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I.

The debate concerning the extent of monopoly welfare loss long dominated by the analysis of Harberger [7] has been revived with the contributions of Tullock [10] and Posner [9] on the one hand, and Cowling [2], Cowling and Mueller [3] and Kay [8] on the other. The thesis proposed by Tullock and Posner is that "profit" is essentially a welfare loss because firms compete for monopoly profits and thus waste resources. Also in this industry of chasing monopoly profits only normal profits are earned on the average. An example is when n firms undertake the same research in order to develop a new product. The first firm to succeed patents the product and has a monopoly. In a crude form, the argument would be that the number of competing firms n would solve the equation: n times the expected research cost equals the expected monopoly profits of owning the patent. Obviously this is an extreme argument. There may be barriers to entry into such competition, and also risk aversion may reduce the equilibrium number of competitors. Also the new commodity may be made available earlier thus increasing welfare the larger the value of n . Nevertheless the Tullock-Posner thesis is useful as a polar case for the discussion of the social costs of monopoly. As Posner ([9], p.823) asserts "D (lost consumer surplus) is only a small fraction of $D + L$ (lost consumer surplus plus monopoly profit) ... The social costs measured by L are unaffected by the existence of second-best problems."

The second development in the literature on the social costs of monopoly emphasises the fact that price-cost margins are endogenous if firms are assumed to maximise profits. In particular, in the case of a single producer and linear demand functions, lost consumers' surplus is one half of monopoly profits, (see Kay [8], p.9). Again this is an extreme argument as the firm may be expected to compromise its price levels in order to inhibit future competition.²

The objectives of the present paper are to combine a general equilibrium approach to monopoly welfare loss with the behavioural assumptions of profit maximisation and with particular formulations of industrial structure. In the analysis below we will develop a simple model using a household utility function. We justify this approach in the usual way: that we are considering problems of efficiency rather than equity (see Bergson [1], p.853, Green [6], p.66-7). Our simple model is then utilised in three areas of analysis.

First in Section II, we will present a formalisation of the Tullock-Posner viewpoint and show that in this polar case:

- (1) An increase in any one price (p_i) of a commodity produced in a monopolised sector will always reduce utility. This is shown by an immediate application of Roy's Identity (see Darrough and Southey [4]).
- (2) The above will be true irrespective of the ratio of D to $D + L$.
- (3) The Tullock-Posner case is formally identical to the case where all firms are owned by foreigners to whom profits are remitted.

Secondly, the model is used, with a particular household utility function in Section III to explore the issue of conglomerate merger and the following result is derived:

- (4) Conglomerate merger may increase (decrease) utility

if the merger is between industries of complementary (substitutable) commodities.

Finally, we consider the significance of our results for the measurement of monopoly welfare loss in aggregate and in response to small adjustments in monopoly positions. In terms of our particular household utility function we reproduce the results of Cowling and Mueller [3] and consider the difficulties of using such measures empirically. We conclude by considering where statements concerning qualitative improvements in welfare due to changing market structure are likely to be justified. We argue that the Tullock-Posner thesis does not remove quantitative estimation problems but that it does imply that more qualitative statements can be made concerning small changes in market structure.

II

We will consider an economy where there is just one resource, the endowment of which is given by the quantity e . The resource is entirely endowed to the household, and can either be consumed directly or exchanged for commodities which are produced by firms. The market for the resource is assumed to be competitive, whereas no such constraint is placed upon the market structure of produced commodities. The firms produce n commodities and the output is given by the n -vector q . Firms use the resource to produce commodities and the total resource required per unit output is described by the n -vector r . This is not initially assumed constant either with respect to output levels or industrial structure; in the next section, we will need to be more restrictive. Firms can also use the resource for competing for monopoly profits or for maintaining monopoly positions or for remitting wealth to foreign shareholders. The total amount of resource used in this way

is z , and the total amount consumed directly by the household is y . The resource is used as the numeraire and so the resource constraint in the economy can be written as (1).

$$e = y + \underline{r}' \underline{q} + z \quad (1)$$

The obvious example of such a resource is manhours. Then the constraint (1) reads that the total stock of manhours (e) is either consumed by the household as leisure (y) or used by firms either to produce commodities in amounts \underline{q} which are then consumed by the household ($\underline{r}' \underline{q}$) or to produce commodities to send abroad as profits to foreign owners or to produce services which compete for monopoly profits (z). The equality in (1) arises from full-employment resulting from the competitive market in the resource.

The household utility function is $U(y, \underline{q})$ and is quasi-concave, and the household's problem is to maximise utility with respect to the choice of y and \underline{q} subject to its budget constraint. The total income of the household is composed of the endowment e plus any profits of firms remitted to the household. Let the latter quantity be R - the aggregate domestically distributed profits. All profits of firms are distributed either to the household (R) or to foreigners if the firms are foreign-owned. We will state the identity:

$$R \equiv \Pi - z \quad (2)$$

$$\text{where } \Pi = (\underline{p} - \underline{c})' \underline{q} \quad (3)$$

The n -vectors \underline{p} and \underline{c} are the prices and unit costs of production of commodities in terms of the resource. Π is thus operating profits. Note that $\underline{c} \neq \underline{r}$ in general as one commodity may use another as

an input and this may not be priced at resource cost. The household's budget constraint is thus (4).

$$e + R = y + \underline{p}' \underline{q} \quad (4)$$

We will assume that R , e and \underline{p} are considered by the household to be invariant to the household's choice of (y, \underline{q}) . If (y^*, \underline{q}^*) maximises utility subject to (4) then (y^*, \underline{q}^*) are functions of $e + R$ and \underline{p} . Substituting (y^*, \underline{q}^*) into the utility function yields the indirect utility function $V(e + R, \underline{p})$. Note that V is not homogeneous of degree 0 in $e + R$ and \underline{p} and is quasi-convex in \underline{p} . Although there are $n + 1$ arguments in the utility function, only n prices are relevant in the indirect utility function due to the form of (4).

Now consider the effect on utility of a change in one price, p_i , holding other prices constant, and denote this as $\frac{dV}{dp_i}$. Then

$$\begin{aligned} \frac{dV}{dp_i} &= \frac{\partial V}{\partial R} \frac{dR}{dp_i} + \frac{\partial V}{\partial p_i} \\ &= \frac{\partial V}{\partial R} \left(\frac{dR}{dp_i} - q_i \right) \end{aligned} \quad (5)$$

$$\text{as } \frac{\partial V}{\partial p_i} = - \frac{\partial V}{\partial R} q_i \quad (6)$$

by Roy's Identity.³ If $\frac{dR}{dp_i} = 0$ then $\frac{dV}{dp_i} = - \frac{\partial V}{\partial R} q_i < 0$ and an increase in price will reduce utility. This will also be the case a fortiori if $\frac{dR}{dp_i} < 0$. It will be useful at this juncture to consider the situations where $\frac{dR}{dp_i} \leq 0$. These are:

- (i) Total collusion by all firms in favour of a

policy of maximising R . In this case $\frac{dR}{dp_i} = 0$ is a first-order condition for optimality.

- (ii) Any increase in Π is counterbalanced by either a similar or greater change in the amount of the resource used for activities which do not yield utility to the household (the Tullock-Posner thesis) or a similar or greater change in the amount of profits remitted to foreigners. Note that it is only the change in R being non-positive that is sufficient for $\frac{dV}{dp_i} < 0$.
- (iii) The increase in p_i may cause some firms' operating profits to increase; others to decrease. Thus although the increase in p_i may be in order to increase some firms' profits, Π may not increase and so clearly R may not increase.

III

In order to investigate a number of issues in the welfare problems of monopoly, we will adopt a particular utility function which will simplify the analysis. Our choice contrasts with that of Bergson [1] who chose a utility function which exhibited constant elasticity of substitution between all commodities. It is more special than the utility function used in Dixit and Norman [5]. We will define our household utility function as:

$$U = y + \underline{a}' \underline{Aq} + \underline{b}' \underline{q} \quad (7)$$

This form of function assumes that (a) there are no income effects on the demand for the output from the monopolised sector and (b) demands for the outputs of the monopolised sector are linear in all prices. \underline{A} is an $n \times n$ negative definite matrix and \underline{b} is an n -vector with sufficiently high elements that there is positive demand for all products at interesting price vectors. Household demands are given by

$$\underline{q}(\underline{p}) = \frac{1}{2} \underline{A}^{-1} (\underline{p} - \underline{b}) \quad (8)$$

$$\underline{y} = \underline{e} + \underline{R} - \frac{1}{2} \underline{P}' \underline{A}^{-1} (\underline{p} - \underline{b}) \quad (9)$$

and the indirect utility function is

$$V(\underline{e} + \underline{R}, \underline{p}) = \underline{e} + \underline{R} - \frac{1}{4} (\underline{p} - \underline{b})' \underline{A}^{-1} (\underline{p} - \underline{b}) \quad (10)$$

If \underline{R} is independent of \underline{p} then (10) is linear in \underline{e} and quadratic in \underline{p} . It is also separable in \underline{e} and \underline{p} . If \underline{R} is linearly related to $\underline{\Pi}$ and if $\underline{\Pi}$ is also quadratic in \underline{p} then (10) has the same properties.

We will now build more structure into the production side.

Suppose that \underline{B} is a constant input-output matrix. Then if the monopolised sector exhibits prices \underline{p} , the unit costs of production are given by the vector \underline{c} as

$$\underline{c} = \underline{B}' \underline{p} + \underline{g} \quad (11)$$

where \underline{g} is an n -vector of the (constant) amounts of the resource required per unit output as direct inputs into production. If $\underline{p} = \underline{r}$ then by definition of \underline{r} :

$$\underline{r} = \underline{B}' \underline{r} + \underline{g} \quad (12)$$

Using (12) to eliminate \underline{g} we can express (11) as

$$\underline{c} = \underline{B}' \underline{p} + (\underline{I} - \underline{B}') \underline{r} \quad (13)$$

Total demand \underline{x} is the sum of household demand and firm demand, and so again using the input-output matrix,

$$\underline{x} = (\underline{I} - \underline{B})^{-1} \underline{q} \quad (14)$$

and aggregate operating profit Π is simply

$$\Pi = (\underline{p} - \underline{c})' \underline{x} = (\underline{p} - \underline{r})' (\underline{I} - \underline{B}) (\underline{I} - \underline{B})^{-1} \underline{q} = (\underline{p} - \underline{r})' \underline{q} \quad (15)$$

as inter-firm prices wash out on aggregation. Substituting (8) into (15) yields

$$\Pi = \frac{1}{2} (\underline{p} - \underline{r})' \underline{A}^{-1} (\underline{p} - \underline{b}) \quad (16)$$

and so this particular production structure, involving as it does constant per unit costs and no pricing which discriminates between households and firms, implies that if R is linearly related to Π , (10) has the same properties (of being separable in \underline{e} and \underline{p} , linear in \underline{e} and quadratic in \underline{p}) as when R is fixed.

From (10) we have if R is independent of \underline{p} :

$$V(e + R, \underline{p}^2) - V(e + R, \underline{p}^1) = \frac{1}{2} (\underline{p}^1 - \underline{p}^2)' (\underline{q}(\underline{p}^1) + \underline{q}(\underline{p}^2)) \quad (17)$$

This states that the utility difference between two price vectors is the difference in cost of a simple average of the commodity bundles purchased under the two price regimes. For completeness the traditional case of Harberger [7], etc. where $R \equiv \Pi$ yields:

$$V(e + \Pi(\underline{p}^2), \underline{p}^2) - V(e + \Pi(\underline{p}^1), \underline{p}^1) = \left(\frac{\underline{p}^2 + \underline{p}^1}{2} - \underline{r} \right) (\underline{q}(\underline{p}^2) - \underline{q}(\underline{p}^1)) \quad (18)$$

which is ambiguous in sign.

Now consider two particular price vectors \underline{p}^m and \underline{p}^n which relate to market structure as described below:

Market Structure m

Aggregate operating profit Π is maximised by the choice of \underline{p}^m .

$$\text{Thus } \frac{\partial \Pi}{\partial \underline{p}} = \frac{1}{2} (\underline{A}^{-1} (\underline{p}^m - \underline{b}) + \underline{A}^{-1} (\underline{p}^m - \underline{r})) = \underline{0}$$

$$\text{and } \underline{p}^m = \frac{\underline{b} + \underline{r}}{2} \quad (19)$$

If this profit is dissipated, so that $R(\underline{p}^m) = 0$, then the monopoly welfare loss is from (17)

$$V(e, \underline{r}) - V(e, \underline{p}^m) = \frac{1}{2} \frac{\underline{b} - \underline{r}}{2} (\underline{q}(\underline{p}^m) + \underline{q}(\underline{r})) \quad (20)$$

$$= \Pi(\underline{p}^m) + \frac{1}{2} (\underline{p}^m - \underline{r}) (\underline{q}(\underline{r}) - \underline{q}(\underline{p}^m)) \quad (21)$$

which is simply monopoly profits plus lost consumers' surplus. If profits are remitted in full to households then:

$$V(\underline{e}, \underline{r}) - V(\underline{e} + \Pi(\underline{p}^m), \underline{p}^m) = \frac{1}{2} (\underline{p}^m - \underline{r}) (\underline{q}(\underline{r}) - \underline{q}(\underline{p}^m)) \quad (22)$$

Market Structure n

The monopolised sector is divided into n sub-sectors each composed of one firm producing one commodity in quantity q_i and setting price p_i . Each firm maximises its operating profit Π_i by choice of p_i^n , assuming that other prices do not respond to its decision. From (13) and (14) we obtain

$$\Pi_i = \frac{1}{2} (\underline{p}^n - \underline{r})' (\underline{I} - \underline{B})_{-i} (\underline{I} - \underline{B})_{i-}^{-1} \underline{A}^{-1} (\underline{p}^n - \underline{b}) \quad (23)$$

where $(\cdot)_{-i}$ and $(\cdot)_{i-}^{-1}$ are the i^{th} column of (\cdot) and the i^{th} row of $(\cdot)^{-1}$ respectively. Maximising (23) with respect to p_i^n for all i simultaneously yields a Cournot-type equilibrium defined by (24).

$$\underline{p}^n = \left[(\underline{I} - \hat{\underline{B}}) (\underline{I} - \underline{B})^{-1} \underline{A}^{-1} + \underline{K}(\underline{I} - \underline{B})' \right]^{-1} (\underline{I} - \hat{\underline{B}}) (\underline{I} - \underline{B})^{-1} \underline{A}^{-1} \underline{b} + \underline{K}(\underline{I} - \underline{B})' \underline{r} \quad (24)$$

where \underline{K} is diagonal with typical element $(\underline{A}_{i-}^{-1} (\underline{I} - \underline{B}')_{-i})^{-1}$, and $\hat{\underline{B}}$ indicates \underline{B} with off-diagonal elements zero.

If $\underline{B} \equiv 0$ and \underline{A} is diagonal then $\underline{p}^m = \underline{p}^n$. Otherwise the price vectors will be distinct. For simplicity consider the case where $\underline{B} \equiv 0$ but \underline{A} is non-diagonal, i.e. there are demand but no supply interdependencies.

Then (24) can be rewritten as

$$\underline{A}^{-1} (\underline{p}^n - \underline{b}) + (\underline{\hat{A}}^{-1}) (\underline{p}^n - \underline{r}) = 0$$

i.e.

$$2(\underline{\hat{A}}^{-1}) (\underline{p}^n - \frac{\underline{b} + \underline{r}}{2}) = \left[(\underline{\hat{A}}^{-1}) - \underline{A}^{-1} \right] (\underline{p}^n - \underline{b}) \quad (25)$$

As \underline{A} is negative definite, so is \underline{A}^{-1} and so therefore is $(\underline{\hat{A}}^{-1})$. If the n firms produce substitutes (complements) then \underline{A}^{-1} will have all positive (negative) off-diagonal elements. Thus if all the n firms produce substitutes (complements) for each other's commodities, the right-hand side of (25) is non-positive (non-negative). As $(\underline{\hat{A}}^{-1}) < 0$, this implies that $\underline{p}^n < \underline{p}^m$ in the substitutes case and $\underline{p}^n > \underline{p}^m$ in the complements case, and from (17) $V(e + R, \underline{p}^n) - V(e + R, \underline{p}^m) \geq 0$ as $\underline{p}^n \leq \underline{p}^m$.

Furthermore it is straightforward to show in this simplified case ($\underline{B} \equiv 0$) that an increase in just one price - say the n^{th} - will result in all other prices being increased if the commodities are all substitutes and we have market structure n . Let $\underline{\bar{A}}, \underline{\bar{A}}^{-1}$ be the matrices $\underline{A}, \underline{A}^{-1}$ without the n^{th} row and column. Then totally differentiating (25) and rearranging yields

$$d \underline{p}^{n*} = - dp_n^n (\underline{\bar{A}}^{-1} + \underline{\hat{A}}^{-1})^{-1} \underline{A}^{-1*}$$

where \underline{p}^{n*} is \underline{p}^n without the n^{th} element and \underline{A}^{-1*} is the n^{th} row of \underline{A}^{-1} without the n^{th} element. $\underline{\bar{A}}^{-1} + \underline{\hat{A}}^{-1}$ is negative definite with non-negative off-diagonal elements. Thus it has a dominant negative diagonal and so its inverse is non-positive, and as \underline{A}^{-1*} is non-negative $\frac{dp_i^n}{dp_n^n} \geq 0$ all i .

Thus in this case, any increase in one price will provoke general price increases all unambiguously reducing utility, given that R does not increase. If $n - 1$ commodities are substitutes of each other but the n^{th} is complementary to all $n - 1$ (but a substitute to leisure) then $A_{-n}^{-1} * \leq 0$ and $\frac{dp_i}{dp_n} \leq 0$ all i . One cannot say, however that if all commodities are complements, prices in \underline{p}^{n*} will either all increase or all decrease.

IV

The special case that we have investigated in the last section based on (7), recreates in a single market ($n = 1$) the results of Cowling and Mueller [3]. With a linear demand curve and profit-maximising behaviour lost consumers surplus is one half of operating profits (Π) in that market. In a multi-market situation this result still holds if all n markets in the monopolised sector are controlled by a single multi-product monopolist. It is also possible to extend the model to consider k identical multi-product firms in a Cournot oligopoly situation. Let $\Pi_\ell = (\underline{p} - \underline{r})' \underline{q}_\ell$ be the profit of the ℓ^{th} firm ($\ell = 1, 2 \dots k$) and \underline{q}_ℓ be the ℓ^{th} firm's net output vector and \underline{p}^k the price vector resulting from this market structure, then $\frac{\partial \Pi_\ell}{\partial \underline{q}_\ell} = \underline{p} - \underline{r} + 2 \underline{A} \underline{q}_\ell$ and the ℓ^{th} firm's optimal output vector is

$$\underline{q}_\ell = -\frac{1}{2} \underline{A}^{-1} (\underline{p} - \underline{r})$$

adding over the k identical firms yields

$$\begin{aligned} \underline{q} &= -\frac{k}{2} \underline{A}^{-1} (\underline{p} - \underline{r}) \\ \text{and } \underline{p}^k &= \frac{k}{1+k} \underline{r} + \frac{1}{1+k} \underline{b} \end{aligned} \quad (26)$$

(26) is a familiar result given the assumptions of the analysis, stating that

if $k = 1$, $\underline{p}^k = \underline{p}^m$ and as $k \rightarrow \infty$, $\underline{p}^k \rightarrow \underline{r}$. From (17), (18) and (26) we have:

$$V(\underline{e}, \underline{r}) - V(\underline{e}, \underline{p}^k) = -\frac{1}{4} \frac{1 + 2k}{(1+k)^2} (\underline{r} - \underline{b}) \underline{A}^{-1} (\underline{r} - \underline{b}) = \frac{1 + 2k}{2k} \Pi \quad (27)$$

and if $R \equiv \Pi$:

$$V(\underline{e}, \underline{r}) - V(\underline{e} + \Pi, \underline{p}^k) = -\frac{1}{4} \frac{1}{(1+k)^2} (\underline{r} - \underline{b}) \underline{A}^{-1} (\underline{r} - \underline{b}) = \frac{1}{2k} \quad (28)$$

as, from (15), operating profits is $-\frac{1}{2} \frac{k}{(1+k)^2} (\underline{r} - \underline{b}) \underline{A}^{-1} (\underline{r} - \underline{b})$.

If an economy can be assumed to have a household utility function such as (7) and if the monopoly structure is of the kind just described then the social costs of monopoly under either or both criteria can be measured using (27) or (28). Indeed if the economy can be separated into a number of groups of commodities for which the demands are only dependent on prices within their own group this will imply \underline{A}^{-1} is block diagonal and will also allow a similar analysis. However, even granted the necessary assumptions, a considerable difficulty lies in the fact that it is R and not Π that is observed, unless $R \equiv \Pi$. If R is merely a normal return on capital (zero in our one-resource model) and all operating profit is competed away before it reaches the household then R itself has no relationship either to lost consumers' surplus or to the social cost of competition for monopoly profits. Some of the latter costs may be observable directly, see Cowling and Mueller [3], p.11-13, but others are unlikely to be so either because the data may not be available or because the expenditure may have a dual purpose -

for instance, advertising may be productive both in increasing barriers to entry and in improving information.

Estimates of (27) or (28) thus require measurement of both k and Π in the economy (or in each separate group of sub-sectors). If estimates of Π are unreliable, valuable insight can still be obtained by considering the ratio of the social costs of monopoly to Π . The results, $\frac{1 + 2k}{2k}$ and $\frac{1}{2k}$ respectively, are both monotonically decreasing in k . Rather more insight can be obtained by expressing the social costs of monopoly relative to maximum value added to welfare which is independent of k . This yields from (10), (27) and (28):

$$\frac{V(e, \underline{r}) - V(e, \underline{p}^k)}{V(e, \underline{r}) - e} = \frac{1 + 2k}{(1 + k)^2} \quad (29)$$

$$\frac{V(e, \underline{r}) - V(e + \Pi(\underline{p}^k), \underline{p}^k)}{V(e, \underline{r}) - e} = \frac{1}{(1 + k)^2} \quad (30)$$

Both (29) and (30) are monotonic decreasing in k . If $k = 1$, $\frac{3}{4}$ of potential utility generated by the existence of the production sector is lost with (29) while $\frac{1}{4}$ is lost with (30). If $k = 3$, the losses amount to $\frac{1}{5}$ and $\frac{1}{25}$ respectively. Thus these measures are highly sensitive to k .

A broad conclusion would be to view with a high degree of scepticism any estimate of the extent of monopoly welfare loss, even assuming this particularly simple market structure. A market structure of type "n", however, yields even more intractable problems. One reason for this is that $\underline{c} \neq \underline{r}$, but the most important reason is that ratios such as (29) and (30) are not independent of the parameters of (7). Furthermore we cannot say that monopoly welfare loss in the n case is less than that of the m case.

For instance if $\underline{B} \equiv 0$

$$V(e + \Pi(\underline{p}^n), \underline{p}^n) - V(e + \Pi(\underline{p}^m), \underline{p}^m) = (\underline{p}^n - \underline{r})(q(\underline{p}^n) - q(\underline{p}^m)) \quad (31)$$

and (31) is certainly negative if $\underline{p}^m < \underline{p}^n$, i.e. commodities are complements of each other. Obviously if all n firms merged together the new firm would wish to reduce the price of one commodity to increase demand for the others, etc. Thus $\frac{1}{2} \Pi$ and $\frac{3}{2} \Pi$ do not define upper bounds on monopoly welfare loss as defined by (27) and (28). Note that (31) is not unambiguously positive if the commodities are all substitutes. This contrasts with the result given in (17) that if R is independent of the market structure and prices $V(e + R, \underline{p}^n) - V(e + R, \underline{p}^m) \geq 0$ as $\underline{p}^m \geq \underline{p}^n$.

Whether or not the exercise of estimating monopoly welfare loss is worthwhile is highly dependent on two things. First, that the Tullock-Posner hypothesis does not hold and that operating profits are revealed profits. Second, the market structure should be broadly distinguished by Cournot oligopolists or monopolists producing all commodities for which the demands and supplies are significantly interdependent. If these points are not met, the general equilibrium approach used in this paper raises considerable problems in measuring monopoly welfare loss either absolutely or relative to observable magnitudes.

However it is when the Tullock-Posner hypothesis does hold, that qualitative statements concerning changes in market structure can be considered. Such changes will result in price adjustments, and Roy's Identity allows us to draw unambiguous conclusions concerning the effects of such adjustments on welfare in a variety of situations. The approach adopted in Section III, although limited as special cases always are, does demonstrate the kind of

changes in structure which will be detrimental to welfare. The model can also be used to shed light on various arguments in the existing literature. For instance Posner [9], p.821) queries Williamson's thesis [11] that economies of scale is a legitimate defence of mergers. Posner's point is that the extra operating profits (Π) post merger are not reflected in extra distributed profits (R), and so reductions in cost made possible by the merger are not used to increase R , but rather to increase z . Given this is correct, it still does not imply that economies of scale might not produce an increase in household utility as a result of a merger, even given the marginal cost and demand curves drawn in Posner's fig. 4 ((9), p.822). Consider the industrial structure shifting from k to $k-1$ identical Cournot oligopolists by two firms merging together and then the market being shared equally among the $k-1$ remaining firms, and let the resource costs and prices under the two regimes be $\underline{r}^k, \underline{p}^k$ and $\underline{r}^{k-1}, \underline{p}^{k-1}$. From Roy's Identity, if $\underline{p}^k > \underline{p}^{k-1}$, utility rises as a result of the merger. Then from (26), we obtain

$$\underline{p}^k - \underline{p}^{k-1} = \frac{1}{k} \{ (k-1) (\underline{r}^k - \underline{r}^{k-1}) - (\underline{p}^k - \underline{r}^k) \}$$

$$\text{Thus } \underline{p}^k - \underline{p}^{k-1} > 0 \text{ if } \underline{r}^k - \underline{r}^{k-1} > \frac{\underline{p}^k - \underline{r}^k}{k-1} \text{ for } k \geq 2,$$

and is ambiguous in other cases. Thus prices will fall (and utility increase) if the fall in resource cost ($\underline{r}^k - \underline{r}^{k-1}$) resulting from the merger is greater than $\frac{1}{k-1}$ times the original price-cost difference.

The welfare effects of mergers resulting in lower costs are thus seen to depend not only on the marginal cost and demand schedules but also on the kind of industrial structure that exists both pre-merger and post-merger.

In fact, a conclusion of this paper is that the nature of the industrial structure is crucial to both quantitative and qualitative measurement of monopoly welfare loss.

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Footnotes

2. It has been argued (Cowling [2], p.7-9) that firms would make their pricing decision to maximise profit flow, and forestall entry by holding reserve capacity. It is unlikely, however, that the reduction in marginal costs, caused by the fact that capital costs are then sunk costs to inhibit entry, would not provoke an increase in output and a concomitant fall in price. In short the pricing decision cannot be considered independently of entry-limiting behaviour.

3. We can derive Roy's Identity simply in this case by using from (4) that $y^* + p' q^* \equiv R + e$ for all e, R and p . Consider a change in p_i with a change in e such that $R + e$ remains unchanged.

Then $\frac{\partial y^*}{\partial p_i} + q_i + p' \frac{\partial q^*}{\partial p_i} = 0$. Also $\frac{\partial U}{\partial y} = \frac{\partial U}{\partial e} = \frac{V}{e}$ and $\frac{\partial U}{\partial q} = \frac{\partial V}{\partial e} p$ at (y^*, q^*) .

Now $\frac{\partial V}{\partial p_i} = \frac{\partial U}{\partial e} \frac{\partial p}{\partial p_i} + \frac{\partial U}{\partial y} \frac{\partial y}{\partial p_i}$ and at (y^*, q^*) this is:

$$\frac{\partial V}{\partial p_i} = \frac{\partial V}{\partial e} p' \frac{\partial p}{\partial p_i} + \frac{\partial V}{\partial e} \frac{\partial y}{\partial p_i} = - \frac{\partial V}{\partial e} q_i$$