HARROD, PROFIT MAXIMISATION AND

NEW ENTRY

by

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Harrod, Profit Maximisation and New Entry (1)

1. Introduction

A recent essay by Harrod [1] has repeated a former argument (2) that the result of inducing new entry and thus new competition if higher than normal profits are made by the firm, may make it optimal for the firm to equate average revenue and average cost, the latter being defined to include both normal profits and rents, even when above normal profits are feasible. Thus full-cost pricing as an optimal policy for the profit-stream maximising firm need not imply that the average revenue and average cost curves are tangential, and a long-run equilibrium may occur where 'excess' profits can be but are not earnedbecause of the new entry (3) that would be induced.

It will be helpful to consider the above statement in two parts. The first is the point that a long-run equilibrium for a profit-stream maximising firm might exist when profits are consistently not maximised. The second part is that the optimal profit level to earn in this situation is 'normal' profits.

Recently a number of papers (4) have appeared which have attempted to investigate the effect of the existence of entry response on the

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(1) The author wishes to acknowledge helpful comments on earlier versions of this paper by Graham Pyatt, Richard Clarke and participants of the Industrial Economics Workshop at the University of Warwick.

(2) See Harrod [2].

(3) Exit of competition may be considered as negative entry.

(4) See Gaskins [3], Pyatt [4] , Kamien and Schwartz [5]. The latter paper adopts a probabilistic rather than a deterministic model and will not be considered here.

optimal behaviour of the profit-stream maximising firm. A paper by Gaskins [3] showed that under certain conditions - mainly a linear reaction function (5) - the optimal price the firm should charge would be a compromise between the profit-maximising price and the 'limit' price, the full-cost price (with an adjustment for a 'cost advantage') which is the lower bound for entry to take place. Eventually the profit-maximising price and the 'limit' price would converge, and then (in long-run equilibrium) profits would be maximised at the limit price, which may or may not be the full-cost price. Thus in this model it is not true that profits are consistently not maximised, as to maximise profits is the optimal policy at an infinite horizon, where the industry is in long-run equilibrium.

Pyatt [4] considers a model where reaction is to profit (6) rather than price. He finds (7) "that in many, but not in all situations, firms may maximise profits in the short run even if they believe that this will lead to new entry. In cases where short-run profits are not maximised, none-the-less profits will be at a level such that entry may be induced. In consequence tangency of demand and average cost will always result in situations where this is a feasible outcome". In fact Pyatt's conclusion is not quite correct as it stands as the reaction function that he uses needs to be more restricted (8).

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(5) If $x_{(t)}$ is the level of rival's sales then $\dot{x}_{(t)} = k(P_{(t)} - \bar{P})$ where $P_{(t)}$ is the price charged and $\bar{P}$ is the limit price and k is a constant.

(6) The question of whether profit level or profit rate is relevant is not so important if a constant shareholding is assumed. I will make this assumption in the rest of this paper. Both profit and maximum profit may be considered as net of normal profit if this is constant.

(7) Pyatt [4] page 254. Pyatt also considers a non-continuous analysis in his paper.

(8) See Pyatt [4] page 248 expression (12). A sufficient specification for Pyatt's conclusion to be correct is that $\pi > 0$ and $\frac{d\pi}{dX} > 0$, $\frac{d^2\pi}{dX^2} > 0$ for $X \geq 0$ where π and X are, in Pyatt's notation, a location parameter in the demand function and the level of excess profits made.

If it is thought that Harrod's work on the theory of the firm lies in part on a desire to explain a long-run non-tangency equilibrium within the context of the profit-maximising model, then the models of Pyatt and Gaskins do not appear to give positive support. These models only allow an adjustment period when the tangency condition is met (unless there is a cost advantage).

It would appear, however, that these models are unduly specific [(9)], and there seems to be no reason why a model which exhibits Harrod's non-tangency equilibrium should not be constructed. It is the purpose of the present paper to construct such a model. This model will have the feature that there are two forces that bring about new entry and that these are counter-acting. One is the desire to take advantages of higher than normal profit <u>opportunities</u> [(10)] on the part of entrant firms. The second force is the sacrifice of present profits by the firm or firms already in the industry in order to discourage such entry. A long-run equilibrium will be found to be a balance of these two forces.

The kind of environment of models of this kind is an obvious difficulty. It seems necessary to assume that the 'firm' whose behaviour is being analysed is either a 'dominant' firm [(11)] in an industry and new entry is from 'fringe' firms, or a cartel of firms in the same situation. The kind of model presented here is not suitable for the analysis of an oligopoly situation.

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(9) See Section 2 of this paper and also Jacquemin and Thisse [6] for a discussion of limit-price models including Gaskins'. They suggest conditions under which Gaskins' result on the eventual optimality of the limit price does not apply.

(10) I believe the model to be novel in this respect. Reaction is in part controlled by profits that can be earnt rather that profit that is earnt. That is response is to what is presently possible in the industry rather than to what is.

(11) See Gaskins [3] for a discussion of the 'dominant' firm.

In the following section a general model of environmental response to the activities of the firm is set up, and it is demonstrated that the results of Pyatt and Gaskin were due to the assumption of a special case. In later sections the more specific model without this assumption, but with response as a result of two counter-acting forces is analysed in detail.

2. A general model and a special case

The environment of the firm determines the maximum profit flow ($\pi_{(t)}{}^{m}$) which the firm can achieve at any time t. The actual profit flow ($\pi_{(t)}$) differs from the maximum profit flow by a profit 'sacrifice' ($S_{(t)} = \pi_{(t)}{}^{m} - \pi_{(t)}$). For given $\pi_{(t)}{}^{m}$ the choice of $\pi_{(t)}$ determines $S_{(t)}$ and vice-versa. The firm wishes to maximise the stream of actual profits discounted at a rate δ. For convenience an infinite horizon will be assumed.

Maximum profit flow will be assumed to respond to $\pi_{(t)}{}^{m}$, $\pi_{(t)}$ and $S_{(t)}$ and without loss of generality we can write:

$$\dot{\pi}_{(t)}^{m} = G(\pi_{(t)}^{m}, S_{(t)}) \qquad (2.1)$$

and we will assume $\dfrac{\partial G}{\partial \pi_{(t)}^{m}} < 0$, $\dfrac{\partial G}{\partial S_{(t)}} > 0$.

The firm therefore wishes to maximise J where

$$J = \int_{0}^{\infty} (\pi_{(t)}{}^{m} - S_{(t)})e^{-\delta t}\, dt \qquad (2.2)$$

subject to (2.1) and

$$\pi_{(0)}{}^{m} = \pi_{0}{}^{m}$$

$$S_{(t)} \geqslant 0$$

by finding an optimal trajectory $S_{(t)}{}^{*}$ and an associated trajectory of the state variable $\pi_{(t)}{}^{m*}$. The problem is solved by applying Pontryagin's Maximum Principle. The current period Hamiltonian is

$$H = \pi_{(t)}^{m} - S_{(t)} + z_{(t)} G(\pi_{(t)}^{m}, S_{(t)}) \qquad (2.3)$$

and the solution is a set of trajectories $S^*_{(t)}$, $\pi_{(t)}^{m*}$, $z_{(t)}^*$ which satisfy the following conditions.

(i) $-1 + z_{(t)} \dfrac{\partial G}{\partial S_{(t)}} \leqslant 0$ and either $S_{(t)} = 0$ or $-1 + z_{(t)} \dfrac{\partial G}{\partial S_{(t)}} = 0.$

(ii) $\dot{z}_{(t)} = -1 + (\delta - \dfrac{\partial G}{\partial \pi_{(t)}^{m}}) \; z_{(t)}$

(iii) $e^{-\delta t} \; z_{(t)} \geqslant 0 \qquad e^{-\delta t} \; z_{(t)} \; \pi_{(t)}^{m} = 0$ as t tends to infinity.

(iv) $\pi_{(0)}^{m} = \pi_{0}^{m}$ and $\dot{\pi}_{(t)}^{m} = G(\pi_{(t)}^{m}, S_{(t)})$

Condition (i) is the condition for H to be a maximum with respect to $S_{(t)}$; condition (ii) is the equation of motion for the co-state variable $z_{(t)}$; condition (iii) indicates the transversality conditions; and condition (iv) is the equation of motion and the initial condition for the state variable $\pi_{(t)}^{m}$.

With sufficiently full information about the function G, the optimal policy for the firm can at least be approximately found. A special case is where the G function can be written:

$$\dot{\pi}_{(t)}^{m} = G(\pi_{(t)}^{m} - S_{(t)}) = G(\pi_{(t)})$$

This response function is formally identical to Pyatt's [4] and if a one to one relationship between profit and price is assumed it is also that of Gaskins [3]. In this special case $\dfrac{\partial G}{\partial S_{(t)}} = - \dfrac{\partial G}{\partial \pi_{(t)}^{m}}$ and conditions (i) and (ii) above become:

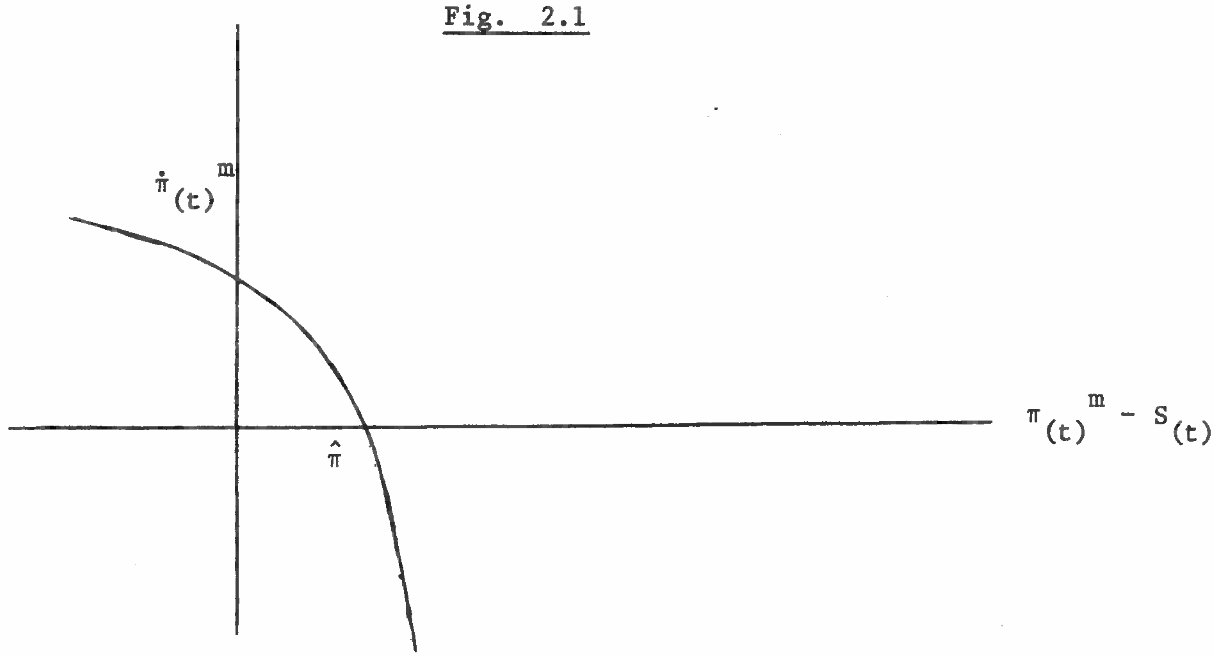
(i)' $-1 - z_{(t)} \; G' \leqslant 0$ and $S_{(t)} = 0$ or $-1 - z_{(t)} \; G' = 0$

(ii)' $\dot{z}_{(t)} = -1 + (\delta - G') \; z_{(t)}$

where $G' = \dfrac{\partial G}{\partial \pi_{(t)}^{m}} = \dfrac{dG}{d(\pi_{(t)}^{m} - S_{(t)})}$

We will assume $G' < 0$ and $G'' < 0$, so that the response function appears as in fig. 2.1. Conditions (i)', (ii)', (iii) and (iv) are necessary and sufficient for an optimal solution [12].

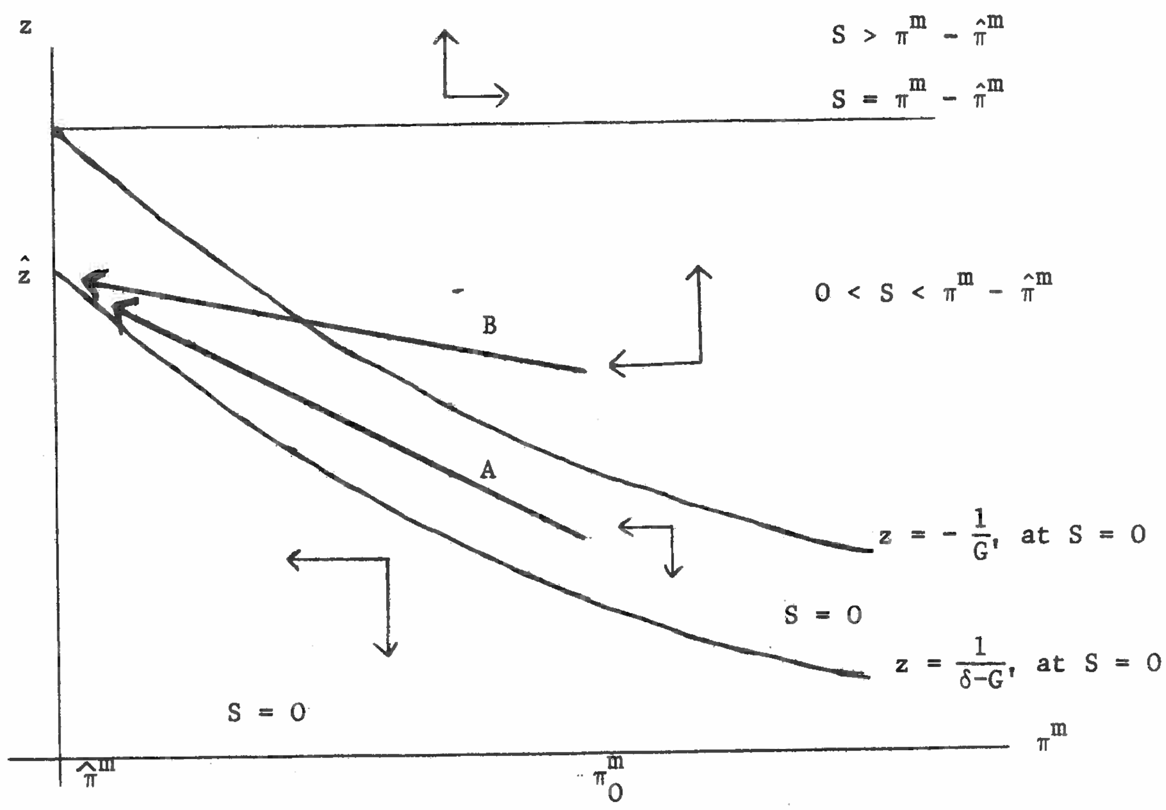
Fig. 2.1



$\hat{\pi}$ in fig. 2.1 might be thought of as the (constant) rate of actual profit that has a neutral response from the firm's environment. If $\pi_0^m > \hat{\pi}$ then the two possible optimal trajectories are drawn on the phase diagram fig. 2.2.

(12) See for instance Arrow and Kurz [7] p. 49.

Fig. 2.2



The steady-state solution $\hat{\pi}^m$, $\hat{z}$ can be arrived at along a trajectory of type A or B. The solution has the following characteristics for $\delta > 0$.

1. The steady-state $\hat{\pi}^m$, $\hat{z}$ has the property $S = 0$, i.e. profits are maximised in the steady-state solution.

2. $S > \pi^m - \hat{\pi}^m$ is never optimal. It is never optimal to make less than $\hat{\pi}^m$ amount of profit.

3. Either trajectory A or B is optimal. Therefore for some finite time before reaching the steady-state it is a property of the optimal path that profits are maximised ($S = 0$ in the neighbourhood of $\hat{z}$, $\hat{\pi}^m$).

4. Trajectory B has the property that $S > 0$ for some period of time.

If G is linear the 2 curves $z = \frac{1}{-G'}$ (at S = 0) and $z = \frac{1}{\delta - G'}$ (at S = 0) in fig. 2.2 are horizontal straight lines and only the one trajectory of type A is optimal.

The steady-state solution in the special case thus has the property that profits are maximised, and whether or not this is a tangency depends on the value of $\hat{\pi}^m$. If $\hat{\pi}^m$ is normal profits then a tangency is established.

In the following section a model relating to a specific G function which differs from the Pyatt-Gaskins special case is developed and solved.

3. A specific model

Consider a 'dominant' firm (or cartel) which has the following characteristics.

i) The firm wishes to maximise its stream of profits discounted at a rate $\delta > 0$, where either the firm has an infinite horizon, or, from some arbitrary point in time, say r, the firm carries out a policy of maximising profits at each moment in time (myopic profit maximisation). In the latter alternative optimal policy up until time r has to be described and the former alternative is a special case of the latter when $r = +\infty$.

ii) At any moment in time (t), there exists a finite maximum profit flow $\pi_{(t)}^m$, so that actual profit at time (t), $\pi_{(t)}$ is constrained to be less than or equal to $\pi_{(t)}^m$.

iii) The maximum profit flow $\pi_{(t)}^m$ will decay at rate $\alpha > 0$ if and only if $\pi_{(t)} = \pi_{(t)}^m$, and in general:

$$\dot{\pi}_{(t)}^m = -\alpha\, \pi_{(t)}^m \quad \text{iff} \quad \pi_{(t)} = \pi_{(t)}^m \quad \text{i.e. iff } S_{(t)} = 0.$$

The rate α will be termed the full-profit rate of decay of maximum profit. The value of α summarizes two stages. Firstly, there is the reaction of fringe firms in entering or exiting the industry if the dominant firm earns $\pi_{(t)}^{m}$, and secondly there is the effect of this on the maximum profits that can be earned in the industry.

iv) The firm may decide not to make all the 'excess' profits possible. In this case there are three possibilities. One is that the firm will choose to operate where marginal cost is not equal to marginal revenue and in particular will tend to take less than full advantage of its monopoly power, by selling at less than its profit-maximising price. The second and third possibilities concern increasing the firm's costs (shifting the total cost curve upwards). One possibility is that the firm spends money on public relations, advertising, patents and other measures in order to slow down the rate of profit capacity decay. The second is that the firm would increase payments to factors of production above the market level or employ more factors of production than are necessary.

All three ways of producing at less than the maximum profit level may have an effect on the rate of profit capacity decay. Possible entrants to the industry may be unaware of profit capacity, but aware of actual profit made. Alternatively they could be aware of both but believe that the difference between actual and maximum profit is a measure of the potential new entry after they have entered.

Thus I will assume that the rate of change of maximum profit can be adequately approximated by the differential equation:

$$\dot{\pi}_{(t)}^{m} = -\alpha\, \pi_{(t)}^{m} + F(S_{(t)}) \qquad (3.1)$$

where $F(0) = 0$ and $S_{(t)} = \pi_{(t)}^{m} - \pi_{(t)}$

$F' > 0$

$F'' < 0$

where the primes denote derivatives with respect to $S_{(t)}$. The F function would therefore look as in fig. 3.1.[13]

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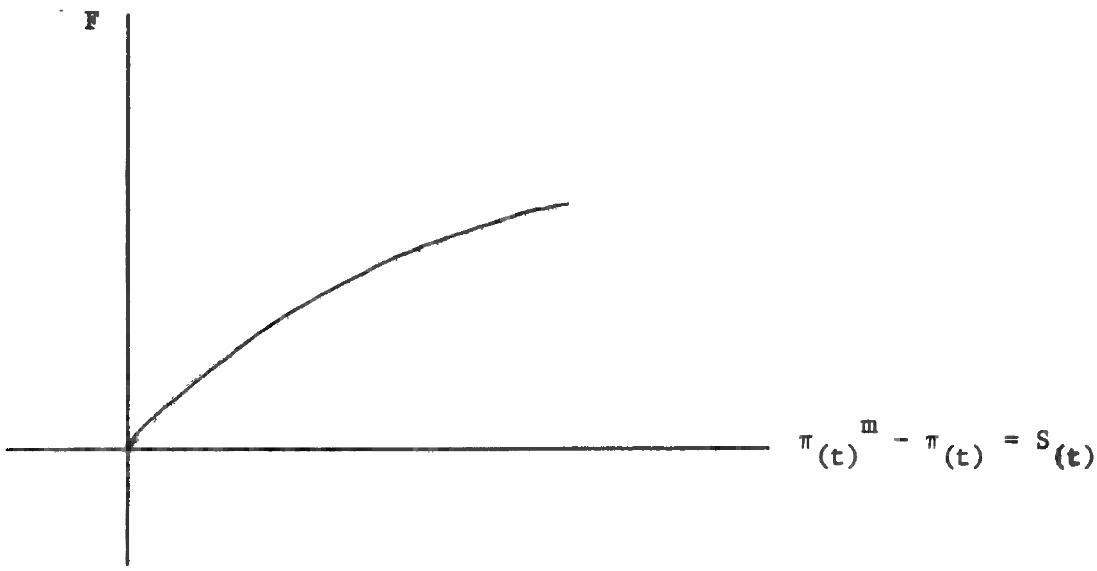
(13) A function that would obey these rules would be a $\tanh(b\, S_{(t)})$ The case where $F'' > 0$ would bring about a 'bang-bang' policy if $S_{(t)}$ had a finite upper bound b. i.e. policy would be found to be some mix in time between $S_{(t)} = 0$ and $S_{(t)} = b$. See Sasieni [9].

Fig. 3.1



The curvature of the F function is such that the sacrifice of just one unit of profit (i.e. $S_{(t)} = 1$) brings more than half as much response in terms of the rate of change of profit capacity flow as a sacrifice of two units of profit.

The analysis of the model in the next section will be designed to find answers to two questions. Firstly, under what conditions is it non-optimal to pursue a policy of continual myopic profit maximisation ($\pi_{(t)}^m = \pi_{(t)}$)? Secondly, is it possible for these considerations to define a long-run equilibrium? That is, is Harrod's concept of a firm being in a position to increase profit but not doing so a possible solution of the model?

4. The Solution of the Model

$$\text{Maximise} \int_0^r \pi_{(t)} e^{-\delta t} dt \; . \; + \int_r^\infty \pi_{(t)}^m e^{-\delta t} dt$$

with respect to $S_{(t)}$, for t is $[0,r]$

subject to $S_{(t)} \geqslant 0$

$$\pi_{(0)}^m = \pi_0^m$$

$$\dot{\pi}_{(t)}^m = - \alpha \, \pi_{(t)}^m + F(S_{(t)})$$

Form the Hamiltonian:

$$H = (\pi_{(t)}{}^{m} - S_{(t)})e^{-\delta t} + z_{(t)}(-\alpha\pi_{(t)}{}^{m} + F(S_{(t)}))$$

If $S_{(t)}{}^{*}$, $z_{(t)}{}^{*}$, $\pi_{(t)}{}^{m*}$ satisfy the following conditions, they define an optimal solution; that is the following conditions are necessary and sufficient for an optimal solution.[14]

i) $-e^{-\delta t} + z_{(t)}\ F' \leq 0$

and either $-e^{-\delta t} + z_{(t)}\ F' = 0$ or $S_{(t)} = 0$ (maximum of the Hamiltonian)

ii) $-e^{-\delta t} + \alpha z_{(t)} = \dot{z}_{(t)}$

iii) $\dot{\pi}_{(t)}{}^{m} = -\ \alpha\ \pi_{(t)}{}^{m} + F(S_{(t)})$ (equations of motion)

iv) $z_{(r)} = \frac{1}{\alpha+\delta}\ e^{-\delta r}$ (transversality condition [15])

v) $\pi_{(0)}{}^{m} = \pi_{0}{}^{m}$ (initial condition)

A solution $S_{(t)}{}^{*}$, $\pi_{(t)}{}^{m*}$, $z_{(t)}{}^{*}$ which satisfies i → v exists and has one of the following two sets of characteristics.

<u>Policy I</u>

$\pi_{(t)}{}^{*} = \pi_{(t)}{}^{m*}$ (continued myopic profit maximisation)

$$z_{(t)}{}^{*} = \frac{1}{\alpha+\delta}\ e^{-\delta t}$$

Policy I is optimal if and only if $F'(0) \leq \alpha + \delta$

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(14) See for instance Arrow and Kurz [7] p. 49.

(15) After $r, \pi_{(t)}{}^{m}$ decays at the full-profit rate . $\int_{r}^{\infty} \pi_{(r)}{}^{m}\ e^{-(\alpha+\delta)t}dt$, differentiated with respect to $\pi_{(r)}{}^{m}$ should be equal to $z_{(r)}{}^{*}$. If $r = \infty$ then $z_{(\infty)}{}^{*}$ should be zero.

Policy II

$$F' = \alpha + \delta \quad \text{and} \quad \pi_{(t)}^{*} < \pi_{(t)}^{m*}$$

$$z_{(r)}^{*} = \frac{1}{\alpha+\delta} e^{-\delta t}$$

Policy II is optimal if and only if $F'(0) > \alpha + \delta$.

Thus myopic profit maximisation (Policy I) is optimal if and only if the sacrifice of one unit of profit does not increase the change of profit flow capacity by more than the rate of discount plus the rate of full-profit decay.

If the sacrifice of one unit of profit produces more than this reaction, then one asks if the second unit of profit does, and so on until, due to the curvature of the F function (see fig.3.1) an optimum (Policy II) is reached.

One of the two policies must be optimal, and given that one particular policy is optimal at time t = 0, then that policy will be optimal for the whole duration of the programme. Both policies define and are defined by an optimal constant profit sacrifice (either of zero (policy I) or a positive constant (policy II). $\pi_{(t)}^{m*}$ is found from conditions iii and v. If policy I is optimal then:

$$\pi_{(t)}^{m*} = \pi_0^{m} e^{-\alpha t}$$

Then $\pi_{(t)}^{m*}$ and $\pi_{(t)}^{*}$ will approach zero as t becomes large.

If policy II is optimal then again from conditions iii and v

$$\pi_{(t)}^{m*} = \frac{F(S_{(t)}^{*})}{\alpha} + \left[\pi_0^{m} - \frac{F(S_{(t)}^{*})}{\alpha}\right] e^{-\alpha t}$$

with $S_{(t)}^{*}$ = constant.

Thus maximum profit will tend to the value $\bar{\pi}^{m*}$ where

$$\bar{\pi}^{m*} = \frac{F(\bar{S}^*)}{\alpha}$$

where $\bar{\pi}^{m*}$ and therefore, from the optimality conditions, $\bar{\pi}^*$ are constants. $\bar{\pi}^{m*}$ is the equilibrium profit flow and $\bar{S}^*$ is the equilibrium profit sacrifice.

For arbitrary initial maximum profit levels, π_{01}^m and π_{02}^m, typical time paths of $\pi^*_{(t)}$ are shown below:

fig 4.1 Policy I

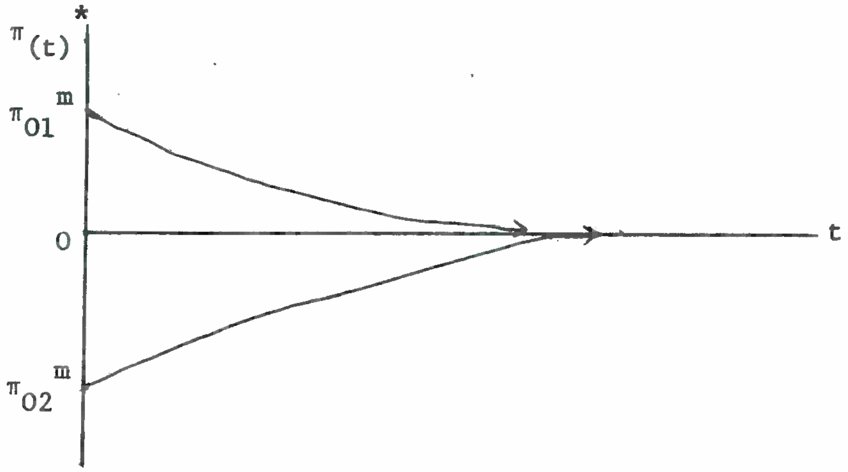
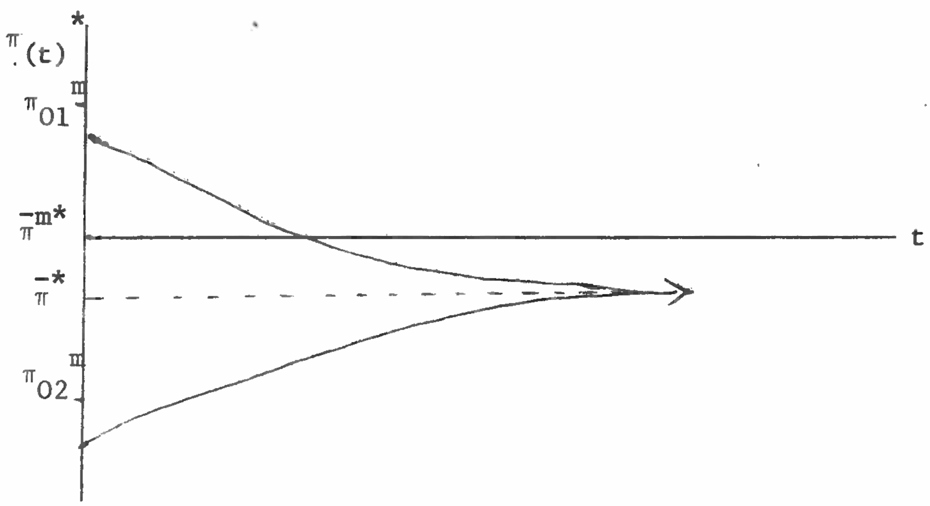


fig 4.2 Policy II



The constant difference between $\pi_{(t)}^{m*}$ and $\pi_{(t)}^{*}$ is given by the optimality conditions. Basically $F'(S_{(t)}^{*})$ shows the marginal benefit of sacrificing a unit of current profit, whereas $\delta + \alpha$ is the marginal cost of doing so in terms of time preference (δ) and decay of the higher maximum profit flow (α). Obviously an increase in either α or δ would make sacrificing profit less attractive.

The solution to the model is reminiscent of models of optimal gross investment or optimal advertising (16). This is to be expected as the problem is a similar one of distributing income flow between now and the future. It should be noted that the choice of r does not effect optimal policy at $t < r$. In the model the optimal r is either any r ($F'(0) \leqslant \delta + \alpha$, i,e, policy I optimal) or $r = \infty$ ($F'(0) > \delta + \alpha$, i.e. policy II optimal). In fact the choice of an arbitrary non-optimal r does not effect the policy decision as the transversality condition (condition iv) is met for any r.

4. Conclusion

In this paper it has been shown that it is possible to construct a model in which under certain circumstances a profit-stream maximising firm will be in long-run equilibrium at a level of profit which is not the maximum profit the firm can obtain. Furthermore the level of actual profit and the level of maximum profit would both be constant. Entry into or exit from the industry will be zero as measured by profit maximum changes. Thus the first point of Harrod's argument is satisfied: that is long-run equilibrium is established in a certain circumstance which is not that of a tangency. This circumstance is that $F'(0) > \alpha + \delta$.

As to the second part of Harrod's thesis, that this level of profit would be normal profits - i.e. price would be equal to average cost - this it is seen is not necessarily so. It might come about that $\bar{\pi}^{*} = 0$, but this would not be the general case, because maximum as well as actual profit flow is involved in the reaction function. However if one defines normal profits as being the level of profit where entry into or out of the industry is zero

(16) See Arrow and Nerlove [8].

then of course this long-run equilibrium satisfies the second Harrod criterion. In different industries there would be different α and δ and different F functions, so that if the form of the reaction function that has been used here is correct, there is no reason why different long-run profit levels should not exist in different industries, providing some present profit sacrifice is optimal [17].

The model that has been presented here is an extremely simple one, and no doubt different interpretations could be put on to the results than the ones stated here. However, it would appear that the model has served its purpose and has gone some small way towards reinstating Harrod's revision of imperfect competition.

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(17) I have ignored a possible change in the optimal share-holding here.

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