LIBERALISM, PARETO PRINCIPLE AND THE CORE

OF A SOCIETY

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Abstract

The paper argues that where individual choice in personal activities is involved, the liberal principle of freedom of choice must include the right for the individual to enter voluntary trade with others regarding the personal activities. Once the problem is so conceived, it can be shown to belong to the general class of n-person games for which Scarf (Journal of Economic Theory, 1971) has proved that there is always a non-empty α-core, on certain convexity assumptions.

Liberalism, Pareto Principle and the Core of a Society

By   S.K. Nath

1. Introduction

It has often been argued recently (see [16], [17] or [18]) that the Pareto principle and the liberal principle of individual freedom of choice in what might be described as personal matters, necessarily conflict with each other in the sense that it is not possible by any rule which obeys both these principles to select a non-empty social choice set from every possible subset of feasible social states - for all possible profiles of individual preferences over the set of all feasible social states. We shall argue (in section 2) that the liberal principle of individual freedom of choice in personal matters must include the right of voluntary exchange regarding these matters; and that, then, the liberal principle necessarily implies the use of the Pareto principle. Once the problem is so conceived, it invites a game theoretic approach. Such an approach is developed in section 4. We point out there that the society we describe meets all the requirements of Scarf's [14] general class of n-person games, so that his theorem about the existence of a non-empty α-core for such games, is immediately applicable. Section 3 briefly reviews Sen's formulation of the Paretian-liberal paradox. In the concluding section, we argue that some liberals' dislike of the Pareto principle may be unjustified.

2. Freedom of Choice and the Pareto Principle

Though it is too much to hope for exact agreement, even among persons of liberal outlook, on a detailed list of actions that the liberal principle of individual freedom of choice should encompass, it is hoped that all such persons would agree that the principle lays down the following general requirement: society should permit each individual an area of autonomy in which he can dispose of the relevant matters - by himself alone, or in voluntary trade with others - according to his own wishes, so long as his exercise of this right is consistent with the others' exercise of their similar rights. (Cf. [12], p.60). Gibbard [7] has also emphasized the importance of the freedom for an individual to engage in voluntary exchange regarding some matter which belongs to his personal sphere. As he says, "There is a strong libertarian tradition of free contract, and on that tradition a person's rights are his to use or bargain away as he sees fit." [7], p.397. We take the view that individual's right to the freedom of choice over some activity is in the nature of a property right - which can be enjoyed on its own, or used as the wherewithal in voluntary exchange to acquire other property rights. (Cf. [5].)

Consider an example. Ralph and Alan live opposite each other across a road, the banks of which rise steeply on both sides. Thus, sitting down in a front room of either house, each can see the front garden of the neighbour opposite but not his own. Let us assume that what each person grows in his front garden is judged to be his personal matter. Ralph loves roses and Alan loves alpines. However, Ralph prefers a state in which Alan's garden has some roses and his own some alpines to one in which his own garden has only roses and Alan's none. Similarly, Alan

prefers a state in which Ralph's garden has some alpines and his own
some roses to one in which his own garden has only alpines and Ralph's
none. There can be no doubt that the liberal principle of freedom of
individual choice in personal matters, must include for each of the two
individuals the right to enter trade with each other, and voluntarily
agree, if it suits each of them, to grow in his garden some or all of
what the neighbour opposite would like, provided that the neighbour does
the same in return, or provides some other quid pro quo.

An individual's actions in what should clearly be regarded as
an area of his personal autonomy are likely often to affect the utility
levels of others, favourably or unfavourably - through external effects.
Compare Robbins' remarks: "There is scarcely anything I do outside the
privacy of my home which has not some overtone of indisciminate benefit
or detriment. The clothes I wear, the shows I frequent, the flowers I
plant in my garden, all directly, or through the mysterious influence of
fashion, influence the enjoyments and satisfactions of others." [13], p.20.
Whenever an economic agent's action gives rise to an external diseconomy,
the question arises: do those who get the external diseconomy have a
right of redress? If they do have such a right, we might say that what
Starret [20] calls the common rule prevails: but if they do not have
such a right, then what he calls the possession rule prevails. Under
the latter rule though those who get the external diseconomy do not have
a right of redress, they may still negotiate with the agent whose
actions produce the external diseconomy, in order to make it worth his
while for him to reduce the level of the activity which gives rise to
the external diseconomy (see [8]); or they may undertake or threaten to
undertake such action in their personal spheres as they hope would induce

the first individual to reduce the level of his personal activity which
gives rise to the external diseconomy.

If an action is deemed to be in an individual's sphere of
personal autonomy, then it follows that the society in question takes
the view that in the event of any external diseconomies arising from
such an action, the possession rule should prevail. We shall describe
the external diseconomies which arise from any individual's personal
activities, and about which the possession rule must necessarily prevail,
as non-essential. For example, though others have preferences about
the length of John's hair, yet - at least if John is an adult - the
length of his hair might be deemed to be John's personal matter in most
societies, because any external diseconomies are considered non-essential.
This suggests that what are declared to be the personal issues of any
individual cannot be chosen arbitrarily. The first requirement that they
must satisfy is that if there are any external diseconomies, then they
are all judged to be of the non-essential type, so that the possession
rule can prevail regarding them; if the possession rule is not considered
appropriate about the external diseconomies arising from some action, then
such an action cannot be deemed to be personal in any meaningfull sense.
Hence what is taken to be the personal issue of an individual does have
to be, according to some acceptable criteria, personal to him. This also
suggests another fundamental requirement: one individual's exercise of
his rights in his personal sphere must be, as Nozick [9] says, "co-possible"
with any other individual's exercise of his rights in his personal sphere.
Or in Seidl's [15] terminology, what are declared to be the personal
activities of a group of individuals must be technologically (one may also
say logically) compatible.

When any two persons i and j trade together and agree to take some joint actions in their personal spheres, some other persons may not like the actions they are contemplating. Though these other individuals do not have a right to directly prevent i and j from their contemplated joint action, they have as much right to form a voluntary trading group, i.e., the coalition complementary to the coalition of i and j, and take such actions in their personal spheres as suit them and which they think might deter i and j from taking their contemplated action. Thus the situation invites a game-theoretic approach.

Turning now th the Pareto principle, we note that it has a strong and a weak form. Its strong form is that if between any two alternatives x and y, all individuals consider x at least as good as y, and some prefer x to y, then x   should be socially accepted as better than y. Its weak form is that if everybody prefers x to y, then x should be socially accepted as better than y. Now, if the social state vectors under consideration are identical in all respects except the particular values of the personal activities of the individuals comprising the society, and if one of these social states is not Pareto optimal, then it can never be a stable social outcome because by definition either all individuals can move to higher levels of utility, or at least some can while nobody's utility level is lowered - by mutual voluntary trade; and the liberal individual right about personal activities, as we have seen, must include the right to mutual voluntary exchange, in order to be a meaningful right. Hence, at least with regard to a set of feasible social states which vary only regarding the particular values of individuals' personal activities, if any social state is to be a stable outcome in any sense, it must be Pareto optimal. The outcomes in the

core of the game model of a society are always Pareto optimal. Our

task in section 4 will be to construct a model of a society where the

core is non-empty, even though it obeys both the strong Pareto principle

and a restriction formulated to protect the individual freedom of choice

in personal activities.

3. Sen's Paradox

Sen proved (see [16], [17] or [18]) that there is no social

decision fuction which satifies the weak Pareto condition (defined in

the previous section), his condition of liberalism (see below), and the

condition of unrestricted domain (that all possible profiles of individual

preference orderings over the set of feasible social states are permitted):

a social decision function being a method of choosing for each possible

profile of individual orderings, an acyclic social preference relation over

the set of feasible social states. Sen's condition of liberalism is: For

every individual, there is at least one pair of distinct alternative

social states, say (x, y), such thet if he prefers x to y, then

society should prefer x to y, and if he prefers y to x, then society

should prefer y to x.

For a number of reasons, we consider Sen's condition of liberalism

unsatisfactory as a representation of the liberal principle of individual

freedom of choice in personal matters. First, as Farrell points out,

Sen "makes no distinction between a man deciding whether to sleep in a

prone or a supine position, and a religious leader dictating whether he

does so" [6], p.5. Secondly as Seidl [15] notes, Sen does not require

that what are regarded as personal activities of different individuals

are at least technologically (or logically) compatible. (In Sen's by

now well-known example (see [16]) of two individuals and one copy of

the book, Lady Chatterly's Lover, which he assumes only one person can

read, if reading (all, any part or none of) the book is regarded as a

personal activity of each of the two persons, then these two personal

activities of the individuals are not always technologically compatible, so

that there is no need to investigate any further the possibility of the

individual freedom of choice in personal matters in that example.)

Thirdly, Sen does not include the right to voluntary exchange regarding

personal matters as part of the liberal individual right. (One consequence

of this is that though the condition of the independence of irrelevant

alternatives is not explicitly imposed by Sen on his social decision

function, it is nevertheless implicitly used in every step of the proof

of Sen's proposition - as Blau [3] points out.)

Fourthly, and this is the most important criticism of Sen's

condition of liberalism, though Sen quite rightly takes individual

preferences to be defined over social states, he grants each individual

too much authority when he makes him dicisive in the social choice over

a pair of social states: though individual preferences must be taken to

be defined over social states (especially when externalities are admitted),

all that the liberal principle of individual freedom of choice in personal

matters requires is that no components of a social state vector which

concern the personal activities of an individual should ever be changed

without his voluntary consent. (Cf [3] and [4]). We shall not discuss

this criticism further here, but its full import will become obvious in

in the next section, in connection with our Definitions 1,2 and 3.

4. The α-Core of a Liberal Society.

A social state is a vector of decisions or activities,
some of which are recognized to be personal activities of the individuals -
in the sense explained in section 2. All other activities are to be
described as non-personal. Our analysis starts at the stage where
particular values for all non-personal activities have been selected some
how or other. After the manner suggested by Farrell [6], we may
envisage that the set of all feasible social states has been partitioned
into non-personal-activities equivalence classes, and that a social
consensus has already emerged on one of these equivalence classes. We
are then left with the further social choice over the social states in
the one non-personal-issues equivalence class. Alternatively, we may
assume that the model to be developed here applies to each one of such
possible equivalence classes, one at a time. Either way, in our model
all non-personal activities are assumed to remain constant; the social
states under discussion vary only in the values of personal activity
vectors.

Let us then consider a set of social states in which they all
have the same values for the non-personal activities; hence we can ignore
those components of each of its social state vectors which specify the
values of such activities. We shall denote such a set of social states
by ξ. Let the society have a set I of n individuals. For us
then a social state vector, $\underline{x} = (x^1, \dots , x^n)$ is an n-tuple of

particular values of vectors x^i of each individual $i \in I$. $X^i \ni x^i$
represents a non-empty activity set for person i, where each X^i is
a subset of a finite-dimensional Euclidian space. Our typical non-
personal-activities equivalence class is then $\xi = X^1 x \ldots x X^n$; and
a typical element of ξ is a social state vector $\underline{x}$.

An activity which is deemed to be personal to an individual,
may be a production, consumption or exchange activity. Of course only
such personal activities are feasible for an individual as are consistent
with his initial endowment of goods and rights. An individual's
property right to his initial endowment is assumed to be exclusive. Any
personal activity of one individual is assumed to be logically compatible
with any personal activity of another person. However, a personal
activity of one person may affect the utility of another person through
non-essential externalities of the kind discussed in section 2. What
was described there as the possession rule prevails regarding these
externalities. Each individual has the liberty to do what he likes
regarding production, consumption or voluntary exchange with respect to
his personal activiteis.

Apart from the usual reasons for voluntary exchange, in this
model there would be the additional reason for individuals to enter into
trade with one another, in response to one another's externalities. If
the number of individuals is greater than two, the possibility arises
for coalitions to form. We shall make the usual (implicit) assumptions
of the theory of co-operative games, namely that there is costless
communication among individuals about possibilities of trade, so that any
mutually gainful coalitions will be formed; and that each individual

assumes the others to be truthful and trustworthy, so that all agreements are expected to be binding. (Notice that these assumptions imply that a prisoner's dilemma like problem cannot arise in our model.) In addition, we shall require the following two assumptions.

Assumption 1: Each X^i is a closed bounded convex set in a finite-dimensional Euclidian space.

Assumption 2: Each individual has a continuous quasi-concave utility function $U^i(\underline{x})$, defined on the product space ξ; or equivalently, each individual has a continuous, convex weak preference pre-ordering over the set ξ.

The essential part of the two assumptions is that about the convexity of the respective sets, Starret [19] has argued that external diseconomies tend to introduce non-convexities in the production and preference sets. We may, however, justify our assumptions by pointing out that the only external diseconomies admitted in our model are of what we have described as the non-essential type, for which what is called the possession rule is assumed to apply. Therefore, external diseconomies admitted in the model are likely to be mild enough not to intruduce any non-convexities.

An activity vector of an individual is a plan of action which can be regarded as a strategy. Recall that any personal activity of an individual is assumed to be compatible with any personal activity of any other individual; hence the same is true of individual strategies. Joint strategies of the members of any coalition cannot be arbitrary; they are constrained by the initial endowments of the members.

A social state is in the α-core if, first, its associated vector

of the utility levels of all individuals is achievable by the members of
the grand coalition, K = I, and, secondly, no coalition, K, can achieve a
higher level of utility for each of its members independently of the
actions of the members of the complementary coalition, ⌐K. (For the
distinction between α-core and β-core see [14]. For a game model which
admits interdependence among the individual utility functions, the concept
of the α-core seems to be more relevant than that of the β-core.)

Since the second condition applies to all possible coalitions -
i.e., all possible subsets of I - a social state in the α-core is
obviously Pareto optimal according to the strong Pareto condition. Thus
the α-core is a subset of the Pareto optimal social states, which consists
of such social outcomes as are stable in the sense defined by the two fore-
going conditions, and as also satisfy the constraint of the initial
endowments that is placed on the joint strategies of the members of any
coalition. In order to see the connection between this usual restriction
on joint strategies, and the restriction required by the individual freedom
of choice in personal activities, we need to formulate the following definitions.

Definition 1: For any person i, T_i, is a binary relation in
ξ, such that $\underline{x} \; T_i \; \underline{x}' \leftrightarrow x^i = x'^i$, where vector x^i is a component of $\underline{x}$,
and vector x'^i is a component of $\underline{x}'$.

Clearly T_i is reflexive, transitive and symmetric. Therefore
T_i is an equivalence relation, which induces a partition of ξ into
non-empty disjoint and exhaustive equivalence classes of ξ, such that
every social state belongs to some equivalence class. We shall call such a class
a personal-activities-equivalence class of person i, and represent it by $\left[\underline{x}\right]^i$.

Definition 2: A personal-activities-equivalence class of a person i,
$\left[\underline{x}\right]^i$, is that subset of ξ, which consists of those social states in which
the values of i's personal activity vectors are the same.

Remark 1: Apart from the activity vectors of person i, there is no restriction on the other components of social states in $\left[\underline{x}\right]^i$; they can vary.

Remark 2: The difference between any two social states in $\left[\underline{x}\right]^i$ from the standpoint of the individual i can only be with respect to the activity vectors of other persons.

Remark 3: There is a partition of the set ξ into personal-activity-equivalence classes, for each of the n individuals.

Remark 4: Given any arbitrary intitial social state $\underline{x}$, a change in the joint-strategy choice of the members of any coalition K, will necessarily shift the society into another social state, say $\underline{x}'$. So long as the new social state, $\underline{x}'$, is in the same personal-activities-equivalence class of every person i who is not a member of K, as the initial social state $\underline{x}$, no individual's liberal right of freedom of choice in his personal activities would have been affected. Of course the utility levels of persons who are not members of K may well be affected, because of externalities, and they may want to make a response to the changes (or the threat of changes) in the personal activities of the members of the coalition K. But that is a different matter.

The last remark suggests the restriction to impose on the joint strategies of any coalition for the individual liberal right to personal activities to be honoured. Clearly, we should require that for the members of any coalition, the joint strategies should be restricted to those which cannot shift the society into a social state that is not in the same personal-activities-equivalence class of every individual who is not a member of K, as is the initial social state.

Definition 3: The liberal principle of individual freedom
of choice requires that the set of joint strategies of any coalition $K \subset I$
be restricted to the following.

$$\xi^K = \{\underline{x} \in \xi : \underline{x} \in \bigcap_{i}^{\sim K} \left[\underline{x}^{\prime}\right]^i \quad \text{for all } i \in \sim K\}$$

where $\underline{x}^{\prime}$ is the initial social state

It now emerges that this restriction which the liberal principle
requires is essentially the same restriction as that which is required
in any theory of co-operative games – namely that members of any coalition
can form their joint strategies only within the constraint of their own
initial endowments; in other words, that members of a coalition are not
to take any of the initial endowments of the complementary coalition
(with whom, by difinition, they are not trading). (see Bergstrom [1]).
Our restriction in Definition 3 requires essentially the same thing:
namely that no joint strategy of the members of any coalition is to change
the endowment of personal activities in the initial social state of
the members of the complementary coalition (with whom, by definition, they
are not trading).

A society which satisfies assumptions 1 and 2, and the restriction
of Definition 3, meets all the requirements of Scarf's [14] general class
of n-person games. Therefore, we can apply his theorem directly to our
society.

Theorem (Scarf): For a society such as that described above,
which satisfies assumptions 1 and 2, there exists a social state which

belongs to the α-core.

Any social state in the α-core is clearly Pareto optimal; its selection also obeys the condition regarding the liberal rights of individuals; and the existence of a non-empty core is proved without imposing any restriction on the possible configurations of individual preferences over the set of all feasible social states. However, since voluntary exchange is permitted in the model, the condition of the independence of irrelevant alternatives is violated, because vuluntary exchange among individuals is influenced by the interpersonal differences in the intensity of preferences (without actually providing a means for measuring these differences). However, nobody has ever argued, nor do we think that a remotely convincing argument can be made, to the effect that voluntary exchange among individuals regarding activities which are judged to be their own concern, should be disallowed because such exchange is influenced by the interpersonal differences in the intensity of preferences.

5. Conclusions

We have argued that the liberal principle of individual freedom of choice over personal issues must include the right for the individual to enter voluntary trade with others regarding them, and that, therefore, we cannot jettison the Pareto principle even in its strong form; because whenever a social state is not Pareto optimal, some individuals can, by definition, through voluntary agreement take certain joint actions which

raise their levels of utility (while others' levels of utility are not lowered) - thus moving the society to a Pareto optimal social state.

We also argued that what are to be regarded as personal activities cannot be chosen arbitrarily. They need to be such that any external diseconomies arising from them can be judged to be what we described as non-essential. Moreover, any personal activity of any individual needs to be logically compatible with any personal activity of any other individual.

Viewed thus, the problem of social choice from among a set of social states which differ from one another only regarding the values of personal activities of individuals, can be treated as that of determining the solution for an n-person co-operative game. We showed that the α-core of such a game is non-empty on certain convexity assumptions.

The Pareto principle needs to be exonerated on two other counts. As Osborne [10] judiciously points out, the Pareto principle - in its weak or strong form - is often misunderstood to be a biconditional normative proposition incorporating "if and only if"; whereas it is only a conditional normative proposition incorporating "if". This means that adherence to the Pareto principle does not preclude ranking a social state x over a social state x', even though x is not superior to x' according to either the weak or the strong version of the Pareto principle. It is true that the principle is sometimes used as a biconditional normative proposition. But one can quite consistently be agaist its use as a biconditional, and for its use as a conditional normative proposition.

Where alternative distributions of initial endowments are to be compared, one needs to know how the individual preference relations are to be interpreted before the Pareto principle can be used. Say between any two alternative distributions of initial endowments x and y, the only difference is that in x the rich are richer than in y, whereas the poor have the same absolute share of initial endowments in both x and y. Now it is possible to so (perhaps implicitly) define the individual binary preference relation that one claims that the rich "prefer" x to y while the poor are "indifferent" between x and y, so that x is socially superior to y according the Pareto principle in its strong form. Though the Pareto principle has been used here only as a conditional normative proposition, yet such a use of it may be obnoxious to a liberal, for - as Peacok and Rowley [11] point out - liberal values and a concern for equity are quite compatible. However, if the individual preference relation is, in this context, more broadly interpreted to reflect all individual values (including those of distributive justice), then one cannot be sure that the rich would necessarily "prefer" x to y, or that the poor would necessarily be "indifferent" between x and y; therefore, one cannot be sure that x would necessarily be superior to y according to the Pareto principle.

Hence if the Pareto principle is understood to be a normative conditional proposition, and if individual preference relation is appropriately interpreted according to the problem being analyzed, a liberal need have no qualms about assenting to it.

References

1) Bergstrom, T.C. "The Core when Strategies are Restricted by Law". _Review of Economic Studies_, 42, (April 1975), 249-257.

2) Bernolz, P. "Is a Paretian Liberal Really Impossible?", _Public Choice_, 20 (Winter, 1974), 99-108.

3) Blau, Julian H. "Liberal Values and Independence", _Review of Economic Studies_, 42, (July, 1975), 395-401.

4) Breyer, F. "Sen's Paradox with Decisiveness over Issues in Case of Liberal Preferences", mimeo, Alfred-Weber-Institut, Universitat Heidelberg; paper presented at the meeting of the Public Choice Society, European Section, Basle, 1976.

5) Buchanan, J.M. "An Ambiguity in Sen's Alleged Proof of the Impossiblity of a Pareto Libertarian", mimeo, Center for Study of Public Choice, Virginia Polytechnique Institute and State University, 1976.

6) Farrell, M.J. "Liberalism in the Theory of Social Choice", _Review of Economic Studies_, 43, (February 1976), 3-10.

7) Gibbard, A. "A Paretian-Consistent Libertarian Claim", _Journal of Economic Theory_, 7, (1974), 388-410.

8) Mishan, E.J. "Pareto Optimality and the Law", _Oxford Economic Papers_, (November 1967), reprinted in _Welfare Economics: Ten Introductory Essays_, (New York: Random House), 1969.

9) Nozick, R. _Anarchy, State and Utopia_, (Oxford: Blackwell), 1974.

10) Osborne, D.K. "On Liberalism and the Pareto Principle", _Journal of Political Economy_, 83, No. 6, (November/December 1975), 1283-1287.

11) Peacock, A.T. and Rowley, C.K. "Pareto Optimality and the Political Economy of Liberalism", _Journal of Political Economy_, 80, (May/June 1972), 476-490.

12) Rawls, J. _A Theory of Justice_, (London : Oxford University Press) 1972.

13) Robbins, L. _The Economic Problem in Peace and War_, (London : Macmillan) 1947.

14) Scarf, H. "On the Existence of a Co-operative Solution for a General Class of N-person Games", *Journal of Economic Theory*, 3, (1971), 169-181.

15) Seidl, C. "On Liberal Values", *Zeitschrift fur Nationalokonomie*, 35, (1975), 257-292.

16) Sen, A.K. "The Impossibility of a Paretian Liberal", *Journal of Political Economy*, 78, (January/February 1970), 152-157.

17) Sen, A.K. *Collective Choice and Social Welfare*, (San Francisco : Holden Day : and Edinburgh: Oliver Boyd), 1970.

18) Sen, A.K. "Liberty, Unanimity and Rights", *Economica*, 43, (August 1976), 217-245.

19) Starret, D. "Fundamental Nonconvexities in the Theory of Externalities", *Journal of Economic Theory*, 4, (1972), 180-199.

20) Starret, D. "A Note on Externalities and the Core", *Econometrica*, 41, (1973), 179-183.