ADVERTISING AND WELFARE

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This paper is circulated for discussion purposes only and its contents should be considered preliminary.

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1. INTRODUCTION

Welfare analysis of advertising usually begins and ends with the remark that there is no fixed standard for value judgement when tastes are variable. Some indirect inferences are drawn from technical issues such as barriers to entry and efficient means of conveying information, but the discussion lacks a satisfactory unifying framework. 1/

We believe that the multiplicity of possible standards should not be an excuse for abandoning analysis. Rather, it should provide the starting point for an examination of the consequences of using each relevant standard, and a comparison of the outcomes. In this paper we do this for each of the two extreme standards of pre-advertising and post-advertising tastes. We would use the former for purely persuasive advertising and the latter for purely informative advertising, if ever we could find an instance of such a clear nature. But, much more importantly, the examination yields a pair of sufficient conditions, rather as in index number theory. Thus, in general, advertising that is beneficial at the margin when judged by pre-advertising tastes will also be beneficial when judged by the post-advertising tastes, and advertising that is harmful according to the latter will also be harmful according to the former.

It may be surprising at first sight that advertising can ever be beneficial when judged by the pre-advertising tastes. However, the reason is exceedingly simple. Advertising for a product will occur when there is some element of monopoly. The equilibrium in absence of advertising will have the price in excess of the marginal cost, making it desirable to shift some resources from the rest of the economy towards the product in question.

Advertising can lead to such a shift, increasing the output of the product and yielding a welfare gain even as judged by the original tastes. It is at least conceivable that the gain exceeds the marginal resource costs of the necessary advertising, thus making some advertising socially desirable.

It must be stressed that this is a 'third-best' proposition. We are not only accepting the monopoly as given, but also ruling out second-best policies of taxation or regulation. Thus we are not seriously proposing advertising as a policy to reduce monopoly welfare losses. The approach merely provides a framework for comparing the private profitability of advertising with its social desirability. For each standard, welfare analysis of a very conventional kind can be applied, and the standard micro-economic theory is available. 2/

Applying this method to a wide range of circumstances, under assumptions that are not contradicted by the available empirical evidence, 3/ we prove two results. First, private profitability is seen to be a necessary but not sufficient condition for the social desirability of a small amount of advertising, when judged by either of the two extreme standards. Secondly, a market equilibrium will be shown to entail a level of advertising that is excessive from the social point of view, even as judged by the post-advertising tastes. This is a strong and precise statement of the vague general idea that a market economy involves too much advertising.

We consider three cases. First, we assume that there is only one product being advertised, and the industry is a monopoly. This neglects some of the most interesting questions which arise from the effects of advertising on the distribution of demand within the industry, and on entry into the industry. However, the basic point can be clarified in the context of monopoly, and a simple geometric explanation given. Next we turn to oligopoly, which introduces the possibility that demand shifts perceived as profitable by each seller fail to materialize when they all advertise together. Finally, we allow entry, and consider Chamberlinian monopolistic competition. This allows erosion of perceived profit-opportunities through new entry. It will be seen that the basic contention of excessive advertising is strengthened at each stage.

We cast the analysis in a model where the notion of consumers' surplus is valid. We do this by assuming that the product, or group of products, being advertised has no income effects. While this is a serious restriction, we are encouraged to believe that the results are fairly robust, since two recent studies of monopolistic competition, using such a setting but with very different assumptions about income effects, reach almost identical conclusions. [4/] We assume that the rest of the economy is competitive, and aggregate it into one good which is then chosen as the numéraire. We also neglect the issue of income distribution, deriving the demand curves from Samuelsonian social indifference curves. Thus the contribution of advertising to monopoly profits is included in the measure of welfare, which makes the result about excessive advertising even more striking.

For analytical convenience, and probably without doing much harm to reality over the relevant range of output, we assume constant marginal costs of production for each good. Some fixed costs exist, and these provide the explanation of the prevalence of monopoly in the first instance.

2. ADVERTISING UNDER PURE MONOPOLY

The basic point is easily stated. Advertising has the effect of shifting demand from other goods to the advertised good. The post-advertising equilibrium must therefore involve a higher price, or a higher output, or both. Given that the demand price exceeds the marginal cost at the initial monopoly equilibrium, some increase in output beyond the pre-advertising level will be desirable, even as judged by the original preferences. Thus, to the extent that advertising leads to an increase in the monopolist's output, there will be a welfare gain associated with some advertising. Such a gain will, of course, have to be weighed against the resource costs of the advertising.

The point is illustrated in Figure 1. The pre-advertising demand curve is D, while D' is the one following upon a small initial dose of advertising. In keeping with the casual empiricism concerning advertising, we assume that the demand curve becomes less elastic as it shifts, in such a way that both the price and the output rise, shifting the equilibrium from E to E'. Neglecting the resource costs of advertising for a moment, the welfare gain as judged by the pre-advertising preferences is the area ABGE, being the area under the demand curve minus the opportunity costs of resources transferred from the rest of the economy. The increase in the monopolist's profit is the difference between the rectangles QBE'P' and QAEP. Neglecting the area HEGE' which is of the second order of smallness, we at once have the relation

$$\Delta W_0 = \Delta \Pi - x \Delta p , \qquad (2.1)$$

where x is output, p is price, Π monopoly profits, W_0 the money equivalent of welfare as measured by the pre-advertising tastes, and the operator Δ denotes differences as usual.

We assume that advertising is supplied at constant cost, thus no rent accrues to the suppliers of this service, and therefore the costs of advertising are the same for the momopolist as for society. The common

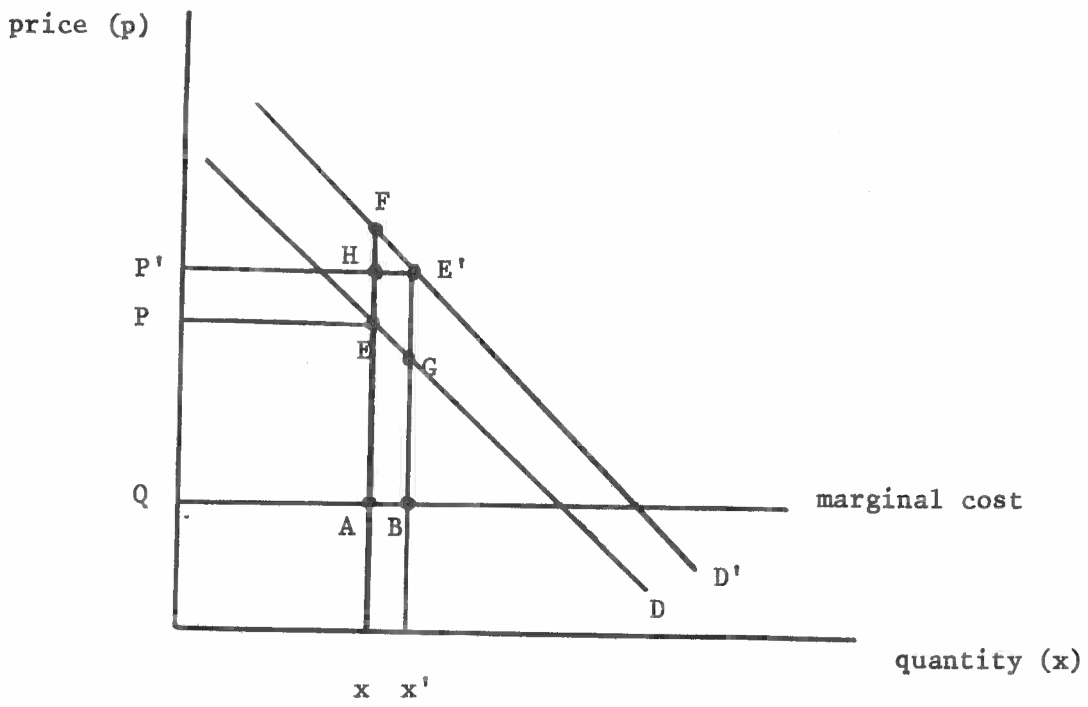

Figure 1

quantity can be subtracted from ΔW_0 and $\Delta\Pi$, and (2.1) is valid even when advertising costs are taken into account. Thus the welfare gain from a small amount of advertising, evaluated using pre-advertising preferences, will equal the increase in monopoly profits brought about by advertising, less the increase in expenditure the consumer would incur in buying the initial quantity at the higher price.

There are two implications of this. First, advertising will never be socially advantageous unless the monopolist finds it profitable. Secondly, in the special case where advertising does not affect the elasticity of demand and therefore does not cause a price increase, such private profitability will also be sufficient to establish that some advertising will be socially desirable.

If we use the post-advertising standard, the corresponding welfare increase is the area ABE'F, less the resource costs of advertising. We must exclude the area between the demand curves D and D' to the left of x: what we want to measure is the effect of a change in output as judged by one standard, and not that of a changing standard on a given level of output. Now we can reinterpret Figure 1 in terms of post-advertising tastes. Let D be the demand curve corresponding to any initial level of advertising (not necessarily zero), and D' that for a small increase in this level. If W is welfare as judged by the post-advertising tastes, neglecting the second-order triangle HFE', we have the relation

$$\Delta W = \Delta \Pi - x \ \Delta p \ . \tag{2.2}$$

This is similar to (2.1); in fact we can think of (2.2) as the general relation and (2.1) as a special case, since $\Delta W_0 = \Delta W$ to the first order when the initial level of advertising is zero.

Now consider the monopolist's profit-maximising choice of advertising. The first-order condition is $\Delta \Pi = 0$, and therefore at this level we have $\Delta W = - x \ \Delta p < 0$. Thus welfare would be increased by reducing the level of advertising below that chosen by the monopolist. This is true using post-advertising preferences, and must also hold _a fortiori_ when we use pre-advertising preferences, as the demand price using the latter is always below the price the consumer becomes willing to pay after he has been subjected to the advertising. Thus we have a strong and precise result concerning excessive advertising in a market economy.

It is useful to set this up formally, to enable easier comparisons with subsequent cases where a simple geometric treatment is not possible. To do so, we suppose that demand is generated by a utility function

$$U = y + u(x, z) , \qquad (2.3)$$

where y is the output of the competitive sector of the economy (aggregated and chosen as the numéraire), x is the monopolist's output, z is his level of advertising, and the function u is increasing and concave in x. Let p denote the price of the monopolised good, then the inverse demand function can be written as

$$p = u_x(x, z) , \qquad (2.4)$$

where subscripts denote partial derivatives.

We can describe shifts of the demand function as z changes in either of two ways. For fixed x, we have

$$\frac{1}{p} \frac{\partial p}{\partial z} = u_{xz}/u_x \qquad (2.5)$$

We abbreviate the right-hand side as δ. It is clearly reasonable to assume $\delta > 0$, for otherwise advertising would be pointless from the firm's point of view. It would also seem reasonable to assume δ to be declining in z, although this is less clear.

Alternatively, for fixed p, we have

$$\frac{1}{x} \frac{\partial x}{\partial z} = - u_{xz}/(x\, u_{xx}) .$$

If we define the elasticity of marginal utility,

$$\gamma = - x\, u_{xx}/u_x , \qquad (2.6)$$

which is also the inverse of the elasticity of demand, this becomes

$$\frac{1}{x}\,\frac{\partial x}{\partial z} = \delta/\gamma \tag{2.7}$$

We assume a linear technology, economies of scale in the monopoly sector being represented by a fixed cost component. Thus

$$y + a + c\,x + h\,z = e, \tag{2.8}$$

where c is marginal cost, a the fixed cost of production for good x, h the unit cost of advertising, and e the resource endowment.

Monopoly profits will be

$$\Pi = p\,x - a - c\,x - h\,z \tag{2.9}$$

The monopolist's profit-maximizing choice of price and output will be given by the condition that marginal revenue equals marginal cost, i.e.

$$p\,(1 - \gamma) = c\,. \tag{2.10}$$

For any given level of advertising, (2.4) and (2.10) determine x and p, and then y can be found from (2.8). We assume $0 < \lambda < 1$.

Assuming optimal pricing and output, the marginal impact of advertising on profit is easily seen to be

$$d\Pi/dz = p\,x\,\delta - h \tag{2.11}$$

The profit-maximizing level of advertising can then be found by setting the right hand side equal to zero. However, we wish to examine the effect of reducing the level of advertising below the profit-maximizing one, and will therefore treat z like a parameter, with the profit-maximizing level as only one of its whole range of possible values. The next question is the shift of the equilibrium price and output in response to an exogenous change in the parameter z.

To simplify the mathematics, we shall impose the additional assumption that the price elasticity of demand is independent of x, and a decreasing function of z. The former is a simplification that is empirically fairly reasonable, while the latter captures part of the casual empiricism concerning advertising. From (2.11), we then see that

$$\frac{1}{p}\frac{dp}{dz} = \frac{1}{1-\gamma}\frac{d\gamma}{dz} \geq 0 \qquad (2.12)$$

We shall use the abbreviation θ for this magnitude. Then, using (2.4), we have

$$\frac{1}{x}\frac{dx}{dz} = (\delta - \theta) / \gamma . \qquad (2.13)$$

Note that these comparative static derivatives, which tell us how the whole equilibrium shifts, are to be clearly distinguished from the partial derivatives in (2.5) and (2.7), which tell us how the demand curve shifts.

A simple interpretation is possible for (2.13): equilibrium output will rise if and only if the increase in the demand price at the initial output level exceeds the price increase which the monopolist finds profitable. As in Figure 1, we assume this to be the case. Finally, turning to the numéraire, we have from (2.8) that

$$\frac{dy}{dz} = - \left\{ \frac{c\,x}{\gamma} (\delta - \theta) + h \right\} . \qquad (2.14)$$

In order to convey a better idea of the magnitudes involved, a digression is warranted. Let us choose a (local) unit of advertising so that a unit increase in its volume raises demand by one per cent at a given price, i.e. $\delta/\gamma = 0.01$. Now use (2.10) to write (2.14) as

$$\frac{dy}{dz} = - p\,x \left\{ \frac{1-\gamma}{\gamma} (\delta - \theta) + \frac{h}{px} \right\} .$$

We are assuming $\theta < \delta$, and we need $\gamma < 1$. Thus the first term in the brackets will be a number of magnitude around 0.01. The second term is now the ratio of the advertising expenditure that induces a 1% increase in demand, to total sales. Casual empiricism would suggest that it is a number of a similar order of magnitude. Thus the effect on the numéraire is a few percent of the monopolist's revenue.

Now we turn to the question of welfare. Begin with the pre-advertising utility function as the welfare index W_0, i.e.

$$W_0 = y + u(x, 0). \qquad (2.15)$$

Differentiating, we have

$$dW_0/dz = dy/dz + u_x(x, 0)\, dx/dz . \qquad (2.16)$$

Write

$$p_0 = u_x(x, 0) \qquad (2.17)$$

Thus p_0 is the pre-advertising demand price for post-advertising output, and not the actual pre-advertising price. We always have $p_0 \leq p$, with equality if and only if $z = 0$.

Substituting from (2.13) and (2.14) into (2.16), we have

$$dW_0/dz = (p_0 - c)\ x\ (\delta - \theta)\ /\gamma\ - h. \qquad (2.18)$$

This says that, apart from the direct resource costs of advertising, the welfare effect depends on (i) whether the pre-advertising demand price exceeds the marginal cost, and (ii) whether the increase in the demand price at the original output exceeds the actual price increase.

We can fortunately go beyond this rather vague conclusion. Recalling from (2.10) that $\gamma = (p - c)/p$, we write (2.18) as

$$\frac{dW_0}{dz} = \frac{p_0 - c}{p - c}\ px\ (\delta - \theta) - h\ .$$

Evaluated at $z = 0$, where $p_0 = p$, this becomes

$$dW_0/dz = (p\ x\ \delta - h) - \theta\ p\ x\ .$$

Recalling (2.11), we have at $z = 0$,

$$dW_0/dz = d\Pi/dz - \theta\ p\ x. \qquad (2.19)$$

This is just the exact analogue of (2.1). Consequently, if advertising initially has no impact on the elasticity of demand, we can say unambiguously that some advertising is beneficial if and only if the monopolist finds it profitable. In the presence of price increases, however, the conclusion is weakened: profitability remains a necessary condition for the social desirability of some advertising, but it is no longer a sufficient one.

Now consider the post-advertising standard of welfare W. This coincides with U, with one difference. When considering changes in z, we do not differentiate directly with respect to z in $u(x, z)$. This is to avoid the effect of using a changing standard, as was explained in the context of Figure 1. Then we have

$$dW/dz = dy/dz + u_x(x, z)\, dx/dz \,, \qquad (2.20)$$

which simplifies to

$$dW/dz = d\Pi/dz - \theta\, p\, x \,. \qquad (2.21)$$

This is the same as the right hand side of (2.19), but is no longer restricted to being evaluated at $z = 0$. Thus (2.21) is the exact form of (2.2).

Now consider the monopolist's profit-maximizing choice of z, given by $d\Pi/dz = 0$. There we have $dW/dz = -\theta\, p\, x < 0$, except for equality in the case where the elasticity of demand is independent of the level of advertising. Thus, as judged by the post-advertising preferences, and so long as advertising leads to some price increase, a monopolist will undertake too much advertising.

Finally, we have from (2.16) and (2.20) that

$$dW/dz - dW_0/dz = (p - p_0)\; dx/dz \,, \qquad (2.22)$$

which is positive so long as $\delta > \theta$, and zero for $z = 0$. Thus, so long as advertising leads to come output increase, any advertising that is beneficial according to the pre-advertising preferences will also be beneficial according to the post-advertising preferences, while any advertising that is harmful according to the latter will also be harmful according to the former. Thus, our results concerning the gain from some advertising according to the pre-advertising standard, and the excess of advertising as judged by the post-advertising standard, are both extended to the other standard as well.

However, if $\delta < \theta$, we have $dW/dz < dW_0/dz$, and it is possible for the former to be negative while the latter is positive. This seems odd, but a little reflection shows how the case can arise. Suppose the effect of advertising on output is positive for a while, carrying the equilibrium into a region where $p_0 < c$. Of course we must always have $p > c$. It is now desirable to reduce the output of the monopolised good as judged by the pre-advertising standard, but it remains desirable to increase it when the post-advertising standard is used. If, at this stage, the marginal effect of advertising on output turns negative, say through a decline in δ, then a further slight increase in advertising can be beneficial according to pre-advertising tastes but harmful according to post-advertising tastes. Of course this is a local result, since what is really optimum according to the pre-advertising tastes is not a slight increase in z but a much larger decrease which will have the same effect on output while saving the resources used up in advertising.

This phenomenon, depending as it does on the marginal effect changing sign, is something of a curiosum in the context of monopoly. A similar effect is more plausible with monopolistic competition, but its local nature remains.

Bearing in mind the relation between W_0 and W, we shall save space in what follows by working only with W. The two are equivalent at $z = 0$, and for the second result we only need the latter.

3. OLIGOPOLY

Intuitively, one should not expect the case of oligopoly to be substantially different from that of pure monopoly. With a fixed number of firms, each of which faces a downward-sloping demand curve, price will exceed marginal cost for each. To the extent that advertising shifts demand to the oligopolistic sector from the competitive sector, we should again expect a potential welfare gain from advertising. We should, however, expect the advertising levels chosen by oligopolists to be even more excessive than that chosen by a monopolist, as advertising under oligopoly to some extent simply shifts demand from one oligopolist to another.

To analyse this case, we shall assume that demand is generated by a utility function of the form [5/]

$$U = y + \phi \left(\sum_{i=1}^{n} u(x_i, z_i) \right), \tag{3.1}$$

where x_i and z_i are the respective output and advertising levels for product i, and n is the number of products involved. The function ϕ is assumed to be increasing and concave. This is a natural generalization of (2.3). Note that by making the functional form of u independent of i, we are introducing an important symmetry for analytical convenience. This will be supplemented with a symmetric cost side later.

An additional point to be noted is that the specification implies that all goods in the industry are substitutes, but not perfect ones. That the goods are not perfect substitutes, i.e. that there is product differentiation, is quite natural in a world of advertising. Further, with economies of scale, we would expect only one of a set of perfect substitutes to survive in the market. Lack of complementarity needs more justification. We would expect that in a world of product differentiation, complementary goods are likely to be sold and advertised as a package. They can then be represented as a sub-aggregate. Thus the product definition implicit in (3.1) is one where a product is a composite good to which a particular advertising effort is attached. In line with this, we also assume that each firm in the industry produces a single product, so that there is a one-to-one correspondence between products and firms.

We concentrate on the case where the behaviour of the firms is non-cooperative, with respect to both output and advertising. Thus the industry equilibrium is of the Cournot-Nash variety. At the end of the section we consider cooperative behaviour briefly.

The inverse demand functions will be given by the first-order conditions of utility maximization, i.e.

$$p_i = \phi'(v) \; u_x(x_i, z_i) \qquad (3.2)$$

where

$$v = \sum_{i=1}^{n} u(x_i, z_i) \, . \qquad (3.3)$$

These equations define all relevant price and advertising effects on demand. It should be noted that output and advertising decisions of any particular firm affect the demand for all firms through v. To the extent that behaviour is non-cooperative, each firm will fail to take these into account, and this is what leads to the extent of advertising in oligopoly being even more excessive than that under pure monopoly. Since the effect of any x_i or z_i on v is of the order $(1/n)$, we can highlight the aspect of non-cooperation by considering the case where n is large, and assuming that each firm totally neglects the effect of its actions on v.

In this setting, for any particular pair of firms, the cross-price and cross-advertising elasticities are negligible. As to the own-effects, the inverse of the elasticity of demand is

$$\gamma_i = - \frac{x_i}{p_i} \frac{\partial p_i}{\partial x_i} = - x_i u_{xx} (x_i, z_i)/u_x (x_i, z_i) , \tag{3.4}$$

and the alternative expressions for the shift of the demand curve as z_i changes can be written

$$\frac{1}{p_i} \frac{\partial p_i}{\partial z_i} = u_{xz} (x_i, z_i)/u_x (x_i, z_i) \equiv \delta_i , \tag{3.5}$$

$$\frac{1}{x_i} \frac{\partial x_i}{\partial z_i} = \delta_i/\gamma_i . \tag{3.6}$$

The notation is a natural extension of that of the previous section.

Of course, even if individual cross-effects in demand are negligible, their sum may be substantial. In particular, a uniform change in output and advertising levels will normally involve significant cross-effects. If we set $x_j = x$ and $z_j = z$ for all j, and then vary these common levels, we see that for any i, the demand price p_i changes in a different way. For example,

$$- \frac{x}{p_i} \frac{\partial p_i}{\partial x} = - (v\phi''/\phi') \cdot (x\, u_x/u) - x\, u_{xx}/u_x \; .$$

This is, in Chamberlinian terminology, the inverse elasticity of the DD-curve. Each of the terms on the right hand side is positive given our assumptions, and the second of them is the inverse of the elasticity of the Chamberlinian dd-curve. Thus, as expected, the dd-curve is the more elastic.

To make notation more compact, we define

$$\varepsilon = - v\phi''/\phi' \tag{3.7}$$

and

$$\alpha = x\, u_x/u \tag{3.8}$$

Then we have

$$- \frac{x}{p_i} \frac{\partial p_i}{\partial x} = \gamma + \alpha\, \varepsilon \tag{3.9}$$

where γ is the common value of the γ_i under our symmetric conditions.

Similarly, the effect of a uniform increase in the level of advertising in the symmetric situation will be

$$\frac{1}{p_i} \frac{\partial p_i}{\partial z} = (v\phi''/\phi') \cdot (u_z/u) + u_{xz}/u_x \; .$$

Letting β denote (u_z/u), this becomes

$$\frac{1}{p_i} \frac{\partial p_i}{\partial z} = \delta - \varepsilon \beta , \qquad (3.10)$$

where δ is the common value of the δ_i. It is not clear whether $(\delta - \varepsilon \beta)$ can be assumed positive, or even non-negative. At a given common initial price, the effect of a uniform increase in advertising on each x_i is similarly given by

$$\frac{1}{x_i} \frac{\partial x_i}{\partial z} = \frac{\delta - \varepsilon \beta}{\gamma + \varepsilon \beta} . \qquad (3.11)$$

Thus we see that the sign of $(\delta - \varepsilon \beta)$ governs whether more advertising by all firms leads to an increase in industry demand. In the borderline case where this expression is zero, advertising merely redistributes demand among firms. We shall see that $(\delta - \varepsilon \beta)$, indicating the effect of advertising on the industry demand, is the magnitude which is relevant for welfare judgements.

Equations (3.2) and (3.3) describe demand. Turning to the supply side, we assume that all products have equal fixed costs of production a, equal marginal costs c, and equal unit costs of advertising, h. Thus the technology is given by

$$y + \sum_{i=1}^{n} (a + c x_i + h z_i) = e. \qquad (3.12)$$

The profit of firm i is

$$\Pi_i = p_i x_i - a - c x_i - h z_i . \qquad (3.13)$$

With non-cooperative behaviour, the first-order conditions for maximum profit become

$$\partial \Pi_i / \partial x_i = p_i (1 - \gamma_i) - c = 0 \qquad (3.14)$$

and

$$\partial \Pi_i / \partial z_i = p_i \, x_i \, \delta_i - h = 0 \, . \qquad (3.15)$$

Given the symmetry of demand and costs, all firms will then decide on the same levels of output and advertising, and hence will charge the same price. The Nash equilibrium will be characterized by a common price p, a common output level x, and a common advertising level z, which, with the auxiliary variables y and v, are solved from the 'supply' relations

$$p(1 - \gamma) = c \, , \qquad (3.16)$$

$$p \, x \, \delta = h \, , \qquad (3.17)$$

the demand relations

$$p = \phi'(v) \; u_x(x, z) \qquad (3.18)$$

$$v = n \, u(x, z) \qquad (3.19)$$

and the balancing condition

$$y = e - n \, (a + c \, x + h \, z) \, . \qquad (3.20)$$

As in the case of monopoly, we will also consider a problem where the level of advertising is a parameter, in order to study the welfare implications of a change in advertising. For part of the discussion, therefore, we will take z to be exogenous, delete (3.17), and determine a conventional Cournot equilibrium from (3.16) and (3.18)-(3.20). Then we can study the comparative statics of this equilibrium as z changes.

The effect of advertising on price is completely analogous to that under pure monopoly. Advertising makes the products appear to be poorer substitutes, making demand for each less price elastic and leading to a price increase. Again, if the price elasticity is (at least locally) independent of the level of output, the price increase will be independent of what happens to output, and can be written simply as

$$\frac{1}{p}\frac{dp}{dz} = \theta \tag{3.21}$$

where $\theta = (d\gamma/dz)/(1 - \gamma)$. As for the output effect, combining (3.18), (3.19) and (3.21), we find that

$$\frac{1}{x}\frac{dx}{dz} = \frac{(\delta - \beta\,\varepsilon) - \theta}{\gamma + \alpha\,\varepsilon} \tag{3.22}$$

This has an interpretation similar to that of (2.13): output increases if and only if the increase in the demand price of any product at the old output level is greater than the actual price increase set by each firm. If we compare (3.22) with the output effect of an increase in advertising by one firm alone, however, the magnitude would be different. If, starting from a symmetric equilibrium, we had increased z_i alone, we would have found that

$$\frac{1}{x_i}\frac{dx_i}{dz_i} = \frac{\delta - \theta}{\gamma} \tag{3.23}$$

The difference here is attributable to the interactions among firms operating through ε. First, a uniform increase in advertising makes the demand price at the old output increase by $(\delta - \beta\,\varepsilon)$ rather than by δ, the demand price increase caused by an increase in advertising by one firm alone. Secondly, since the DD-curve is less price elastic than the dd-curve, a given increase in demand price will correspond to a smaller increase in quantity demanded along the former curve. Consequently, the denominator is larger in (3.22) than in (3.23). Note that if z_i alone changes, for any $j \neq i$ we have

$$\frac{1}{x_j} \frac{dx_j}{dz_i} = -\frac{1}{n} \frac{\varepsilon \beta}{\gamma} ,$$

a magnitude of order (1/n), but the total effect of all these changes is not negligible since there are (n - 1) such firms.

We are now in a position to discuss the welfare implications of advertising. Consider first the effect of a change in the common level of advertising of all the firms. This is a natural setting to examine, since in the symmetric equilibrium all firms have the same incentives. For the criterion W, remembering that we are not to differentiate with respect to z directly in u, we find

$$dW/dz = dy/dz + \phi'(v)\, n\, u_x (x, z)\, dx/dz .$$

This simplifies to

$$dW/dz = n \{ (p - c)\, dx/dz - h \} . \tag{3.24}$$

Writing Π for the uniform profit level in a Cournot equilibrium, we at once have

$$d\Pi/dz = (p - c)\, dx/dz + x\, dp/dz - h . \tag{3.25}$$

Using (3.21) and (3.22), we have

$$dW/dz = n \{ d\Pi/dz - p x \theta \} . \qquad (3.26)$$

This is an immediate generalization of (2.21). The similarity is misleading, because (3.25) gives the ex post effect on a given firm's profit of a uniform increase in advertising. This is not directly relevant to a non-cooperative equilibrium, where what matters is the perceived effect of a firm's own advertising on its profits. This is found by evaluating the expression for $\partial\Pi_i/\partial z_i$ from (3.15) at the initial symmetric equilibrium. Let μ denote this perceived impact of advertising on profit for any one firm, and we have

$$\mu = p x \delta - h. \qquad (3.27)$$

Substituting from this and (3.22) in (3.24), we have

$$\frac{dW}{dz} = n \left\{ \mu - \frac{px}{\gamma + \alpha \varepsilon} \left[\varepsilon (\delta \alpha + \gamma \beta) + \gamma \theta \right] \right\} \qquad (3.28)$$

Consider first the point $z = 0$, where $dW_0/dz = dW/dz$ as in the case of monopoly, and we can use (3.28) for either criterion. Again, a necessary but not sufficient condition for a little advertising to be socially desirable is that each seller should perceive it to be profitable. In this case, however, the decrease in the elasticity $(\theta > 0)$ is no longer the sole factor that prevents the condition from being sufficient: the interaction among firms working through ε has the same effect.

Secondly, at the Nash equilibrium for advertising, $\mu = 0$ and then $dW/dz < 0$. Thus, once again, the market equilibrium will involve excessive advertising as judged by the post-advertising standards. Because of the interaction among firms, this will be so even when advertising has no price effects, i.e. when $\theta = 0$.

If the expression in (3.22) is positive, this conclusion is valid a fortiori for the pre-advertising tastes. Else the kind of local exception discussed under monopoly can arise. We see that owing to the interaction term, this is somewhat more likely than it would be for pure monopoly. However, as explained in the previous section, the exception really involves excessive advertising when we consider discrete changes and global optima.

We can also examine what happens if, starting from a symmetric equilibrium, only one firm changes its level of advertising. We shall omit the calculation and merely state the result that for any i, we have

$$\partial W/\partial z_i = \mu - p\,x\,(\varepsilon\,\beta + \theta)\ . \tag{3.29}$$

Thus we see that, starting from the Nash equilibrium where $\mu = 0$, it is also socially desirable to reduce the level of advertising for any one firm. This excess of advertising in a market equilibrium again arises from the price rise term involving θ, and from the interaction effects that are not taken into account by any one firm, which arise through ε.

As a general conclusion, therefore, we find that the results for the case of monopoly are strengthened for oligopoly, i.e. the case for advertising is further weakened. It should also be noted that there is a continuity here, in the sense that elements of cooperative behaviour among the oligopolists will lead to cases intermediate between those of pure monopoly and the Cournot-Nash non-cooperative oligopoly. For example, we could suppose that each firm at least partially anticipates the reaction of other firms to its output change, thus producing models of apparent collusion. [6/] Further, we have two policy variables, output and advertising, and it is possible to conceive of cooperation or competition with respect to each. Thus, with cooperative pricing but non-cooperative advertising, (3.28) is replaced by

$$dW/dz = n \{ \mu - p\, x\, (\varepsilon\, \beta + \theta) \}$$

Similarly, if advertising levels are determined cooperatively but pricing is non-cooperative, we have

$$dW/dz = n \{ \mu - p\, x \left[(\delta - \beta\, \varepsilon)\, \alpha + \gamma\, \theta \right] / (\gamma + \alpha\, \varepsilon) \}.$$

In both instances, the case for advertising is 'better' than that for a Cournot-Nash oligopoly but 'worse' than with a pure monopoly.

4. MONOPOLISTIC COMPETITION

Now we allow entry into the non-cooperative large group of the previous section, thus obtaining a Chamberlinian monopolistically competitive industry. This seemingly minor change has some important consequences. As in the case of monopoly and oligopoly, there is a sense in which the initial equilibrium in this section involves underproduction relative to the rest of the economy. However, there are now to dimensions to the problem. The sectoral output can be expanded by increasing the output of each firm, or the number of firms, or some combination. The direction in which the equilibrium errs relative to the optimum is not immediately clear, and recent research has shown that much of the accepted conjecture concerning excessive product diversity in monopolistic competition is mistaken. [7/]

On the contrary, there is strong reason to believe that a wider range of products would be available if production were organized to achieve a first-best Pareto optimum. In such a setting, advertising can increase welfare by increasing the range of products. Similarly, if the optimum entails a greater output for each firm, and some advertising leads to some such increase, there is the possibility of a welfare gain. It is even possible for one of these two to move in the wrong direction but for the total effect to be beneficial. Of course in each case the relevant consideration is the net gain, subtracting the resource costs of advertising.

These remarks apply to the desirability of a small amount of advertising. We can also consider the equilibrium level of advertising in relation to the optimum, using methods that are similar to those used before.

In formal terms, it is necessary to make only a small alteration to the model of oligopoly of section 3 in order to handle monopolistic competition. We make n endogenous, and determine it from the condition that each active firm at least breaks even, while no firm contemplating entry can see the prospect of breaking even. In the case of the symmetric large group, the condition boils down to requiring that each active firm makes exactly zero profit, i.e.

$$p\,x = a + c\,x + h\,z \qquad (4.1)$$

in our notation. Now, for any given value of z, the equilibrium values of p, x, n and v are given by (3.16), (3.18), (3.19) and (4.1), while the Nash equilibrium value of z itself can be found by appending (3.17) to these.

Having characterised equilibrium for given z, we examine how it changes in response to exogenous changes in this level. The effect on p is completely analogous to that under monopoly or oligopoly. Assuming that the price elasticity is (at least locally) independent of the level of output, we find that

$$\frac{1}{p}\frac{dp}{dz} = \theta \qquad (4.2)$$

in the same notation as before.

The effect on the output of each firm now works differently, since the price and output are constrained by the zero-pure- profit condition (4.1). Thus we have

$$\frac{1}{p}\frac{dp}{dz} + \frac{1}{x}\frac{dx}{dz} = \frac{c}{p}\frac{1}{x}\frac{dx}{dz} + \frac{h}{px}.$$

Using (4.2) and solving,

$$\frac{1}{x}\frac{dx}{dz} = \frac{1}{\gamma}\left\{\frac{h}{px} - \theta\right\}. \qquad (4.3)$$

Now θ is the percentage increase in revenue brought about by the price increase induced by a unit increase in advertising if output is held constant, while (h/px) is the added cost of this advertising as a fraction of the total revenue. If the latter exceeds the former, then holding output constant will mean a loss for each firm, so there must be a rise in the output of each firm (accompanied by some exit if the total demand shift to the industry is not large enough) to maintain zero profits.

The new effect of interest is the impact of advertising on the number of firms or products, i.e. on the supply of 'variety' in the industry. From (3.18) and (3.19) we have

$$\frac{1}{p}\frac{dp}{dz} = -\varepsilon\left\{\frac{1}{n}\frac{dn}{dz} + \alpha\frac{1}{x}\frac{dx}{dz} + \beta\right\} - \gamma\frac{1}{x}\frac{dx}{dz} + \delta .$$

Substituting for the price and output effects, we obtain

$$\frac{1}{n}\frac{dn}{dz} = \frac{\delta - \beta\varepsilon}{\varepsilon} + \frac{\alpha\theta}{\gamma} - \frac{h}{px}\frac{\gamma + \varepsilon\alpha}{\gamma\varepsilon} . \quad (4.4)$$

To interpret this, consider the case where the price increase happens to be just enough to cover the extra advertising cost, so that the output effect is zero, and the expression reduces to $\{(\delta - \beta\varepsilon) - \theta\}/\varepsilon$. Then the number of goods will increase if and only if the demand price increase for each good at the old equilibrium exceeds the price increase that actually occurs. Comparing this with (3.22), we see clearly how entry takes over some of the burden of adjustment in the case of monopolistic competition.

Thus it is important to look at a combination of the number of firms and the output of each. It is natural to base such a measure on v, which can be thought of as a sectoral sub-utility. Of course we continue to neglect the direct effect of z on u when considering changes in v, in order to find an index of aggregate output change according to some fixed standard. Since $v = n\,u(x, z)$, the index of proportionate change is $dv/v = dn/n + \alpha\, dx/x$, and we have

$$\frac{1}{v}\frac{dv}{dz} \equiv \frac{1}{n}\frac{dn}{dz} + \alpha\frac{1}{x}\frac{dx}{dz} = \frac{1}{\varepsilon}\left\{(\delta - \beta\varepsilon) - \frac{h}{px}\right\} . \quad (4.5)$$

Thus aggregate industry output will rise provided the increase in the demand price at the old equilibrium exceeds the price increase necessary to cover the added advertising costs. It may seem surprising that the actual price increase is irrelevant, but this has a simple explanation. The proper price dual to the index based on v is of course $\phi'(v) = q$, say. Now $p = q\, u_x$, and differentiating and ignoring the direct effect of z on u, we have

$$\frac{1}{q}\frac{dq}{dz} = \frac{1}{p}\frac{dp}{dz} + \gamma\,\frac{1}{x}\frac{dx}{dz}\ ,$$

which simplifies to

$$\frac{1}{q}\frac{dq}{dz} = \frac{h}{px}\ . \qquad (4.6)$$

This can be interpreted by saying that the zero-pure-profit condition ensures that the effective price of the aggregate must rise to cover the added advertising costs. Then (4.5) says that the aggregate output will rise if the increase in demand price exceeds the actual increase in the effective price for the aggregate.

A rather remarkable comparative static effect can be read off from (4.5). At the Nash equilibrium level of advertising, we have (3.17), and then $(1/v)\ dv/dz = -\beta < 0$. Thus, at the equilibrium, the marginal effect of advertising must be to reduce the aggregate sector output. This can come about through a reduction of x, or n, or some combination. This shows that, quite apart from welfare considerations, advertising in monopolistic competition is excessive in the simple sense of being self-defeating.

Turning to the welfare aspects, we use (4.1) to write

$$W = \phi(v) + e - n\,p\,x\ .$$

Differentiating, and ignoring the direct effect of z on u as usual,

$$dW/dz = \phi'(v)\ dv/dz - (n\ p\ dx/dz + n\ x\ dp/dz + p\ x\ dn/dz).$$

This simplifies to

$$\frac{dW}{dz} = n\ p\ x\ \left\{ \left(\frac{1}{\alpha} - 1\right) \frac{1}{n} \frac{dn}{dz} - \frac{1}{p} \frac{dp}{dz} \right\}.$$

Now we can substitute from (4.2) and (4.4), and simplify further. Recall that we are assuming $\gamma = - x\ u_{xx}/u_x$ to be independent of z. Integrating twice, the general form of the utility function that satisfies this is

$$u(x, z) = f(z)\ x^{g(z)} + h(z),$$

where f, g and h are arbitrary functions. The additive term $h(z)$ does not affect u_x, but does affect demand through v. At the risk of a slight loss of generality, we set $h(z) = 0$. Then we find that $\alpha = g(z)$ and $\gamma = 1 - g(z)$, so that $\alpha + \gamma = 1$. Then the expression for dW/dz becomes

$$\frac{dW}{dz} = n\ p\ x\ \frac{\gamma}{\alpha\ \varepsilon} \left\{ \frac{\mu}{px} - \beta\ \varepsilon - \frac{h}{p\ x} \frac{\alpha\ \varepsilon}{\gamma} \right\}. \tag{4.7}$$

It is then clear that the two results from the previous sections carry over. Private profitability is a necessary but not sufficient condition for the social desirability of a small amount of advertising, and the equilibrium amount of advertising is socially excessive. The same two forces are at work: an interaction effect and a price increase. Moreover, the increase in the sectoral price index is now a necessary feature in order to maintain zero profits, and there is no possible escape clause of $\theta = 0$.

One new special feature emerges. Since dv/dz is negative at the equilibrium, dx/dz and dn/dz cannot both be positive. This makes it more likely that the local exception to the relation between dW/dz and dW_0/dz will arise. Thus the aggregate output of the monopolistic sector may be too high when judged by the pre-advertising standard, and a little advertising beyond the equilibrium level, by reducing the output, may increase welfare. As was explained earlier, the global considerations then suggest that a greater reduction, rather than a slight increase, in advertising is what is really called for, and the result concerning excessive advertising remains.

5. CONCLUDING COMMENTS

Of the results proved in this paper, probably the most significant is the precise formulation of a common view, and the outcome is even stronger than most economists would have suspected. Even when advertising is given every chance, judging welfare by the post-advertising tastes, and including the monopoly profits contributed by advertising in the measure of welfare, it turns out that the market equilibrium in a very wide range of circumstances involves excessive advertising. The result arises from two forces, namely the price increase that can follow increased advertising, and the effects of one firm's actions on other firms' demand curves.

In the course of proving our results, we have developed a framework for modelling equilibrium with advertising, and have obtained some comparative static results which we believe to be new. We would particularly like to point out the result that under monopolistic competition, advertising is always carried on to the point of being self-defeating from the point of view of the sector as a whole.

We hope that further progress is possible by using this framework both in positive and normative analyses of advertising. Most importantly, we hope we have shown that fairly conventional welfare analysis can be fruitfully applied to the question.

FOOTNOTES

1/ For a resume of the literature, see Schmalensee (1972, pp. 4-9).

2/ A less orthodox approach, using achievement relative to aspirations, is taken by Weckstein (1962).

3/ E.g. Comanor and Wilson (1967), McGuinness and Cowling (1975), McGuinness (1975), Schmalensee (1972).

4/ Dixit and Stiglitz (1975), and Spence (1975).

5/ For a similar formulation without advertising, see Spence (1975).

6/ E.g. Cubbin (1975), Cyert and DeGroot (1973).

7/ See Dixit and Stiglitz (1975) and Spence (1975).

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