REGIONAL INPUT-OUTPUT MODELS IN THE U.K.:

A RE-APPRAISAL OF SOME TECHNIQUES*

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ABSTRACT

Three aspects of regional input-output models are examined. First, the use of the Leontief system unadjusted for the particular needs of regional planning is examined. In particular, the difficulty of deriving meaningful final output projections for the region is discussed. Secondly, the results obtained in a recent paper by HEWINGS, Reg. Studies 5, 11 - 22, using badly derived estimates of final output are shown to yield serious errors in his results. Some tractable improvements to his techniques are offered. Thirdly, the model produced for Wales (NEVIN, ROE and ROUND, The structure of the Welsh Economy) University of Wales Press, 1966, is set alongside and shown to be similar in type to the non survey techniques employed by him and also, it is shown to have decided advantages.

1. Introduction

In a recent and provocative article HEWINGS (1971) pinpointed the use of the input-output technique within the framework of regional analysis. In particular, he outlines the relevance of its use in the United Kingdom before examining further a series of techniques which have been used in order to simulate regional input-output tables in the U.S.A. He did so by applying these techniques to a minimum amount of data for the West Midlands. On the basis of the range of tables so produced he felt able to conclude that these techniques give rise to some promise in the simulation of regional input-output tables. In view of the quality of the methods employed to date, it will be argued here that any statement even approaching the tone of this conclusion is questionable. That is not to say that studies of this kind are of no use. On the contrary, they do serve to narrow the range of possibilities; albeit that the remaining spectrum of possibilities is forbiddingly large.

In this article, three issues are re-examined. In the first place, the usefulness of regional input-output models in their present form as an aid to regional policy is questioned. Secondly, the assumptions of the non-survey techniques used by Hewings in order to simulate a table for the West Midlands are critically investigated, and some workable amendments are suggested. Thirdly, the model used to simulate a table for the Welsh economy[1] (NEVIN, ROE and ROUND, 1966) is set alongside the particular models adopted by Hewings, so as to reveal some distinct advantages of the former approach.

2. How useful are regional input-output tables ?

It is undeniable that in recent years, there has been a great deal of effort poured into the production of a variety of input-output tables at the regional level. Most certainly, Tiebout's early statement, that these studies have been carried out almost predominantly in the United States still holds true (TIEBOUT, 1957) although not exclusively, as is indicated by the many bibliographies on input-output studies (TASKIER, 1967). It would seem however that the time is imminent, if it has not arrived already, when a thorough appraisal of the uses of these regional and small-area tables in their present form should be carried out. It is not the purpose of the present paper to carry out this appraisal although it is necessary in the present context to make a few pertinent remarks.

Consider the basic equations of the Leontief system as applied to the region. They may be written as:

$$(1) \qquad \sum_{j=1}^{n} a_{ij}^{\;r} \; {}_{r}g_{j} + {}_{ro}f_{i} \equiv {}_{r}g_{i} \qquad i = 1, \ldots, n$$

where $a_{ij}^{\;r}$ = current sales from industry i (region r) per unit output of industry j (region r)

${}_{r}g_{j}$ = gross output of industry i (region r)

${}_{ro}f_{i}$ = final output of industry i (region r)

For any base period, the system depicted by (1) is an identity relationship, and for that period alone, will involve no assumptions for it to hold good. Only when deviations from this base period set-up are considered is there any need to depart from this identity relationship. The usual departure is to consider changes in the final output vector, $\Delta\,{}_{ro}f\,(\theta)$, where the new final output vector ${}_{ro}f\,(\theta)$ represents either intertemporal projections for the region, or alternatively policy impacts in the base period, or even a combination of both. In any case, one may obtain a set of resulting gross outputs in the usual way.

$$(2) \qquad {}_{r}g\,(\theta) = (I - {}_{r}A)^{-1} \; {}_{ro}f\,(\theta)$$

where ${}_{r}A$ is the nxn matrix of regional input-output co-efficients, ${}_{r}g(\theta)$ and ${}_{ro}f(\theta)$ are vectors of gross outputs and final outputs respectively for state θ.

Now consider the precise composition of the final output vector ${}_{ro}f$. Following the usual social accounting classification of the components of final output, five broad groupings can be isolated.

${}_{ro}f\,(1)$: private consumers' current expenditure on commodities produced by region r

${}_{ro}f\,(2)$: private fixed capital formation using commodities produced by region r

${}_{ro}f(3)$: government (central or local) current and capital expenditure on commodities produced by region r

${}_{ro}f(4)$: exports to other regions <u>within</u> the national boundaries of commodities produced by region r for intermediate use

${}_{ro}f(5)$: exports <u>outside</u> of the national boundaries of commodities produced by region r

Thus, quite simply

$${}_{ro}f = \sum_{k=1}^{5} {}_{ro}f(k)$$

It should be specifically noted however, that each of these elements of the regional final output vector ${}_{ro}f$ are <u>origin-oriented</u>.

With the exception of ${}_{ro}^{f}(4)$ and ${}_{ro}^{f}(5)$ they reveal nothing about the ultimate destination of the outputs of region r. With this in mind, the elements of each of the vectors ${}_{ro}^{f}(1)$, ${}_{ro}^{f}(2)$ and ${}_{ro}^{f}(3)$ can be subdivided into two components.

One component relates to sales of region r's commodities for final use within the region and the second component relates to sales of region r's commodities for final use outside of the region (but within the nation)[2]. Designating collectively all other regions which comprise the rest of the nation by prefix s then for each of the vectors ${}_{ro}^{f}(k)$ (k = 1, 2 and 3) the two components may be isolated correspondingly as follows:

$${}_{ro}^{f}(k) = {}_{rr}^{f}(k) + {}_{rs}^{f}(k)$$

If the regional economy is a small part of the nation (and here, the regions are thought to be one of the 10 standard regions of the U.K.) then the degree of openness will be high and hence the elements of the vectors ${}_{rs}^{f}(k)$ may well be relatively large.

There is, of course, an exactly analogous breakdown for final outputs of regions. The final outputs of the two regions r and s (which, as defined here, constitute the entire national economy) can therefore be decomposed as shown in Table 1.

Table 1 - Breakdown of final outputs

	Final demands by region r			Final demands by region s			(4)	(5)
	(1)	(2)	(3)	(1)	(2)	(3)		
Final output of region r	${}_{rr}^{f}(1)$	${}_{rr}^{f}(2)$	${}_{rr}^{f}(3)$	${}_{rs}^{f}(1)$	${}_{rs}^{f}(2)$	${}_{rs}^{f}(3)$	${}_{rs}^{f}(4)$	${}_{ro}^{f}(5)$
Final output of region s	${}_{sr}^{f}(1)$	${}_{sr}^{f}(2)$	${}_{sr}^{f}(3)$	${}_{ss}^{f}(1)$	${}_{ss}^{f}(2)$	${}_{ss}^{r}(3)$	${}_{sr}^{f}(4)$	${}_{so}^{f}(5)$

This two-way classification of the final use sector serves to facilitate a further analysis. The point to be emphasised here is that one should make a careful and clear distinction between final outputs and final demands.

The prime purpose of constructing regional input-output tables is essentially to assess the impact of local oriented policies, or perhaps, of national policies with a regionally differentiated impact. Long gone are the days when the sole aim was to show that such tables can be produced. With well over 40 such tables now in existence, this latter purpose has now lost its appeal[3]. Given that impact analyses are the end products in regional input-output studies it can be seen that a great deal depends upon the construction of a reliable set of vectors of the type $_{ro}f(\theta)$. It is also true that these final output vectors are not easily obtainable.

In order to illustrate this point, consider two simple examples of impact analyses in the regional sphere.

(a) Suppose it is desired to assess the likely impact of some public works expenditure (such as the construction of a new hospital) within region r. This expenditure will create direct demands for goods and services on final account. But these final demands will not necessarily - either exclusively, or even largely - be derived from industries operating within region r. The smaller the size of the region relative to the nation, the more open the regional economy is likely to be and hence the more likely a significant portion of goods and services will be imported from other regions. In just the same way, a new hospital in region s may well create direct expenditures on final outputs of region r.

(b) Suppose there is proposed by the central government, some regionally differentiated changes in income tax[4], so that in particular, personal incomes in region r are affected. A fall in these taxes would tend to stimulate personal consumption by residents of the region. Only if the marginal propensities to consume out of 'home-produced' (region r) goods and services and the marginal propensities to import goods and services from region s are known, will the precise level of direct impacts on region r and region s be calculable. There is no *a priori* reason whatsoever for believing either one of these marginal propensities to be greater than the other.

It is quite clear that the marginal and average propensities to consume locally produced goods will depend upon a multitude of factors, not least of which will be the product composition and mix within each industry of the region. The point here is easily stated. In order for interesting policy questions to be answered of the type posed above, it is essential that some notion of the magnitude of the propensities to consume locally is at hand. Without them, it is impossible to obtain a vector of final outputs ${}_{ro}f(\theta)$ in order to start the round of calculations. What normally is far more readily available is a set of regional final demands ${}_{or}f(\theta)$ where, dropping the θ for convenience

$$(3) \qquad {}_{or}f \equiv \sum_{k=1}^{3} {}_{rr}f(k) + \sum_{k=1}^{3} {}_{sr}f(k) + {}_{or}f(4) + {}_{ro}f(5)$$

These differ from final outputs to the extent of a vector of net exports to region s, since:

$$(4) \qquad {}_{ro}f \equiv \sum_{k=1}^{3} {}_{rr}f(k) + \sum_{k=1}^{3} {}_{rs}f(k) + {}_{ro}f(4) + {}_{ro}f(5)$$

For the base year, the final output vector ${}_{ro}f$, may well be available for those tables produced by full surveys. The problem usually reappears when ${}_{ro}f(\theta)$ is required.

A relatively recent input-output model at the regional level which avoids this difficulty of translation from demands to outputs has been derived by LEONTIEF and STROUT (1963). The basic premises of this model require that the regional impacts be applied to the model as vectors of final demands, ${}_{or}f$, rather than final outputs, ${}_{ro}f$. This feature therefore alleviates the need for difficult empirical gymnastics to convert final demands into final outputs. However, the model is designed for, and only works for, a multiregional system (where the number of regions is much greater than two) and as such it is perhaps not comparable with the system at hand. Nevertheless, the point should be made that in order to implement a regional input-output model, a set of final outputs for the region have to be derived and that this empirical derivation (even with good survey data) is by no means trivial.

The individual components of the regional final output vector can give rise to difficulties of interpretation too. One large question mark hovers over the component ${}_{ro}f_{(4)}$ $(= {}_{rs}f_{(4)})$. This is quite simply, the gross exports of intermediate commodities to region s, and is likely to be quite a significant part of total final output. It is one matter to state that these sales will be treated exogeneously, even acknowledging that they will really be endogenously determined, but it is quite a separate matter to be able to say anything sensible afterwards about the effects on gross outputs of a vector of final outputs constructed in this way. By the very nature of the regional problem, interregional interindustry feedback effects are likely to be significant. It is questionable therefore whether or not it is legitimate to treat interregional flows and services on intermediate account as being exogenous, whilst at the same time one looks at intraregional flows and their multiplier effects with such minute precision.

The first contention to be borne out in this paper is therefore, that simple regional input-output tables in their present form present formidable conceptual problems, and may require further rethinking before many more tables are produced.

3. Non-survey techniques and the West Midlands Study - can we do better ?

In the previous section, just one aspect of the difficulty of applying the Leontief input-output system wholesale to a regional economy was emphasised. One conceptual improvement would be to work within the framework of a pure interregional model of the type proposed by ISARD (1951). In so doing, the vectors of the type ${}_{ro}f_{(4)}$ would no longer be exogenous but instead, they would be endogenously determined. Moreover, one would take a more realistic account of the interregional multiplier effects which are so essential to understanding the ramifications of regional policy in the U.K.

With this in mind, the attempt by HEWINGS (1971) to construct a regional table for the West Midlands will now be examined. Hewings re-opened the question first posed by others in a U.S.A. context (SCHAFFER and CHU, 1969; CZAMANSKI and MALIZIA, 1969) as to whether non-survey methods may simulate regional input-output tables, at least to an acceptable degree of accuracy, and thereby reduce (and possibly eliminate) expensive, full scale surveys of the interregional system. The answer may well be in the affirmative - but one can only say that it is unlikely that the present batch of techniques, which are set out so well by Hewings, should perform with corresponding excellent as simulators of a West Midlands input-output table. Whilst in no way undermining the extreme difficulties of this task, this section will be devoted to assessing some of the results of Hewings' work.

3.1. The Final output estimates

His first serious obstacle is that there are no final output estimates directly available for the West Midlands. It is slightly unclear as to how he surmounted this obstacle. He states in his assumption 3.1. c:

> "the percentage of sales to the final demand sections contained in the national tables was the same in the region for each industrial sector".

It would seem therefore, that for the base period he takes

(5)
$$_{ro}f_i(54) = \frac{f_i(54)}{g_i(54)} \cdot {}_{r}g_i(54) \qquad \text{(all i)}$$

However, he does not in this case account for interregional sales on intermediate account $_{ro}f_{(4)}$ and moreover, nor is it guaranteed that his row sums will agree with his sector gross outputs.[5] This he appears to correct by making the assumption that:

> "Interregional sales were estimated from the models as residuals".

So that it would now seem that

(6)
$$_{ro}f_i(54) = {}_{r}g_i(54) - \sum_{j=1}^{45} a^r_{ij} \; {}_{r}g_j(54) \qquad \text{(all i)}$$

otherwise base year row balance would not be achieved. In any case his last point on this issue is quite correct:

"total final (output) for each sector was not necessarily the same in each model".

This point is crucially important because it does lead on to show that his final table 4, which shows the variations between predictions of total outputs for 1958 according to his seven methods of obtaining a^r_{ij} may be nothing more than a table of computational errors. Now consider why this is so.

3.2. The Projections

Hewings constructed estimates of final outputs for 1958 by using a straightforward ratio as follows:

$$(7) \qquad {}_{ro}f_i(58) = {}_{ro}f_i(54) \frac{{}_{r}g_i(58)}{{}_{r}g_i(54)} \qquad \text{(all i)}$$

From these final output estimates and for each of the seven regional input-output coefficient matrices ${}_rA$ obtained from his non-survey techniques, he attempts to derive a set of estimates of gross outputs for 1958. Thus for any particular ${}_rA$ he obtains

$$(8) \qquad {}_{r}g^*(58) = (I - {}_rA)^{-1} \; {}_{ro}f(58)$$

where the vector ${}_rg^*(58)$ denotes the predicted gross outputs for 1958.

But ${}_{ro}f(58) = \hat{\alpha} \; {}_rg(58)$ where $\hat{\alpha}$ is the diagonalised vector[6] of α_i's where, using (7)

$$\alpha_i = \frac{{}_{ro}f_i(54)}{{}_rg_i(54)}$$

now,

$${}_rg(54) \equiv (I - {}_rA)^{-1} \; {}_{ro}f(54)$$

since for each model producing ${}_rA$, ${}_{ro}f(54)$ must be chosen so as to achieve an accounting balance. Thus

$${}_r\hat{g}(54) \equiv (I - {}_rA)^{-1} \; {}_{ro}\hat{f}(54)$$

and $$I \equiv (I - {}_rA)^{-1} \; {}_{ro}\hat{f}(54) \; {}_r\hat{g}(54)^{-1}$$

$$\equiv (I - {}_rA)^{-1} \; \hat{\alpha}$$

But from (7) and (8)

$$ {}_rg^*(58) = (I - {}_rA)^{-1} \; \hat{\alpha} \; {}_rg(58)$$

therefore ${}_rg^*(58) = {}_rg(58)$ so it is indeed surprising that any of the seven alternative predictions reveal any discrepancies from the column of actual outputs whatsoever. In other words, if the final output vectors were defined in an internally consistent manner, each of the gross output vectors in Hewings' Table 4 should be identical with that showing the actual values for 1958. Moreover, and more importantly, the predictive test (which was central to his main argument) should be totally discounted as evidence for further promise of non-survey techniques.

3.3. The remaining assumptions of the West Midlands Study

Hewings' assumptions 3.1. a and 3.1. b. are apparently not as disastrous as his assumption 3.1. c. already dealt with, but it will now be shown that even these may be improved with readily available data and a very little additional ingenuity.

As a repliminary to this discussion and essentially following MOSES (1955), let us more precisely define the breakdown of the regional input-output coefficients as follows. Consider the (i,j)th element of the matrix ${}_rA$ as being the product of two components. So that

(9) $a^r_{ij} = {}_rt_{ij} \; b^r_{ij}$ where b^r_{ij} defines the

requirement of the ith commodity (what (whatever the region of origin) per unit output of the jth industry situated in region r. This coefficient is termed the technical coefficient since it reflects pure technology with a regional orientation. The coefficient ${}_rt_{ij}$ on the other hand is termed the trading coefficient and it merely shows the percentage of commodity i, used by industry j in region r, which is itself produced within r. The coefficients ${}_r t_{ij}$ and b^r_{ij} are not of course likely to be independently determined. Locational factors will simultaneously determine trading patterns and technical structures. However, viewed in this way, the entire notion of embodying non-survey techniques becomes clearer.

In the first place, all of the seven techniques employed by Hewings may be thought of as ways of measuring the trading coefficients. The method is essentially simple. For each pair of industries (i,j) each of the quotient methods yield a numerical value, denoted by α_{ij}, which measures in some sense the relative size of industry i in the region to that of industry j.

If $\alpha_{ij} < 1$ then industry i may be considered to be the relatively less prevalent in the region (as compared with the nation) and the region is said to import the products of i when supplying industry j. For the lack of anything better

${}_r t_{ij}$ is set equal to α_{ij}.

If $\alpha_{ij} > 1$ then the corresponding argument labels i as an exporter to industry j (or more precisely, as not an importer of i to j) so that here ${}_r t_{ij}$ is set equal to 1.

Clearly this is a pragmatic approach and leaves much to be desired. The choice amongst the various quotients so far devised will be the subject of the final section and will not therefore be discussed any further here.

In the second place there is the question of estimating the regional technical coefficients b^r_{ij}. In particular, the

assumption of replacing b^r_{ij} by the national coefficient must be examined more closely.

The national coefficient will in practice, in fact, be some weighted average of all corresponding regional technical coefficients. The

b^r_{ij}'s may vary between regions for a great variety of reasons. Regional variations in the product mix, product prices, wage rates and primary factor prices generally, will all contribute significantly to the final coefficients. One piece of information is readily available to quantify some part of these coefficient variations. Data for wages and salaries are available by region by industry group for each region of the United Kingdom and for the nation as a whole from the Census of Production returns (BOARD OF TRADE, 1958) Analogous to Stone's fabrication effect (CAMBRIDGE, 1963) (which was an intertemporal phenomenon) one may define a regional fabrication factor defined by the vector ρ^r, for each region r, which for the lack of anything better may be applied to each column coefficient in the same way as the fabrication factors in the RAS method. For the (i, j)th regional technical coefficient

$$(10) \qquad b^r_{ij} = \rho^r_j \quad a_{ij}$$

Implicitly, the procedure incorporates the assumption that all input coefficients are affected by the same percentage amount, which is reasonable if one remembers that the b^r_{ij}'s relate to a technological phenomenon. Thus for example, one machine tool worker is required per machine say, in order to produce combustion engines even though his wages as a percentage of total output may vary from region to region. In effect, this device says that if returns to labour and other primary factors combined are proportionately less in region r than the national average then proportionately more inputs per unit of output of the jth industry (measured in money terms) will be absorbed from the intermediate sectors.

This is just one modification which might be employed to refine national coefficients as estimates of regional technical coefficients. It was devised for and utilised in the Welsh model. Clearly however, no claim can be made for this refinement accounting for more than a small part of total regional technical coefficient variation. In this instance it does serve to reduce Hewings' assertion (in his assumption 3.1.b.) that one simply must have more information about industry mix at the regional level and more detailed sectioning in the interindustry tables before one is able to make headway with devising separate estimates for the regional technical coefficients.

There are other features of the Welsh model, quite apart from the inclusion of regional fabrication effects, which compare very favourably with any of the techniques utilised in the West Midlands Study.

This comparison will be made in the next section. In so doing, consideration will only be given to the methods of arriving at values of the trading coefficients since this was the only aspect discussed by Hewings. To simplify algebra, it will be assumed from henceforth that the regional fabrication factors are each equal to unity, although this assumption will amount to no loss of generality in the analysis.

4. The Welsh Model - A Comparative Exercise

4.1. The model restated

It will facilitate the analysis if a small amount of notation is introduced at the outset. Since the model relates to a two-region interregional model the regions will be denoted by r (as before) and s.

(i) $u \equiv$ n-vector of national total intermediate outputs.

$_{r}u \equiv$ n-vector of region r total intermediate outputs.

Total intermediate outputs of a region are defined paradoxically in a national context. Flows of goods and services are intermediate providing they are current flows used by other industries for further production within the boundaries of the nation as a whole. In the interregional model, flows are still considered intermediate if they constitute current flows to industries outside the boundaries of the region but within the boundaries of the nation. These interregional intermediate flows are included in the vector $_{r}u$ and amount to no more than the elements of vector $_{rs}f(4)$ disaggregated by industry of destination.

(ii) $v \equiv$ n-vector of national total intermediate inputs.

$_{r}v \equiv$ n-vector of region r total intermediate inputs

A similar explanation applies to that given in (i)

(iii) $W \equiv$ nxn matrix of national intermediate flows.

$_{R}W \equiv$ 2nx2n matrix of interregional intermediate flows.

$$\equiv \left[{}_{rs}w_{ij} \right]$$

(so that $u \equiv Wi$ and $v \equiv W'i$

$$\begin{bmatrix} {}_{r}u \\ {}_{s}u \end{bmatrix} \equiv {}_{R}Wi \qquad \begin{bmatrix} {}_{r}v \\ {}_{s}v \end{bmatrix} \equiv {}_{R}W'i \qquad).$$

The model is dependent on three assumptions, which, together with their implications are set out below.

Assumption I. Total intermediate output of each industry in region r is distributed as inputs into national (U.K.) industries in the same relative proportions as the distribution of the total intermediate output of the corresponding U.K. industry amongst U.K. industries.[7]

An nxn matrix W_r may be defined to be a matrix of intermediate flows of goods and services from region r industries to U.K. industries according to assumption I.

$$\text{Thus} \qquad W_r \equiv \left[w_{r.ij}\right] = {}_r\hat{u}\,\hat{u}^{-1}\,W.$$

Assumption II. Intermediate inputs into each industry in region r are obtained from national industries in the same relative proportions as each corresponding industry in the nation as a whole obtains its intermediate inputs from national industries.

Similar to W_r, an nxn matrix W_c may be defined which depicts intermediate flows of goods and services to industries in region r from industries in the nation as a whole.

$$\text{Thus} \qquad W_c \equiv \left[w_{c.ij}\right] = W\,\hat{v}^{-1}\,{}_r\hat{v}$$

Assumption III. As far as possible, the industries in region r obtain their inputs from supplying industries located within the region, and likewise, industries in region r sell their outputs to industries within the region.

Thus the intra-regional interindustry flows of region r are given by:

$$_{rr}w_{ij} = \min\,(w_{c.ij},\, w_{r.ij})$$

For flows in the remaining three quadrants, three cases are possible.

(i) If $w_{r.ij} > w_{c.ij}$

then region r is a net exporter to region s of the intermediate output of industry i when supplying industry j.

$$_{rs}w_{ij} = w_{r.ij} - w_{c.ij}$$

$$_{sr}w_{ij} = 0$$

$$_{ss}w_{ij} = w_{ij} - w_{r.ij}$$

(ii) If $w_{c.ij} > w_{r.ij}$

then region r is a net importer from region s of the intermediate goods of industry if for use in industry j.

$$_{rs}w_{ij} = w_{c.ij} - w_{r.ij}$$

$$_{sr}w_{ij} = 0$$

$$_{ss}w_{ij} = w_{ij} - w_{c.ij}$$

(iii) If $w_{r.ij} = w_{c.ij}$

then on balance, region r neither exports nor imports intermediate output of i to or from region s, so that

$$_{rs}w_{ij} = {}_{sr}w_{ij} = 0$$

$$_{ss}w_{ij} = w_{ij} - w_{r.ij}$$

4.2 A reformulation

The direct algebraic interpretation of three, clearly defined assumptions is capable of being reformulated into a form which is more suitable for comparison with the quotient-type models. Typical elements in the interregional intermediate flow matrix can be expressed as follows:

$$
\begin{aligned}
{}_{rr}w_{ij} &= {}_{r}t_{ij}\ b^{r}_{ij}\ {}_{r}g_{j} \\
{}_{sr}w_{ij} &= (1 - {}_{r}t_{ij})\ b^{r}_{ij}\ {}_{r}g_{j} \\
{}_{rs}w_{ij} &= (1 - {}_{s}t_{ij})\ b^{s}_{ij}\ {}_{s}g_{j} \\
{}_{ss}w_{ij} &= {}_{s}t_{ij}\ b^{s}_{ij}\ {}_{s}g_{j}
\end{aligned}
\tag{11}
$$

However, for simplicity (and no loss of generality) it will be assumed that the regional fabrication effects are unity, so that for all i,j

$$b^{r}_{ij} = b^{s}_{ij} = a_{ij}$$

Now remembering that

$$w_{c.ij} = w_{ij} \frac{{}_{r}v_{j}}{v_{j}}$$

(12)

and $$w_{r,ij} = w_{ij} \frac{{}_{r}u_{i}}{u_{i}}$$

then expressions can easily be found for ${}_{r}t_{ij}$ and ${}_{s}t_{ij}$.

<u>Consider case (i)</u> where $w_{r.ij} > w_{c.ij}$ then from (12) it follows that

$$\frac{{}_r u_i}{u_i} > \frac{{}_r v_j}{v_j}$$

Now

$$
\begin{aligned}
{}_{ss}w_{ij} &= w_{ij} - w_{r.ij} \\
&= w_{ij}\left(1 - \frac{{}_r u_i}{u_i}\right) \\
&= w_{ij}\,\frac{{}_s u_i}{u_i} \\
&= a_{ij}\,\frac{{}_s u_i}{u_i}\,g_j
\end{aligned}
$$

Comparing with (11) this means that

$$
\begin{aligned}
{}_s t_{ij} &= \frac{{}_s u_i}{u_i} \Big/ \frac{{}_s g_j}{g_j} \\
&= \frac{{}_s u_i}{u_i} \Big/ \frac{{}_s v_j}{v_j} < 1
\end{aligned}
$$

Also,

$$ {}_{rr}w_{ij} = w_{ij}\,\frac{{}_r v_j}{v_j} $$

$$ \therefore \quad {}_r t_{ij} = \frac{{}_r v_j}{v_j} \Big/ \frac{{}_r g_j}{g_j} = 1 $$

Consider case (ii) where $w_{s.ij} > w_{r.ij}$ then from (12) it follows that

$$\frac{{}_r v_j}{v_j} > \frac{{}_r u_i}{u_i} .$$

Now

$$ {}_{rr}w_{ij} = w_{ij} \frac{{}_r u_i}{u_i} $$

$$ = a_{ij} \frac{{}_r u_i}{u_i} \cdot g_j $$

Comparing with (11) this means that

$$ {}_r t_{ij} = \frac{{}_r u_i}{u_i} \Big/ \frac{{}_r g_j}{g_j} $$

$$ = \frac{{}_r u_i}{u_i} \Big/ \frac{{}_r v_j}{v_j} < 1 $$

Also,

$$ {}_{ss}w_{ij} = w_{ij} - w_{c.ij} $$

$$ = w_{ij} \left[1 - \frac{{}_r v_j}{v_j} \right] $$

$$ = w_{ij} \left[\frac{{}_s v_j}{v_j} \right] $$

$$ = a_{ij} \left[\frac{{}_s v_j}{v_j} \right] g_j $$

so that

$$ {}_s t_{ij} = \frac{{}_s v_j}{v_j} \quad \frac{{}_s g_j}{g_j} = 1. $$

Consider case (iii) where $w_{s.ij} = w_{r.ij}$.

Here, the values for ${}_s t_{ij}$ and ${}_r t_{ij}$ are trivial since

$${}_{rs}w_{ij} = {}_{sr}w_{ij} = 0$$

Thus ${}_r t_{ij} = {}_s t_{ij} = 1.$

All of this may be summarised in Table 2, by defining two related quotients as follows:

$$\alpha_{ij} = \frac{{}_r u_i}{u_i} \bigg/ \frac{{}_r v_j}{v_j}$$

and $$\beta_{ij} = \frac{{}_s u_i}{u_i} \bigg/ \frac{{}_s v_j}{v_j}$$

and it is clear that precisely the same quotient/trading coefficient procedures pertain to the Welsh model as applied in the simple regional case.

Table 2: Procedures for defining trading coefficients

	Value of α_{ij}	${}_r t_{ij}$	Value of β_{ij}	${}_s t_{ij}$
Case (i)	> 1	1	< 1	β_{ij}
Case (ii)	< 1	α_{ij}	> 1	1
Case (iii)	$= 1$	1	$= 1$	1

4.3 A comparison with other quotient techniques

The purpose in representing the Welsh model in this form is to facilitate a direct comparison with the quotient techniques utilised by Hewings in his West Midlands study. There are three essential points.

(a) All methods, including the Welsh model belong to the same class of techniques. In no sense therefore can one claim that the quotient techniques are a more recent development; the Welsh model is just as much a quotient technique as any of the others.

(b) The Welsh model has additional appeal. In the first place the scheme is based upon a set of well-defined assumptions. Good or bad, they exhibit a precision which is lacking in other techniques.

(c) A second advantageous feature is that the method of defining a value for the trading coefficient in region r directly defines also a symmetrical value for the corresponding trading coefficient for region s and more importantly, which also preserves row and column balance of the intermediate flows. Thus the Welsh model is completely internally consistent.

The extension of the other quotient techniques to the full interregional system has never been performed. One may hazard a guess at one reason; analysts have quickly recognised a balance problem and have abandoned any attempt. The Welsh model overcomes this difficulty in the most logical way, and eliminates the need to treat large residual flows out of the regional table as net exports without any firm reasoning to support the procedure.

Nothing has been said about the final output estimation. This was never claimed to be a strong aspect of the Welsh model, but it is enough to recall that the method of arriving at final output for the regions obeyed the same set of assumptions which set up the interindustry flow table. As has been shown earlier, Hewings had no sensible method available to derive final outputs for the West Midlands so that it is not necessary to dwell on this aspect at this juncture. Clearly there are some published data for the regions, and not only for Wales, so that one should do much better than resort to gross indicators of the type suggested by Hewings.

5. Conclusions

It is difficult to see how Hewings' results can add significantly to that which was learned by the Welsh exercise in 1966. Certainly, the errors of his analysis lead one to reject as without evidence, his optimistic views on the production of regional tables by non-survey methods. This paper has shown that on theoretical grounds, the techniques set out by Schaffer and Chu and re-worked in a West Midlands region exercise are not very different from the technique set out in the Welsh model, whilst the latter would seem to have decided additional advantages.

It is difficult to believe that the highly complex pattern of interregional trade can be adequately described by a set of such extremely simple constructs. It is clear from the earlier tests of the Welsh model and more recent tests, as yet unpublished, that a great deal more research is yet to be carried out before one is in a position to assess the prospects of utilising non-survey techniques in regional input-output analysis.

Footnotes:

* I wish to record my appreciation to Professor Edward Nevin for stimulating my interest in multiregional input-output analysis, whilst acknowledging that all the usual disclaimers must apply.

1. Hereafter referred to as the Welsh model.

2. Referring to the usual national (and regional) accounting conventions of distinguishing between normal residents and non-residents within a region, it is clear that a sale of a commodity from an industry within region r to a normal resident of that region would constitute an intra regional sale on final account.

3. See TASKIER (1967).

4. In the U.S.A. these would correspond to the State taxes which are levied independently from State to State, and separately from Federal taxes.

5. Since both intra regional intermediate sales $\sum_{j=1}^{45} a^r_{ij} \; {}_r g_j$ and final outputs ${}_{ro}f_i$ have been estimated independently.

6. The n elements of vector α comprise the n diagonal elements of an nxn matrix $\hat{\alpha}$ whose other (off-diagonal) elements are zero.

7. For ease of definition, a somewhat spurious distinction is made between regional industries and national industries. Strictly of course, national industry i is no more or less than the sum of activity of industry i in region r and activity of industry i in region s.

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