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Do Larger Committees make Better Majority Decisions with Costly Expert Information?

*Jonathan Newman**

I present a two-stage model of committee voting with costly expert information. For every member of the committee to observe and synthesise independent testimony of some fixed and known quality, a majority of the agents must contribute to its acquisition. When testimony is observed with positive probability, I show that adding agents to the committee depresses the probability with which any single agent contributes - due to free-riding - and demonstrate how, with some careful assumptions, the probability of reaching the correct decision should correspondingly fall with the committee size. Moreover, I show individuals will make more accurate decisions than all groups whose aggregated signals are, collectively, inferior to the expert testimony. In keeping with Mukhopadhaya's (2003) seminal work on the acquisition of private signals, these findings argue against arbitrarily enlarging committees to improve the quality of majority decisions but instead propose the dichotomous choice between individual decision-makers, and collectives whose aggregated signals are more accurate than the expert signal. Further research might permit agents to choose the amount of information they acquire, or model both private and expert information as costly.

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1 Introduction

The potentially adverse consequences of highly influential expert opinions in the democratic, legal and technocratic processes is of special concern in today's political climate. (Bussiere & Stracca 2010), for instance, argue European Central Bank policy-makers in the decade prior to the Credit Crunch were over-reliant on professional economists' DSGE models for informing policy decisions, contributing to heightened instability in the financial sector.

Similarly, the World Bank has historically faced sharp criticism for its imposition of privatisation and public sector cuts on developing countries under the guise of economic and financial expertise, especially in healthcare (Abbasi 1999). It was perhaps in light of these institutional shortcomings that in the month prior to the 2016 Brexit referendum, then-Lord Chancellor Michael Gove famously exclaimed Britain had "had enough of experts" (Mance 2016).

The bulk of game theoretic analyses of decision-making - especially committee voting - with expert and private information treat expert opinions as cost-less (see Austen-Smith & Banks (1996); Kawamura & Vlaseros (2017) Morris & Shin (2002). Even those which endogenise the sources of expertise (see Liu (2019) and Jeong (2019)) impose no onus on agents to acquire testimony themselves.

I argue it is unreasonable to assume expert information is always free. As a motivating example, consider an enquiry by the House of Commons' Standards Committee into a revived expenses scandal. The committee is tasked with deciding whether to suspend senior ministers over financial infringements in a majority vote, but can first decide to commission a Cabinet Office report into their conduct if a simple majority elect to do so. Those who request the report risk retaliation from their colleagues and parties for highlighting misconduct, but would they rationally vote to commission the account if it enhanced their decision-making capacities?

By adapting Austen-Smith & Banks' (1996) jury voting game and Palfrey & Rosenthal's (1984) model of public good provision, I present a dynamic game of committee voting with costly expert information to explain strategic behaviour in these settings. To acquire expert testimony, a majority of participants must contribute. My primary findings show that so long as expert information is welfare-enhancing, decision accuracies are decreasing in the committee's size and individuals make the most accurate decisions. This is due to exacerbated incentives to free-ride on other agents acquiring expertise in larger groups, consistent with Olson (2009).

Alongside Mukhopadhaya (2003) seminal analysis of strategic voting with expensive private information and Kawamura & Vlaseros (2017) experimental study with free public information, my analysis further refutes Condorcet's famous Jury Theorem (Condorcet 2014): a common argument by which larger committees are thought superior to small ones. In the remainder of this paper, Section 2 evaluates the relevant literature. In sections 3 and 4 I explain the salient features of my model and solve for its symmetric equilibria. Section 5 investigates how different committee sizes affect contribution and signal acquisition probabilities, as well as decision accuracies. Finally, 6 concludes.

2 Literature Review

2.1 Information Aggreggation

An extended non-asymptotic version of Condorcet's Jury Theorem (Condorcet 2014) posits that when committees make a dichotomous choice between two alternatives by a majority vote, if every agent votes independently for the correct alternative with probability exceeding $\frac{1}{2}$ then i) multi-agent groups make more accurate decisions than any randomly drawn committee member and ii) the the joint decision's accuracy is increasing in the number of agents (see Miller (1986) cited in Ladha (1992) for a revived exposition).

Austen-Smith and Banks' (1996) seminal strategic analysis of committee voting with private information first demonstrated how Condorcet's assumption of independent or 'sincere' voting in majorities is generally neither consistent with following one's private information or voting rationally: it does not maximise everyone's utility subject to having the casting vote. Intuitively, prior biases toward either state can outweigh any further evidence to the contrary. The noteworthy exception is when states are equally likely ex ante and private information is equally precise in both states; in which event, the assumptions and tenets of Condorcet's Jury Theorem hold (ibid.).

In jury settings with both private and expert information, freely observed and high-quality expertise is a poisoned chalice in larger committees if agents are over-reliant on its testimony without concern for aggregating their own private signals (Kawamura & Vlaseros 2017). Experimental subjects in 7-person groups were found to obey expert opinion 97% of the time, in spite of the authors trying to promote coordination on the game's unique, welfare-maximising mixed equilibrium by making private and public signals sufficiently similar in quality (ibid.). Decision accuracies are thus fixed to the quality of expertise, and do not improve with size.

(Morris & Shin 2002) contrarily find that with strategic complimentarities (a feature not shared with Kawamura and Vlaseros' set-up), expert information actually enhances collective welfare when agents have no prior private information. Whether it improves decisions with private information as well is, however, ambiguous. My set-up and model are nonetheless products of Austen-Smith and Banks' binary choice setting, and I will more closely compare the findings with Kawamura and Vlaseros' work.

Liu (2019) and Jeong (2019) are among the few to have tackled the endogenous sourcing and presentation of expert opinions, showing biased information controllers can strategically hide public signals which indicate against their preferred alternative. Neither, however, emphasise committee agents' role themselves in acquiring expertise: a gap in the literature this paper aims to fill.

2.2 Information Acquisition

Insofar as committee decisions are non-rivalrous and non-excludable public goods (Olson, 2009b), the effort agents collectively exert in ameliorating these decisions deteriorates with each additional member as a consequence of exacerbated free-riding in larger groups (Olson 2009). When private information is obtained at agents' personal expense rather than freely observed, not only does every agent contribute less frequently, but fewer signals are acquired in larger majority-rule committees. This yields less accurate majority decisions, thereby confirming Olson's thesis (Mukhopadhaya 2003).

Persico (2004) employs a mechanism design approach to corroborate Mukhopadhaya's findings, demonstrating the optimal committee size is always finite no matter the k-threshold voting rule. More advanced studies develop optimal rules for acquiring and aggregating agents' private information, highlighting the failure of ex post efficiency in ex ante efficient procedures (Gerardi & Yariv 2008; Gershkov & Szentes 2009).

I contemplate committee design in Section 5 but my approach is closest to Mukhopadhaya (2003) in its concern, not for developing efficient procedures, but evaluating how to maximise welfare just from adjusting the committee size in an environment where everyone acquires and observes information simultaneously.

3 Set-Up

3.1 Preliminaries

Out set-up adapts Austen-Smith & Banks' (1996) Model I* voting game with private information (to which I credit much of the notation) and Palfrey & Rosenthal's 1984 fixed threshold model of discrete public good provision without refunds. I call these Γ^1 and Γ^2 respectively and quote their set-up in appendices A and B.

Consider the set $N = \{1, ..., n\}$ of agents (such that n is odd) tasked with making a collective decision $d \in \mathcal{D} = \{L, R\}$. There is a true state of the world $w \in \mathcal{W} = \{L, R\}$ and a common prior θ such that $Pr(w = L) = Pr(w = R) = \theta = \frac{1}{2}$. The timing is thus:

*Model III in Austen-Smith & Banks (1996) also explicitly models the combination of public and private signals but permits two private signals per agent, thus adding diversity to the committee (which I do not consider).

1. Agents choose whether they contribute A or not B to acquiring an expert signal $s_e \in \{0, 1\}$ of precision $Pr(s_e = 0|w = L) = Pr(s_e = 1|w = R) = q \in (\frac{1}{2}, 1)$. Contributions everywhere incur the cost of effort $c \in \Re^+$ agents incur in seeking and distributing the signal's content - which is non-refundable. Everyone observes s_e if a majority contribute. Denote i's action in this 'acquisition' stage as $x_i \in X = \{A, B\}$

2. Every $i \in N$ is then endowed with a private, conditionally independent and identically distributed private signal $s_i \in \{0, 1\}$ whereby $Pr(s_i = 0|w = L) = Pr(s_i = 1|w = R) = p \in (\frac{1}{2}, q)$

3. Agents cast their votes for $d_i \in \{L, R\}$ irrespective of whether they observe s_e. The joint decision d is made under majority rule.

Using the payoff structure in Γ^1 I specify symmetric preferences $U_i(d, w)$ whereby agents care only about matching d with w. For simplicity, let $U_i(L, L) = U_i(R, R) = 1$ and $U_i(L, R) = U_i(R, L) = 0$. From Γ^2 I determine total payoffs $\Pi_i(x_i, d, w)$:

$$\Pi_i(x_i, d, w) = \begin{cases} 1 - c & d = w,\ x_i = A \\ 1 & d = w,\ x_i = B \\ -c & d \neq w,\ x_i = A \\ 0 & d \neq w,\ x_i = B \end{cases}$$

Define agent i's strategy by the vector $\sigma_i = (\psi_i, \nu_i)$ such that $\psi_i \in [0, 1]$ is their (mixed) acquisition strategy: the probability they contribute to acquiring s_e. i's voting strategy $\nu_i : \{0, 1\} \times \{0, 1, \varnothing\} \to \{L, R\}$ maps from i's private and expert information to their vote for either alternative L or R. Note when s_e is not acquired, ν_i takes $\varnothing$ in argument; which is uninformative about w.

The solution concept is Sequential Equilibrium due to Kreps & Wilson (1982): an assessment (σ, μ) requiring the strategy profile σ is sequentially rational given the belief system μ, and that beliefs are consistent with σ.

3.2 Assumptions

I treat the committee's welfare gain from acquiring the expert signal as a discrete public good by fixing its precision. This is most appropriate for scenarios where expert information is indeed of known quality. For instance, in the introductory example it is well known senior civil servants are tasked with investigating ministerial misconduct: consider the Sue Gray report into lockdown parties at Downing Street (Cabinet Office 2022). I further argue that the majority contribution threshold can more accurately model *votes* to acquire testimony than a fixed threshold, as well as describing the more complex, proportional dissemination of expertise between agents in larger groups.

Finally, equally precise private signals and *ex ante* equally likely states are assumed for simplicity, as are cost-less private signals. I assume the latter also because it reasonable that in settings like parliamentary committees (as in my example) the reputational and political costs of acquiring one's private information are small. Further assumptions relevant to the voting stage are outlined in the next section.

4 Equilibrium Analysis

4.1 Symmetric Equilibria

I restrict my attention to symmetric equilibria: everyone has a common strategy. This will precludes the analysis of strategy profiles in which, say, $\frac{n+1}{2}$ agents always contribute and everyone else never contributes (thus acquiring the expert signal). This is important for coordination purposes: supposing agents are identically drawn from the same population and cannot repeat the game, they cannot then identify themselves as distinct from the general population. In which event, only symmetric strategies are admissible (Weibull 1997).

Definition. *The following voting strategy constitutes informative voting:*

$$v_i(s_i) = \begin{cases} L & s_i = 0 \\ R & s_i = 1 \end{cases}$$

Let the mapping $\phi : \mathfrak{N} \times \mathfrak{R} \to (0, 1)$ denote the collective decision's accuracy when everyone votes informatively. Since private signals are equally precise in either true state, $\phi(n, p)$ is the probability at least $\frac{n+1}{2}$ signals truthfully indicate w[†]:

$$\begin{aligned} \phi(n,p) &= \Pr(w = L)\Pr\left(\underset{i=0}{\overset{n}{\#}} s_i = 0 > \frac{n}{2} \middle| w = L\right) + \Pr(w = R)\Pr\left(\underset{i=0}{\overset{n}{\#}} s_i = 1 > \frac{n}{2} \middle| w = R\right) \\ &= \frac{1}{2}\sum_{j=\frac{n+1}{2}}^{n} p^j(1-p)^{n-j} + \frac{1}{2}\sum_{j=\frac{n+1}{2}}^{n} p^j(1-p)^{n-j} := \sum_{j=\frac{n+1}{2}}^{n} b(j;n,p) \end{aligned} \tag{1}$$

Wherein $b(j; n, p)$ denotes the probability of j successes occurring independently and with equal probability in a sequence of n Bernoulli trials.

Definition. *The following voting strategy constitutes obedient voting:*

$$v_i(s_e) = \begin{cases} L & s_e = 0 \\ R & s_e = 1 \end{cases}$$

[†]I borrow the notation $\underset{i=0}{\overset{n}{\#}} s_i = 0$ to mean "the number of signals with content $s_i = 0$" from Fox (2015)

That is, i follows the expert signal. When everyone votes obediently, the probability d matches w is just s_e's precision:

$$\begin{aligned} \Pr(d = w) &= \Pr(w = L)\Pr(s_e = 0|w = L) + \Pr(w = R)\Pr(s_e = 1|w = R) \\ &= \frac{1}{2}q + \frac{1}{2}q = q \end{aligned} \tag{2}$$

Notice this is fixed in n; obedient voting in large committees yields no more accurate decisions than in smaller ones. In what follows, I only admit pure voting strategies as defined above and also preclude profiles in which agents' votes ignore both their private and expert signals. Not only is this essential for tractability, but it also aligns with the experimental results due to S Guarnaschelli & Palfrey (2000) and Kawamura & Vlaseros (2017), who show everyone voting informatively in the simultaneous game without expert information (approximately 94% of the time), and obediently with outside testimony (approximately 97% of the time with seven agents) respectively, are strong predictions of equilibrium behaviour.

To satisfy sequential rationality, if v_i is a best response in the voting stage it has to maximise i's expected utility conditional on other agents' voting and acquisition strategies. By the following lemma however, I simplify the analysis:

Lemma 1. *ψ_i is uninformative about w, $\forall i \in N$. Formally,* $\Pr(\psi_i = \alpha|w = L) = \Pr(\psi_i = \alpha|w = R) = \frac{1}{2}, \forall \alpha \in [0, 1]$

Proof. Obvious - agents have symmetric preferences and the common prior puts equal probability on either state, $\theta = \frac{1}{2}$ ■

Essentially, the game possesses no signalling devices because agents do not observe any private information before deciding whether they contribute. Henceforth, voting-stage beliefs effectively condition only on agents' private information, the expert opinion (if observed) and everyone else's voting strategy.

Lemma 2. *Consider i. When everyone else votes obediently it is always sequentially rational for i to vote obediently*

Proof. See Proposition 2 in Kawamura & Vlaseros (2017).

Intuitively, when everyone else votes obediently, i is indifferent between either alternative under majority rule, since their vote d_i cannot change d. It is therefore always a best response to also vote in accordance with s_e. Though the experimental validity of everyone voting obediently is important for justifying this paper's analysis it is, intriguingly, strategically frivolous. This is because agents effectively ignore their private signals, making it indistinguishable whether everyone is playing an equilibrium or just voting sincerely, independent of other agents (ibid.)

Lemma 3. *Consider* i*. Conditional on* not *observing* s_e*, if everyone else votes informatively it is always sequentially rational for* i *to vote informatively*

Proof. The result follows from Theorem 2 in Austen-Smith & Banks (1996) and is separately shown in Appendix C

Since informative voting is rational if and only if states are *ex ante* equally likely, signals are equally precise in either state and the committee is majoritarian (ibid.), the voting threshold herein is an explicit modelling choice. I am especially concerned by the intractability of computing expected decision accuracies when agents adhere to symmetric mixed strategies proposed by Feddersen & Pesendorfer (1998) in, for example, super-majorities. That said, I also sought to emulate Mukhopadhaya's (2003) majority-rule environment for comparison.

Let $\beta(n,p,q) = q - \phi(n,p)$ be the committee's welfare gain from raising the number of contributions from $\frac{n-1}{2}$ to $\frac{n+1}{2}$ and let $\tilde{c}(n,p,q) = \binom{n-1}{\frac{n-1}{2}}\frac{1}{2}\beta(n,p,q)$; uniquely so. Before proceeding to analysis in committees of size $n \geq 3$, consider first the single-agent or 'singleton' case. My first proposition outlines how randomly selected agents would *sincerely* behave in others' absence.

Proposition 1. *Assume* $c \in (0, q-p]$*. Then the singleton contributes and votes obediently.*

Proof. For $n = 1$, β is the difference in signal qualities $q-p$, so if $c \leq q-p$ contributing is trivially rational. In the voting stage, the agent matches s_e against s_i and sincerely follows the more precise signal. ■

For the remainder of this section assume $n \geq 3$. In multi-agent committees, it is clear the following holds:

Lemma 4. *It is never sequentially rational for everyone to contribute*

Proof. Consider i. When everyone else contributes, the signal is automatically acquired. i's best response is then to not contribute, thus not incurring c ■

Consequently, the only symmetric equilibria in which s_e can be acquired with positive probability must have everyone mixing between contributing and not contributing. My next proposition establishes that when the benefit of acquiring s_e is weakly negative and/or the cost of acquiring expert information is too high, strategic agents will never contribute.

Proposition 2. *Assume one or both of* $\beta(n,p,q) \leq 0$ *and* $c > \tilde{c}(n,p,q)$*. If everyone votes obediently when observing* s_e *and informatively when not, then in the unique sequential equilibrium no one contributes*

Proof. See Appendix D

Since s_e is never acquired, the information set at which agents observe s_e is necessarily offpath. Nonetheless, beliefs are still updated according to Bayes' Rule at each information set, which makes this a sequential equilibrium (Kreps & Wilson 1982) and I derive these in Appendix F. The prescribed strategy profile and associated beliefs will always also constitute an equilibrium when $\beta > 0$ and $c \leq \tilde{c}$, because when no one else contributes i is not pivotal to acquiring s_e. Hence, their best response is to not incur c. I now establish the conditions under which symmetric positive contributions are sustained in equilibrium.

Proposition 3. *Assume both $c \in (0, \tilde{c}(n, p, q))$ and $\beta(n, p, q) > 0$. There are exactly two values of $\psi \in (0, 1)$ for which everyone contributing with probability ψ, voting obediently after acquiring s_e and informatively otherwise - with beliefs updated using Bayes' Rule - is a sequential equilibrium*

Proof. See Appendix E

In Appendix D, I further show that as a function of ψ, the probability any given agent is pivotal to acquiring s_e - and henceforth the contribution cost - is single-peaked on $(0, 1)$, attaining its maximum when $\psi = \frac{1}{2}$. Henceforth, one of these probabilities lies on the interval $(0, \frac{1}{2})$ and the other on $(\frac{1}{2}, 1)$. This was expected, since there are two equilibrium contributing probabilities in Γ^2; one to the right of the ratio $\frac{k-1}{n-1}$ (where k is the required number of contributions) and one to the left. I refer generally to symmetric sequential equilibria where everyone follows the strategy prescribed in Proposition 2 as *non-contributing*. Equally, those where everyone follows the strategy prescribed in Proposition 3 are *contributing* equilibria.

4.2 An Example

For illustrative purposes, let $p = 0.55$, $q = 0.95$ and $c = 0.05$. Recall the necessary and (together) sufficient conditions for the existence of two contributing equilibria are $\beta > 0$ and $c \in (0, \tilde{c})$. $\beta(9, 0.55, 0.95)$ is $0.95 - \sum_{j=5}^{9} b(j; 9, 0.55) = 0.95 - 0.6214 = 0.3286 > 0$. The maximum cost $\tilde{c}$ is given by $\binom{8}{4}\frac{1}{2}^{8} \cdot 0.3286 = 0.0899 > 0.05$. Hence, two contributing equilibria are guaranteed. As always, there trivially exists the non-contributing equilibrium with everyone voting informatively on-path and obediently off-path. The two contributing equilibria have everyone contributing with $\psi = 0.752$ and 0.247. See Appendix G for a full derivation and associated beliefs.

4.3 Existence

The conditions $\beta > 0$ and $c \in (0, \tilde{c})$ restrict the parameters for which two contributing equilibria exist. For example, let $p = 0.75$ and $q = 0.80$. Even when $n = 3$, it is never sequentially rational to contribute regardless of cost, because $\beta(3, 0.75, 0.80) =$

$-0.0438 < 0$. I first evaluate the conditions on n, p and q for which $\beta(n, p, q) > 0$ is satisfied. This elaborates on an arguably unique mechanism by which agents are disincentivised from contributing in the class of contribution games first modelled in Palfrey & Rosenthal (1984): the diminishing benefit of acquiring s_e.

Lemma 5. *The inequality* $\beta(n) > \beta(n+2)$ *holds*

Proof. By Condorcet's (non-asymptotic) Jury Theorem (Condorcet 2014) the inequality $\phi(n+2) > \phi(n)$ holds and of course $\beta(n) = q - \phi(n)$. ∎

Proposition 4. *For every pair* (p, q) *of signal qualities* $q \in (\frac{1}{2}, 1)$ *and* $p \in (\frac{1}{2}, q)$ *there exists a finite* $\bar{n}(p, q) \in \mathfrak{N}$ *for which* $\beta(n, p, q) \leq 0$ *in committees of size* $n > \bar{n}$. *Furthermore,* $\bar{n}$ *is non-decreasing in* q

Proof. By Condorcet's (asymptotic) Jury Theorem, $\lim_{n\to\infty} \phi(n) = 1$, guaranteeing finite existence given $q \in (\frac{1}{2}, 1)$. See Appendix H for monotonicity.

That is to say, better quality expert information sustains the existence of contributing equilibria in larger committees compared with when q is small. I illustrate this in Table 1, demonstrating also how $\bar{n}$ is both decreasing in p for any fixed q as well as for the *same* difference between signal qualities. This implies greater doubts as to agents' private decision-making competencies should inhibit the speed at which committees start to disregard the value of expert testimony, as its accuracy approaches that of their collected signals.

		q							
		0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
p	0.55	5	13	25	43	69	105	161	267
	0.60	-	3	5	9	15	25	39	65
	0.65	-	-	1	3	5	9	15	27
	0.70	-	-	-	1	3	5	7	15
	0.75	-	-	-	-	1	3	5	7
	0.80	-	-	-	-	-	1	3	5
	0.85	-	-	-	-	-	-	1	3
	0.90	-	-	-	-	-	-	-	1

Table 1: $\bar{n}$ for an array of p and q

I later refer to $\tilde{n}$ as the committee size above which no equilibria exist because the cost of acquiring s_e is too high relative to β (i.e. when $c > \tilde{c}$). I do not elaborate on its precise features but immediately observe, since $\tilde{c}(n, p, q) = \binom{n-1}{\frac{n-1}{2}} \frac{1}{2} \beta(n, p, q) \iff \tilde{c}(n, p, q) \geq \beta(n, p, q)$ for $n \geq 3$, that $\bar{n}$ is always weakly larger than $\tilde{n}$. Henceforth, by Proposition 4 I conclude $\tilde{n}$ always exists and is finite. Finally, the following claim is needed for refining the assumptions made in later propositions:

Claim 1. *The inequality* $\tilde{c}(n+2) < \tilde{c}(n)$ *holds*

Proof. See Appendix I

5 Comparative Statics and Discussion

5.1 The Contribution Probability

I now examine whether equilibrium predictions with costly testimony yield more accurate majority decisions in larger committees. Note that because n is odd, size increases are in increments of two. I first assess how the equilibrium contribution probability changes with committee size.

Regarding equilibrium selection, I defer to the 'acute' contribution probability larger than $\frac{1}{2}$ since, as highlighted in Section 4, the equilibrium cost function $c(\psi)$ is single-peaked and attains its maximum at $\psi = \frac{1}{2}$. Henceforth c is non-increasing on $(\frac{1}{2}, 1)$ and as is well known its inverse is therefore also non-increasing. As such, the acute probability falls when the cost increases, as one expects. By contrast and the same logic, the equilibrium probability smaller than $\frac{1}{2}$ rises with cost - an evidently unintuitive prediction. In what follows, assume non-singletons adhere to the acute probability (now generally denoted ψ) when costs are less than the threshold for two contributing equilibria. For legibility, I write $\psi(n)$ as ψ_n.

Proposition 5. *Assume* $c \in (0, \tilde{c}(n+2))$ *and* $\beta(n+2) > 0$. *Then the inequality* $\psi_{n+2} < \psi_n$ *holds.*

Proof. See Appendix J

Simply put, agents are less likely to contribute in bigger committees. Indeed, it is well established that *both* contribution probabilities in Γ^2 are negatively associated with group size (see Offerman 1997; Hindriks & Pancs 2002; Nöldeke & Peña 2020). Both there and in this model, such behaviour is consistent with deteriorating individual effort put towards the improvement of common goods - and committee decisions in particular - due to exacerbated incentives to free-ride on other agents' contributions in larger collectives (Olson 2009).

Figure 1 graphically illustrates this important result for $n \leq 35$ and an array of cost parameters with signal accuracies $p = 0.55$ and $q = 0.95$. Note further how, as demonstrated in Appendix J, the smaller benefit to acquiring s_e (noted in Lemma 5) strictly depresses ψ in larger groups compared with if β were fixed. I propose this further highlights the important role the restriction $\beta > 0$ plays in 'driving' contributing equilibria from existence.

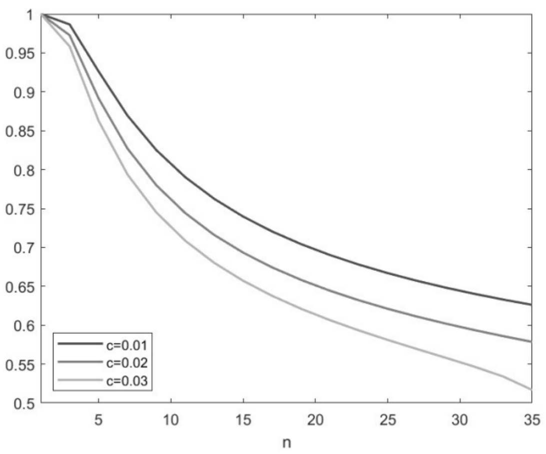

Figure 1: Plot of the Contribution Probability ψ

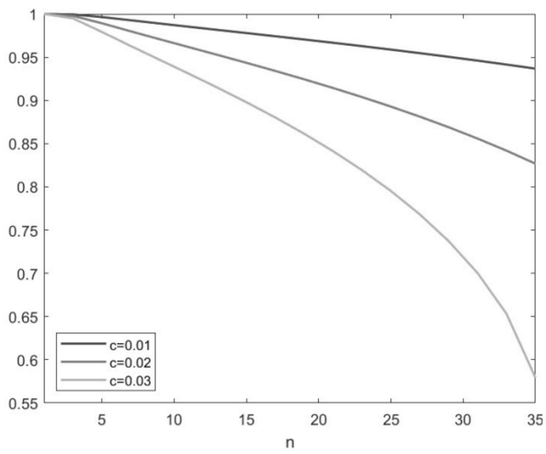

Figure 2: Plot of the Acquisition Probability Ψ

5.2 The Acquisition Probability

The acquisition probability denoted Ψ is the probability at least $\frac{n+1}{2}$ agents contribute, and thus with which the committee observes s_e:

$$\Psi_n = \sum_{j=\frac{n+1}{2}}^{n} b(j; n, \psi_n) \tag{3}$$

Given the singleton always contributes with probability $\psi = 1$ (assuming $c \leq q - p$), it is obvious they will always acquire s_e if costs are small; it is also clear, from Lemma 4, that multi-agent committees always acquire with probability strictly less than 1. Supposing $n \geq 3$ however, even if I assume $\beta(n+2) > 0$ and $c \in (0, \tilde{c}(n+2))$ it is not obvious Ψ gets smaller with the committee size. To see why, note how by Proposition 5 $\psi_n > \psi_{n+2}$ holds, and it is well known the binomial cumulative distribution function (with n fixed) is strictly decreasing in the probability of success (see Schmetterer, 2012: Theorem 32.2). By contrast, recall also that by Condorcet's (non-asymptotic) Jury Theorem $\sum_{\frac{n+3}{2}}^{n+2} b(j; n+2, p) > \sum_{\frac{n+1}{2}}^{n} b(j; n, p)$; meaning if ψ were fixed in n, adding more agents would make Ψ larger.

Henceforth, the sign of the change in Ψ from adding two members is apparently ambiguous. Similarly, in Γ^2 ambiguity in the effect of group size on the probability of providing discrete public goods is observed in Hindriks & Pancs (2002) by analogous reasoning[‡]. Instinctively, in Palfrey and Rosenthal's model, whenever n increases there are more agents to supply the same number of contributions despite each agent being less likely to do so.

Nöldeke & Peña (2020) nonetheless prove the Γ^2 provision probabilities deteriorate unambiguously with group size and there are two significant reasons the same should

[‡]More generally, it is well known the binomial cumulative distribution function's upper tail $\sum_{j=k}^{n} b(j; n, p)$ is non-decreasing in n

hold in this model. First, recall that since β falls with n the acute probability is necessarily smaller in bigger groups than if the benefit were fixed. Furthermore, although the majority threshold shrinks as a proportion of group size when 2 agents join the committee, it also increases in magnitude. This should exacerbate the consequences of accentuated free riding at the individual level in larger groups, who must now aggregate more contributions to acquire s_e.

Since both the success probabilities and threshold vary with size, it was computationally infeasible to show $\Psi_n > \Psi_{n+2}$ holds. However, I simulated the acquisition probability for an array of reasonable cost parameters and found, as expected, that Ψ was correctly decreasing in n. These are plotted in Figure 2, again for $p = 0.55$ and $q = 0.95$. The pattern is indeed further consistent with Olson's (2009) proposed ramifications of individual free-riding for the quality of common goods: agents in larger groups should collectively exert less effort in ameliorating their joint objectives.

5.3 The Decision Probability

Finally, consider now the primary variable of interest: the probability, in expectation *ex ante*, that the committee's decision d matches w, which I denote Φ:

$$\Phi_n = q\Psi_n + \phi(n)[1 - \Psi_n] \tag{4}$$

Proposition 6. *Assume* $c \in (0, q - p]$. *Then* $\Phi_1 > \Phi_n, \forall n \in \{3, ..., \bar{n}\}$. *Furthermore,* $\Phi_1 \leq \Phi_n, \forall n > \bar{n}$.

Proof. The singleton's acquisition probability is always 1 when $c \leq q-p$, and therefore $\Phi_1 = q$. Furthermore, since it is never rational for everyone to always contribute in committees of size $n \geq 3$, the acquisition probability in multi-agent committees is strictly less than 1. So long as $n \leq \bar{n}$ this implies $\Phi_n < \Phi_1$. Finally, from Proposition 2 everyone votes informatively and therefore makes decisions with accuracy $\phi(n) > q$ when $n > \bar{n}$ ■

This is a powerful result. So long as the contribution cost is less than the difference in signal qualities (and hence the singleton's benefit from acquiring s_e), it tells us *every* committee smaller than $\bar{n}$ will perform worse than the singleton in a majority vote. It is thus evident Condorcet's Jury Theorem does not generally hold with costly expertise, since if costs are sufficiently small the singleton will make strictly better decisions than all other committees for whom expert information is welfare-enhancing.

Because of ambiguity in the sign of $\frac{\Delta\Psi}{\Delta n}$, I will not try to sign the change in Φ for every parameter configuration. Furthermore it is is not obvious, even if one assumes Ψ is decreasing in n whenever contributing is undominated, if Φ follows suite. Whilst agents would make the correct decision with accuracy $q > \phi$ less often, ϕ itself gets larger with more agents. Henceforth, the terms in 4 work against each other. However, in Figure 3 I illustrate that for an appropriate set of cost parameters (for $p = 0.55$ and $q = 0.95$), Φ is correctly decreasing in n so long as $\beta(n) > 0$ and $c \in (0, \tilde{c}(n))$ hold - further contravening Condorcet's (non-asymptotic) Jury Theorem. This suggests the negative free-rider consequences of transitioning to larger committees (thus depressing Ψ) outweigh the advantages of more agents piecing together their private signals whenever s_e is not acquired. Indeed, this decline in Φ with committee size mimics the deteriorating decision accuracies in Mukhopadhaya's (2003) seminal work with expensive *private* signals. Similarly, their paper also unveils tension between the free riding and 'pure numbers' effect in determining decision accuracies. Nonetheless, as with our work an array of simulations suggest the former phenomenon is strongest.

However, it is evident this pattern only holds if contributing is undominated. Since, whenever $n > \tilde{n}$ only the non-contributing equilibrium abounds, decision accuracies necessarily rise with group size, in accordance with Condorcet's Jury Theorem, when everyone votes informatively. Indeed, as stated in Proposition 5, when $n > \tilde{n}$ these exceed the singleton's accuracy. Figure 4 illustrates this 'cut-off' and subsequent resurgence in Φ when n exceeds $\tilde{n}$ (whereby $\tilde{n} = 35$ for $p = 0.55$, $q = 0.95$ and $c = 0.03$), demonstrating how - unlike in Kawamura & Vlaseros' (2017) model with free public information - expert testimony is at least never *binding* on decision accuracies.

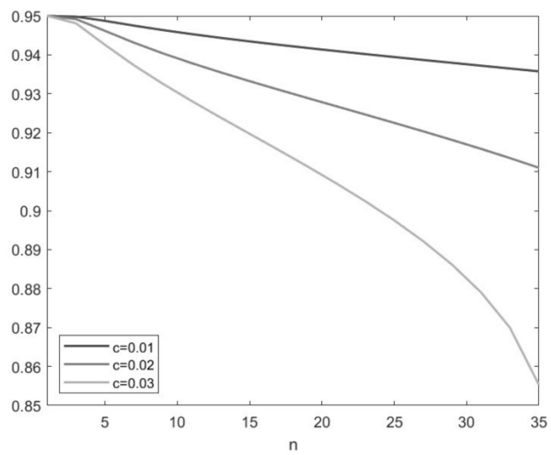

Figure 3: Plot of the Decision Probability Φ

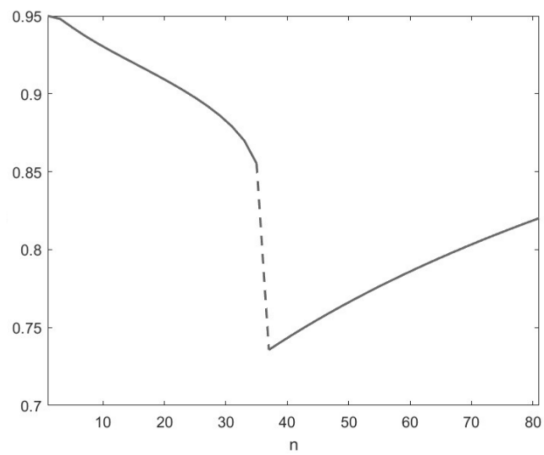

Figure 4: Plot of the Decision Probability 'Cut-Off'

In practice therefore, when agents can acquire outside information at cost these results should deter so-called 'social planners' from selecting committees barely larger than the singleton, in the belief their aggregated private signals are a boon to their decision qualities. However, since s_e is not binding on Φ whenever not contributing is dominant, it presents social planners with the dichotomous choice between singleton committees, and collectives larger than $\bar{n}$. In which instance, I propose planners defer to the wider range of criteria proposed in, for instance, Karotkin & Paroush's 2003 "quality versus quantity" debate. In particular, if social planners need to acquire agents themselves at cost (ibid.), this should favour the smaller alternative.

6 Conclusion

I have sought to define the basis on which rational agents would acquire costly expert testimony, and how this impacts the quality of collective decisions in committee votes. When obedient voting with expert testimony is welfare-enhancing, if agents conform to the intuitive 'acute' equilibrium contribution probability the committee should make decisions with poorer accuracy when it has more agents, due to free-riding. Moreover, individuals are not disadvantaged against marginally larger groups.

These findings largely concur with Mukhopadhaya's (2003) early analysis of voting with costly private signals, insofar as they highlight the shortfalls of arbitrarily enlarging committees for the benefit of aggregating more private signals when information sources are costly. Social planners may nonetheless choose between singleton committees, who invariably acquire and obey testimony when acquisition costs are small, and a much larger group who entrust their private capacities when expert information is of no extra benefit.

Further research should take advantage of this work's limitations. One clear extension is to present agents with a menu of costly private and public signals, thus combining our set-up with Mukhopadhaya's. I anticipate decisions will deteriorate in quality as the two original models suggest, but it would be noteworthy to observe whether sufficiently expensive expert information could promote coordination on acquiring private information with greater probability. In addition, though I justified the application of fixed-quality expertise for when opinion is indeed of known quality, this should not be the case in competitive markets (for instance, consultancy). In the spirit of Martinelli (2006) one could specify the expert signal's precision as an increasing function of agents' total contributions: more accurately modelling expertise-seeking when testimony varies in quality.

Permitting heterogeneous preferences is another attractive adjustment. If signals in different states were not equally precise and agents observed their private information before deciding to contribute, this would make the number of contributions a signal about the true state. Whilst this would make calculating best responses in the voting stage more complex, it would arguably better describe differences in opinion between politicised agents as described in the introductory example. Finally, I cited the intractability of computing expected decision accuracies when agents adhere to symmetric mixed strategies as a reason for my majority threshold. My results cannot, therefore, explain behaviour in super-majorities or unanimous groups like juries. I propose future work correct for this oversight, perhaps facilitating asymmetric pure strategies without expert information as adopted in Persico's (2004) mechanism design approach.

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