# PROFIT MAXIMISATION AND THE THREAT OF NEW ENTRY

by

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This paper is circulated for discussion purposes and its contents should be considered preliminary.

Introduction(1)

A recent volume of essays in honour of Edward H. Chamberlin contains a contribution by Sir Roy Harrod(2) in which he returns to the subject of his earlier essay 'The Theory of Imperfect Competition Revised'.(3) This earlier essay is well known for its challenging argument that a firm is unlikely to set price and output levels without regard to the effect these may have in inducing new entry into an industry. Harrod argued that because of the threat of entry, price will most likely be set to equate average revenue and average cost, the latter being defined to include both normal profits and rents. If price is determined in this way there will be no induced entry; the position is stable and does not imply that the average revenue and average cost curves are tangential. Such behaviour suggests a possible explanation of the widespread practice of full-cost pricing. It can also imply that a firm building a new plant will select a scale of operation such that average costs will be a minimum at the point where these costs are equal to average revenue. Since this is to be the point at which the firm will choose to operate under the assumed pricing policy there will be no excess capacity.

The challenge of Harrod's arguments was taken up by Hicks [4]. The latter's analysis introduced the notion of 'Stickers' and 'Snatchers', being a distinction based on the relative importance placed by a firm on profits in the short period, when new entry is not possible, and in the longer term, when entry can occur. This analysis was clarified and advanced by Hahn [2] who concluded (a) that firms will operate in the long term with excess capacity unless they place no value on short-run profits and, (b) that full-cost pricing merely describes behaviour in an industry which has reached equilibrium and is of little moment in explaining how this equilibrium comes about.

In contrast to Hicks, Chamberlin [1] did not accept much of Harrod's revision of imperfect competition theory and, in particular, he resisted Harrod's rejection of tangency of average costs and revenue in equilibrium. We now have, in Harrod's latest essay on the subject, his reply to Chamberlin's criticism of his earlier paper. In it Harrod confirms his rejection of tangency as necessary for equilibrium, and presents his theory in a more rigorous and developed form than was previously available. In this theory price is set equal to average cost, new entry is not induced, and there is no excess capacity in the long run.

Unfortunately, Harrod's theory does not seem to resolve the issues. The justification for this view is given in the next section where the Harrod model is closely examined. Yet Harrod must surely be correct in focusing our attention on a firm which knows that future entry may be induced by its current actions. This is an interesting, if not the most general case. And Harrod must again be correct in arguing that there is no reason why the shapes of the average revenue and long run average cost curves should necessarily permit a tangency solution. The cost curve could be horizontal, for example. It follows that there is scope for further analysis, and this is attempted in the remainder of this paper.

It emerges from the analysis that if the present value of net profits is the objective function according to which firms regulate their actions, then two distinctions are critical. The first is whether excessive profits determine the rate at which the demand curve shifts or the extent to which it shifts. The second is whether capital is continuously variable within the plant or is fixed for all time once the plant has been built. The importance of these distinctions is demonstrated through three models, in all of which time is a continuous variable. In part the analysis runs parallel to a fascinating paper by Wan [6] and has in common the use of the calculus of variations. Among the conclusions to be reached is that in cases which permit a tangency solution, a tangency solution will come about. In cases which do not permit tangency, there may well be no pricing policy which can prevent entry. In this event we can say something about the number of firms in the industry. All this depends, of course, on careful definition of normal profits and freedom of entry. Such definitions precede discussion of the three models referred to. Meanwhile, we first examine Harrod's formulation of the problem.

Harrod's Formulation

Harrod [3] assumes that a firm is faced with a downward sloping demand curve for its product. Its average cost curve is shown as being U-shaped in his main diagram, which is reproduced as Figure 1 below. It is not readily apparent whether the average cost curve refers to the long-run or the short-run : the arguments that are associated with the figure are stronger if the cost curve refers to the long-run, and this will therefore be assumed. It is not assumed that the average cost curve must necessarily intersect the demand curve twice. Notwithstanding the fact that Harrod's essay is titled 'Increasing Returns', average costs have a minimum at a finite rate of output and the analysis does not require that the firm should be producing in an environment which economies of scale are possible.

The main pillar of Harrod's argument is that a firm will not ignore the implications of its policy for potential entrants. Specifically he considers a situation which 'brings into the picture once more a tangency condition (which is) necessary only if we postulate absolutely free entry in the fullest sense.' Moreover this situation is not claimed 'to be true in all, or even most, cases.'(5)

The situation envisaged is one in which

'if the producer charges more than full cost, this will damage his future position; that if he charges less than full cost this may even improve it, being a sort of sales-pushing device; and that charging full cost will be neutral in relation to future prospects.... The curve entitled 'average net proceeds (long period)' in (Figure 1) shows what, for any given price charged, is the value of present proceeds per unit (i.e. price) plus or minus the present value of the increase or decrease of future proceeds, also per unit of output.'

Thus if p denotes price, if $\dot{p}$ is its derivative with respect to time, and $\lambda$ is a rate of discount, we have

$$\text{average net proceeds (long period)} = p_o + \int_o^\infty \dot{p}e^{-\lambda\tau}\,d\tau \qquad (1)$$

where $p_o$ is the initial price. We can now assume that the firm is restrained by a demand function

$$p = p(q,\pi) \qquad (2)$$

where q is the rate of output, and $\pi$ is an argument of the demand curve such that an increase in $\pi$ will lower demand, and conversely, i.e.

$$\frac{\partial p}{\partial \pi} < 0 \tag{3}$$

Thus if $\pi$ is raised by new entry, the firm will subsequently be obliged to sell at a lower price for any given output. Further, we have of course that

$$\frac{\partial p}{\partial q} < 0 \tag{4}$$

From (1) and (2) it follows that for a given value of q

$$\text{average net proceeds (long period)} = p_o + \int_0^\infty \frac{\partial p}{\partial \pi} \dot{\pi} e^{-\lambda t} dt \tag{5}$$

Accordingly the average net proceeds curve lies below the demand curve for those rates of output for which $\dot{\pi}$ is negative, and conversely. Now in Figure 1 and in the quotation given above we have

$$\dot{\pi} \lessgtr 0 \text{ depending on } p \gtrless \Omega(q) \tag{6}$$

where $\Omega(q)$ is average cost, including normal profit. No further restriction is placed on the specification, so that it would seem to be legitimate to write

$$\dot{\pi} = \dot{\pi}(m) \tag{7}$$

where
$$\frac{\partial \dot{\pi}}{\partial m} > 0 \text{ for all } m \tag{8}$$

and
$$m = p - \Omega(q) \tag{9}$$

Thus m is the excess of price over full cost, and such excess, if positive, raises $\pi$ and hence lowers the demand price at each level of output. Consequently the average net proceeds curve has the desired relationship with the demand curve under this formulation.

But Harrod's diagram involves a stronger specification than his text. In the diagram the average net proceeds (long period) curve is bounded from above by the average cost curve, $\Omega$, and is tangential to it when $m = 0$. This is not implicit in the formulation given above. According to the formulation, if a firm starts off earning super-normal profits then, keeping output fixed, demand will decrease until such time as the super-normal profits are eliminated. Consequently, starting from an initial value of $m > 0$, $m$ declines until such time as $m = 0$, and at this juncture we reach the traditional long-run equilibrium state. Since for all t we have

$$m > 0$$

and therefore $$p > \Omega$$

and for some t $$p > \Omega$$

it follows that

$$\text{average net proceeds (long period)} = p_o + \int_o^\infty \dot{p}\, e^{-\lambda t}\, dt$$

$$= \frac{1}{\lambda} \int_o^\infty p\, e^{-\lambda t}\, dt \qquad (10)$$

$$> \Omega$$

i.e. average net proceeds exceeds average cost. A converse result is obtained for rates of output at which m is negative initially. This together with the earlier result yields the following conclusion :

> If average revenue equals average cost, both are equal to average net proceeds (long period)
>
> Otherwise average net proceeds (long period) lie between average revenue and average cost.

The second of these two statements contradicts Harrod's conclusion as expressed both in the text and his diagram. It is not correct to claim that if marginal net proceeds (long period) is set equal to marginal cost, then average revenue, and hence price, will equal average cost. The tangency of average net proceeds (long period) and average cost which this requires does not exist unless there is an error among the equations (1) to (10).

The only possibility for reconciling Harrod's conclusion with the above formulation of his model is that the effect of entry expressed in equation (7) is incorrect. Harrod states that '(tangency is) necessary only if we postulate absolutely free entry in the fullest sense'. Now in terms of demand, free entry in the fullest sense should mean access to the same demand curve without extra marketing costs. In this event a new entrant might not simply cause a movement in the demand curve as specified in (7), but could have the effect of satisfying all demand

and so making demand for the existing firm vanish. It is by no means apparent that this is the case that Harrod had in mind. If it is then the analysis remains subject to certain shortcomings, of which not all are minor.[(6)] But meanwhile it can be noted that this attempt at reconciliation puts a new light on the controversy with Chamberlin. If the effect of new entry is to make (a part of) the demand curve for a firm vanish, then there is no possibility in general for a tangency of average revenue and average costs. Thus the tangency issue does not hinge on whether excessively high prices or profits induce entry, but rather on whether new entry, when it occurs, results in a continuous movement or a discreet jump in the demand curve.

## Three Conventional Models

For further analysis, and in contrast with Harrod, it will be assumed that it is the level of profit, rather than of profit per unit or price that might be influential in inducing new entry. This seems to be the more natural assumption to make if entry is a quest for profit opportunities. In so far as it leaves room for doubt as to whether this implies imperfect competition or oligopoly, the definitions will be left ambiguous. The operational distinctions are what matter : we here restrict ourselves to a firm which knows that if it earns excessive profits, then new entry will take place.

Since the behaviour of a firm depends in the first instance on its conception of its environment, it is important to define this clearly. In the first two of the three models to be discussed, the firm considers that new entry will imply a rate of change in the location of its demand curve. Thus if X denotes excess profits, the firm is of the view that its price, p, and output, q, are restricted by a demand curve of form

$$p = p(q, \pi) \tag{2}$$

where $\pi$ is a location parameter the rate of change of which depends on X, i.e.

$$\dot{\pi} = \dot{\pi}(X) \tag{11}$$

It follows that the value of $\pi$ depends not only on the current value of X but also on the whole time-profile of previous values of X.[7] By contrast, in the third model, new entry results in a discreet change in $\pi$ and could be such as to entirely eliminate demand for the existing firm.

Obviously some compromise between these two extreme formulations of the effect of entry can be envisaged. For example new entry could result in a discreet shift of demand without implying its total elimination. However, such formulations lead straight to the indeterminancies of the type found in duopoly theory, and I have wanted to avoid these here.

It must be assumed, of course, that there are no legal obstacles to entry. Entry may however be restricted by the fact that the existing firm will be earning rents which are not available to a new entrant. These rents are defined as the difference between the actual costs of the existing firm and the costs that would be incurred by a new firm which entered in the sense of having the effects on demand discussed above. Thus the marketing costs of product innovation are included in the rentals, as well as peculiar advantages of location, etc.

Total costs are to be defined as including all factor costs and rentals. In consequence of the definition of rentals, this means that the new entrant will have the same cost curves as an existing firm. Since capital as well as labour costs are included, an existing firm charging a price equal to average cost will earn a normal rate of return on capital, plus rents. In this sense it may be earning super-normal profits. But it will not be inducing entry. The excessive profits which induce entry, i.e. X, are defined as the excess of total revenue over total costs as specified.

If X is negative there is no reason to assume in models I and II that $\dot{\pi}$ will be negative. If a firm is absorbing its rental income through inefficiency there is no reason to think that this will cause the demand for its product to grow. If the firm does not earn normal profits, for whatever reason, this is unlikely to eliminate any competition. It is assumed, therefore, that

$$\begin{aligned} \dot{\pi} &> 0 \text{ for } X > 0 \\ \dot{\pi} &= 0 \text{ for } X < 0 \\ \frac{d\dot{\pi}}{dX} &> 0 \text{ for } X > 0 \end{aligned} \qquad (12)$$

In all three models there is no technical progress in the sense of a shift in the long-run cost curves. These curves remain stationary throughout. In model I capital is continuously variable without an investment lag and its services are assumed to be hired by the day. The firm can therefore move continuously along its long-run cost curves. Models II and III differ in that they assume that a firm builds a plant which has its own particular short-run average cost curve which is tangential to the long-run curve at a particular point. Once a plant is built only its particular cost curve is relevant until such time as it is replaced. Since physical deterioration is to be ignored, there will be no incentive to scrap a plant and build a new one other than through changes in demand.

The objective function for a firm is taken to be the present value of revenue in excess of total costs. It will be recalled that these costs include rents and a normal rate of return on capital. If rents are a fixed item of costs, then the objective function is a linear transform of the present value of trading surplus less capital costs. The objective is written

$$\int_0^\infty X\, e^{-\lambda t}\, dt \qquad (13)$$

where $\lambda$ is the discount rate. From this it is immediately apparent that if the firm has the opportunity of earning positive X at some moments in time without necessarily incurring negative X in future, then the opportunity is sure to be ceased.

## Model I

In this first model the firm can instantaneously change its capital input and is able to move along its long-run average cost curve. Denoting average costs by $\Omega(q)$, the decision problem facing the firm is to set price and quantity so as to maximise the objective function (13) subject to (a) the demand function (2), (b) the relationship (11) between changes in the location parameter and X, and (c) the definition of X which is

$$X = pq - q\,\Omega(q) \qquad (14)$$

Introducing these restraints under dynamic Lagrangean multipliers yields as the integrand of the objective function

$$Xe^{-\lambda t} + \alpha(p-p(q,\pi)) + \beta(\dot{\pi}-\dot{\pi}(X)) + \gamma(X-q(p-\Omega(q))) \qquad (15)$$

where $\alpha$, $\beta$ and $\gamma$ are the dynamic Lagrangean multipliers. The Euler conditions for the objective function to be stationary are

$$\begin{aligned} e^{-\lambda t} + \beta\frac{d\dot{\pi}}{dX} + \gamma &= 0 \\ \alpha - \gamma q &= 0 \\ -\alpha\frac{\partial p}{\partial q} + \gamma\left\{\Omega(q) + q\frac{d\Omega}{dq} - p\right\} &= 0 \\ -\alpha\frac{\partial p}{\partial \pi} &= \beta \end{aligned} \qquad (16)$$

Elimination of $\alpha$ from these equations reduces them to

$$\gamma \{ \Omega(q) + q \frac{d\Omega}{dq} - p - q \frac{\partial p}{\partial q} \} = 0 \qquad (17)$$

$$\left.\begin{aligned} e^{-\lambda t} + \gamma &= \beta \frac{d\dot{\pi}}{dX} \\ \dot{\beta} &= - \gamma q \frac{\partial p}{\partial \pi} \end{aligned}\right) \qquad (18)$$

Equation (17) is of great interest. It states that either $\gamma$ is zero or the expression in brackets is zero. Since this expression is the difference between marginal revenue and marginal cost we have at once that either $\gamma = 0$ or the firm maximises X at each and every moment in time.

If $\gamma = 0$ then it follows from (18) that $\beta$ is a constant and

$$\frac{d\dot{\pi}}{dX} = \frac{1}{\beta} e^{-\lambda t} \qquad (19)$$

The alternative solutions of maximising X and following (19) have in common that they result in $X > 0$ if this is feasible. It will be feasible if for some range of output demand price exceeds average cost, and we have therefore a demonstration of the proposition that if excess profits can be earned then they will be, and this will cause a reduction in demand. Clearly a position of tangency will be reached eventually.

The fact that the demand curve is to move from its initial position to one of tangency implies that the optimisation problem can be set up as

$$\text{maximise} \int X e^{-\lambda t} dt \text{ subject to} \int \dot{\pi}(X)\, dt = \Delta\pi \text{ and } X < X^* \qquad (20)$$

where $\Delta\pi$ is the change in the location parameter corresponding to the initial position of the demand curve and its ultimate position of tangency to the average cost curve. The inequality restraint states that profits X must always be less the maximum that could be obtained, denoted by $X^*$. If the inequality restraint is not binding, then the integrand of the objective function, allowing for the integral restraint, can be written as

$$X e^{-\lambda t} - \frac{1}{\beta} \dot{\pi}(X) \tag{21}$$

where β is a constant. The Euler condition applied to this integrand yields at once equation (19). It follows that (19) is the solution of the reformulated maximisation problem, provided it yields $X < X^*$. If $X = X^*$, then X is maximised and marginal revenue is set equal to marginal cost.

Since (19) is the solution of a less constrained formulation than $X = X^*$, it is obviously to be preferred when it is feasible. And feasibility can be explored in the following way. A choice of β in (19) determines a particular time path for X and for $\dot{\pi}$. The time path for $\dot{\pi}$ must satisfy

$$\int_0^\infty \dot{\pi}(X)\, dt = \Delta\pi \tag{22}$$

and hence β can be determined (but not necessarily uniquely). Given β, a time sequence for X is established. If this satisfies $X < X^*$, then (19) gives the optimal solution. Otherwise we have $X = X^*$.

But whether $X = X^*$ or (19) is the optimal solution, it remains true that we have $X > 0$ in the short-run, and in the long-run the average revenue and average cost curves will be tangential.

Model II

This second model is the same as the first on the demand side. Now, however, the production technology is assumed to be more conventional. The firm chooses a plant size which determines a fixed amount of capital. The plant is operated with respect to its short-run (i.e. fixed-capital) cost curves throughout its life.

This model is analysed in two stages. First we examine how a given plant would be operated. We then look at the question of choosing a plant size.

The operation of a given plant can be derived directly from Model I. All we need is to rename Ω as the short-run average cost curve and the rest of the analysis follows immediately. Hence the firm will realise short-run profits where possible, demand will decrease, and this process will continue until the demand curve is tangential to the short-run average cost curve. Subsequently the firm will continue to earn normal profits at least. But now the demand curve no longer shifts : a tangency equilibrium position has been reached.

In equilibrium demand must be tangent to short-run average cost but need not be tangent to long-run average cost. In this sense, which is illustrated in Figure 2, there can be excess capacity in the long-run. Whether or not there is such excess capacity will depend simply on whether plant A or some plant B in Figure 2 is chosen : if A is chosen there is no long-run excess capacity, if $B_2$ is chosen then there is excess capacity, and if $B_1$ is chosen then there is a capacity deficiency.

The choice of plant will be determined by the objective function. For each plant the objective function will have a value depending on the level of X and time period through which optimal behaviour results in $X > 0$. Once the tangency equilibrium has come about the choice between plants is a matter of indifference : none of the plants A or $B_1$ to $B_3$ earns excessive profits and all cover normal capital costs and rents. There is, therefore, no reason to prefer one to any other within the framework of the model, since none of them would be faced with the threat of new entry as they perceive it.

In practice the plant which maximises the objective function is not likely to be very different from A. A plant far to the left of A is likely to yield a lower value of X for any given location of the demand curve, and similarly for a plant far to the right. To be preferable to A a plant must yield a higher value of X for at least some location of the demand curve. This implies a degree to which the plant will lie in the region of A, but in no sense establishes A as the uniquely best choice.

This analysis could be further extended by allowing for the possibility that a firm, having built a plant, could subsequently scrap it and build another. In such a model capital costs would have to be treated somewhat differently since it would imply that each plant had a finite life. However, such reformulation would not change the two basic conclusions of this model, namely that whatever plant is built the firm will operate with $X > 0$ whenever possible, and that the first plant built is not necessarily A.

Model III

In the first two models the firm is able to control the rate and extent to which its demand curve shifts. Both models yield a form of tangency solution with demand eventually tangential to long-run average cost in Model I and to short-run average cost in Model II. We now turn to what is perhaps the most interesting case where a new entrant could result in the elimination of the existing firm by commanding the whole of product demand. The technology of Model II is retained here so that a firm is choosing a plant to operate for all time and has fixed capital, and therefore fixed capital costs once the choice has been made.

If a new entrant is potentially capable of satisfying the demand facing an existing firm, it must be capable of producing the identical product. Given this, the new entrant will make a successful entry if it can earn normal profits with a price-output combination which cannot be undercut by the existing firm if the latter is itself to earn normal profits. In such a situation the existing firm might accept less than normal profits in the hope of eliminating a new entrant. But our concern here is not with what happens after entry, but rather with the prior thinking of the existing firm in deciding what plant to build. And the hypothesis is that the existing firm will not envisage entry if it can permanently achieve normal profits with a price-output combination which leaves no room for a new entrant to do the same.

Since the possibility of the existing firm earning normal profits but no rents now enters the analysis, it is necessary to define a new long-run average cost curve which excludes such rents. This is labelled LRAC in subsequent diagrams. Hence the cost curve which includes rent is the cost curve that represents normal profits for a new entrant : that which excludes rents covers normal costs for the existing firm.

Both cost curves feature in Figure 3 which depicts the case of increasing returns. In the Figure $p^*$ is the lowest price that a new entrant could charge and still earn normal profits. To achieve this would involve building a plant with capacity $q^*$, i.e. the plant A. If the existing firm is to be able to earn normal profits at least while charging a price not greater than $p^*$ and satisfying all demand, it must build to a scale somewhere between that of plants $B_1$ and $B_2$. Any plant within this interval eliminates the possibility of entry as defined.

In the present model there is no threat of entry provided that an appropriate plant is built. Hence once a plant is built profits will be maximised at each moment in time : new entry is deterred by the capability of the existing firm to meet demand at a price not greater than p*, and the fact that this firm may choose to behave differently is irrelevant given that the capability exists. Thus the choice of plant in the range $B_1$ to $B_2$ depends on the profits yielded by short-run profit maximisation in each. Unless the rental income of the existing firm is very large, the preferred plant will always be $B_1$, i.e. the smallest plant in the relevant range. However if the rents were so large that $B_1$ was a smaller plant than the one for which short-run profits were a maximum, then clearly the latter would be preferred.

It follows that whichever plant is built, it will be operated so as to maximise short-run profits. Price will exceed p*. To the extent that existing firms earn rents, the plant will be smaller than A. If there is no rental income however, plant A will be built.

The case of no rental income can be thought of as 'freedom of entry in the fullest sense', and the choice of Plant A corresponds to Harrod's prescription as discussed earlier. However choice of A does not imply full-cost pricing, since the firm will maximise short-run profits as we have seen. But the analogy to Harrod's result is quite strong.

In the case of decreasing returns, it may not be possible to build a plant which will preclude entry. If the existing firm is to earn normal profits then the lowest price it can charge at each level of output is given by its short-run average cost curve. Hence the demand facing a new entrant is at least the excess of product demand over short-run average cost of the existing firm. And if there is no plant, i.e. no short-run average cost curve, which yields a residual demand lying wholly below the long-run average cost curve, then new entry cannot be precluded in this model

Such a situation is illustrated in Figure 4. The firm is assumed to have chosen a plant corresponding to the minimum point on the long-run average cost curve. In the case illustrated demand is sufficiently large to permit the new entrant to choose the same scale of plant; in this case any plant between $B_1$ and $B_2$ could conceivably be chosen. But the range $B_1$ to $B_2$ does not limit the new entrant. All that is relevant for our purposes is that if a range such as $B_1$ to $B_2$ exists, then new entry cannot be prevented, i.e. the existing firm will expect new firms to enter in competition for market demand. The case illustrated is one where the market is significantly bigger than the optimal scale of production. Accordingly more than one plant is to be expected.

## Conclusions

The models explored in this paper are extremely simple. Yet their logical conclusions are sufficient to demonstrate that the influence of potential new entry on pricing and investment decisions does not permit of any neat description. We have found that in many, but not in all situations, firms will maximise profits in the short-run even if they believe that this will lead to new entry. In cases where short-run profits are not maximised, none-the-less profits will be at a level such that entry may be induced. In consequence, tangency of demand and average cost will always result in situations where this is a feasible outcome. To this extent Harrod's contention that tangency might be incompatible with recognition of the threat of entry is disproved. But a second part of Harrod's solution is retained, albeit in a modified form. For we have found that when new entry can take the form of a threat to replace a firm, then the threatened firm will tend to build a plant of such scale as to produce most efficiently in the region where long-run average costs approximate the demand price if the former are falling. Whether or not this is a typical case is at present beside the point. My own view is that it may well be. But typical or not, the case implies that the firm can earn excess profits for as long as demand remains stationary. If demand were to grow for exogeneous reasons then the firm would still have to expect new entry or to plan for the construction of a new plant.

FIGURE 1

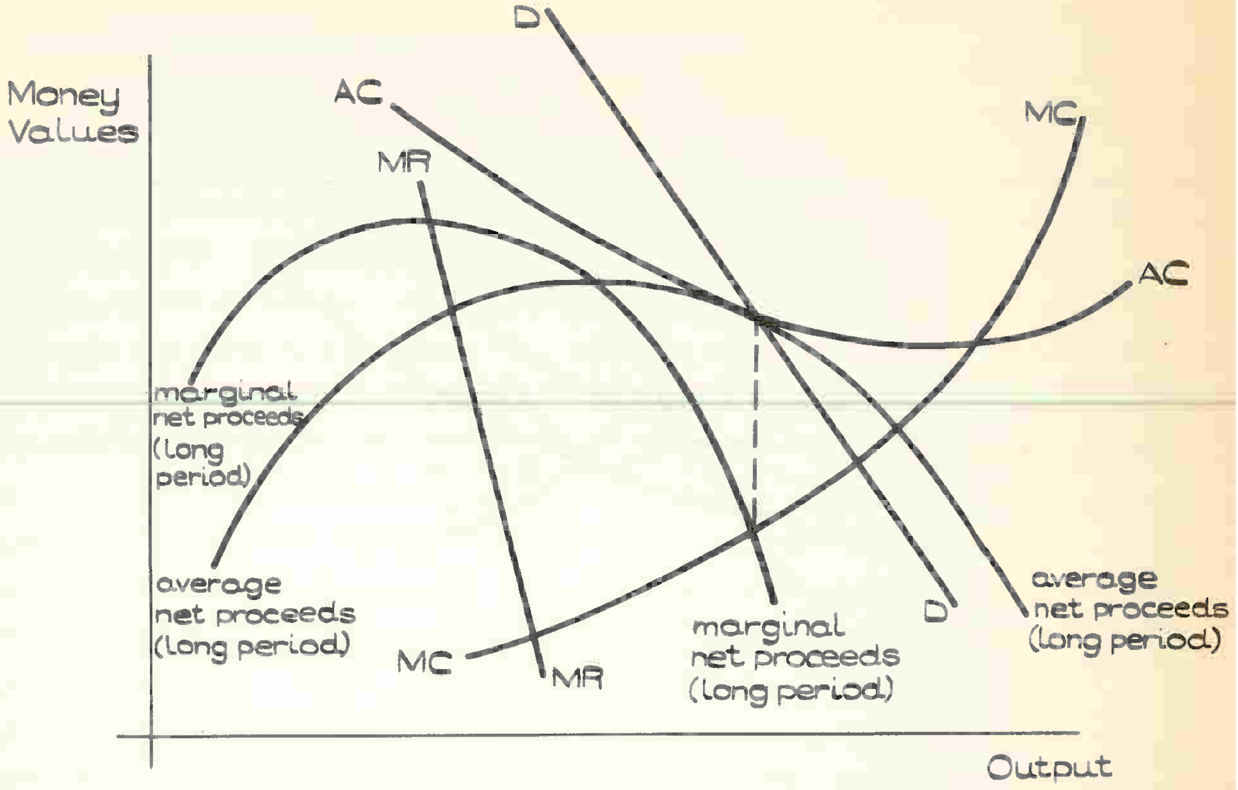
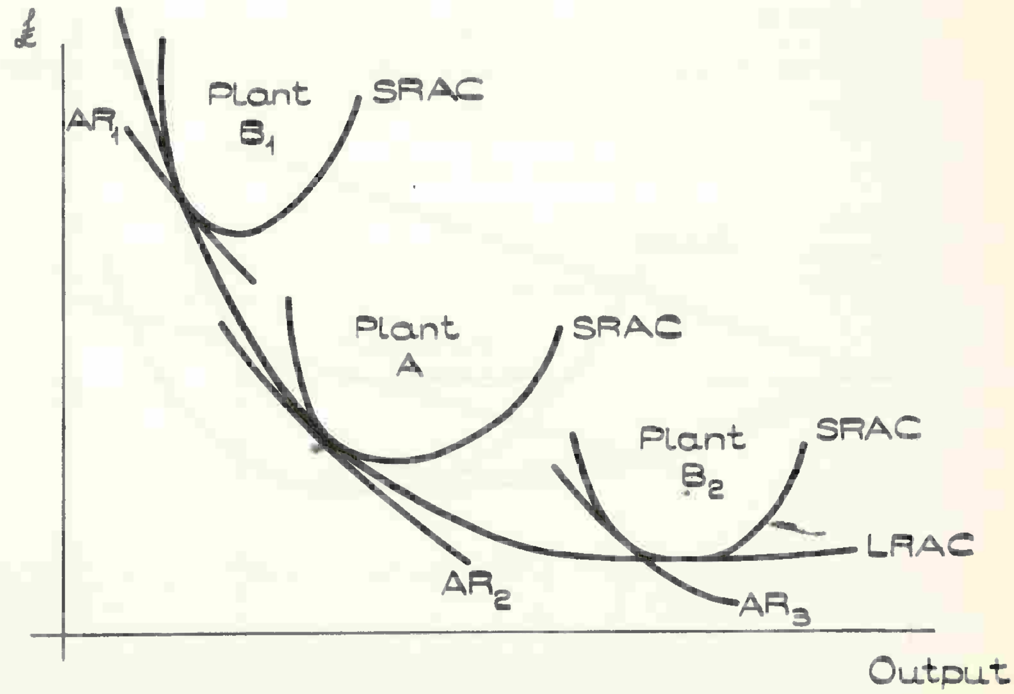



FIGURE 2



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Footnotes

1. This paper has been written in part while spending leave at Nuffield College, Oxford, and I am happy to record my debt to the College for facilities provided. I am also indebted to Mr. Max Corden for stimulating discussion of the subject matter, and to members of the Oxford Quantitative Methods seminar who heard an earlier version of this paper and made several helpful comments on it.

2. Harrod's essay is titled 'Increasing Returns' and is Chapter 3 of Kuenne [ 5 ].

3. Published as Essay 8 in Harrod [ 3 ].

4. See 'Increasing Returns', pp. 69 - 70.

5. Both these quotations and that which follows are from Harrod's essay 'Increasing Returns'

6. An example of the remaining difficulties is that both average net proceeds (long period) and average cost must depend on the rate of discount, $\lambda$. If these curves were tangential for one value of $\lambda$ there would seem to be no reason why they should be tangential for some other value.

   As a second example, it is not clear what would be the outcome if demand and average cost have only one point of intersection, at which point demand cuts average cost from below.

7. Some form of discounting previous values of X could be introduced without materially affecting the conclusions.