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Dynamic Personalized Pricing with Active Consumers

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Jeremy Smith (Head of the Department of Economics, University of Warwick) and Michael Ward (Head of the Department of Economics, Monash University)

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Dynamic Personalized Pricing with Active Consumers

Xiaolei Wang*

Abstract

We study a two-period duopoly model where firms gather consumer data from first-period customers then use them for second-period personalized pricing, with a focus on active consumers who can bypass price discrimination with identity management (IM). As a result, IM weakens competition and allows firms to adopt perfect price discrimination which gives massive profit for firms in the personalized-pricing stage. Anticipating this, firms engage in below-cost pricing in the first stage to compete for consumer data. This strategy is similar to predatory pricing not only because of below-cost pricing but firms can also recoup losses later, however, we show that in this case below-cost pricing is driven by competition and beneficial to consumers.

Keywords: Personalized pricing, behavior-based price discrimination, identity management

JEL classifications: D43, L13, L5

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1 Introduction

Increasingly firms can price discriminate consumers base on their purchase history or other records, due to the availability of large individual-level datasets. In addition, with the rapid advancement in information technology firms gain high accuracy at predicting the willingness to pay of every customer, therefore conditioning on consumers' prior behavior personalized pricing has become feasible. There are many ways a firm can acquire consumer data in the first place, for example tracking browsing histories with cookies, directly looking up purchase records in customer accounts, etc. One example of utilizing such tracking tools for price discrimination is charging higher hotel prices for Apple users ('On Orbitz, Mac users steered to pricier hotels', *The Wall Street Journal*, August 23, 2012), this can be done by simply looking up the HTTP request sent by the customer. It is also well-known that Uber implements "route-based pricing", meaning it prices customers base on predicted demand for a certain route ('Is your friend getting a cheaper Uber fare than you are?', *Forbes*, April 14, 2018). Although online shopping is genuinely easier for price discrimination to carry out, it is evident that supermarkets use loyalty program and mobile apps to collect individual data and recommend personalized coupons ('Individualized coupons aid price discrimination', *Forbes*, August 21, 2012).

Although firms can take advantage of customer characteristics and purchase history for price discrimination, consumers are not at all defenseless, they can use various technologies to remain untraceable. It only takes minutes for one to disable cookies in the browser. Cryptocurrency is a widely accepted payment method, and paying with them is mostly anonymous. Installing an ad blocker can also help, because online advertisements have switched from one-to-many to one-to-one and they trace you all over the web. Or, simply reject to sign up for loyalty programs. We refer to such kind of behaviors taken by consumers as *identity management* (Acquisti, 2008), henceforth IM for simplicity. As consumers become more aware of data-driven price discrimination, they will choose to engage in IM for a better deal, or just for privacy concerns.

There is a large strand of literature on behavior-based price discrimination (BBPP), that is discriminating consumers base on their past purchase history. The general agreement of the literature on competitive price discrimination is that, more consumer information is utilized for pricing strategy leads to more intense competition between firms. In the competitive BBPP environment, it was shown that third-degree price discrimination makes firms worse off than when they compete without segmenting consumers (Shaffer and Zhang, 1995; Chen, 1997; Villas-Boas, 1999; Fudenberg and Tirole, 2000; Pazgal and Soberman, 2008; Esteves, 2010). In addition, personalized pricing leads to more intense competition than third-degree price discrimination (Fudenberg and Tirole, 2000; Choe et al., 2018). The reason why price discrimination intensifies competition is rather intuitive, when firms adopt finer pricing strategies they have a better means to protect their targeted customers (those that they can price discriminate), at the same time poaching from rivals with a low price does not affect targeted customers, so better protection essentially intensifies competition.

The results from non-BBPD literature mostly agree with the finding that a higher degree of discrimination leads to a higher degree of competition (Thisse and Vives, 1988; Chen and Iyer, 2002). But Chen et al. (2020) showed that this is not true when IM is taken into account, hiding identity weakens competition and consumer surplus can be exactly zero. The reason of this opposite finding is that, when consumers are active in IM, firms cannot freely poach from rivals with a low price anymore, because their targeted consumers have access to this low price as well. Therefore, firms may choose to not poach at all which allows them to perfect price discriminate, this resembles tacit collusion between firms. However, non-BBPD models are essentially static, whether this result holds in a more general and dynamic context is uncertain. We show that by incorporating IM in the BBPD environment personalized pricing still intensifies competition, but this happens in the data-collection stage rather than the price-discrimination stage.

In this paper, we study competitive pricing strategies in a dynamic context where firms collect data in the first period then use them for second-period personalized pricing, with a focus on consumers who can be active in IM. Thus, we fill the gap in the literature between behavior-based personalized pricing and identity management. Formally, we consider a two-period duopolistic model with firms reside at two ends of a Hotelling linear city, consumers' position represents their brand preferences. Following the BBPD literature, we assume firms compete with uniform prices in the first period to collect data, then in the second period, they set personalized prices for those targeted consumers who have purchased from them and a uniform poaching price for non-targeted consumers. On the other hand, consumers can gain access to the uniform (poaching) price from the targeting firm by hiding identity. So, unlike passive consumers in traditional price discrimination studies who can only choose between the discriminated price from the targeting firm and the uniform price from the non-targeting firm, consumers in our study have an additional third choice.

Before showing our main findings, we start with an overview of personalized pricing and IM in the static competition. When there are active consumers in the market, a firm is happy to not to poach its rival if it is endowed with sufficiently large data on consumers. Because active consumers have access to the uniform poaching prices, so if one firm decides to poach, its targeted consumers will prefer this poaching price as well since they value this price more than those non-targeted consumers. Causing the firm to give up all personalized pricing profit, clearly it is not willing to lose this profit when it has a large amount of data. Therefore, when both firms have collected a sufficient amount of data on consumers, they are both happy without poaching which allows them to perfect price discriminate consumers. On the contrary, if a firm is endowed with a small amount of data, it chooses to poach and give up personalized pricing profit, because the small amount of data does not generate higher profit than poaching. We show that only the perfect price discrimination (PPD) equilibrium in the static personalized pricing competition exists in our dynamic context. Because in the second period, PPD profit is much higher than the poaching profit, also giving up data collection in the first period to serve a small segment of consumers is not sensible, hence firms compete for more data in the first period to achieve PPD later.

The most important finding from our study is that competition in the data-collection stage is highly intensified and it is possible for firms to engage in below-cost pricing, leading the total industry profit to be lower than the non-discriminatory benchmark. This result matches the general agreement in the BBPD literature that, a higher degree of price discrimination intensifies competition, however, our finding differs in the sense that competition is intensified in the data-collection stage rather than the price-discrimination stage. The reason why competition is intensified is as follows. In the data-gathering stage, a firm faces the trade-off between the current profit and the later PPD profit. Given the rival's first-period price, it can choose a price to maximize the current profit or lower it for more data and higher PPD profit, since raising it makes the firm strictly worse off (does not maximize first-period profit and collects less data). The question is then how much the firm is willing to charge below the optimal price for current profit, if consumers have a high valuation for its product hence high PPD profit, then the firm finds second-period PPD profit more appealing and will charge a significantly lower price.

In addition, the finding of below-cost pricing has an important policy implication because it is related to predatory pricing. A dominant firm can engage in predatory pricing by lowering its price to drive out competitors in the short run, then raise its price back to the monopoly level to recoup the previous loss, this action can be illegal when it harms consumers. Since below-cost pricing is not profitable in the short run and it can only happen for other strategic purposes, it has become an important test for predatory pricing in courts. However, we have shown that below-cost pricing can be the natural result of competition by considering personalized pricing and IM. Below-cost pricing in this case does not harm but benefit consumers, in fact, a naive above-cost regulation in this case can hurt consumers since it only puts an artificial threshold on the competition level between firms.

We contribute to Chen et al. (2020) by directly extending their static model to a dynamic one using BBPD. Showing that their salient result holds in a more general context, however contrary to their finding, consumers can gain massive surplus in the data-collection stage as firms compete with below-cost prices. Also, we significantly extend the current literature on BBPD in an important way by considering active consumers who can avoid price discrimination by hiding identity. As consumers gain experiences in price discrimination and concerns for privacy, IM becomes the natural choice and is inevitable in the discussion of price discrimination. By incorporating this factor, we show that the period in which competition takes place is reversed. Existing literature shows that competition is intensified in the period of price discrimination due to better protective strategy, but we show that IM completely weakens competition in the price-discrimination stage and intensifies competition in the data-collection stage due to the trade-off between intertemporal profits.

2 Model

Consumers are uniformly distributed on $[0, 1]$ in a Hotelling linear city, two firms A and B are located at 0 and 1 respectively. Firms produce homogeneous products with the same constant marginal cost c, they do not discount future profits, but consumers are myopic such that they discount future utility completely.

There are two periods in this model, denote by $\tau = 1, 2$. If a consumer buys from firm i in $\tau = 1$ then firm i can charge a personalized price for her in $\tau = 2$, and we will call her a targeted consumer of firm i. Each firm sets a uniform price r_i in $\tau = 1$, then in $\tau = 2$ it sets a uniform price q_i and personalized prices $p_i(x)$. The utility of the consumer locates at x (or consumer x for simplicity) from buying one unit of firm A's product in $\tau = 1$ is defined as:

$$U(x, r_A) := v - r_A - tx$$

and similarly the utility of buying from firm B is defined as:

$$U(x, r_B) := v - r_B - t(1 - x)$$

Where v is the perfect brand valuation for all consumers, and t is the travel cost. Second-period utilities $U(x, q_i)$ and $U(x, p_i(x))$ are defined in analogous ways.

Following the standard in the literature (Thisse and Vives, 1988; Choe et al., 2018; Chen et al., 2020), the timing of the model is as follows. In $\tau = 1$, firms simultaneously set uniform prices, then each consumer buys one product from either firm A or B. In $\tau = 2$, there are two stages, in the first stage firms simultaneously set uniform prices, in the second stage firms simultaneously set personalized prices for each consumer, finally, consumers make the second-period purchase decision. When a consumer is indifferent between a uniform price and a personalized price from two firms, we assume the consumer prefers the personalized price as it can always be reduced by an arbitrarily small amount.

We focus on the situation which all consumers are active in identity management, and assume they incur 0 costs for doing so. Then consumers have three choices in $\tau = 2$, for example, if consumer x is targeted by firm A then clearly she can choose from $p_A(x)$ and q_B, additionally by hiding her identity she has access to q_A as well.

For future reference, we include two benchmark results adapted to our setting. First, in the standard Hotelling model where firms do not price discriminate, in both periods firm A serves $[0, 1/2]$ and firm B serves $[1/2, 1]$ with the same price $t + c$, hence the total profit of both firms are $\Pi_A = \Pi_B = t$ and consumer surplus is given by $2v - 2c - 5t/2$. Second, in Choe et al. (2018) firms engage in behavior-based personalized pricing which is the same as our setup except consumers are passive in IM, in this case there exist two asymmetric equilibria, firms charge $(r_i = \frac{10t}{13} + c, r_j = \frac{8t}{13} + c)$ in $\tau = 1$, they earn $(\Pi_i = 0.572t, \Pi_j = 0.645t)$ as total profits and consumer surplus is given by $2v - 2c - 1.726t$.

3 Fully-covered market

In this section, we consider equilibrium outcomes when all consumer purchase, the case when some consumers do not purchase is analyzed in the next section of partially-covered market. Because the market is assumed to be fully-covered, there can only be two cases in $\tau = 1$, (i) there is an indifferent consumer δ such that consumers to the left of δ is targeted by firm A, and consumers to the right of δ is targeted by firm B, or (ii) there is a segment of consumers $[b, a]$ that is targeted by both firms, $[0, a]$ is targeted by firm A and $[b, 1]$ is targeted by firm B. However, consumers only buy one product in each period, hence (ii) is not feasible. Then, solving $U(\delta, r_A) = U(\delta, r_B)$ gives the indifferent consumer $\delta = (r_B - r_A + t)/(2t)$. The concept of equilibrium we will adopt is subgame perfect Nash equilibrium (SPNE), we will solve the game by backward induction starting from the second period.

3.1 Second period

We begin with an important observation that differentiates active and passive consumers. When consumers are active in IM, if a firm wants to poach some non-targeted consumer with a low uniform price, it will need to give up all personalized pricing profit. Because its targeted consumers have access to its uniform price as well and since intrinsically they have a higher valuation for the closer firm, they will prefer it more than any non-targeted consumer.

Since v is the highest possible utility any consumer can obtain, v can be viewed as a threshold of the price that consumers are willing to pay. Then we can formally define perfect price discrimination (PPD) as both firms charge a uniform price above v so that no consumer is willing to pay for it, and they set personalized prices such that their targeted consumers derive 0 utility from purchasing. Then, intuitively two types of equilibrium can happen $\tau = 2$.

□ *PPD equilibrium.* This type of equilibrium is possible when both firms have a sufficiently large segment of targeted consumers, or equivalently when δ is around $1/2$. Both firms are happy with the initial data collection and the profit they can make with PPD. Neither of them has an incentive to deviate, lowering the uniform price below v to poach non-targeted consumers is less profitable then PPD. Then, in this case, the profit of firm

A can make from consumer x is exactly equal to her valuation $v - tx$, subtracting off the marginal cost and integrate from 0 to the indifferent consumer δ we have firm A's PPD profit,

$$\pi_A^{PPD} = \int_0^\delta v - c - tx \; dx = \frac{\delta\left(-\delta t - 2c + 2v\right)}{2}$$

similarly, firm B's PPD profit is given by,

$$\pi_B^{PPD} = \int_\delta^1 v - c - t(1-y) \; dy = -\frac{\delta^2 t}{2} + \delta\left(c + t - v\right) - c - \frac{t}{2} + v$$

□ *One-way poaching equilibrium.* This type of equilibrium is possible when the majority of consumers are targeted by a single firm, or equivalently when δ is close to 0 or 1. For instance, suppose δ is close to 0, clearly firm A does not find PPD profitable because it has a very small targeted segment of consumers. Then, it can give up personalized pricing profits and poach with a low uniform price q_A. Responding to this, firm B cannot keep its PPD pricing because it will lose all customers to firm A's low poaching price. Instead, firm B can try its best to make its targeted consumers indifferent between $p_B(y)$ and q_A, that is $p_B(y) = max\{c, q_A + t(2y-1)\}$. So, consumers close to 1 will be indifferent and firm B competes with firm A for consumers that are far away from 1. Similarly, when δ is close to 1, firm B poaches with a low uniform price, and firm A responses by optimizing personalized prices.

Formally, the second-period equilibria are described in the following lemma.

lemma 1 *Let* $L = \frac{2v-2c-\sqrt{(2v-2c+t)(2v-2c-t)}}{t}$ *and* $R = 1 - L$. *When the market is fully covered in* $\tau = 1$, *base on the first-period indifferent consumer* δ *there are three equilibria in* $\tau = 2$*:*

1. *PPD equilibrium. When* $L \leq \delta \leq R$, *firms engage in PPD. That is,* $p_A(x) = v - tx$, $p_B(y) = v - t(1-y)$ *and uniform prices above* v. *The equilibrium profits are* $\pi_A = \frac{\delta(-\delta t - 2c + 2v)}{2}$ *and* $\pi_B = -\frac{\delta^2 t}{2} + \delta\left(c + t - v\right) - c - \frac{t}{2} + v$

2. *One-way poaching by firm A. When* $\delta \leq L$. *Firm A finds it profitable to lower its*

uniform price below v hence by lemma 2 we can ignore its reasonable personalized price, and firm B keeps its uniform price above v. Then, $q_A = \frac{t}{2} + c$ and $p_B(y) = max\{c, \frac{t(4y-1)}{2}\}$. Profits are $\pi_A = \frac{t}{8}$ and $\pi_B = \frac{3t}{16}$

3. *One-way poaching by firm B. When $\delta \geq R$. Firm B finds it profitable to lower its uniform price below v hence by lemma 2 we can ignore its reasonable personalized price, and firm A keeps its uniform price above v. Then, $q_B = \frac{t}{2} + c$ and $p_A(x) = max\{c, \frac{t(3-4x)}{2}\}$. Profits are $\pi_B = \frac{t}{8}$ and $\pi_A = \frac{3t}{16}$*

proof *See Appendix A.*

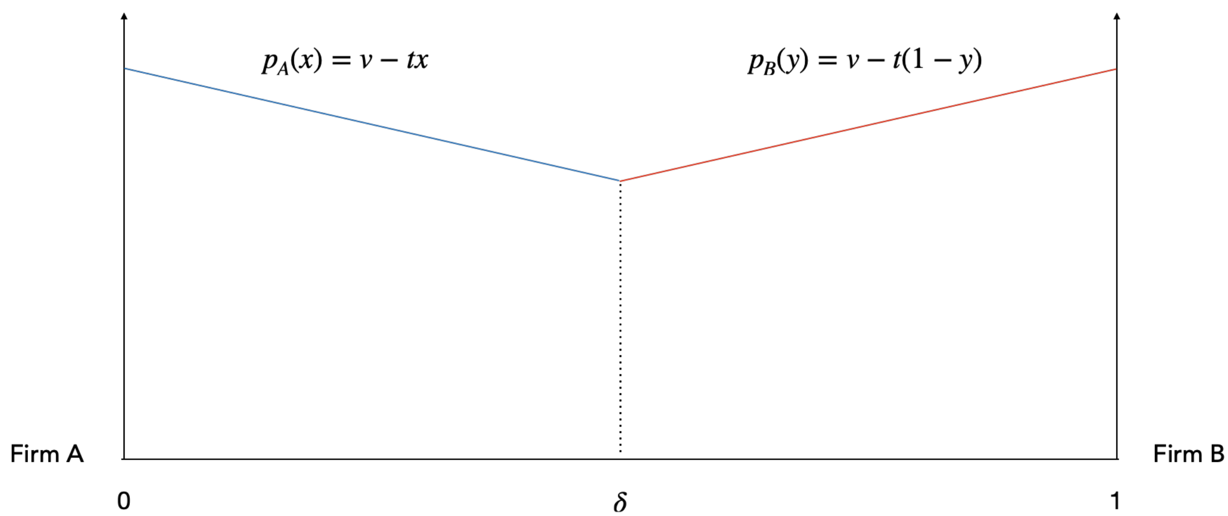

Figure 1: The PPD equilibrium in $\tau = 2$.

3.2 First period

The strategy for solving the first-period game is as follows, calculate the total profit over two periods of both firms, derive the best responses and see if there is a match between them, if there is a match then check for deviation conditions.

Suppose one-way poaching happens in $\tau = 2$ in equilibrium, since second-period profits are "constant" in the sense that they do not depend on the indifferent consumer δ from

$\tau = 1$, and hence they do not depend on the first-period prices r_A and r_B either. Therefore, the best responses derived from total profits are the same as those derived from only the first-period profits, because derivatives ignore "constant" profits. So, the potential equilibrium prices are the same as those in the standard Hotelling equilibrium, that is $r_A = r_B = t + c$. However, we would have $\delta = 1/2$ which violates the condition for second-period one-way poaching equilibrium, therefore one-way poaching cannot happen in $\tau = 2$.

Then, we are left with the case of PPD equilibrium in $\tau = 2$. Following the proposed procedure, the first-period best responses derived from total profits are:

$$BR_i = \frac{4c}{5} + \frac{3r_j}{5} + \frac{3t}{5} - \frac{2v}{5}$$

for both firms. By symmetry, solving for the potential equilibrium prices we have $r_A = r_B = 2c + \frac{3t}{2} - v$, and $\delta = 1/2$ which satisfies the condition of PPD equilibrium in $\tau = 2$. Accounting for the deviation conditions, the equilibrium of the game is formally described in the following proposition.

proposition 1 *When $c+t \leq v \leq c+\frac{9t}{2}$, there exist a unique equilibrium of the game. Firms implement PPD in $\tau = 2$, and set $r_A = r_B = 2c + \frac{3t}{2} - v$ in $\tau = 1$. Total profits of firms are $\Pi_A = \Pi_B = \frac{5t}{8}$, consumer surplus is give by $CS = 2v - 2c - \frac{7t}{4}$.*

proof *See Appendix A.*

corollary 1 *When $v > c + \frac{3t}{2}$, firms engage in below-cost pricing in $\tau = 1$.*

proof $r_i = 2c + \frac{3t}{2} - v < c \Rightarrow v > c + \frac{3t}{2}$

corollary 2 *There is no other equilibrium even if we consider a partially-covered market*

proof *See section 4.*

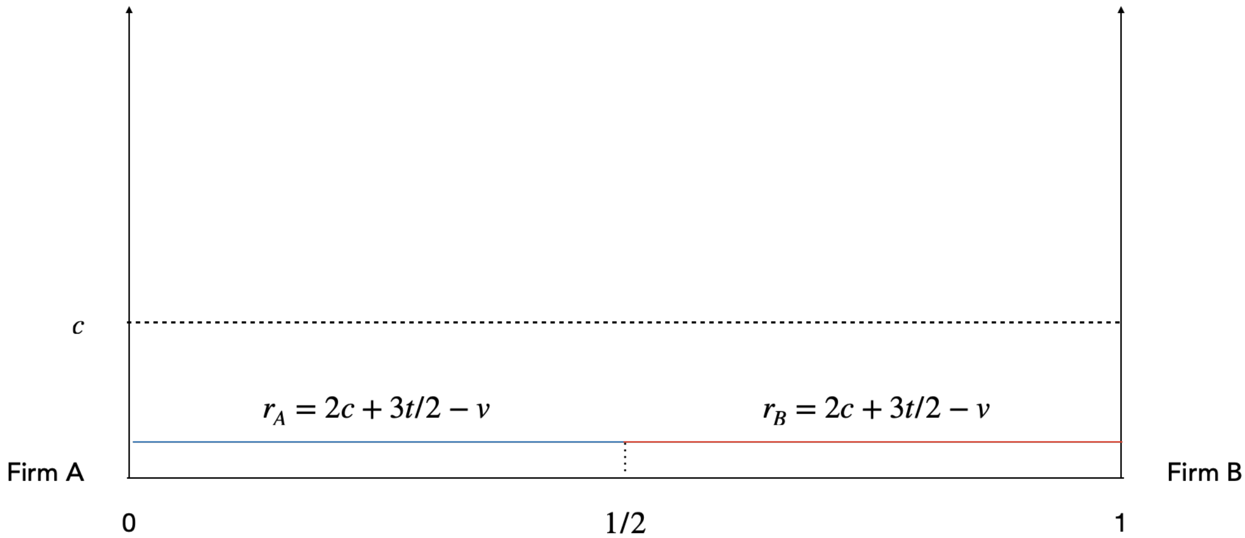

Figure 2: First-period equilibrium as in proposition 1.

3.3 Discussion

There are two interesting findings from proposition 1. First, as presented in corollary 1, firms find it the most profitable to lure consumers in with below-cost prices in $\tau = 1$ when they have a high valuation v. This finding has an important policy implication as it is highly related to predatory pricing, and it will be discussed in the next section. Second, although the equilibrium involves firms adopting PPD in $\tau = 2$, the total profits over two periods are $\Pi = \frac{5t}{8}$ for both firms which does not depend on consumers' valuation v.

The reasons behind these findings are simple and essentially the same. Note that a firm's data collection is negatively correlated with its first-period price r_i, because consumers prefer lower prices so for any price the lower it is the more data it can collect. Suppose firm A do not care about future profit, then given the rival's first-period uniform price, it has an optimal one-shot price to maximize its current profit. Raising to a price above this one-shot price does not benefit firm A because it collects fewer data and by definition it does not maximize the current profit. Then, anticipating the second-period PPD profit firm A's only reasonable strategy is to choose some price lower than the one-shot price because this allows it to collect more data even though this can hurt its current profit. The same logic also applies to firm B, so we can see firms face a trade-off between current and future profits, and they can control this trade-off with the first-period uniform price. In addition, since firms engage in PPD in the second period, their total profits are positively correlated with consumers' valuation v. Therefore, higher v increases potential second-period profits, and attracts firms to lower their first-period price to collect more data. As a result, this trade-off neutralizes the effect of consumer valuation v in the equilibrium profits. When v is large enough, second-period profit dominates first-period profit, and firms are willing to charge below-cost prices. This can also be verified by the best response functions in section 3.2, they indicate that a higher v shifts out both best responses. Then, when v is sufficiently large the best responses intersect each other in the third quadrant, and the equilibrium first-period prices are both negative which is shown in figure 3.

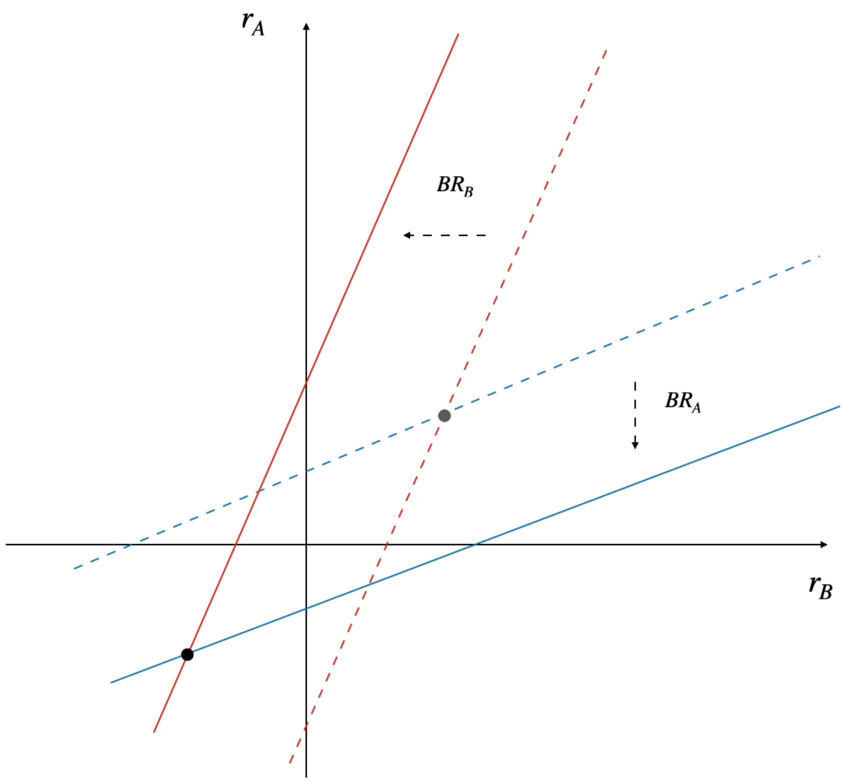

Figure 3: An increase in v shifts out the best responses.

One of the main results from Chen et al. (2020) is that IM weakens competition and allows firms to engage in PPD which leaves consumers with 0 surpluses. This is exactly what is happening in $\tau = 2$, and can be a terrifying result for consumers, while they try to protect themselves from price discrimination, the behavior of hiding identity softens competition between firms and causing them to lose all surplus. However, we found that by extending their model to a more realistic scenario using BBPD, consumers gain massive surplus because the competition is intense in the first period. The higher consumers are willing to pay for the products the higher the total consumer surplus, this shows IM can be beneficial to consumers.

In the standard Hotelling model, total profits of firms are t, while in our setup total profits of firms are $\frac{5t}{8}$. Although firms can perfect price discriminate, they earn a lower total profit overall. Consumer surplus in the Hotelling model is given by $2v - 2c - 5t/2$, which is lower than the consumer surplus $CS = 2v - 2c - \frac{7t}{4}$ in our setup. That is, changing from the non-discriminatory Hotelling environment to our setup, firms derived a lower profit but consumers have a higher surplus. So, comparing to the standard Hotelling competition, overall IM creates a higher level of competition between firms and benefits consumers.

We then compare to the case in Choe et al. (2018) which is identical to our model except consumers are passive in IM, this comparison allows us to discuss the effect of IM alone. Due to the nature of asymmetric equilibria in their result, we consider the total industry profit which is given by $0.572t + 0.645t = 1.217t$, it is lower than the total industry profit in our setup $10t/8 = 1.25t$. Consumer surplus in the case without IM is given by $2v - 2c - 1.726t$, which is higher than the consumer surplus with IM $2v - 2c - 1.75t$. So, in the context of BBPD, consumers are worse off when they engage in IM while firms are better off. This shows a prisoners' dilemma between consumers, each consumer is better off by hiding her identity, however, they are worse off when everyone tries to do so.

3.3.1 Policy implication

As shown in corollary 1, firms can engage in below-cost pricing by taking IM into account, however, below-cost pricing is very sensitive to antitrust authorities as it is often tied to predatory pricing. Predatory pricing is the act of a dominant firm sacrificing short-term gain with an unprofitable price to eliminate competitors so that it can maintain monopolization in the long run which can harm consumers and is anti-competitive. There is a lot of discussion on predatory pricing in both of the economic and legal literature (Areeda and Turner, 1975; Brodley and Hay, 1980; Edlin, 2002), most courts have adopted the Areeda-Turner test that a price is predatory when it is below the short-run marginal cost because it is just not profitable. Also, Elhauge (2002) has pointed out that above-cost pricing should not be deemed as predatory, because less efficient firms will be driven out when regulation expires, and the more or equally efficient firms can still enter the market. Therefore, below-cost has been a key indicator of predatory pricing. The US Supreme Court has required two conditions for predatory pricing in the famous case *Brooke Group Ltd. v. Brown & Williamson Tobacco Corp.* (1993): (i) price must be well below cost, and (ii) predator is able to recoup its short-run loss.

These two conditions are completely met in our model, however, we have shown that below-cost pricing can be a pure result of competition between firms. Under personalized pricing and identity management, anticipating the massive personalized pricing profit, firms want to collect more data at the beginning and hence willing to give up short-term profits which is similar to the predatory action. In this case, if a naive regulation occurs such that the firms have to charge a price above their marginal cost then consumer surplus is reduced. Because given the regulation, firm i's profit-maximization problem in the first period becomes:

$$\max_{r_i} \ \Pi_i(r_i, r_j) \quad \text{s.t. } r_i \geq c$$

where Π_i is the total profit over two periods for firm i. Suppose its rival firm j chooses $r_j = c$ then firm i's optimal response is to choose some r_i below c if the regulation does not exist, but with this constraint, its best response is $r_i = c$. That is, given the rival choosing the marginal cost as the uniform price to collect data, the best response for either firm is to choose the marginal cost as its price as well, so their best responses match and $r_A = r_B = c$ constitutes an equilibrium. Also, $r_A = r_B = c$ implies the indifferent consumer $\delta = 1/2 \in [L, R]$ hence the condition for second-period PPD equilibrium is not violated, and PPD with marginal cost as prices to collect data is a valid SPNE. Then, the competition is artificially weakened with this regulation and consumer surplus is reduced, previously consumers can enjoy a large surplus from below-cost pricing but now they cannot. Therefore, when determining whether below-cost pricing is predatory we need to take into account the feasibility of personalized pricing and identity management, and when they are indeed feasible, below-cost pricing is beneficial for consumers and should not be regulated.

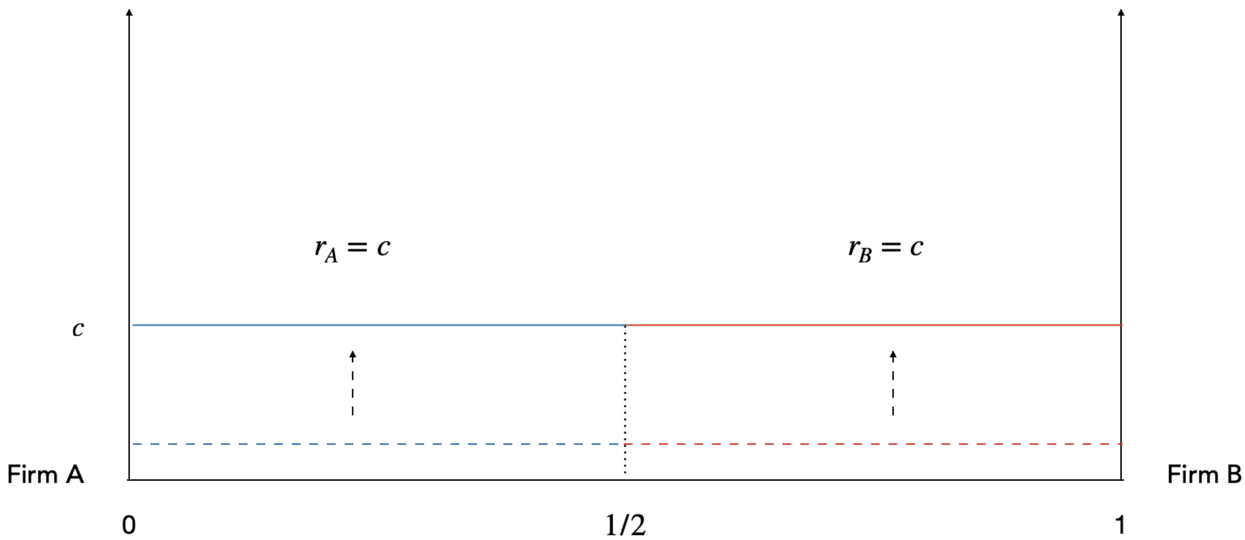

Figure 4: The equilibrium in $\tau = 1$ with naive regulation.

4 Partially-covered market

Now we consider equilibrium outcomes when there exist consumers do not purchase. Since we have shown that when all consumers purchase in $\tau = 1$, the equilibrium of the whole game involves all consumers purchase in $\tau = 2$ as well, then for the market to be partially-covered there must exist some segment $[a, b] \in (0, 1)$ which is not targeted by both firms. So, let $[0, a]$ be the targeted segment of firm A, $[b, 1]$ be the targeted segment of firm B, and a is strictly less than b.

4.1 Second period

In this case, there are four types of possible equilibrium in $\tau = 2$.

☐ *PPD without full coverage.* This is similar to the PPD equilibrium in the fully-covered market, which requires both firms to have sufficiently large segments of targeted consumers. Then, both firms are satisfied with the PPD profit and do not want to deviate which leaves the non-targeted consumer in $[a, b]$ to be unserved. Profit of firms can be easily derived as,

$$\pi_A^{PPD}(a) = \int_0^a v - c - tx \; dx = \frac{a\,(-at - 2c + 2v)}{2}$$

$$\pi_B^{PPD}(b) = \int_b^1 v - c - t(1-y) \; dy = -\frac{b^2 t}{2} + b\,(c + t - v) - c - \frac{t}{2} + v$$

☐ *One-way poaching equilibrium.* This is exactly the same as its counterpart in the fully-covered market. When b is close to 0 firm A poaches with a low uniform price, and when a is close to 1 firm B poaches with a low uniform price.

☐ *PPD and a uniform price.* This is similar to the one-way poaching equilibrium, but interestingly there is no competition between firms. The firm with a large targeted segment finds profitable to exercise PPD, while the other firm with small targeted segment chooses to serve the remaining segment with a uniform price without poaching. This is likely to happen when the advantageous firm targets about half the market, so it is not beneficial for the other firm to poach. For example, suppose b is close to $1/2$ and a is close to 0, firm B's profit is still $\pi_B^{PPD}(b)$. Firm A then serves $[0, z]$ for some indifferent consumer z, if $z < b$ then $q_A = (v + c)/2$ and in this case $\pi_A^U = (v - c)^2/(4t)$. Or if $z = b$ then $q_A = v - tb$, and firm A's profit is given by $\pi_A^U = b(v - tb - c)$.

☐ *Hotelling equilibrium.* This classic equilibrium can happen when both firms have small targeted segments, that is a is close to 0 and b is close to 1. So, both firms do not find personalizing profitable, and they engage in uniform-price competition. The outcome is exactly the same the benchmark Hotelling equilibrium, $q_A = q_B = t + c$ and $\pi_A = \pi_B = t/2$.

Formally, by considering the deviation conditions, the second-period equilibria involving a partially-covered market are presented in the following lemma.

lemma 2 *When the market is partially covered in $\tau = 1$, there are cutoffs $(L, R, L^{PPD}, R^{PPD}, L^U, R^U, L^{OW}, R^{OW}, l^{PPD}, r^{PPD})$ as functions of (v, c, t, a, b), such that the following equilibria in $\tau = 2$ exist:*

1. *PPD without full coverage if (i) $L^{PPD} \leq a < b \leq R^{PPD}$, or (ii) $L^{OW} \leq a < b \leq l^{PPD}$, or (iii) $R^{OW} \leq a < b \leq r^{PPD}$.*

2. *One-way poaching equilibrium. (i) One-way poaching by firm A if $b \leq L$ and $a \leq L^{OW}$, (ii) one-way poaching by firm B if $a \geq R$ and $b \geq R^{OW}$.*

3. *PPD with a uniform price. (i) PPD by firm B if $a \leq L^{PPD}$ and $b \leq L^U$, (ii) PPD by firm A if $b \geq R^{PPD}$ and $a \geq R^U$.*

4. *Hotelling equilibrium if $a \leq 1 - \frac{\sqrt{2}}{2}$ and $b \geq \frac{\sqrt{2}}{2}$.*

proof *See Appendix A.*

4.2 First period

Building on the result from the second period, we can go back to the first period and solve for the whole game, and we will consider each potential equilibrium separately.

□ *PPD without full coverage.* Note that consumer a is the last consumer that is willing to buy from firm A, solving $U(a, r_A) = 0$ we have $a = \frac{v - r_A}{t}$, similarly we have $b = 1 - \frac{v - r_B}{t}$. Then, the first-period profits are $\pi_A^1 = a(r_A - c) = (c - r_A)(r_A - v)/t$ and $\pi_B^1 = (1-b)(r_B - c) = (c - r_B)(r_B - v)/t$. Together with second-period profits $\pi_A^{PPD}(a)$ and $\pi_A^{PPD}(b)$ we have the total profits: $\Pi_A^{PPD} = (r_A - v)(4c - 3r_A - v)/(2t)$ and $\Pi_B^{PPD} = (4cr_B - 4cv - 3r_B^2 + 2r_B v + v^2)/(2t)$. First order conditions give $r_A^{PPD} = r_B^{PPD} = \frac{2c}{3} + \frac{v}{3}$, hence equilibrium total profits are

$$\Pi_A^{PPD} = \Pi_B^{PPD} = \frac{2(v - c)^2}{3t}$$

Also, we need the condition $a \leq b$, that is $v \leq c + \frac{3t}{4}$. So, we have a candidate equilibrium of the whole game which involves PPD without full coverage in $\tau = 2$.

$\square$ *PPD and a uniform price.* Suppose firm A charges a uniform price and firm B exercises PPD in $\tau = 2$. Then, firm A serves $[0, z]$, we will consider the unconstrained case $z < b$ since the unconstrained profit is always higher than the constrained profit, so if we can show the unconstrained profit is dominated by the previous candidate equilibrium then the constrained profit is also dominated. Since the second-period profit with a uniform price does not depend on a, the optimal first-period profit is given by $\pi_A^1 = \pi_A^U$, then we have

$$\Pi_A^U = 2 \times \pi_A^U = \frac{(v-c)^2}{2t} < \frac{2\,(v-c)^2}{3t} = \Pi_A^{PPD}$$

So, the candidate equilibrium involves PPD without full coverage strictly dominates the candidate equilibrium involves PPD and a uniform price.

$\square$ *The Hotelling equilibrium.* In $\tau = 2$, $q_A = q_B = t + c$ and profits are $t/2$. Consider firm A, if it deviates to serve targeted consumers with personalized pricing $p_A^d(x) = t + c + t(1-2x) = 2t(1-x) + c$, then its profit is $\pi_A^d = \int_0^a 2t(1-x) + c - c\, dx = at\,(2-a)$. Firm A does not deviate when $t/2 \geq \pi_A^d$ or $a \leq 1 - \frac{\sqrt{2}}{2}$. Since the second-period profit does not depend on a, so $r_A = \frac{v+c}{2}$, hence $a = \frac{v-c}{2t}$. So, for this candidate equilibrium to exist, we need the condition $a = \frac{-c+v}{2t} \leq 1 - \frac{\sqrt{2}}{2} \Rightarrow v \leq c + (2-\sqrt{2})t$. However, that means

$$U(x, q_A) = v - t - c - tx < 0, \text{ for all } x$$

and no consumer will purchase in $\tau = 2$, hence this is not a valid equilibrium.

$\square$ *The One-way poaching equilibrium.* By lemma 1, second-period profits in this case does not depend on first period prices, so $r_A = r_B = \frac{v+c}{2}$, and by symmetry $a \leq 1/2 \leq b$ which violates the condition for one-way poaching equilibrium in $\tau = 2$.

Therefore, PPD without full market coverage strictly dominates all other possible equilibrium. However, firms can still deviate in $\tau = 1$ to poach its rival. Suppose firm A deviates, base on (v, c, t) there are two potential outcomes of the indifferent consumer δ, namely $\delta \geq R$ or $\delta \in [L, R]$ where L and R are defined in lemma 1. Suppose $\delta \in [L, R]$, then we have PPD equilibrium in $\tau = 2$, hence firm A does not deviate when $\Pi_A = \frac{2(v-c)^2}{3t} \geq \Pi_A^d$ (the deviation profit is derived in the total profit function of firm A in the proof of proposition 1) or $v \geq c + \frac{3t}{11} + \frac{3\sqrt{15}t}{22} \approx c + 0.801t$. Repeat the same analysis for $\delta \geq R$ we have the condition, $v \geq c + \frac{3t}{47} + \frac{3\sqrt{474}t}{94} \approx c + 0.759t$. Both of these conditions are disjoint with the previous condition $v \leq c + \frac{3t}{4}$ for $a < b$, hence firm A always deviates whenever PPD without full coverage is feasible. Therefore, there is no equilibrium involving a partially-covered market as stated in corollary 2.

4.3 Discussion

Together with section 3, we have provided an exhaustive analysis on BBPD with IM. More importantly, although there can be as much as a total of 9 equilibria in the second-period as shown by Chen et al. (2020), by analyzing how firms collect data we narrow down the possibilities to a single equilibrium. That is the salient PPD equilibrium with full market coverage presented in proposition 1, firms engage in below-cost pricing to collect data then use PPD extract full surplus in the later period. So, by considering how firms collect data we drastically improved the robustness and predictability of the PPD equilibrium.

5 Conclusion

This paper has studied a duopoly model where firms can use the first-period price to collect data, then use those data to discriminate consumers in the second-period with personalized prices, also allowing consumers to protect themselves with identity management (IM). In the second period, there are a total of 9 possible equilibria, but one of them stands out which we called the perfect price discrimination (PPD) equilibrium, this allows firms to charge personalized prices exactly at consumers' willingness to pay and is the result from IM weakening the competition between firms. By analyzing how firms collect data in the first place, we narrowed down the 9 possibilities to only the PPD equilibrium, and firms choose below-cost pricing at the beginning to compete for data. Below-cost pricing in this case is due to the anticipation of high future PPD profit, to achieve this high profit firms want to collect more data and to collect more data they need to set lower prices.

One important policy implication arises from the result of below-cost pricing since it has always been the test for predatory pricing. However, we have shown that by considering personalized pricing and IM, below-cost pricing can be the result of natural competition and not a strategic act to drive competitors out of the market. In fact, in this case an above-cost regulation will only weaken the competition between firms and hurt consumers.

By endogenizing the choice of IM in personalized pricing competition, our paper adds insight to the existing literature. However, some important limitations still remain. In reality, not all consumers know about IM, let alone understand how to protect themselves from privacy disclosure and price discrimination with it. Moreover, the cost of hiding one's identity is most likely different for everyone, for example, an existing customer with positive account balance has a higher hiding cost than a new customer. While these are important factors of consumer behavior when modeling personalized pricing, they still remain for further research.

Appendix A

proof (lemma 1) *Consider the candidate equilibrium when both firms engage in PPD so that they set uniform prices above v, and personalized prices which make consumers gain 0 utility when buying. First we need to derive the PPD profits of firms which depends on the indifference consumer δ from $\tau = 1$.*

$$\pi_A^{PPD} = \int_0^{\delta} v - c - tx \; dx = \frac{\delta\left(-\delta t - 2c + 2v\right)}{2}$$

$$\pi_B^{PPD} = \int_{\delta}^{1} v - c - t(1-y) \; dy = -\frac{\delta^2 t}{2} + \delta\left(c + t - v\right) - c - \frac{t}{2} + v$$

If they decide to deviate from PPD, then by lemma 2 they serve all existing consumer with the uniform price and poach some consumers from the rival. Observing a deviation of the rival in stage 1, a firm would reduce its personalized price in stage 2 to avoid losing too many consumers, and keep the uniform price sufficiently high. Optimally, it sets the personalized price exactly making consumers have the same utility from either choices, that is, from purchasing with the uniform price of the other firm and purchasing with its reduced personalized price.

Suppose firm A finds it profitable to deviate, then given its lowered q_A^d, firm B sets $p_B(y) = max\{c, q_A^d + t(2y-1)\}$. So, the indifference consumer here is $z = \frac{c - q_A^d + t}{2t}$, given z maximizing firm A's profit gives $q_A^d = \frac{t}{2} + c$. Hence, $z = \frac{1}{4}$ and firm A's deviation profit is $\pi_A^d = \frac{t}{8}$. Then it is only profitable for firm A to deviate when $\pi_A^{PPD} \leq \pi_A^d$, that is, when

$$\frac{\delta\left(-\delta t - 2c + 2v\right)}{2} \leq \frac{t}{8}$$

$$\Rightarrow \delta \leq \frac{2v - 2c - \sqrt{\left(2v - 2c + t\right)\left(2v - 2c - t\right)}}{t} = L$$

Similarly, we can show that it is only profitable for firm B to deviate from PPD when $\delta \geq 1 - L = R$.

So, we have found two types of equilibria in $\tau = 2$. First, when $L \leq \delta \leq R$, firms exercise PPD in equilibrium. Second, when $\delta \leq L$ or $\delta \geq R$, the firm with less data deviates by setting a uniform price equals ti $\frac{t}{2} + c$, the deviation profit is given by $\frac{t}{8}$ and the rival's profit is given by $\frac{3t}{16}$.

proof (proposition 1) *From lemma 2, the total profit function for firm A is:*

$$\Pi_A = \begin{cases} \frac{t^2 - 4(c - r_A)(-r_A + r_B + t)}{8t}, & \frac{r_B - r_A + t}{2t} \leq L \\ \frac{(-r_A + r_B + t)(-8c + 5r_A - r_B - t + 4v)}{8t}, & L \leq \frac{r_B - r_A + t}{2t} \leq R \\ \frac{3t^2 - 8(c - r_A)(-r_A + r_B + t)}{16t}, & \frac{r_B - r_A + t}{2t} \geq R \end{cases}$$

FOC:

$$r_A(r_B) = \begin{cases} r_A^L(r_B) = \frac{c}{2} + \frac{r_B}{2} + \frac{t}{2}, & \frac{r_B + t}{4t} \leq L \\ r_A^{PPD}(r_B) = \frac{4c}{5} + \frac{3r_B}{5} + \frac{3t}{5} - \frac{2v}{5}, & L \leq \frac{-c + r_B + t + v}{5t} \leq R \\ r_A^R(r_B) = \frac{c}{2} + \frac{r_B}{2} + \frac{t}{2}, & \frac{r_B + t}{4t} \geq R \end{cases}$$

Solving the condition $\frac{r_B + t}{4t} \leq L$ for r_B gives:

$$r_B \leq -4c - t + 4v - 2\sqrt{4c^2 - 8cv - t^2 + 4v^2}$$

denote this expression by L^d, solving the condition $L \leq \frac{-c + r_B + t + v}{5t}$ for r_B gives:

$$r_B \geq -4c - t + 4v - \frac{5\sqrt{4c^2 - 8cv - t^2 + 4v^2}}{2}$$

denote this expression by L^{PPD}, clearly for $r_B \in [L^{PPD}, L^d]$ there are two local optimal responses for firm A. Solve for the critical value L_c such that $r_B \geq L_c$ implies $\Pi_A(r_A^{PPD}) \geq \Pi_A(r_A^L)$ gives:

$$L_c = -4c - t + 4v - \sqrt{20c^2 - 40cv - 5t^2 + 20v^2}$$

Repeat this argument for r_A^{PPD} and r_A^R shows no local optimal conditions exist there.

FOC of firm B's total profit gives its best responses $\{r_B^L, r_B^R, r_B^{PPD}\}$, where $r_B^{PPD}(r_A) = \frac{4c}{5} + \frac{3r_A}{5} + \frac{3t}{5} - \frac{2v}{5}$. Solving the pairs (r_A^L, r_B^L) and (r_A^R, r_B^R) both gives $r_A = r_B = c + t$, hence $\delta = 1/2$ and violates the condition for one-way poaching equilibrium in $\tau = 2$. But solving r_A^{PPD} and r_B^{PPD} gives a candidate equilibrium $r_A = r_B = 2c + \frac{3t}{2} - v$. Then given $r_B = 2c + \frac{3t}{2} - v$, r_A^{PPD} is optimal when $r_B \geq L_c$, that is when

$$c + \frac{t}{2} \leq v \leq c + \frac{9t}{2}$$

The whole argument can be applied to firm B.

Also, we need to make sure with $r_A = 2c + \frac{3t}{2} - v$, firm A serves all consumers on $[0, 1/2]$ in $\tau = 1$ (then by symmetry firm B serves $[1/2, 1]$), that is

$$U(r_A, 1/2) = v - (2c + \frac{3t}{2} - v) - \frac{t}{2} \geq 0 \Rightarrow v \geq c + t$$

hence when $c + t \leq v \leq c + \frac{9t}{2}$ we have a unique equilibrium $r_A = r_B = 2c + \frac{3t}{2} - v$.

Then, we can derive the total consumer surplus over two periods, since consumer surplus in $\tau = 2$ is 0 due to PPD, by symmetry we have

$$CS = 2 \times CS_A^1 = 2\int_0^{1/2} v - r_A - tx \; dx = 2v - 2c - \frac{7t}{4}$$

proof (lemma 2) *First, we analyze the case $a < 1/2 < b$, the cases $a < b \leq 1/2$ and $1/2 \leq a < b$ will be discussed later. For simplicity we will assume $v \geq c + 2t$, when $v < c + 2t$ the proof exactly the same but involves too many cases.*

Consider the equilibrium of PPD without full market coverage. Suppose firm A wants to deviate, then it serves $[0, z]$ with a uniform price q_A^d for some indifferent consumer $z = \frac{v - q_A^d}{t}$. Then profit maximizing gives $q_A^d = \frac{v+c}{2}$, hence $z = \frac{v-c}{2t}$, however $v \geq c + 2t \Rightarrow v \geq c + 2tb \Rightarrow z = \frac{v-c}{2t} \geq b$. Therefore firm A can only serve $[0, b]$ with $q_A^d = v - tb$ and $z = b$. So

$\pi_A^d = b(v - tb - c)$, and firm A does not deviate when $\pi_A^{PPD}(a) \geq \pi_A^d$ or

$$a \geq \frac{-c + v - \sqrt{2b^2t^2 + 2bct - 2btv + c^2 - 2cv + v^2}}{t} = L^{PPD}$$

Similarly, firm B's deviation is given by $q_B^d = v - t(1-a)$ and $\pi_B^d = (1-a)[v - t(1-a) - c]$, so it does not deviate when $\pi_B^{PPD}(b) \geq \pi_B^d$ or

$$b \leq \frac{c + t - v + \sqrt{2a^2t^2 - 2act - 4at^2 + 2atv + c^2 + 2ct - 2cv + 2t^2 - 2tv + v^2}}{t} = R^{PPD}$$

Next, consider the equilibrium with firm B exercises PPD and firm A charge a uniform price. The deviation condition for firm A is the exact opposite in the previous equilibrium of PPD without full market coverage. Then consider firm B, given $q_A = v - tb$ it can serve $[a, 1]$ with $q_B^d = q_A - t + 2ta = v - t(1 + b - 2a)$, hence $\pi_B^d = (1-a)[v - t(1 + b - 2a) - c]$ and it does not deviate when $\pi_B^{PPD}(b) \geq \pi_B^d$ or

$$b \leq \frac{-at + c + 2t - v + \sqrt{5a^2t^2 - 4act - 10at^2 + 4atv + c^2 + 4ct - 2cv + 5t^2 - 4tv + v^2}}{t} = L^U$$

Similarly, when firm A exercises PPD and firm B charge a uniform price, the deviation condition for firm B is the exact opposite as in case of PPD without full coverage. Then, firm A can deviate to serve $[0, b]$ with $q_A^d = q_B + t - 2tb = v - t(2b - a)$, and it does not deviate when

$$a \geq \frac{-bt - c + v - \sqrt{5b^2t^2 + 4bct - 4btv + c^2 - 2cv + v^2}}{t} = R^U$$

Now consider the Hotelling equilibrium with $q_A = q_B = t + c$ and $\pi_A = \pi_B = t/2$. Firm A can deviate to serve $[0, a]$ with $p_A^d(x) = q_B + t(1 - 2x)$, hence $\pi_A^d = a\left(-at + c + 2t\right)$ and it does not deviate when $a \leq 1 - \frac{\sqrt{2}}{2}$. Similarly, firm B does note deviate from the Hotelling equilibrium when $b \geq \frac{\sqrt{2}}{2}$.

Finally, the equilibrium of one-way poaching does not exist when $a < 1/2 < b$ because it can only happen if $a < b \leq L$ or $R \leq a < b$ where L and R are defined in lemma 1.

So, consider the case $a < b \leq 1/2$. Clearly the Hotelling equilibrium and the one-way poaching by firm B cannot happen. If $b \leq L$, then one-way poaching by firm A is possible, in this case firm B has no incentive to deviate, while firm A can deviate to PPD. So, $\pi_A^d = \pi_A^{PPD}(a)$ and firm A does not deviate when $\pi_A = t/8 \geq \pi_A^{PPD}(a)$ or

$$a \leq \frac{-c + v - \frac{\sqrt{-(-2c+t+2v)(2c+t-2v)}}{2}}{t} = L^{OW}$$

Also, for the equilibrium of PPD without full coverage, firm A's optimal deviation can be $q_A^d = t/2 + c$ if $t/8 \geq b(v - tb - c)$ or

$$b \leq \frac{-\frac{c}{2} + \frac{v}{2} - \frac{\sqrt{4c^2-8cv-2t^2+4v^2}}{4}}{t} = l^{PPD}$$

So, the equilibrium of PPD without full coverage can also exist if $L^{OW} \leq a < b \leq l^{PPD}$.

Finally, the case $1/2 \leq a < b$ is symmetric to the previous case. The equilibrium of one-way poaching by firm B exists if $b \geq 1 - L^{OW} = R^{OW}$. The equilibrium of PPD without full coverage can also exist if $R^{OW} \leq a < b \leq 1 - l^{PPD} = r^{PPD}$.

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