ON THE EFFECTS OF ENTRY*

Jesus Seade

NUMBER 122

WARWICK ECONOMIC RESEARCH PAPERS

DEPARTMENT OF ECONOMICS

UNIVERSITY OF WARWICK
COVENTRY

ON THE EFFECTS OF ENTRY*

Jesus Seade

NUMBER 122

November 1977.

ON THE EFFECTS OF ENTRY

1. Introduction

The problem of entry receives a great deal of attention in present-day Industrial Economics. The main question typically asked in this connection, ever since the work of Bain and Sylos-Labini, is what the best strategies are for oligopolists facing the threat of entry into their industry, that is, the implications of potential entry on their optimal policies regarding pricing, investment, R & D, advertising and so on. Were entry to occur, conventional wisdom says, the effects would be unambiguous: profits per firm, and perhaps also output per firm would fall, while the industry as a whole becomes "more competitive" in some sense, in particular expanding output. These effects are commonly taken for granted in discussions on entry, as obvious truths or, at best, as underlying assumptions. The natural question arises of whether this deep-rooted piece of conventional wisdom is in fact correct for the general case, as the behaviour of oligopoly is, alas, complex enough to keep many surprises in store.

Of course, these remarks are not meant to apply to the limit case where barriers to entry are removed altogether, thus breaking entirely the oligopolistic set-up. The effect on profits, in particular, would in this extreme case be necessarily unambiguous, as they would need to be zero in the new equilibrium, be it perfect or monopolistic competition. This is no more than a definition of equilibrium, but perhaps our intuition draws too heavily on this trivial consideration.[1]

Some of the effects of entry we shall be looking at, in particular those on output, have been studied before, albeit in a rather limited form.

Frank [1] and Ruffin [3] found that certain "reasonable" conditions were sufficient for aggregate output to rise [both authors] and firm-output to fall [Ruffin] as entry occurs in the simple Cournot model of oligopoly.[2] However, these authors do not examine what happens when these non-trivial requirements are not imposed, nor do they look into the behaviour of other related variables. We shall see, for example, that much stronger sufficient conditions for "normal" behaviour of aggregate output can be given than those they assume - in fact we shall find that output always expands with entry, given only observability of equilibrium, but we shall argue too that it is not quite safe to disregard the cases they exclude.

Our aim here is to look at the effects of entry systematically, (i) discussing movements in profits as well as in outputs, at the firm and industry levels; (ii) allowing the conditions for various forms of behaviour to emerge from the analysis, with no a-priori requirement other than profit-maximisation, and (iii) relaxing somewhat the strong Cournot assumption, by allowing firms to have a certain degree of "collusion", that is, awareness of their interdependence via the equilibrium effects, of their individual actions. We shall identify certain demand (and cost) conditions under which entry (or exit) would cause the equilibrium values of some of the variables studied to move in the opposite direction to conventional wisdom. There is a close connection between these comparative-statics results and those of [4] on stability, however, and this will allow us to rule out some possible "perverse" effects of entry, on the grounds that the corresponding equilibria are (virtually) unobservable.

2. Framework

Let homogeneous output Y be produced by n firms, producing y^f each, with price p determined by the inverse demand function $p = p(Y)$. Profits for a firm are

$$\Pi^f = y^f\, p(Y) - c^f(y^f),$$

where $c^f(\cdot)$ is the cost function. Profit-maximisation requires

$$\frac{d\Pi^f}{dy^f} = p + y^f p' \frac{dY}{dy^f} - c^{f\prime} = 0$$

for all f, where dY/dy^f is the conjectural change in total output Y for firm f, relative to a given small change in its own output y^f. This conjectural change can take on any value in general, which renders the whole exercise in principle indeterminate. It is this game-theoretic indeterminateness of the conjectural variations that make oligopoly theory difficult to handle.

In order to make things manageable, I shall assume each firm conjectures others would follow, perhaps only partially, its own expansions or contractions of output around a given joint-optimum, a reflection of their common desire to protect their market shares, in such a way that the value of dY/dy^f is some factor λ^f which would probably be taken by the firm to be, and shall be treated here as a constant in the relevant ranges. This way of modelling interactions within an industry is a convenient standard practice. An extreme instance of this is the Cournot assumption, when each firm conjectures the rest of the industry would not follow it at all were it to expand its own output, hence $dY/dy^f = 1$ then.

At the other end, if all firms were fully aware of their interactions, any change by a given firm would be exactly matched by similar changes by the rest, all of them reacting to the same environment. We would then get $dY/dy^f = n$. This is tantamount to having full collusion of all firms concerned, as if they had an explicit monopolistic agreement to choose the level of aggregate output that maximises joint profits, subject only to the condition that these profits be equally generated and distributed amongst them. This provides an intuitively plausible upper bound on the change in Y that would follow a firm's changing its own output. In sum, we shall treat the reaction-factor λ^f as a constant λ common to all firms, and deem its interesting values to be $1 \leq \lambda \leq n$.[3] The larger λ is, the more "collusive" the behaviour it captures.

Finally, we shall assume, for great convenience and at low cost for the purpose of this paper, that firms are identical and equilibria symmetric. We can then write first and second order conditions for an equilibrium as

$$p + \lambda y p' - c' = 0, \qquad (1)$$

$$\lambda^2 y p'' + 2 \lambda p' - c'' < 0. \qquad (2)$$

For convenience, I use (2) in this strong version, which with (1) is <u>sufficient</u> for an equilibrium, instead of the customary <u>necessary</u> condition with a weak inequality.

3. Modelling entry

Let us now let a new producer enter the industry. It would be of great help to be able to treat the number of firms n as a continuous variable and differentiate all relevant variables with respect to it. This in fact is common practice, usually with an apology rather than a justification. But this expedient can easily be made rigorous. We simply allow the number of firms n to be an actual continuous variable on which everything depends differentiably, but we restrict attention to integer realisations of this variable. Then, if ξ is any dependent variable defined on the number of firms n, its change when one firm enters is $\Delta \xi = \int_n^{n+1} \xi'(\nu)\, d\nu$. It is clear that (sign $\Delta \xi$) = (sign $\xi'(\nu)$) whenever the latter sign does not change in the relevant range $(n, n + 1)$; otherwise the sign of $\Delta \xi$ is ambiguous. I shall assume away cases where this ambiguity arises and hence work with sign $\xi'(n)$ directly. It is essentially this single-signedness assumption, which one can check, that underlies the common continuous treatment of discrete variables in problems of the present sort.

4. Output per firm

The "normal", or conventional-wisdom result whose robustness we want to examine is that output per firm declines as entry occurs: $dy/dn < 0$. Differentiating (1) with respect to n:

$$p'\left(n \frac{dy}{dn} + y\right) + \lambda\, y\, p''\left(n \frac{dy}{dn} + y\right) + \lambda\, p' \frac{dy}{dn} = c'' \frac{dy}{dn}$$

which, assuming $p' < 0$, yields

$$\eta_{yn} = -(E + m)/(E + m + k) \qquad (3)$$

(if $E + m + k \neq 0$), where $\eta_{yn} \equiv (n\, dy/dn)/y$ is the elasticity of y

with respect to n, whose sign interests us, and

$$m \equiv n/\lambda \ ;$$

$$E \equiv Y\,p''/p' \ ;$$

and $$k \equiv 1 - c''/\lambda p' .$$

The use of these variables will considerably simplify our notation and be an aid to intuition. It will be seen that, in fact, m and E are the key explanatory variables in most of what follows. The former, $m \equiv n/\lambda$, can be regarded as the number of "effective" firms in the industry, i.e. the inverse of a measure of effective concentration quite distinct from the standard measure of observed concentration n, which is here adjusted by the degree of collusion λ.[4] When $\lambda = 1$ (no collusion, the Cournot case), the market has n "effective" (and actual) firms, whereas when $\lambda = n$, the market behaves in a fully collusive manner, $m = 1$ (as a monopolist, subject only to symmetry).[5]

Apart from m, which in the Cournot model reduces to n and has no special interest in itself, the central variable to look at turns out to be $E \equiv Yp''/p'$, the elasticity of the slope of demand, negative (positive) for convex (concave) curves. This will be discussed in some detail in the following section.

Finally, the variable $k \equiv 1 - c''/\lambda p'$ will usually have a value not far from 1 and at any rate will normally be positive, since $k < 0$ is equivalent to $c'' < \lambda p'$, i.e. marginal costs not only falling but doing so faster than perceived demand, a rather uninteresting possibility. I shall mostly assume this case away, imposing $c'' > \lambda p'$, hence $k > 0$.

With this notation we can rewrite the second order condition (2) as

$$E + m + mk > 0. \tag{4}$$

We immediately get the following result:

<u>R1 (Ruffin) : $E > -m$ is sufficient for $\eta_{yn} < 0$.</u>

<u>Proof</u>: $E > -m \Rightarrow E + m > 0$. Hence $(E + m + k) \leq 0 \Rightarrow k < 0$. Thence, since $m \geq 1$, $(E + m + mk) < 0$ which contradicts (4). It follows that $(E + m + k) > 0$ and $\eta_{yn} < 0$ ||.[6]

The following result (together with footnote 6) gives the sign of dy/dn (= sign η_{yn}) for the general case.

<u>R2. : Given $c'' > \lambda p'$, output per firm behaves in a "perverse" form as entry occurs ($\eta_{yn} > 0$) if and only if $-(m + k) < E < -m$; elsewhere, $\eta_{yn} < 0$.</u>

"Elsewhere" here, given the second-order condition (4), means <u>either</u> $-(m + mk) < E < -(m + k)$ <u>or</u> $-m < E$. It is somewhat surprising that the range of values for E which yield $dy/dn < 0$ is not an interval but the union of two disjoint ones. It is shown in [4] however, that stability requires[7]

$$E > -(m + k), \tag{5}$$

i.e. "excessive convexity" of the demand curve at a given point is excluded. It is only natural to disregard unstable equilibria as being <u>unobservable</u>, particularly when performing comparative-statics exercises, where any possible initial unstable equilibrium would be lost when perturbed and would not be regained subsequently. We shall then impose (5), a stronger requirement than (4) (for $k > 0$). The "perverse" possibility of R2, however, is compatible with this condition.

Notice that as λ increases, the "perverse interval" for E in R2 becomes less unlikely to be hit by E: the size of this interval remains roughly constant, k, while the whole of the interval moves to the right, thus requiring less negative E's (less curvature, roughly) for perverse results to arise. In the full-collusion limit $m = 1$, the critical interval for E is close at hand, $[-(1 + k), -1]$, so that entry of a new producer into a fully collusive industry may even more easily result in increased output by the typical firm ! Industry-output will of course have to rise substantially then, to accommodate more and bigger firms than before. Notice that second order conditions and necessary conditions for stability still impose no problem here: both require $E \geq -(1 + k)$ when $\lambda = n$.

5. Interpretation

The elasticity of the slope of demand $E \equiv Yp''/p'$, is (negatively) related to the curvature of $p = p(Y)$. When $E < -m$, the slope is falling in absolute value "too" quickly, i.e. we have a "very" convex curve. This is the critical value of E in R2. The condition $E < -m$ can be transformed into

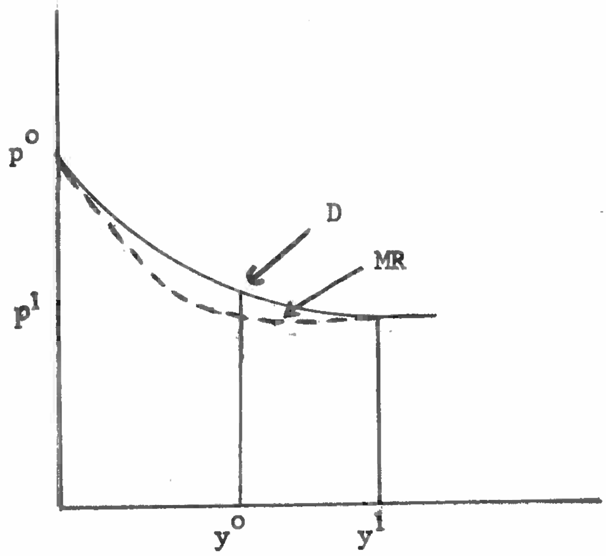
$$2\lambda p' + \lambda^2 y p'' < \lambda p' \qquad (6)$$

Now, each firm conjectures that $dY/dy = \lambda$ and so that $Y = \lambda y + Q$ (for some constant Q), locally. Hence perceived demand for the firm can be written as $p = p(\lambda y + Q)$ as a function of y, D(y), whose slope is $\lambda p'$

and whose marginal-revenue curve is $p + \lambda y\, p'$. The slope of the latter, finally, is $2\lambda p' + \lambda^2 yp''$. Hence (6) (and so $E > -m$) requires marginal revenue to be steeper than perceived demand.

The relation between these relative slopes and the interpretation of E in terms of the curvature of demand is easily shown diagrammatically. In Fig. 1 perceived demand meets the price-axis at p^o and becomes perfectly elastic at p^1. Clearly, MR < D whenever D' (i.e.$\lambda p'$) < 0, while MR = D both when D is flat and at p^o. It obviously follows that MR must be flatter than D for some values of y, say at y^o, before merging with D at y^1. Hence an equilibrium at y^o does not satisfy $E > -m$.

Figure 1



Yet another interpretation of the condition $E > -m$ is helpful. We know marginal revenue for the producer is given by $MR = p + \lambda yp'$, where the argument of $p(\cdot)$ is $\lambda y + Q$. Let us now perform the thought-exercise of increasing everyone else's output while keeping "our" producer's output y constant, hence increasing total output $Y = \lambda y + Q$ via Q alone. From the expression for MR we get that $MR_Q = p' + \lambda y p''$, which is negative if and only if (6) holds. That is, the sufficient condition for "normality" of results $E > - m$, can be re-read as saying that $MR_Q < 0$, that MR for a firm should fall as other firms expand output. Viewed in this light, the perverse result in R2 is not so surprising: suppose a new firm enters an industry producing initially the same output the typical firm was producing before entry. This extra output increases Q ("output by all others") for each of the previous firms. But if $E < - m$, this expansion-by-others increases MR for each individual firm, which obviously induces all previously established firms to expand.

A simple example will suggest that "perverse" possibilities should generally not be ruled out a-priori. Suppose demand has constant elasticity ϵ : $p = AY^{-\frac{1}{\epsilon}}$, and that $c'' = 0$. In this case $E \equiv Yp''/p'$ is also a constant : $E = - (\frac{1}{\epsilon} + 1)$. Hence, the second order condition reads $\epsilon > \frac{1}{2m - 1}$, while the critical interval for E indicated in R2 corresponds to $\frac{1}{m} < \epsilon < \frac{1}{m-1}$. The lower of these limits is also the (lower) limit for stability of equilibria. Remember that the number of effective firms $m = n/\lambda$ will usually be lower, perhaps considerably so, than n, the crude number of oligopolists in the given industry. If m is close to 1, practically every value of ϵ consistent with stable equilibria yields $dy/dn > 0$. But as m grows larger, say beyond 4 or 5, it becomes virtually certain that conventional wisdom will reign.

Critical interval (1/m, 1/n-1) for the price-elasticity ϵ

m	1	1.5	2	2.5	3	5
1/m	1	2/3	1/2	2/5	1/3	1/5
1/(m-1)	∞	2	1	2/3	1/2	1/4

6. Profits per firm and total output

Conventional wisdom suggests $d\Pi^f/dn < 0$, $dY/dn > 0$. From $Y \equiv ny$, we immediately get $\eta_{Yn} = 1 + \eta_{yn}$.

Hence, from (3),

$$\eta_{Yn} = k/(E + m + k) \tag{7}$$

It is clear that $k > 0$ and $(E + m) > 0$ are sufficient for $\eta_{Yn} > 0$. This is the main result of Frank [1] and Ruffin [4] on limited entry. We have just argued that $E > - m$ is not necessarily a good assumption. But (7) allows us to read directly necessary and sufficient conditions for $\eta_{Yn} \gtrless 0$:

R3 : Given stability of equilibria (at n, n+1), total output will always expand (contract) as entry occurs, provided only $k > 0$ (< 0).

Hence, when $k < 0$, the negative effect on y of footnote (6) overcompensates for the addition to total output by the new entrant.

Similarly, differentiating $\Pi^f = yp\,(ny) - c(y)$ we obtain, after some manipulations,

$$\frac{d\Pi^f}{dn} = \frac{y^2p'}{m} \left[\frac{E + m + mk}{E + m + k} \right], \tag{8}$$

which from $p' < 0$, second-order (4) and stability (5), is definitely negative, for any k.

R4 : Profits per firm will always fall with entry, given only stability of equilibria.

I found these results stronger than I expected, as they apply to the general Cournot case and more, without any special conditions.

7. Total Profits

The effect of entry on profits is less clear-cut than those on previous variables. This is what one would expect, however, as levels previously irrelevant, notably the fixed cost $c(0)$, are now important. Some special cases can be considered, an interesting one being when profits per firm are negative before entry occurs (a short-run possibility, provided only running-profits remain positive), with "entry" being now interpreted as of negative sign presumably, i.e. exit. For this case, since

$$\frac{d\Pi^T}{dn} = \frac{d(n\Pi^f)}{dn} = n\frac{d\Pi^f}{dn} + \Pi^f, \tag{9}$$

we immediately get (using R4),

R5 : $\Pi^f \leqslant 0$ is sufficient for $d\Pi^T/dn < 0$, given only stability of equilibria.

Hence "exit" from an ailing industry is always beneficial not only to the firms that remain but also to the industry as a whole. But, of course, if $\Pi^f > 0$, the positive profits made by the new entrant may compensate for the loss of profits by previous firms, rendering the sign of $d\Pi^T/dn$ indeterminate. This is made clearer by the following alternative form of (9) (derived, e.g., using (8)):

$$\frac{d\Pi^T}{dn} = \frac{(n - \lambda)y^2 p'k}{E + m + k} + (yc' - c). \tag{10}$$

The first term in (10) depends essentially on demand conditions, and can be thought of as representing the change in the extent to which consumer's surplus is captured by producers as entry occurs. The second term, on the contrary, is purely related to cost conditions, and corresponds to the extent to which producers are collective cost-minimizers for given total output. As entry occurs, costs fall by yc', the total marginal cost saved by established firms as they reduce output by y units altogether, minus c, the total cost of production incurred by the new entrant. Clearly, the two terms in (10) are rather independent of each other and the second one can take any sign.

Let us note, without emphasis, two special cases from (10).

(i) Under full collusion, total profits increase (fall) as average cost for the firm increases (decreases) with output. The reason is clear: under increasing costs, the total-cost curve for the group is lowered by entry, as a more efficient scale is used per plant, while Y is simply chosen collectively (fully-collusively) so as to maximise total profits. But this shows how easily one can have $d\Pi^T/dn > 0$, merely by having, say, increasing costs and a high degree of collusion in the industry, a pair of not too unlikely conditions.

(ii) Non-increasing-costs industries necessarily have their total profits lowered by entry.

8. A variable degree of collusion

The conjectural variation λ has been assumed not only to be (well defined and) constant at and around a given equilibrium but, what is probably more debatable, it has been treated as a constant across equilibria, as entry takes place. It might be argued, however, that λ does depend on the number of firms n, as the greater ease of communication amongst firms when they are few, say, may facilitate more collusive policies to be pursued. This could have easily been incorporated in the above analysis at great expense in terms of simplicity of the equations. However we do not need full details of these new equations to get qualitative conditions. Writing $\xi = \xi(n, \lambda)$ for any of the variables considered above, we get $\frac{d\xi}{dn} = \frac{\partial\xi}{\partial n} + \frac{\partial\xi}{\partial\lambda}\frac{d\lambda}{dn}$, so that we need only look at $\partial\xi/\partial\lambda$ here, and briefly put this effect together with the $\partial\xi/\partial n$ of previous sections. Proceeding as before, one easily finds:

$$\eta_{y\lambda} = -1/(E + m + k), \tag{11}$$

and

$$d\Pi^f/d\lambda = -y^2p'(m-1)/(E + m + k). \tag{12}$$

Hence, since $\eta_{Y\lambda} = \eta_{y\lambda}$ and $d\Pi^T/d\lambda = nd\Pi^f/d\lambda$, and imposing $(E + m + k) > 0$, the effect of increased collusion turns out to be unambiguous in all cases: y and Y fall, while Π^f and Π^T rise with λ. Collusion works. And if $d\lambda/dn < 0$, as one would assume, the total effect on Π^f and Y remains "normal" always,[8] while the ambiguous effects on y and Π^T remain of course ambiguous, but are somewhat "pulled" towards their "normal" behaviour when λ falls with entry.

Footnotes

* My interest on this problem arose from a conversation with James Mirrlees, some time ago. I would like to thank Keith Cowling, Avinash Dixit and participants at the 1977 Warwick Summer Workshop on Oligopoly for helpful comments.

1 Even then, however, the limit behaviour of output and price is not obvious [1,3] .

2 Otherwise, these authors mainly discuss the limit behaviour of the Cournot model as unrestricted entry is introduced. McManus [2] derives similar results diagrammatically, assuming linear demands and costs, and Telser [5] wrongly derives a result on aggregate output (p. 136), again for the Cournot case.

3 Nothing that follows actually depends on these values of λ, which are only given to fix ideas. One might perhaps want to allow for λ's in $[0,1)$, a regime of "struggle", rather than "collusion" as in $(1,n]$. The story would go that, were I to reduce my output trying to move up along my perceived demand curve, others would jump in, expanding output and taking part of my market share. Or one might argue for lower values of λ for reductions than for increases in y^f, a general kinked-demand-curve-type phenomenon.

4 The non-identical-firms counterpart of this m would be $1/H\lambda$, where $H \equiv \Sigma w_i^2$ is the Herfindahl index of <u>observed</u> concentration, the w_i's being the market shares of individual firms.

5 Similarly, when $\lambda = 0$ (c.f. note 3) the number of effective firms is $m = \infty$, just like in perfect competition. Here, no individual firm can change aggregate output (hence price), because of the voracious practices of its competitors.

6 <u>Corollary : $c'' < \lambda p'$ ($\Leftrightarrow k < 0$) $\Rightarrow \eta_{yn} \leq 0$.</u> Proof: $k < 0 \Rightarrow E + m > 0$ (by (4)). Hence $\eta_{yn} < 0$ by <u>R1</u>. ||

7 Assumptions <u>0'</u>, <u>I'</u> and <u>II'</u> in [4] are equivalent to $k > 0$, $E > -m$ and $E < -(m + k)$ respectively.

8 Assuming here $k > 0$, for Y.

References:

[1] Frank, C.R. "Entry in a Cournot Market", Review of Economic Studies 32 (1965), 245-250.

[2] McManus, M. "Equilibrium, Numbers and Size in Cournot Oligopoly", Yorkshire Bulletin of Economic and Social Research 16 (1964), 68-75.

[3] Ruffin, R.J. "Cournot Oligopoly and Competitive Behaviour", Review of Economic Studies 38 (1971), 493-502.

[4] Seade, J.K. "The Stability of Cournot Revisited", Warwick Economic Research Paper No. 121 (November 1977).

[5] Telser, L.G., Competition, Collusion and Game Theory (MacMillan, London, 1971).