CONTROLLING THE ECONOMY

An Exercise in Collaboration

by

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1. INTRODUCTION

In 1936 Keynes book "The General Theory of Employment, Interest and Money" [1] was published. Previously it was held that an economic system would tend to converge towards a full employment equilibrium and that it was therefore the role of government to assist this convergence by encouraging mobility of labour and capital from one market to another and by supporting sound trading and financial institutions which would provide a framework in which competitive forces could interact. Just as today there are those who still maintain this view, so in the early 1930's there were some who swam against the tide of opinion. At that time the crucial question concerned how best might full employment be restored. The orthodox view blamed the high level of wages : trades unions resisted wage reductions, yet how but by lowering wage rates could labour expect capitalists to want to employ more men?

The Keynesian revolution consisted of first, a demonstration that even given competitive conditions in the labour market, a state of permanent under-employment was possible, and secondly, a prescription for getting out of such a state. The demonstration was based implicitly on a set of six simultaneous equations which are discussed by Klein [2]. Their properties need not concern us here beyond noting their implication that the demand for labour is derived from a demand for goods, so that unemployment is to be cured by raising this latter demand. There is some ambiguity about whether or not this can be achieved through increased wage rates, but there is no question that among the alternative strategies which can work are lower taxation and increased government spending, i.e. a government budget deficit. To Keynes therefore the crux of the problem of regulating the level of unemployment was to use the budget as a control variable : run a deficit to decrease unemployment, run a surplus to decrease the inflationary pressures of high employment. Somewhere in between the extremes is an optimal level of unemployment which Beveridge put at about 2-3%.

It is not difficult to discern many of the ingredients of Britain's stop-go policies in the Keynesian prescription. Thanks to Keynes we have, to use a phrase of Sir Denis Robertson, been able to replace booms and slumps by "sloons and bumps", i.e. the amplitude of fluctuations in output and employment has been diminished.

In economics, as in any other science, we move from one problem to the next. First, we can note that while allowing the budget surplus to fluctuate may stabilise the private sector of the economy, it does not facilitate the development of the public sector. It could be argued that some instability in the private sector is a price worth paying for the steady development of programmes for schools, hospitals and motorways. It would be even better to have the best of both worlds by finding some other control variable, and to this end the prices and incomes policy has been conceived.

Full employment and growth of the public sector are not the only objectives of our economy. Growth in general is a primary target, and while our post-war growth rate has been impressive if seen in an historical context, it does not look so good in comparison with the contemporary performance of other nations. Indeed growth could be said to be the only objective in the long term. Persistent unemployment can be cured by lowering the length of the working week provided that incomes are not reduced thereby, and growth of the public sector is made easier if the rest of the economy is also growing.

But we cannot have everything. Britain already exports more goods per head than any other comparable nation, yet finds herself with recurrent Balance of Payments problems. For the last ten years some economists have found it reasonable to argue that devaluation is the answer, but politicians have not been prepared to regard the exchange rate as a major control variable. Accordingly we come back to the prices and incomes policy as the main alternative to using government expenditure to regulate our affairs : its efficacy in this capacity cannot yet be regarded as proven.

The purpose of this paper is to describe our work in building a model of the economy which will show the interdependence of growth in the public and private sectors, the level of unemployment and the balance of payments. It does not pretend to be a neat description of the British economy. Rather the model describes a simple economic system which has several of the principle features of our economy but little of its sophistication. Our concern has been, and remains, to focus on some essentials in the hope that the implications will not be irrelevant for subsequent more detailed work. Meanwhile, the simplicity is potentially helpful in producing intelligible analytic results in terms of control theory on the one hand and economics on the other.

The application of control theory methods to economic concepts is by no means new. Once Keynes had pointed the way a few pioneers set out to explore the possibilities, among whom Tustin [3] and Phillips [4] were outstanding. Subsequently there was a lull in activity until quite recently, so that now there are several workers in the field. This renewed enthusiasm no doubt has many causes, but two would seem to be most important. First there has been rapid progress in control techniques since Tustin wrote his book in 1953. And secondly, economics has advanced in its ability to formulate mathematical models and in the estimation of their parameters. There is probably as yet still a long way to go, but the future trend is apparent. Accordingly we have embarked on a few first steps now, not least because we are conscious that economists and control theorists have a lot to learn from each other before the fruits of co-operation can all be harvested. Our first exercises have already been reported [5]. The present paper is based on essentially the same model and covers subsequent efforts and some related studies by Hulme [6].

2. THE BASIC MODEL

The basic model is taken from the present authors' recent paper [5].

(i) The Accounting Framework

Underlying every economic model there is an accounting framework which defines the level of detail. Such a framework is shown in the form of a social accounting matrix in Figure 1, this being at once the most concise and instructive method of display.

The entries recorded in the matrix refer to money flows. Money received by an account appears in the row for that account: money spent appears in the corresponding column. By accounting convention, the diagonal elements of the matrix are zero: transactions within an account are of no interest.

The matrix shown as Figure 1 distinguishes eight accounts. The first is the production account which is the consolidated trading account of all production activities. The next three accounts are the current accounts of the domestic institutions, namely households, companies (which together make up the private sector) and government, which is also the public sector. The fifth, sixth and seventh accounts are the individual domestic capital accounts. In the remainder of this paper we treat these as a single consolidated capital account for all domestic institutions. The last account is the combined current and capital account of the rest of the world.

Tracing through the accounts, we see that production activities receive money from households in payment for consumer goods, C, from government, on account of its current expenditure, G, from the combined domestic capital accounts for capital goods, I, and from the rest of the world for exports, X. On the expenditure side, an amount W is paid to households as wages, and an amount M to the rest of the world for imports. The remaining item Π, represents profits which go to companies. This is a balancing item, so that within the production account total receipts equal total revenue.

Households have two sources of income additional to wages. These are D, which represents profits distributed (as dividends and interest) by companies, and U, which stands for transfers from government. This would normally include pensions, child allowances, etc., but is here restricted to unemployment benefits. Out of this income, households spend C on consumption goods as we have already seen. They also pay an amount T_H in taxes to government. The remainder, S_H, is savings. This item balances receipts and revenue and goes to the household capital account as a form of income.

Companies have an income made up of profits in domestic production Π, and returns on foreign investment, R_C. Some of this income is paid out to households as distributed profits, D, and to government in the form of taxes, T_C. The balancing residual is S_C which is company savings.

Government receives income in the form of taxes from households and companies. It also receives some income from abroad on its foreign investment (which is largely overseas lending). Expenditure on goods and services, G, goes to the production activities, and in addition there are transfers, U, to households. The residual is the budget surplus in our model (which might be negative), which is also government saving by a different name.

At this point we see that the combined domestic capital account has three sources of income which are the savings of the three domestic institutions. This money is invested at home by buying capital goods, I, or abroad in the form of foreign investment by companies, I_{FC}, and by government, I_{FG}.

The surplus in our current transactions with the rest of the world, known as the balance of payments surplus, is represented by the excess of $X + R_C + R_G$ over M. To the extent that this surplus exists, it is offset by a deficit on the capital account given by overseas investment $I_{FC} + I_{FG}$.

The fact that for each account the total receipts and total revenue must be equal gives the first six of the equations of our model. Of these, one is redundant, since if five accounts are balanced, the sixth automatically falls into line.

The social accounting matrix yields 6 equations of which only 5 are linearly independent. We consider these with 13 other equations derived from econometric considerations and these are shown in matrix form in Figure 2. In Fig. 2 equations 1-5 are obtained from the social accounting matrix. The remaining equations 7-19 are explained below :

Equation (7) $\quad T_H = \gamma_1(W + D)$

Taxes are paid by households at a rate γ_1 on that part of their income which is not received as a direct transfer from government.

Equation (8) $\quad U = \gamma_2(c_4K - (W+\Pi))$

The transfers received by households from government are proportional to the excess of c_4K over $W + \Pi$. The new variable K which enters here is the value of the productive capacity of the economy, and

c_4K is the value of the goods which the economy could produce if it were operating at full stretch. $W + \Pi$, on the other hand, is the value of goods produced within the economy, i.e. the domestic product. Accordingly $c_4K - (W+\Pi)$ is excess capacity and U is assumed to be proportional to it.

Equation (9) $\dot{C} = a_1(C+S_H) + a_2C$

This equation relates consumption expenditure by households to their income after tax. The form of the equation is such that the trend value of this income, rather than its actual value is the determining variable.

Equation (10) $R_C = \lambda_1 K_{FC}$

Companies receive an annual return at a rate λ_1 on their overseas assets, K_{FC}.

Equation (11) $I_{FC} = \dot{K}_{FC}$

I_{FC} is the change in overseas assets K_{FC}.

Equation (12) $T_C = \gamma_3\Pi - \gamma_4 I$

Taxes are paid by companies at a rate γ_3 on their profits in excess of investment allowances, i.e. on $\Pi - \frac{\gamma_A}{\gamma_B} I$

Equation (13) $S_C = b_1(I + I_{FC})$

Company savings are set as a proportion, b_1, of the total investment requirements of the company sector.

Equation (14) $R_G = \lambda_2 K_{FG}$

Government receives an annual return at a rate λ_2 on its overseas assets, K_{FG} (c.f. Equation 10).

Equation (15) $I_{FG} = \dot{K}_{FG}$

I_{FG} is the change in overseas assets K_{FG}.

Equation (16) $M = c_1(W+\Pi)$

Imports are proportional to the domestic product.

Equation (17) $I = \dot{K}$

I is the change in domestic capital K.

Equation (18) $W = c_2(W+\Pi) + c_3(c_4K - (W+\Pi))$

Wage payments to labour are a proportion, c_2, of the domestic product plus an amount which increases with the amount of excess capacity.

Equation (19) $\dot{X} = c_5X + c_6(c_4K - (I+C+G))$

The increase in exports is proportional to the level of exports plus an amount which is proportional to the extent to which capacity, c_4K, exceeds domestic demand, I+C+G.

This equation has been modified in later tork to :

Equation (19A) $\dot{X} = c_5X + c_6(c_4K - (W+\Pi))$

In connection with developments of this basic model we shall use also some additional equations (not shown in Fig. 2) :

Equation (20) $\tau\dot{G} + G = k(c_4K - (W+\Pi))$

Two special cases of this equation are of interest : 20(a) τ positive, k positive; making government expenditure a filtered form of excess capacity; 20(b) τ negative, k = 0; government expenditure increases exponentially at a rate $-1/\tau$

Equation (21) $D = c_7K$

Dividends are proportional to the value of productive capacity K.

Equation (22) $I = gK$

Investment is proportional to capacity K. Together with Equation (17) this implies that K grows at a constant rate g.

A most important feature of this model is that all its variables are expressed in value terms. Accordingly, it has nothing to say about the separate behaviour of prices and quantities. In particular, it has nothing to say about the separate behaviour of the quantities of labour employed and unemployed. For this reason the variable U is not well defined since it depends on unemployed capital and not labour. This difficulty could be overcome by further extension of the model.

Throughout the model, coefficients which normally depend on behaviour in the private sector are designated as a, b or c. Those coefficients which government directly controls are designated γ.

The units we shall use will be £ for K, K_{FC}, K_{FG} and £ per year for the other variables.

3. BEHAVIOUR OF THE MODEL

When investigating the behaviour of the model it is helpful to have in mind some criteria of desirable behaviour. We can suggest that these are :

(i) to achieve an expanding economy, so that variables such as K, W, Π increase exponentially;

(ii) to keep U, which represents surplus capacity, small in relation to c_4K.

(iii) to avoid a persistent surplus or deficit of the balance of payments, that is to keep $I_{FC} + I_{FG}$ (= $X+R_C+R_G-M$ from Eq. 6) close to zero.

In 5 we investigated the behaviour of the basic model (1) to (19) together with Equation 22 and the objectives above. Two important conclusions were reached in that study :

(i) the model (1) to (19) plus Equation (22) is consistent with an exact balance of payments and U not increasing exponentially provided that g equals c_5 in Equation (19), i.e. the rate of investment in productive capacity is set by the growth rate of exports

(ii) the tax rates γ_1 do not appear in the exponential solutions for the key variables K, I, X, W, Π and U.

In an extension to this work Hulme [6] has studies the effect of making government expenditure sensitive to excess capacity $c_4K-(W+\Pi)$ by use of Equation (20) (a) instead of Equation (22), and at the same time dropping the variable U and the equation 8 which defines it. Reduction of the equations of this new system leads to a characteristic equation for the vector (G K C X)'

$$\begin{vmatrix} \frac{1}{\tau}+s & \frac{-kc_4}{\tau} & 0 & \frac{k}{\tau c_1} \\ -1 & -s & -1 & \frac{1}{c_1} \\ -\gamma_4'R & \gamma_3c_3c_4R & a_2-R\gamma_4'-s & \frac{QR}{c_1}+\frac{\gamma_4'R}{c_1} \\ 0 & c_6c_4 & 0 & \frac{c_5-c_6}{c_1}-s \end{vmatrix} = 0$$

where $Q = 1-\gamma_3 + \gamma_3c_2 - \gamma_3c_3$, $R = a_1(1-\gamma_1)$, $\gamma_4' = \gamma_4 - b_1$.

The influence of the coefficient k in equation (20) may be found by plotting a root locus diagram as shown in Fig. 3. With the exact balance of payments situation defined by $I_{FC} = I_{FG} = 0$, it becomes apparent that once again the positive root coming out of the root locus diagram, and thus governing the exponential growth, should be equal to c_5 if the percentage of excess capacity

$$\delta = \frac{c_4K-(W+\Pi)}{c_4K}$$

is to be zero : however this is only possible for a very large k, when oscillatory transients will also be encountered.

One difficulty with the studies above is that the models are not determinate : the number of unknowns exceeds the number of equations, and additional assumptions are required to make the system full determined. For example we can introduce a new equation such as (21), together with (20) and (22), thus making 21 equations in 21 unknowns. If this procedure is followed and we take the opportunity of substituting (19a) for equation (19), equations (17) and (22) immediately give an exponential for K, $K = K_o \exp gt$, and reduction of the other equations yields the following matrix equation for C, X and G

$$
\begin{bmatrix}
\frac{d}{dt} - a_2 + \frac{\gamma_2 a_1}{1+c_1} + \frac{(c_3-c_2)(1-\gamma_1)a_1}{1+c_1} & \frac{\gamma_2 a_1}{1+c_1} + \frac{(c_3-c_2)(1-\gamma_1)a_1}{1+c_1} & \frac{\gamma_2 a_1}{1+c_1} + \frac{(c_3-c_2)(1-\gamma_1)a_1}{1+c_1} \\
\frac{c_6}{1+c_1} & \frac{d}{dt} - c_5 + \frac{c_6}{1+c_1} & \frac{c_6}{1+c_1} \\
\frac{k}{1+c_1} & \frac{k}{1+c_1} & \tau\frac{d}{dt} + 1 + \frac{k}{1+c_1}
\end{bmatrix}
\begin{bmatrix} C \\ X \\ G \end{bmatrix} =
$$

$$
K_o e^{gt}
\begin{bmatrix}
ga_1 - \frac{\gamma_2 a_1 g}{1+c_1} - \frac{(c_3-c_2)(1-\gamma_1)a_1 g}{1+c_1} + (1-\gamma_1)a_1 c_3 c_4 \\
c_4 c_6 - \frac{gc_6}{1+c_1} \\
kc_4 - \frac{kg}{1+c_1}
\end{bmatrix}
$$

The situation of an exact balance of payments and zero δ is only possible once again for $g = c_5$ and root locus plotting will show that this requires $k \to \infty$ assuming $\tau > 0$. However an alternative approach is to assume k and c_6 both zero and that τ is negative (hence G increasing exponentially). Neglecting also I_{FC} and I_{FG} by putting $\lambda_1 = \lambda_2 = 0$ we can write down solutions for X-M and c_4K-W-Π. It is apparent that once again g must equal c_5, and it is also important that $-\frac{1}{\tau}$ is less than or equal to $g = c_5$. If $-\frac{1}{\tau}$ is less than $g = c_5$ then the final asymptotic form of X-M and c_4 K-W-Π will depend only on the initial conditions X and K at time $t = 0$. The diagram shown in Figure 4 illustrates these points using estimated parameters for the British economy. If actual values of X and K are plotted one sees that the points are in the asymptotic surplus region. However G has been increasing more rapidly than X or K (as shown clearly in Fig. 5) so that the conditions above are not met. If g was restrained to increase at a rate $(-\frac{1}{\tau})$ equal to $g = c_5$, a modified diagram would be required in terms of initial values for X, K and G. The effect of g would be to improve δ but to make the balance of payments surplus smaller. This is probably politically a more realistic situation.

4. PARAMETER AND VARIABLE ESTIMATION

In earlier studies the following numerical values were used :

$c_1 = 0.22$	$a_1 = 0.3$	$\tau = 0.5$
$c_2 = 0.6$	$a_2 = -0.33$	$k = 1$
$c_3 = 1.0$	$b_1 = 0.6$	
$c_4 = 1/3$	$\lambda_1 = \lambda_2 = 0.05$	
$c_5 = 0.03$	$\gamma_1 = 0.1$	
$c_6 = 0.1$	$\gamma_2 = 0$	
$c_7 = 0.05$	$\gamma_3 = 0.4$	
	$\gamma_4 = 0.1$	

Values for the variables C, G, I, X, M, W, S_H, D, T_H, S_G, $I_{FC} + I_{FG}$ and R_C were obtained from the 1965 edition of the National Income and Expenditure blue book published by the Central Statistical Office, using constant price figures (and therefore neglecting inflation). Difficulty was experienced with the time series for K (given in Table 61 of the Blue Book) on account of $c_4K - W - \Pi$ becoming negative. To avoid this difficulty a new series was produced which involves keeping K for 1958 the same but scaling up of I. The two plots are shown in Fig. 5.

Using certain of the parameters as given in the previous table, least squares estimates of others have been obtained, viz :

$c_1 = 0.232$	$c_6 = 0.02$
$c_2 = 0.599$	$k = 0.83$
$c_3 = 0.197$	$a_2 = -0.26$
$\gamma_1 = 0.069$	

More work is required on these estimates and programmes are being written for this purpose.

5. CONCLUSIONS

From the basic model and the modifications certain results emerge :-

(i) the growth is limited by the export growth rate c_5 for ideal behaviour, with government expenditure also being held in check to this rate.

(ii) making government expenditure proportional to excess capacity is not a very effective method of control, as root-locus diagrams reveal that a very high k is necessary to achieve (i), and oscillatory transitories will be encountered.

(iii) more work on parameter estimation and on accurate estimates of K is required to improve the reliability of the results (i) and (ii) and to provide a basis for further work.

Finally, it is clear to us that the field is ripe for development and in particular for the exploitation of the techniques of modern control theory, as illustrated by Noton's paper [7] on optimal control. There is a need for approaches by different groups of people using models of varying complexity and for trying out other new techniques in system identification and analogue and digital computation. Here is a real chance for British economists and control engineers to develop a highly important interdisciplinary topic.

6. REFERENCES

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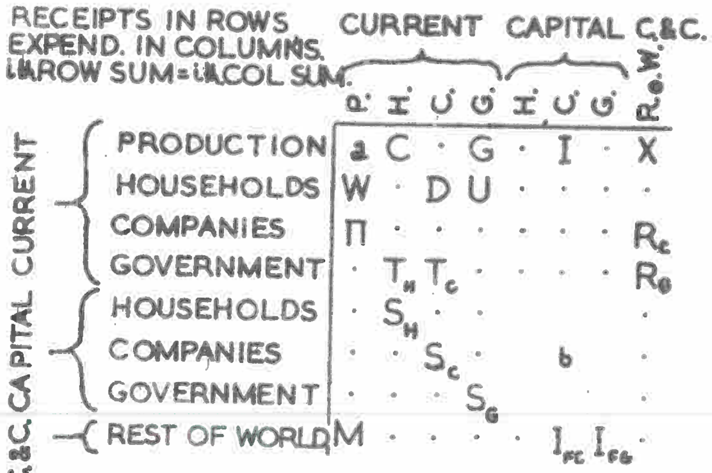
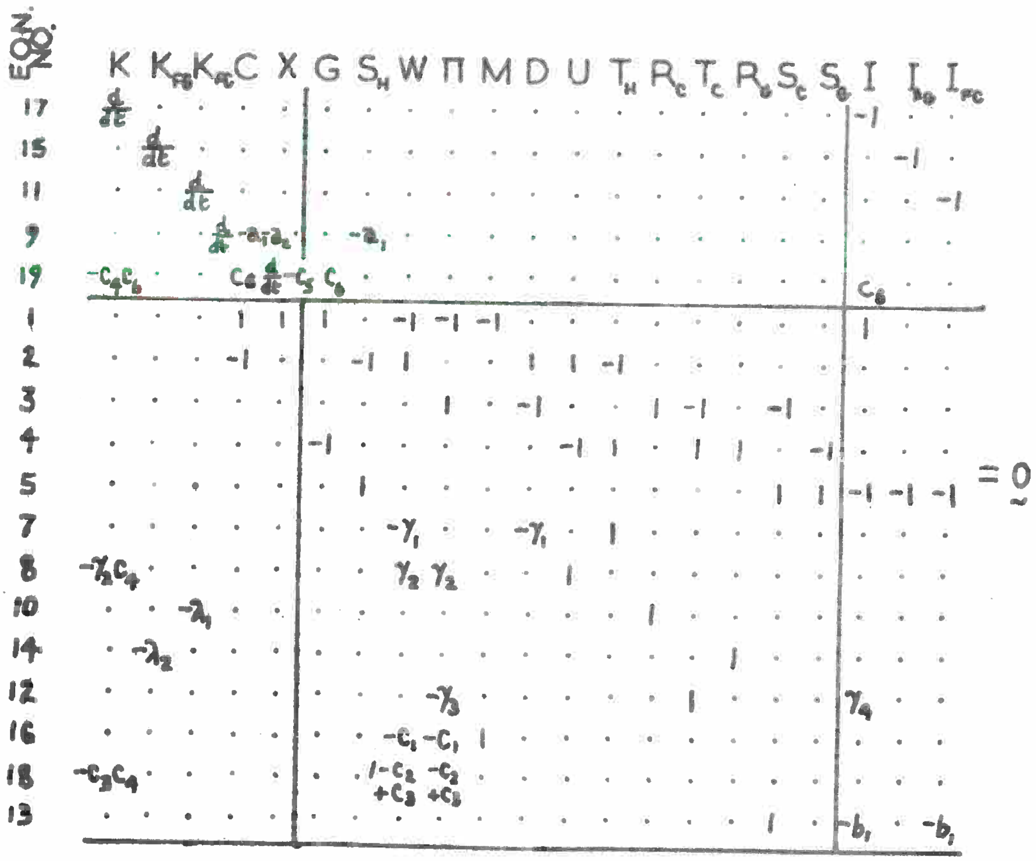


a = RAW MATERIALS FROM HOME

b = NET ACQUISITIONS OF CLAIMS

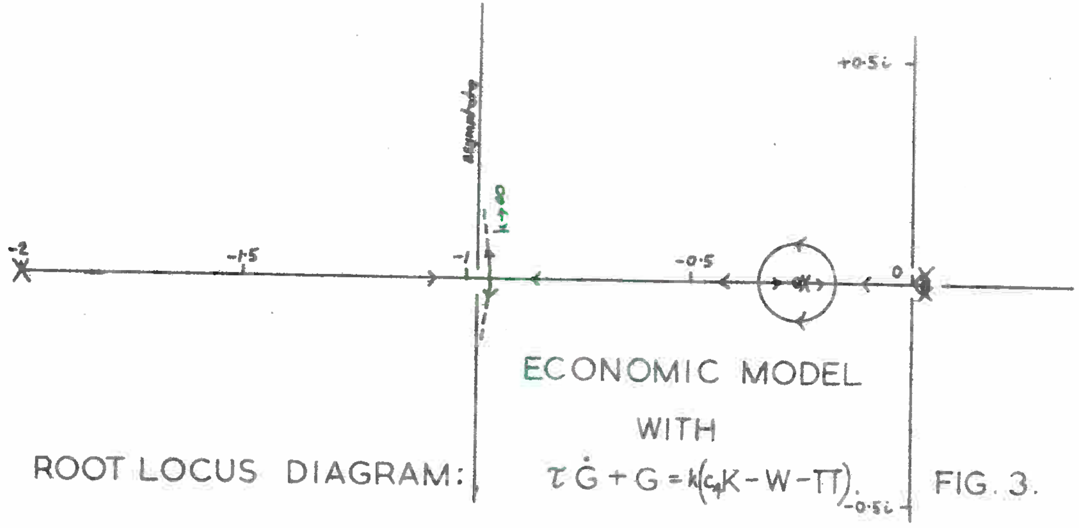
SOCIAL ACCOUNTING MATRIX.

FIG. 1.



ECONOMIC MODEL: MATRIX EQUATION.

FIG. 2.



ROOT LOCUS DIAGRAM: $\tau\dot{G} + G = k(c_4K - W - T\Gamma)$. FIG. 3.

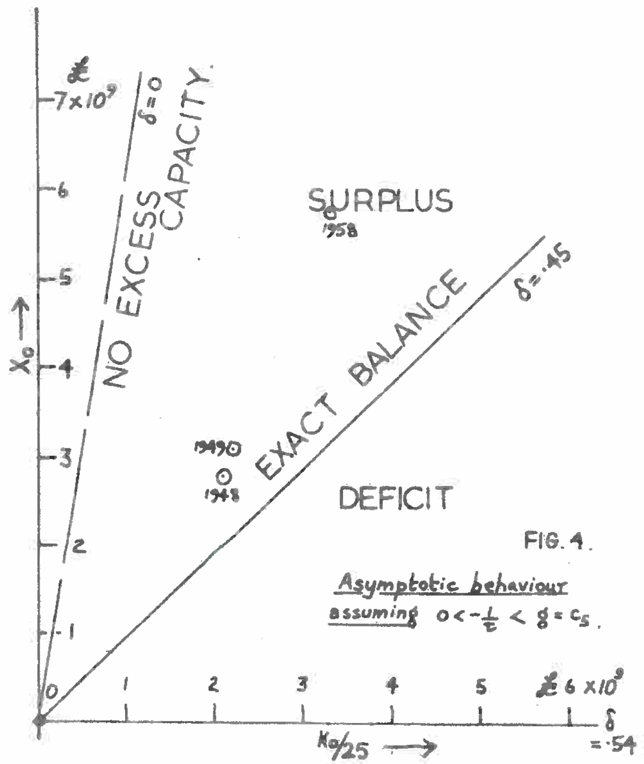


FIG. 4.

Asymptotic behaviour assuming $0 < -\frac{1}{\tau} < g = c_5$.

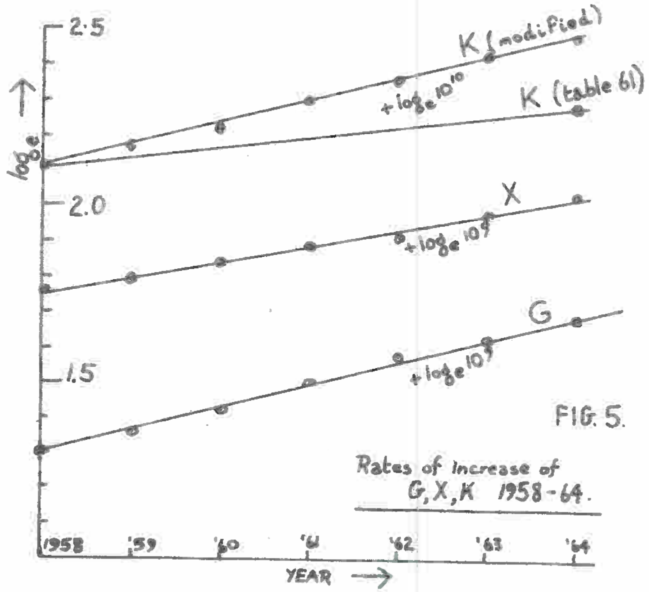


FIG. 5.

Rates of Increase of G, X, K 1958-64.