factor. A common language and culture means costs arising from distance are insignificant.

$$\Pi_I(R) = t \int_{x-R}^{x+R} 1dR + e \int_{x-R}^{x+R} 1dR \tag{13}$$

$$\Pi_I(R) = 2Rt + 2Re \tag{14}$$

This cost function is linear in nature, reflecting that distance is no longer a serious factor in extracting revenue from any given city. However, a greater fixed cost is required as a language that can be disseminated to the whole ruling class is needed as opposed to using scribes in the different courts. Furthermore, buildings and cultural touch points that propagate and educated newly conquered peoples in this culture must be built.

$$c_I(R) = aR + \Psi \tag{15}$$

3.5 Depiction of Control Strategies

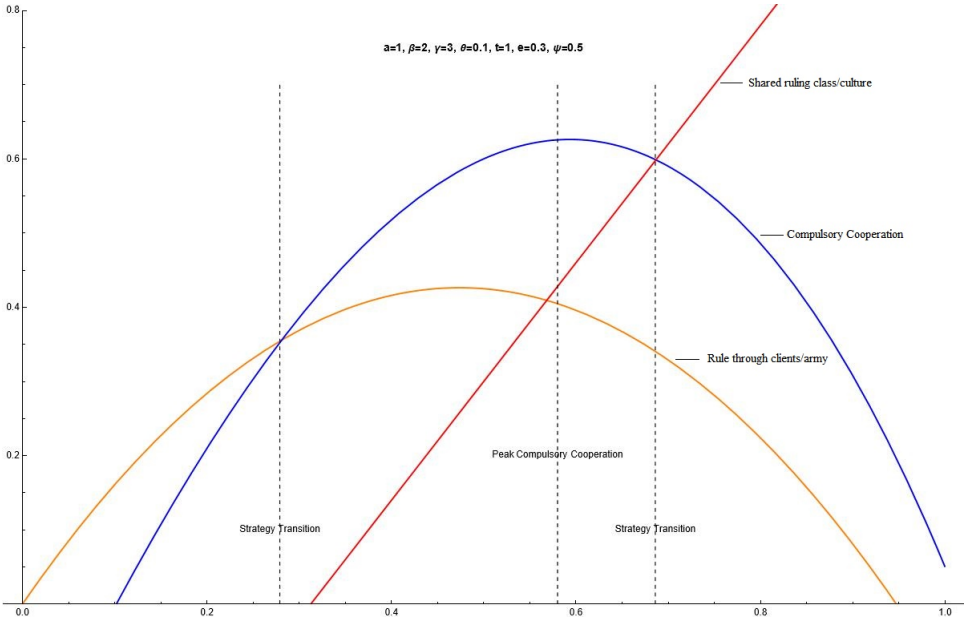

Figure 6. *Low Cost Control Strategy Comparison*

IMPLICATION 3 (MINIMUM SIZE OF EMPIRE) - *Any state can establish an empire.*

Mann (1986) emphasises that ruling through clients and ruling through army are the simplest and first utilised strategies of control. However, the strategy is inefficient due to rebellion and limited extraction of wealth. Figures 6 and 7 support that contention as the limited source of revenue paired with the chance of rebellion, i.e. chance of no revenue, mean that the strategy has the lowest ceiling. Equally, the strategy is the first to become unprofitable as its turning point is at the smallest value of R.

There are some interesting additions to Mann's (1986) theory though from Figures 6 and 7. While inefficient there is no minimum size required for the strategy to be profitable so any state is able to utilise it. That is to say, it has a very low barrier of entry. Second, the strategy is still effective when it comes to profitable control of a region, the strategy in these Figures can support an empire of about half the line. Admittedly this result changes as the parameters change but it does suggest that the other strategies are not always strictly better or the only method to establishing a large empire.

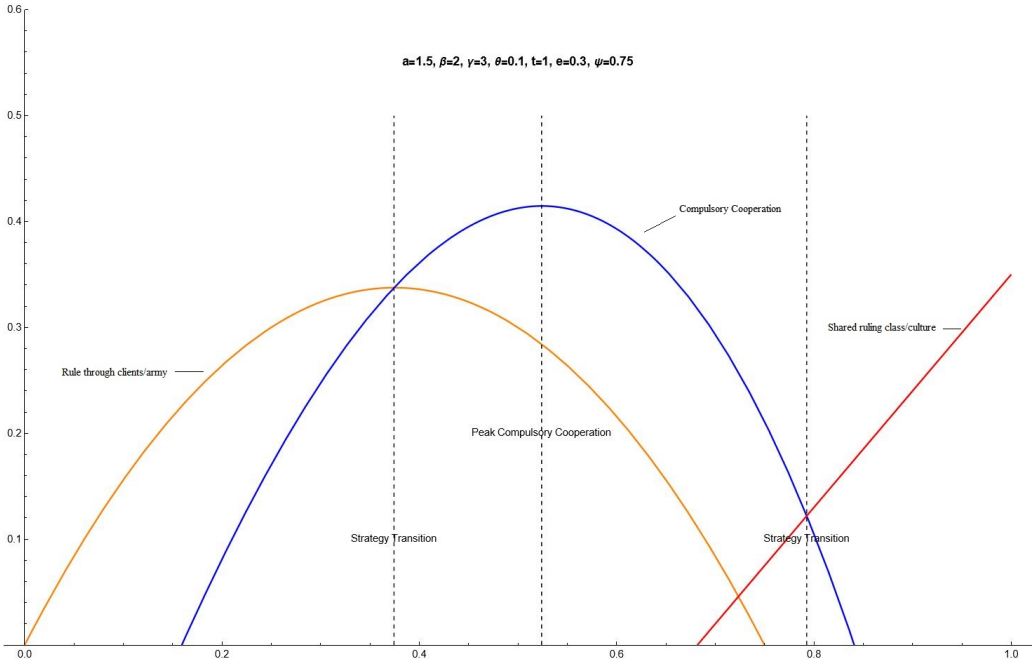

Figure 7. *High Cost Control Strategy Comparison*

IMPLICATION 4 (LIMITS OF CONTROL) - *Empires have a natural equilibrium where they will not expand beyond.*

Figures 6 and 7 show a gray area between peak compulsory cooperation and a strategy transition point where the size of empire is such that moving from compulsory cooperation to common ruling class makes sense. To an extent this reinforces implication 3 as it suggests that strategies are not strictly better than each other, i.e. it does not always make sense to adopt common ruling culture. More importantly, this implication highlights that an empire may choose to conquer less territory than it can conquer as it is less profitable to do so. The extra land would stretch their state capacity too far. This implication supports the findings of Yu Ko, Koyama & Sng (2018) in that the costs of empire and protecting land from external invaders meant it made sense to settle for smaller borders that the kingdoms or empires could theoretically control.

IMPLICATION 5 (COMMON RULING CLASS) - *Common ruling class is designed for large empires.*

While common ruling class is theoretically the best strategy given it has the best revenue extraction function, it is only an option for the largest of empires. The fixed cost required to establish a common culture is prohibitive and is only

offset when the empire is sufficiently large to extract enough revenue to afford the vast infrastructure and cultural development to make it possible. However, we argue that due to its linear property as it bypasses the logistics costs which limits the profitability of empire. Thus, we provide a degree of robustness to Mann's (1986) argument as we identify that it is the fixed cost of setting up the strategy that limits its use.

4 Technological Investment

4.1 Technology Investment with a Guaranteed Return

Our final model development is state's tendency to invest in technology for a price. The previously discussed models are limited to one period and individual lords take their military power as given, whereas Mann (1986) argues that throughout history new technologies, like iron weapons or more advanced cavalry tactics, reshaped regional power balances. To test this we consider two actors; the current marcher lord (or dominant state) and an independent state outside the control of other states.

First, the current marcher lord we assume to have at least some revenue from their empire, i.e. $\Pi(R) > 0$, control an amount of land with the range $1 > R > 0$ and have a non-zero amount of military power, $M(x) > 0$.

Secondly, a lord or city that falls outside the controlled territory of the marcher lord and therefore still maintains their independence. Like the current marcher lord they are taken to have a non-zero amount of revenue, land under their control and military power but the amount is always less than the current marcher lord or dominant state.

To test whether there is a rational calculus to adopt technology we say that these lords can pay an amount δ to substantially increase their military power. If they pay δ they will have $M_n(x)$ where $M_n(x) > M_o(x)$.

IMPLICATION 6 (INVESTMENT IN TECHNOLOGY) - *A lord will choose to invest in a new technology if the increase in revenue outweighs the cost.*

A lord will pay δ if the condition set out in Equation 16 is met. Where R_n is the new amount of land they can control and meets the following condition $R_n > R_o$. Therefore, if the revenue increase from the larger amount of land that a lord can conquer is greater than the cost of the technology, a lord will choose to invest in the technology.

$$\Pi(R_n) - \Pi(R_o) \geq \delta \tag{16}$$

We expand this Equation to consider to add additional insights by showing how the expansion of land may create the possibility for different strategies of control.

First, a state will pay the price of technology if a lord can conquer more land such that the increase in profit from the same strategy is greater than or equal to the technology cost. This is the same as Equation 16, just that it is applied to an empire ruling through army.

$$\Pi_A(R_n) - \Pi_A(R_o) \geq \delta \tag{17}$$

Second, a state will pay the price of technology if a lord can conquer more land such that the lord can upgrade their strategy and generate a profit greater than or equal to the technology cost. For this condition to hold, the factor limiting a state's strategy of control must be that they had a lack of military power to conquer a sufficient number of cities to justify a profitable transition of control strategies. An example of this is highlighted in Figure 6 where it is more profitable to adopt a strategy of compulsory cooperation unless the state can conquer a large enough realm to reach the 'strategy transition' point.

$$\Pi_{CC}(R_n) - \Pi_A(R_o) \geq \delta \tag{18}$$

There are a few important outcomes from these conditions. First, in the case of a single marcher lord they will never invest in the new technology as their total military power is not limiting their expansion, instead they are limited by the revenue and costs of the various control strategies. Conversely, where there are two marcher lords, a marcher lord will choose to invest as their expansion and controllable territory are limited by the presence of an equally strong marcher lord. Finally, an autonomous lord will also choose to invest as any increase in military power will allow them to expand, however, given their low income the price of the technology is particularly limiting. This would suggest that as long as a lord can afford the cost of the technology (that is $\Pi(R_o) \geq \delta$) they will invest in it unless they are the only dominant state.

This finding tends to support Mann's (1986) intuition, that new marcher lords arise out of the periphery of older empires. There is a lack of incentive of powerful empires to continue investing in new types of technology as there is

no return. However, lords who have capacity to grow and would but for their lack of military power will choose to invest in new types of technologies. These are the lords that reside on the periphery of empires, outside the land the old marcher lords can affordably control. Thereby setting the foundation for a new marcher lord to invest in a new type of military technology which could enable them to become dominant.

4.2 Military Technology with Probability

Unlike the first model of military technology, where investment in technology lead to a guaranteed increase in military power, we instead assume that paying $\bar{\delta}$ leads to a probability of σ that military power, and therefore size of empire, will increase. Conversely, there is a $1-\sigma$ chance that military power does not shift and the empire retains the same size. Thus the profit function is $\sigma\Pi(R_n) + (1-\sigma)\Pi(R_o)$.

A lord will choose to invest in this technology if the following condition is met. This condition is a variation of the one in the above section.

$$\sigma\Pi(R_n) + (1-\sigma)\Pi(R_o) - \Pi(R_o) \geq \bar{\delta} \tag{19}$$

This condition can be converted into a ratio.

$$\sigma \geq \frac{\bar{\delta}}{\Pi(R_n) - \Pi(R_o)} \tag{20}$$

Which represents the following terms.

$$\text{Probability of successful military development} \geq \frac{\text{Cost of technology}}{\text{Increase in empire revenue}} \tag{21}$$

IMPLICATION 7 (PERIPHERAL LORDS ARE MORE INNOVATIVE) - *States with the capacity to grow are more likely to invest in new and uncertain technologies*

The ratio in Equation 21 indicates that an empire will choose to spend the required investment in the military technology if the probability of successful military development is greater than the ratio of cost to increase in revenue. If the numerator is greater than the increase in revenue, even if the probability

of successful military development is 100%, an empire will never invest as costs outweigh the benefit. More interestingly, the smaller the cost or the greater the increase in empire profit the lower the probability needs to be. Suggesting that states will invest in very dubious technologies if they are cheap enough or offer a large increase to empire revenue. We argue that similar to the results above, it is the empires that have the greatest capacity to grow that will invest in technology most often. If we assume that a technology will result in a certain fixed amount of new land falling under the control of a state, we find that smaller empires will be more likely to adopt that technology as the increase in their realm has a higher marginal revenue than larger empires. That higher marginal revenue, and therefore revenue from the same amount of land, means that smaller empires are prepared to accept a higher cost of technology or a lower probability of success. While this is a similar result to above, implication 7 argues that a larger empire is less likely to take risks and will prefer to invest in technologies that are more *likely* to be effective and more *substantially* increase their military power such that they can conquer more land. This finding is in line with the literature from Chase-Dunn et al. (1988) and Wallerstein (1974) who argue that established empires lose the same expansionary incentive as marcher lords due to the burdens of empire.

4.3 Economic Technology

We also consider another variation where a lord or city can pay the price δ to upgrade the amount of revenue earned through their economy. This results in the following two conditions where the increase in economic revenue must outweigh the cost of technology.

If using the strategy of compulsory cooperation.

$$(2R - R^2)e_n - (2R - R^2)e_o \geq \delta \tag{22}$$

If using the strategy of common ruling culture.

$$Re_n - Re_o \geq \delta \tag{23}$$

IMPLICATION 8 (ECONOMIC INVESTMENT AMONG CITIES AND STATES)

Notably, the strategy of rule through army or rule through client will never lead to investment in these kinds of technology as there is no economic benefit/value extracted. As with the other two strategies, a lord will only invest

if the benefit outweighs the cost as changing the amount of economic revenue extracted does not change the amount of land controlled. Which raises a few cases to discuss if we assume that the decision to invest in economic technology is mutually exclusive with the decision to invest in military technology. We will assume all cases are using the common ruling class strategy.

1. If $\Pi(R_n) - \Pi(R_o) \geq \delta$ does not hold but $Re_n - Re_o \geq \delta$ then the state will choose economic technology over military as an improvement in military technology is not inhibiting the state because it results in the conquest of an unprofitable amount of land either because the new level of military power is insufficient to conquer any land or results in the conquest of an amount of land where the costs of empire outstrip the revenues generated.

2. If the opposite to case one is true then the state will choose military technology over economic technology as the economic technology is insufficient to justify its costs.

3. If $\Pi(R_n) - \Pi(R_o) \geq Re_n - Re_o$, which can also be written as $\Pi(R_n) + Re_o \geq Re_n + \Pi(R_o)$, then increasing the realm is superior to improving the economic revenues of the land under the state's control.

4. If the inverse is true, $Re_n - Re_o \geq \Pi(R_n) - \Pi(R_o)$, or $Re_n + \Pi(R_o) \geq \Pi(R_n) + Re_o$, then the improvement to the realm already under control is superior to conquering more land.

These series of cases provide some insight to Mann's (1986) discussion of Greece and Phoenicia where he notes the economic focus of these two cultures. He argues they become more mercantile due to their interstitial point between other empires. Our line of reasoning supports a similar conclusion. Greece and Phoenicia became more mercantile and invested in economic technologies, i.e. the development of currency, as the returns were greater than the same investment in military technology, the military technologies on offer were insufficient to enable them to transition to a militaristic empire, or their individual costs of empire were too great thus justifying development of the land/cities they already possessed.

5 Case Studies

We briefly consider two case studies. First, we consider Akkad which is

Mann's preeminent example of the marcher lord, arguably it is history's first marcher lord. Second, we consider Egypt which is a special case where there is no marcher lord.

5.1 Akkad

The region of Akkad sat on the edge of Mesopotamia, in the highlands around the agricultural heartland tied to the Tigris and Euphrates rivers (Mann, 1986). The people of this region developed expertise in archery as additional food sources were required to supplement relative the lack of agricultural production. The region of Akkad was not one of the first states nor were they a major player in the region till the rise of Sargon. Their relation with the more dominant cities was one of subservience (seen in the exchange of hostages) and their role as border guards to protect the trade routes of the core Mesopotamian cities (Mann, 1986). Sargon revolutionalised the area by combining a more powerful military (utilising a combination of predominately infantry paired with an archer cohort) and the use of compulsory cooperation that enabled his empire to spread further than any previous Mesopotamia empire.

The dominance of the Akkadians arises from the situations suggested by implications 1 and 2. Akkad's location relative to the periphery enabled them to control a more powerful army than their neighbours. Sargon's infantry was relatively more valuable than the use of archers (Mann, 1986) but the use of archers added significantly to Akkad's military power as other Mesopotamian lords lacked these soldiers. Our model supports this argument as Akkad's location gave Sargon better access to a type of soldier that was in relatively low supply due to the highlands being in only one part of Mesopotamia.

Eventually, the Akkadians fell to other Mesopotamian cities, the details as to why are unclear so it is hard to draw any specific conclusions. However, we do know that the region was conquered by a foreign peoples that brought new and advanced chariot technology enabling them to win battles against Mespotamian lords. Mann claims that this technological development was exogenous or effectively the result of 'luck'. Out model provides a possible incentive as this new chariot technology may have been seen as having the potential to conquer the whole region of Mesopotamia but with a low probability of doing so. Thus explaining why such a peripheral people developed the technology and why more developed states adopted the technology after seeing the technology's effectiveness.

5.2 Egypt

Egypt provides a very different case to Akkad and Mesopotamia and is only

briefly considered by Mann (1986). Egypt was heavily circumscribed around the Nile river, i.e. a fertile area surrounded by desert, allowing rulers to centralise to an extent that no other state could at the time (Carneiro, 1970, Mann, 1986, Allen, 1997, Mayshar et al., 2017). A coincidental effect of the circumscription due to desert was the lack of another abutting region that specialised in a type of warfare that was on par with what was practiced around the Nile. As such, being closer to the periphery did not lead to increases in military power rather peripheral lords actually were less powerful. Thus making Egypt a special case as the marcher lord is the strongest lord at the center or core of a soldier producing region, rather than always being slightly offset from them. In later periods, the deserts were able to produce another type of solider that would empower lords at the periphery but in ancient history this was not so (Mann, 1986). Levine & Modica (2013) note that Egypt was only pressured to change its institutions when threatened by external forces from beyond the neighbouring desert. Our model supports this reasoning as Egypt could only be threatened by external threats that were beyond its own region as the militarily strongest lords will always be located at the centre. Therefore, due to the lack of peripheral threats there was no need to continue developing new military technologies which led to Egypt being conquered by invaders at a similar time to Mesopotamia (Mann, 1986).

6 Conclusion

The illustrative models we consider support the arguments made by Mann (1986) and provide additional insights to his theories. We find the strongest lord is closer to the periphery in order to amass a diverse military by taking advantage of the soldier spcialisations across multiple regions. We extend on Mann's hypothesis with two main implications. First, the relative value and supply of each type of soldier is important to finding the location of the marcher lord as there is a trade off of accessing each type of solider. Second, the marcher lord sits in a range of locations which is always closer to the periphery or intersection between regions than the strongest infantry or cavalry lord. Mann (1986) only considered the case where the marcher lord must sit exactly between two regions because he assumed the relative value of each type of soldier was the same. We show that location varies due to implication 1 meaning Mann's definition of a marcher lord is a special case.

Modelling Mann's strategies of empire highlights the substantive trade-offs between different strategies and size of empire. While we find that broadly common ruling class is better than compulsory cooperation which is better than ruling through army, we add depth to this initial claim. Implication 3 shows

that any lord with sufficient military power to conquer other cities can create an empire as ruling through clients or army is a very cheap strategy. However, it is overshadowed by the effectiveness of compulsory cooperation which is only viable for empire's of sufficient size to afford it. This finding suggests that the strategy of control impacts state capacity as each has different revenue raising capabilities and thus the budget available to states. It is worth noting that ruling through army or client is not a totally ineffective strategy as it does enable a relatively large realm to be controlled.

Implication 4 is consequential in the literature regarding the size of empires or countries. The transition from ruling through army or client to compulsory cooperation makes sense in most cases as the strategy transition reflects the ability for a state to control more land and extract more revenue. Transitioning from compulsory cooperation to common ruling class is less clear cut as there is a gulf where even if the state can conquer the land it is less profitable to do so as it taxes their current strategy too much but is also insufficient to fully utilise the advantages of a more expensive strategy. Finally, implication 5 highlights the true advantage of common ruling class, it enables states to control the largest amount of territory as it overcomes the logistics of control that mean there are diminishing returns to scale of empire.

Lastly, our consideration of technology has two important implications. We extend Mann's analysis by introducing a degree of endogeneity and choice to individual actors as to why they would or would not adopt technology. Mann's assumption is that a state will always adopt new technologies but they are hampered by exogenous technological advancement. Implication 6 and 7 reflect a very simple finding - states will invest in technology if the increase in revenue outweighs the cost. The importance of this conclusion is when it is considered with our findings from control strategies. The fact that a state is very unlikely to be able to control the whole line due to the cost of empire or having insufficient military power means there are independent states with capacity to grow but for their lack of military power. The adoption of technology gives these states a means to grow and as such they are more likely to be innovative than established empires who may choose to not invest in technology as it does not allow them to increase the size of their empire. Implication 8 attempts to contribute to Mann's analysis of Greece and Phoenicia by considering cases where a city or lord may choose to invest in economic technology and improve the territory they own rather than invest in military capacity to facilitate expansion.

Our insight into the ancient world suggests that moving first could lead to stagnation as peripheral lords were either endowed with a superior location to amass a stronger army or able to invest in new technologies due to their capacity for expansion. Furthermore, when empires had achieved a period of dominance

over their neighbours they must face the burden of running an empire which changes their incentives and might encourage a state to remain smaller than their current military power might suggest.

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