ON CONTRACTS WITH CONTINGENT PAYMENT FUNCTIONS

by

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This paper is circulated for discussion purposes only and its contents should be considered preliminary.

1. Introduction.

One firm[1] may enter into a contract to supply a commodity or commodities to another firm before production has taken place or at least before it has been completed. The costs of production and value upon completion are uncertain a priori. This is particularly relevant when the production involves a significant research and development input or when the prices of factors of production or the commodity market situation may alter to an uncertain extent during the period between contract and completion. If the contract involved a fixed price for the completed commodity, the supplier might respond to higher than expected costs by using different and perhaps inferior raw materials, by not achieving the agreed technological performance, by delivering late,in short measure, or not at all. To counter this the contract would specify the commodity in terms of inputs, performance and delivery date, etc., and state penalties to be paid to the buyer for not producing a commodity according to the specification, either explicitly in the contract or implicity within the judicial interpretation of the law of contract. Nevertheless, the contracted specification may not be realised, as repeated reassessments by the supplier during the production period would still be possible and may be desirable to take advantage of information concerning costs as it became available.

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(1) A 'firm' here has a broad interpretation. It is any economic agent potentially able to fulfill the role defined in the contract. 'Commodity' has a similarly broad interpretation.

These factors allow us to consider a sequence of production plans terminating with the completion of the realised specification. Even if the penalty costs were enormous, the contracted specification may be replaced by that of "no commodity" during the production period. More generally, changes in plan which involve small adjustments in specification may be to the supplier's advantage, despite large penalties. A labour dispute quickly settled might mean high wage costs and no penalties for late delivery, but it might be to the supplier's advantage to settle only after a stoppage has taken place, thus incurring less high wage costs but some penalties of this kind.

As an alternative to a "fixed price" contract, the price may relate to the actual costs incurred - perhaps even to the extent of being equal to all costs plus a profit margin. This, in a sense, is a penalty on the buyer payable to the supplier if the production costs were higher than estimated.

Although I am not aware of any analysis specifically of penalties in contracts, recent and not unrelated papers by Ross (1973) and Stiglitz (1974) represent analysis of similar systems. They have been concerned with the problem of fees contingent upon both random events and behaviour in relation to those events. Also the theory of optimal insurance is obviously related. The agent-principal[(2)] relationship analysed by Ross views a contract wherein the agent is to be paid by the principal a fee which is dependent upon the size of the principal's pay-off.

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(2) See also Ross (1972)

In particular the importance of the property of similarity (S) is asserted. If a fee schedule has this property, then the agent will, in maximising his own utility, coincidently maximise the principal's. Also if the agent's reaction to a random event θ is to produce a Pareto efficient (P.E.) allocation of utility, then the two properties (S and P.E.) imply that the fee schedule is linear (L). More generally any two of the three properties of a fee schedule (S, P.E., and L) will imply the third.

Stiglitz' paper is concerned with the theory of sharecropping(3). Much of his analysis is concerned with obtaining important general equilibrium results. However if there were just one labourer and one worker, then the analogy with the agent-principal problem is close, with the pay-off the value of the crop and the 'fee' a linear function of this. One distinction, however, is that the worker has a (negative) pay-off of his own - the effort or money he puts into the production process, and this is under his control(4). The model of inter-firm penalties treated in this paper will be seen to have a similar characteristic. It is argued that this phenomenon can be analysed as a two-way agent-principal problem.

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(3) See also Reid (1974). He stresses the class of problems of which sharecropping is just one member. He states (p.22) "Sharecropping in agriculture is analogous to piece rates, commissions, promotion from within and stock options or 'performance shares', joint participation in franchise operations and proportional fees."

(4) Stiglitz considers sharecropping produces an incentive effect to increase effort as compared with the 'pure wage system'. This must be qualified by statements about how 'controlled' the labour is under the wage systems (e.g. hours of work) and the kind of payments further utilised under the sharecropping system.

The analysis in the next Section will also investigate how a need for recontracting may occur in situations where random events and the agent's reaction to them, take place over a finite time interval and not simultaneously. Then a sequence of agent's decisions are relevant and not just a single reaction to random events. Neither of the papers just cited are concerned with this complication. Stiglitz (1974) (p.238) lists decisions to be made such as "what crops to grow, irrigation techniques, fertilisation techniques, etc." However, he continues to assume that for all decisions the choice reduces to the selection of one element of a constant set (T) of means and standard deviations of output per worker ($T(\mu,\sigma)$). This constancy implies that the decisions are simultaneous and not sequential. This may be a reasonable assumption in some cases, but consider the following simple counter-example. The decision of how much to water a crop is dependent on the rainfall at that time. If water is bought by the gallon by payment from the harvested crop and $T(\mu,\sigma)$ is considered as the set of mean/standard deviation outcomes net of this, and a decision is made everyday as to the amount of watering, then $T(\mu,\sigma)$ will in general change from day to day. From this the existence of a constant "linear relationship between workers' mean income and the standard deviation of the income from the contract" (Stiglitz (1974) (p.238) is unlikely to hold true <u>over the production period.</u>

In this paper, we need be concerned only with a simple bi-lateral problem. The distribution of expected utility at the time of the signing of the contract is a result of relative bargaining power (5). The contract is enforceable, except that the terms can be renegotiated if both firms prefer some new set of terms to the old. Within this provision, the distribution of gains in expected utility is again the result of relative bargaining power. Note that whatever the original market situation, as soon as a contract has been signed, a bi-lateral monopoly relationship exists between the two parties.

In Section II we set out a model of a contract between two firms consisting of a price with penalties for deviations from the contracted specification and cost of the commodity. Attention is confined to linear penalty systems and a class of linear penalty systems is found which have properties S and P.E. when decisions are taken instaneously. Approximations to the firms' utility functions are taken by means of the 'risk premium' techniques, and an initially-optimal linear penalty system is derived and it is seen that this may become sub-optimal over the production period so that, for sufficiently low recontracting cost, it becomes desirable for both firms to recontract. Some conclusions and applications are presented in Section III. In particular it is argued that penalty clauses, sharecropping agreements and agents' fees are applications of a common model, where payment is contingent upon the pay-offs of the two firms. Some notions of optimal insurance result from a further application.

(5) We are not concerned here with the determinants of this bargaining power.

II. A Model of Penalty Systems.

Let the buying firm be firm B and the selling firm, firm S. Both maximise strictly concave monotonically increasing utility functions (U_B and U_S), the only argument of each function being their respective profit (X_B and X_S) from the transaction.(6) X_B is the money equivalent of the commodity when received (R) minus the (net) payment of firm B to Firm S (P). X_S is P minus the production cost C. R and C are uncertain before production. Both will result from a sequence of decisions by firm S in response to and in conjunction with random events. Both firms are assumed to have the same subjective joint probability distribution of the random events. P will encompass both a base price and penalty payments. This is constructed with reference to the agent-principal analysis in the following way.

(i) Consider firm S as the agent and firm B as the principal. Firm B will pay a fee F_1 equal to a linear non-decreasing function of its pay-off (R), i.e.

$$F_1 = a_1 + b_1 R \qquad b_1 \geqslant 0 \tag{1}$$

(ii) Consider firm B as the agent and firm S as the principal. Firm S will pay a fee F_2 equal to a linear non-decreasing function of its pay-off (-C), i.e.

$$F_2 = a_2 - b_2 C \qquad b_2 \geqslant 0 \tag{2}$$

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(6) U_S, U_B are independent of the 'state of the world'. For a contrasting view see Parkin and Wu (1972) and Salkever (1974)

In (i) firm (S) is a production agent and in (ii) firm B is a buying agent. The 'net' payment by firm B to firm S is $P = F_1 - F_2$ i.e.

$$P = a_1 - a_2 + b_1 R + b_2 C \tag{3}$$

(3) could be written :

$$P = a_1 + b_1\bar{R} - (a_2 - b_2\bar{C}) + b_1 (R-\bar{R}) + b_2 (C-\bar{C}) \tag{4}$$

when $a_1 + b_1\bar{R} - (a_2 - b_2\bar{C})$ is interpreted as the price given R,C are equal to their 'norms' $\bar{R}$, $\bar{C}$, and the rest of the equation represents the effect on price of deviations away from these norms.

If both b_1 and b_2 are zero then $a_1 - a_2$ is the simple non-variable price of the commodity. If $b_1 = o$, $b_2 > o$, then the price reflects a contribution to firm S in relation to the cost of production. In particular, if $b_2 = 1$ then the price equals the full cost and $a_1 - a_2$ is the profit for firm S. If $b_1 > o$, $b_2 = o$, the payment represents a contribution to firm S in relation to the value of the commodity when produced.

Both firms are assumed to maximise expected utility. Firm S has to make decisions during the production period, given the function P to maximise

$$EU_S = EU_S(a_1 - a_2 + b_1 R - (1 - b_2)C) \tag{5}$$

while firm B has to find that function P given firm S's optimising behaviour to maximise

$$EU_B = EU_B(a_2 - a_1 + (1 - b_1)R - b_2 C) \tag{6}$$

subject to $EU_S \geq \bar{U}$ where $\bar{U}$ is given by relative bargaining power.

<u>Theorem I.</u> The linear payment function (3) has the properties of S and P.E. if

(i) $b_1+b_2=1$

(ii) $b_1>o$

(iii) Firm S takes all of its decisions simultaneously after and in reaction to the total set of random events

<u>Proof</u> Firm S will chose $(R-C)^* \varepsilon [R-C \mid \theta]$ where $(R-C)^*$ is the maximum element of this set. $(R-C)^*$ will imply X_S^*, X_B^* such that

$$U_S(X_S^*) \geqslant U_S(X_S)$$
$$U_B(X_B^*) \geqslant U_B(X_B) \qquad (7)$$

This establishes similarity (S).

$(R-C)^*$ is the maximum value of $[X_S+X_B \mid \theta]$ i.e. in response to the random events θ, decisions are taken which maximise joint profit. No different set of decisions could increase both U_S and U_B. This establishes Pareto efficiency (P.E.) (7)

<u>Theorem 1a.</u> If condition (ii) does not hold in Theorem 1, then S and P.E. hold only if firm B has control over the production decision, i.e. firm B will then choose $(R-C)^*$.

<u>Proof</u> as in Theorem 1.

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(7) We use the term Pareto efficiency despite the partial equilibrium nature of the model.

Theorem 1b. If condition (iii) does not hold in Theorem 1, then S and P.E. will not in general be properties of the linear payment function (3).

Proof Divide the random events into two groups θ_1 and θ_2. Some decisions are taken after the first group has occurred but before the second. The firm S maximises EU_S with respect to those decisions where

$$EU_S = \int_{\theta_2} U_S(X_S | \theta_1) \; f(\theta_2) d\theta_2 \tag{8}$$

whereas

$$EU_B = \int_{\theta_2} U_B(X_B | \theta_1) \cdot f(\theta_2) d\theta_2 \tag{9}$$

The decisions that maximise (8) will not in general maximise (9). Therefore P will not in general have the property S. Also the decisions made to maximise (8) above will not in general prove to be the same as would have been taken if θ_2 had been known, i.e. they would not produce $(R-C)^*$, and S would not have the property P.E.

Theorem 1 simply extends the inter-relationship between L, S, and P.E. to two-way fee schedules [(8)]. Theorem 1a considers the case where firm B agrees to pay all the costs of production. Here, if Firm B does not have any control over the production period decisions, $(R-C)^*$ does not result as firm S has no incentive to make any particular decision; its profit is entirely non-stochastic.

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(8) It is easily demonstrated that any two of L, P.E. and S imply the third.

Theorem 1b states that different degrees of risk-aversion will mean that firm S and firm B will have different preferences for the various courses of action possible. Firm S may decide not to water the crop today in the hope that it will rain tomorrow, even though there is a risk to the crop in doing so. If firm B was more risk-averse it might disapprove of this decision. Of course, if future rainfall was known with certainty, there could exist a pattern of crop-watering that would be Pareto efficient.

It seems likely, however, that some linear payment functions would produce less possibility of divergent attitudes towards risk than others. After all, the choice of the parameters b_1 and b_2 allocate risk between the two firms. As the distribution of expected utility can be adjusted by changing a_1-a_2, a reasonable course of action is to find b_1, b_2 which approximately equates the attitudes towards risk. To do this, it is necessary to approximate EU_B and EU_S in terms of the means and variances of R and C. Assume at some point of time R and C have mean values of μ_R and μ_C, and a variance-covariance matrix M where

$$M = \begin{bmatrix} \sigma_R^2 & V \\ V & \sigma_C^2 \end{bmatrix} \qquad (10)$$

μ_R, μ_C and the elements of M are a result of applying firm S's optimising behaviour to the effect of random variables on R and C. They will be dependent on the values of the parameters (a_1-a_2), b_1 and b_2 of P. We can approximate EU_S and EU_B as :

$$EU_S \simeq U_S(EX_S-\phi_S(b_1^{2}\sigma_R^2 + (1-b_2)^2\sigma_C^2-2b_1(1-b_2(v)/2) \quad (11)$$

$$EU_B \simeq U_B(EX_B-\phi_B((1-b_1)^2\sigma_R^2 + b_2^{2}\sigma_C^2-2b_2(1-b_1)v)/2) \quad (12)$$

when $EX_S = a_1-a_2 + b_1\mu_R-(1-b_2)\mu_C$

$$EX_B = a_2-a_1 + (1-b_1)\mu_R-b_2\mu_C$$

and when ϕ_S, ϕ_B are the coefficients of risk-aversion $-\frac{U''(\ )}{U'(\ )}$ evaluated at EX_S, and EX_B, respectively. The argument of the utility function is thus the 'riskless-equivalent' profits of the firm - that is the expected profit minus a risk premium. U_S and U_B are now expressed as functions of a linear combination of μ_R, μ_C, σ_R^2, σ_C^2 and v. Firm S will choose from a feasible set $T(\mu_R, \mu_C, \sigma_R^2, \sigma_C^2, v)$. An approximately optimal payments function will maximise the sum of the two firms' 'riskless-equivalent' profits. (a_1-a_2) will then serve to allocate such profits.

<u>Theorem 2</u>. The approximately initially-optimal payments function will have parameters $b_1^* = \frac{\phi_B}{\phi_S+\phi_B}$; $b_2^* = 1 - b_1^*$

<u>Proof</u> Consider $F_1(h'x)$, $F_2(k'x)$ where F_1 and F_2 are monotonic increasing functions. x^* will only coincidentally maximise F_1 and F_2 if $h \equiv tk$ when t is a scalar. Therefore by direct application of

this

$$b_1^* = \frac{\phi_B}{\phi_S+\phi_B} \quad \text{and } b_2^* = \frac{\phi_S}{\phi_S+\phi_B}$$

With b_1^*, b_2^* both firms will wish to maximise monotonic increasing functions of y where

$$y = \mu_R - \mu_C - \frac{\phi_S\phi_B}{\phi_S+\phi_B}(\sigma^2_R + \sigma^2_C - 2v) \qquad (13)$$

Optimal decisions can be found by applying dynamic programming.(9) Also $o<b_1^*,b_2^*<1$ as strict concavity is assumed. The approximately optimal payments schedule defined in Theorem 2 will soon become non-optimal during the production period. The reason is that b_1^*, b_2^* are evaluated at expected profits. As expected profit changes with new information, so a divergence will occur between the two firms' attitudes towards risk. Thus b_1^*, b_2^*, although initially optimal, will become non-optimal over the production period unless ϕ_B, ϕ_S are constants over the relevant range of profits. A renegotiation of contract to restore b_1, b_2 to their optimal values in response to a change in expectations would yield extra 'riskless-equivalent' aggregated profits compared with continuation with the old contract. If this gain exceeded the cost of renegotiation there would be an incentive to renegotiate. The distribution of the net gain would be decided by relative bargaining power. Finally, a change in the parameters of R-C would alter the ability of the mean-variance equivalent utility function to approximate the true utility function. Over the production period, it is reasonable to suppose that the variance of R-C would tend to zero - improving the approximation. However, this may not occur monotonically. Note that b_1^*, b_2^* satisfy the first two conditions of Theorem 1.

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(9) For an application of dynamic programming to find optimal decisions in a similar sequential problem see Bowden (1974)

III. Conclusions and Applications.

The analysis in this paper has been concerned with a two-way penalty system yielding a payments function which consisted of a linear function of 'value' and cost. It was shown that a subset of such functions had the properties of similarity and Pareto efficiency if production decisions were not taken sequentially. It was argued that these properties were not present when decisions were taken contingent upon future random events. It was also argued that a penalty system that was initially optimal, in the sense that both firms would agree decisions given initial means and variances of R and C, would not in general retain this property throughout the production period unless the risk-aversion of both firms was independent of the argument of the utility function.

The following propositions follow immediately from the theorems proved in Section II.

(i) Optimal allocation of risk will produce a mixed ($b_1, b_2 > o$) payments system unless one of the firms is risk neutral.

(ii) The relative initial standard deviations of the profits of the two firms should be equal to the reciprocal of their relative coefficients of risk aversion :

$$\frac{SD(X_S)}{SD(X_B)} = \frac{\phi_B}{\phi_S} \qquad (14)$$

R and C are statistics which summarise the impact of random events upon the two firms. Either or both R and C can be directly affected by these events or indirectly via decisions taken in response to them. By giving various interpretations to R and C, applications more specific than that of penalties and risky costs can be considered.

(a) Sharecropping.

Let R be the value of output and C the cost of production which in addition to the costs of fertilisers, water, tools, etc. incorporates the opportunity (piece-work) labour cost. The following results are immediate

(i) Pure wage system exists when $b_1 = o$, $b_2 = 1$. This is only optimal if the landlord has full control over the production decisions and is risk neutral.

(ii) Pure rental exists when $b_1 = 1$, $b_2 = o$. This is only optimal if the labourer is risk neutral.

(iii) A fee schedule where $o<b_1<1$ and $b_2 = o$ is not optimal unless C is not under the labourer's control. This will only occur if the contract specified plans for all contingencies or if C is completely constant. Otherwise the marginal opportunity cost of the labourer's time would be equated with the fraction b_1 of the value of marginal product [10].

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(10) If risk sharing was accomplished by 'mixed' contracts (see Stiglitz (1974)), then agreed "modifications of production plans" and "commonality of factor interests" (Reid) (1974) (p.22) remain strong arguments for the efficiency of sharecropping, even if the payments function is not 'optimal'.

(b) Agency.

A pure agency problem must be considered as one where C is either zero or a given constant. Then write $a' = a_1 - a_2 + b_2 C$, and (3) becomes $P = a' + b_1 R$. The usual results of the analysis of this problem follow providing sequential decisions or decisions in terms of expectations are not involved. The latter implies b_1^* and the former the probability of accumulated error.

(c) Moral Hazard.

All the applications considered so far are essentially 'moral hazard' problems: the 'value' and 'cost' of a decision have to have the same weights, (Theorem 1), and these weights have to be such that risk is efficiently proportioned (Theorem 2). If the weights are not the same then a moral hazard problem will occur. Consider the following example of medical insurance. Let R be the value to the patient of medical services consumed and C the cost of providing medical services. Firm S is the insurer and receives a premium P. His profit is P-C. The insured's 'profit' is R-P and he decides the level of consumption measured by the cost C. The optimal premium by Theorem 2 is :

$$P = a + b_1^* R + (1 - b_1^*) C \quad \text{where } b_1^* = \frac{\phi_B}{\phi_S + \phi_B} \tag{15}$$

although any value $o < b_1 < 1$ (9) will ensure P.E. The insured would then equate marginal value with marginal cost. If the insurer is much less

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(11) Note here that the decisions are taken by 'firm B'. Theorem 1 is easily modified to accomodate this change. If $b_1^* = 1$, then the lack of an incentive will prevent a pareto efficient solution. Obtaining the best allocation of risk when the insurer is risk neutral is not thus compatible with the maintenance of incentives.

risk averse than the insured, then $b_1^* \simeq 1$, and a small proportion $(1-b_1^*)$ of the excess of R over C (the consumer's surplus) is paid to the insurer.

It can thus be argued that the existence of moral hazard is due to the difficulties of measuring R and C. If no such difficulties were present then a payments function could be found to eliminate moral hazard.

Similar measurement problems inhibit the widespread use of optimal payments functions in any general bi-lateral contracts. Although this is understandable when the contracts involve small transactions, it is more surprising in the case of large-scale transactions when the appointment of an unbiased referee to measure R and C and to indicate the impact of production period decisions upon their distribution might seem warranted. Obviously on some occasions the firms may internalise the referee by active collusion.

It is hoped that the concept of the two-way payments function (3) contingent upon ultimate values of pay-offs for the two firms provides a simple and general framework for the analysis of bi-lateral situations involving risk.

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