SECOND-BEST POLICIES IN IMPERFECT COMPETITION:
HOW IMPROVED INFORMATION MAY LOWER WELFARE

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ABSTRACT

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A consumer choosing among brands of a differentiated product gathers information over the relative performance characteristics and prices of the various brands. Even taking prices as known, since information on performance characteristics is costly to gather and process, the consumer will typically base his brand choice on only limited information.

In a perfectly competitive market, selecting a brand with only limited information unequivocally lowers consumer welfare for risk-averse consumers. *Ex ante*, expected satisfaction falls since performance is now uncertain. *Ex post*, the "wrong" brand may be purchased. In imperfect competition, however, there is a second effect--prices may rise or fall. Equilibrium prices are jointly determined by the interactions of demands and costs. If firms are Nash competitors, for example, price for a particular brand is given by the familiar markup equation

$$P(1 - \frac{1}{\varepsilon}) = MC$$

where ε represents the absolute value of the firm's own price elasticity at the equilibrium prices of itself and its competitors. If imperfect information makes demand for a brand more elastic then the price of that brand will fall. Thus, in imperfect competition information has two effects on consumer welfare; net welfare may rise or fall. In this paper, this issue is analyzed in some models of monopolistic competition, and the role of informative advertising in this process is discussed.

HOW IMPROVED INFORMATION MAY LOWER WELFARE

Steven C. Salop*

I. Introduction

An extensive literature has developed in recent years analyzing the effect of price information in commodity markets. In a series of papers beginning with Diamond (1971), it has been shown that in markets for homogeneous commodities, limited price information endows even small firms with informationally based monopoly power. As a result the equilibrium is often characterized by price dispersion.[1]

Even if all prices are known however, the consumer is also concerned with the quality and variety characteristics of the various brands. By quality, we mean characteristics about whose value all consumers agree, dimensions such as breakdown probability, durability, and ease of operation; that is, a brand may be of higher or lower quality than another brand. By variety, we denote variations across brands for which there is no consensus, but rather for which consumers preferences differ. Variety includes dimensions such as sweetness, color and size. Since quality variations affect value per dollar equally for all consumers, imperfect price information models are easily extended to cover this case. The same is not true for the variety differences. Price information may always be gathered

1/ See Salop-Stiglitz (1977), Butters (1977), and von zur Muehlen (1976). A recent survey is Salop (1976a).

* See References, page 29.

by prepurchase search activities such as direct inspection, consumer publications and consultations with acquaintances. This may also be true with respect to variety, but one is more likely to require actual personal experience to gauge the relative value of different varieties.

In a perfectly competitive industry, the risk-averse consumer who makes his brand choice on the basis of limited information perceives a decrease in satisfaction *ex ante*, relative to perfect information and lowers his demand price accordingly. His *ex poste* satisfaction is also lowered relative to perfect information if, in fact, he does not choose the brand most suited to him. If he purchases multiple units of the brand, satisfaction is also decreased since *ex poste* marginal rate of substitution differs from price. It is these potential utility losses that leads the consumer to gather variety information.

In imperfect competition there is an additional effect on monopoly power; the consumer's *relative* declines in *ex ante* satisfaction for brands may not decrease equally, relative to perfect information; since it is the relative demand prices that determine the elasticity of an imperfect competitor's demand curve and, hence profit-maximizing price, imperfect variety information may change equilibrium prices.

If limited variety information causes firms' demand curves to become more elastic, then prices will fall. Similarly, they will rise if demand becomes more inelastic. In the latter case, net consumer welfare falls; for both higher prices and incorrect decisions both lower utility. On the other hand, if equilibrium prices fall, and the gain from the price effect outweighs the loss from the utility effect, then consumers benefit from limited information.

In the shortrun, the price effect is offset by a change in producer profits. If the industry began in a perfect information zero profit equilibrium, then a negative price effect causes some firms to exit the industry until a new zero profit equilibrium at lower prices and decreased variety is achieved, relative to the perfect information equilibrium. Now total welfare does not involve producers profits, but rather represents the tradeoff between price and variety in the industry. Monopolistic competition does not generally produce the optimal compromise of the price-variety tradeoff. 1/ If the original equilibrium is non-optimal, and if the new equilibrium is closer to the optimal allocation, then the increased uncertainty may raise total welfare.

Viewed in this way, it is not surprising that limited information can improve the allocation in an imperfectly competitive industry. If an equilibrium is inefficient, then the theory of the Second-Best suggests that additional distortions may be used to offset rather than magnify the initial inefficiency. 2/

1/ See Lancaster (1975), Salop (1976), Spence (1976) and Dixit-Stiglitz (1977).

2/ Diamond (1977) makes a similar point in a model of price dispersion and optimal search.

Similarly, this notion may be utilized to analyze the effect on welfare of any non-price variable, such as service quality or durability, as well as informative advertising. The social value of a non-price quality variable includes both its direct welfare effect and its effect on the ultimate price-variety allocation at the ultimate long-run equilibrium.

In this paper, this view of non-price variables is studied on a number of examples of imperfectly competitive equilibrium. Section II presents two examples of the effect of information on equilibrium and welfare. Section III sets outs a general model which is extended to non-price competition in Section IV. Finally, equilibrium with imperfect information and the effects of limiting information are analyzed in Section V.

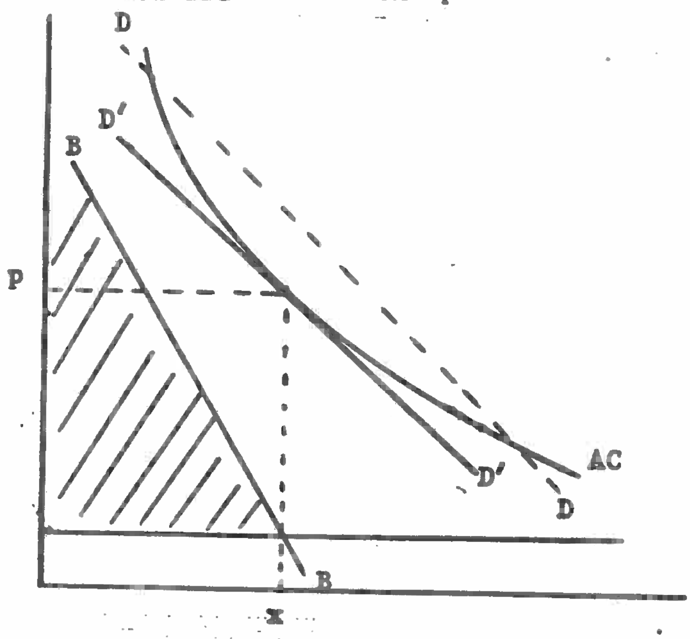
II. Examples

Example 1: Welfare Improving (Biased) Misperceptions in Product Selection

The formal results in the body of the paper concern the role of uncertainty for commodities that are produced. In this example, we focus for a moment on the interaction of biased perceptions with the entry decision. Biased perceptions may increase total welfare in an imperfectly competitive industry by permitting the profitable production of brands with total social benefits in excess of total costs, yet whose total revenue (under certainty) does not cover total costs.

Consider a commodity with the perfect information net benefits (willingness-to-pay) schedule labeled BB in Figure 3.1 below. Since BB lies everywhere below the average cost curve-AC, this commodity cannot be profitably produced even by a monopolist unable to discriminate.[1/]

However, suppose that net benefits (the shaded area under BB less-fixed costs) are positive at the optimal production level x. Thus, an optimal allocation would include the commodity, if a producer subsidy could be arranged. In the absence of subsidies or price discrimination, if consumers over-estimated their preferences and perceived the willingness-to-pay schedule given by DD above AC, then a private producer could produce the commodity at a profit. If perceptions were given by D´D´, then an entrant willing to earn normal (to zero) profits would produce the quantity-x at price-p, as illustrated, and the optimal allocation would obtain. Thus biased perceptions raise welfare in this example.



1/ See Spence (1976) for a general discussion of this product selection problem.

Advertising has an interesting, yet quite perverse effect in this example. Advertising that is truthful in the sense that it provides information that enables consumers to learn their true demands, lowers welfare by eliminating this brand from the set of products offered. On the other hand, deceptive advertising that raises aggregate demand is welfare improving by allowing the brand to be produced.

Example 2: Ignorance versus Perfect Information

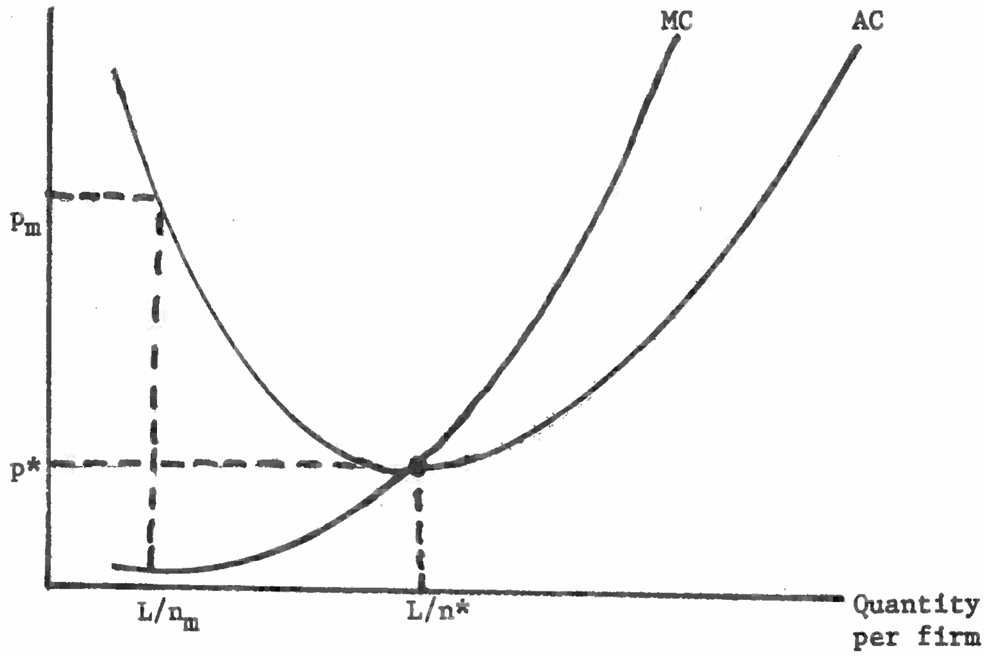
Suppose the ice cream industry consists of n - flavors, each produced by a single firm at zero marginal cost. Every consumer purchases exactly one pint per period. Each consumer values his favorite flavor at \$1 per pint and all other flavors at zero. Preferences are symmetric, that is, $(1/n)^{\underline{th}}$ of the consumers prefer each of the n - flavors. There are L - consumers in all.

If consumers are perfectly informed of their preferences and firms compete with respect to price, each producer may act as a monopolist in his own flavor and extract the entire consumer surplus at a price of \$1 per pint.

On the other hand, suppose consumers are perfectly ignorant of their relative flavor preferences, *ex ante*, and instead view each brand as giving equal satisfaction of \$(1/n) per pint. Alternatively, suppose that the ice cream were sold in unlabeled containers that could not be opened before purchase or resold after purchase. In this case, consumers treat all containers (and hence all flavors) as perfect substitutes. Price competition among producers drives the equilibrium price to the marginal cost of zero. Revenue falls to zero. If each flavor is purchased $(1/n)^{th}$ of the time, average consumer surplus rises to \$(1/n), above the perfect information level of zero. Aggregate welfare remains constant, of course; the price decrease represents a pure transfer from producers to consumers.

In the long run however, this income transfer does have allocative effects through the process of entry competition. For example, if we generalize by having production per flavor be characterized by U-shaped average costs and begin at an n - firm zero profit monopolistically competitive equilibrium, then as price falls to the competitive price, some producers will exit and those remaining will each produce more output. In the diagram below, suppose we began at price - p_m and an equilibrium number of firms - n_m in which each firm sells one unit to L/n consumers. At the competitive price - p*, zero profits entails a higher output per firm or a fewer number of firms in equilibrium - n.*

Thus, ignorance decreases price from p_m to p* and the number of firms from n_m to n*. Since the number of firms equals the number of flavors produced in this model, the new equilibrium entails less variety, but at lower price.



In the long run then, there are two effects of ignorance on aggregate welfare. First, consumers bear a deadweight loss from ignorance of purchasing a flavor different from their most preferred flavors. Second, the price-variety compromise achieved by the market entails lower price and variety. If the perfect information equilibrium (p_m, n_m) were at the optimal price-variety point, ignorance surely lowers welfare. However, if the perfect ifnormation equilibrium entailed excess variety, then the new price-variety point (p*, n*) <u>may</u> be preferred to (p_m, n_m) even to the extent of more than offsetting the deadweight loss due to incorrect choices under ignorant. If so, then a policy of limiting information might raise aggregate welfare.

III. Equilibrium and Welfare Policies in Monopolistic Competition

The previous example contrasted two policy settings, perfect information and perfect ignorance. It was shown that if the perfect information equilibrium were sub-optimal, then a policy of enforced ignorance has the potential for improving aggregate welfare. In this section, a general model of monopolistic competition is discussed with a focus on policies designed to improve welfare.

Beginning fairly general, aggregate welfare consists of consumer welfare - W less production costs. Let welfare depend positively on the degree of variety in the market, which we formalize as the number of brands produced - n. In addition, suppose consumer welfare depends on the setting of some policy variable - θ. Eventually, θ will be identified with limits on consumer information. For now, we simply assume that $\theta = 0$ denotes no policy and increases in θ create some sort of deadweight consumer welfare loss. If the market consists of L - consumers who each purchase one unit of some brand, marginal production cost is a constant - m and there are fixed costs per brand - F, then the first-best policy problem is to choose a number of brands - $\hat{n}$ and a policy-setting $\hat{\theta}$ to maximize aggregate surplus $S(n, \theta)$ given by

$$(3.1) \qquad S(n, \theta) = W(n, \theta) - mL - nF$$

where $W(n, \theta)$ denotes consumer welfare with partial derivatives

(3.2) $W_n > 0,\ W_\theta < 0$ 1/

Differentiating (3.1) with respect to (n, θ) the optimum clearly entails $\hat{\theta} = 0$ and $\hat{n}$ according to

(3.3) $W_n(\hat{n}, 0) - F = 0$

That is, the gain in welfare from the addition of a brand (W_n) equals the cost of adding the brand (F). The planner sets $\hat{\theta} = 0$ since it entails a deadweight loss (W_θ) with no offsetting welfare gain.

In a moment we will compute the equilibrium number of brands in the market, and we will wish to compare the equilibrium to the optimum given by (3.3). Looking ahead, if the equilibrium number of firms - n entails a greater degree of variety than optimal, this will be signaled by (3.3) as follows.

(3.4) $W_n(n,0) - F < 0 \Rightarrow n > \hat{n}$

The inequalities are reversed for an underprovision of variety by the market, with equalities for optimal variety at equilibrium.

The equilibrium price - p and degree of variety - n in the market is determined by the interaction of demand elasticities and costs in the market. Since consumer welfare depends on the policy setting θ, then so will the elasticity of demand - ε for a representative firm in the market. Thus, we may write $(p(\theta), n(\theta), \varepsilon(\theta))$ as the equilibrium price, variety and elasticity, if the policy is set at θ, and the mark-up equation of profit-maximization is given by

1/ We assume W is concave in n and θ.

(3.5) $$p(\theta)\{1 - \frac{1}{\varepsilon(\theta)}\} = m$$

where we write elasticity as a positive number. If free entry by competing brands continues until every brand earns zero profits,[1] then equilibrium price equals average cost. If demands in the market are symmetric across brands, then each brand sells to $(1/n)^{\underline{th}}$ of the market of L - consumers or L/n - units. Thus, price equal to average cost may be written as

(3.6) $$p(\theta) = m + \frac{F}{L} n(\theta)$$

Differentiating (3.5) and (3.6) with respect to θ, we may calculate the effect of changes in the policy setting on the equilibrium degree of variety. It is easy to show that

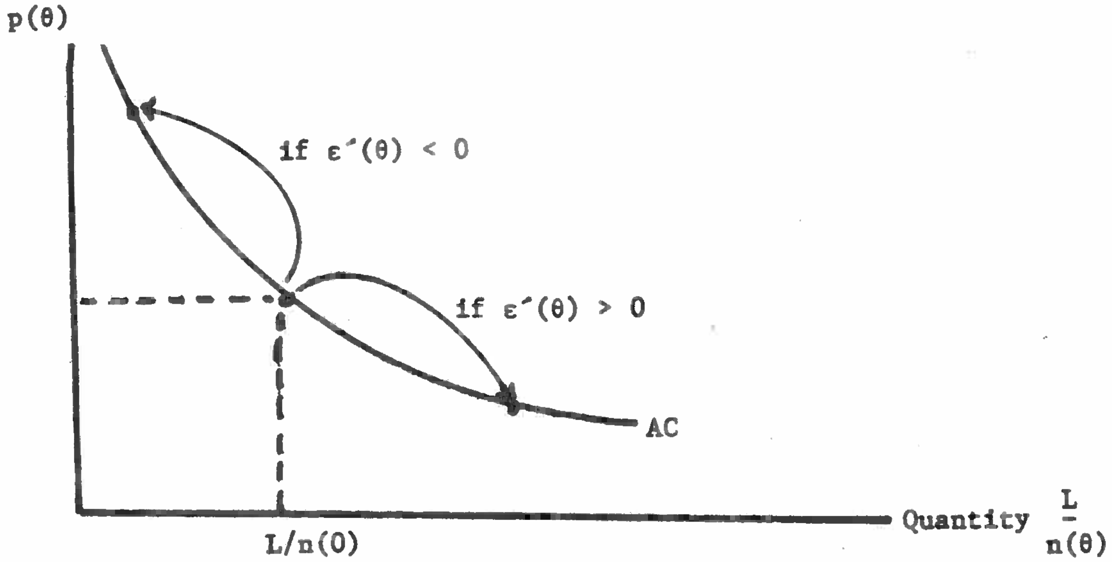
(3.7) $$p'(\theta) < 0 \Longleftrightarrow \varepsilon'(\theta) > 0$$

That is, if increases in θ make demand for the representative firm more elastic, then equilibrium price falls. Noting from (3.6),

(3.8) $$n'(\theta) = \frac{L}{F} p'(\theta)$$

then a decline in price entails a decline in variety - n. In the ice cream example, ignorance-policy made demands perfectly elastic. In general, policy may make demand more or less elastic with price and variety changes summarized in the diagram below.

1/ We ignore the requirement that n(θ) be an integer.



Thus, changes in θ affect the equilibrium degree of variety in the market. It is now well-known[1/] that monopolistically competitive equilibrium generally entails too little or too much variety. We now see how a policy - θ might affect the equilibrium. We may now combine the two notions and consider a Second-Best policy problem of choosing θ to maximize surplus, given the constraint that variety is the outcome of a monopolistically competitive equilibrium price and entry process, or

$$\max_{\theta} S(\theta) = W(n(\theta), \theta) - m\,L - n(\theta)\,F \tag{3.9}$$

$$\text{s.t.} \quad p(\theta) = m + \frac{F}{L}\,n(\theta)$$

$$p(\theta)\{1 - \frac{1}{\varepsilon(\theta)}\} = m$$

1/ See for example Spence (1976), Lancaster (1975), Dixit-Stiglitz (1977)

Ignoring the constraints explicitly and taking them into account through their effect on $n(\theta)$, differentiating (3.9) with respect to θ, we have

$$(3.10) \qquad S'(\theta) = (W_n - F)n'(\theta) + W_\theta = 0$$

where $n'(\theta)$ is given from (3.7) and (3.8)

To get a feel for the policy tradeoff, consider the local problem of increasing θ in the neighborhood of $\theta = 0$.

$$(3.11) \qquad S'(0) = (W_n(n(0), 0) - F)n'(0) + W_\theta(n(0), 0)$$

Rewriting the first-best optimum condition from equation (3.3) as

$$(3.12) \qquad W_n(\hat{n}, 0) - F = 0$$

then the following necessary condition obtains.

Condition I:

For a policy $\theta > 0$ to improve welfare over no policy ($\theta = 0$), it must be true that either

(a) $\hat{n} < n(0)$ and $\varepsilon'(0) > 0$

or

(b) $\hat{n} > n(0)$ and $\varepsilon'(0) < 0$

The proof and interpretation are straightforward. Since policy entails a deadweight loss ($W_\theta < 0$), then surplus rises only if the degree of variety moves closer to the optimum. If equilibrium entails too many brands ($\hat{n} < n(0)$), then the policy must decrease the number of brands ($n'(0) < 0$), which itself occurs if policy increases demand elasticity ($\varepsilon'(0) > 0$). Finally, from (3.4), excess variety is signalled by $W_n - F < 0$. From (3.11), $S'(0) > 0$ requires $(W_n - F)$ and $n'(0)$ to have identical signs.

This is the essence of the model. Beginning from a sub-optimal equilibrium, even policies with deadweight losses may improve welfare by pushing the equilibrium closer to the first-best optimum. As in all problems of the second-best, adding additional distortions to the economy (the policy) might offset rather than amplify the original distortion.

IV. Non-Price Competition and Welfare

In the next section, the framework just set out will be used to analyze the effects on equilibrium and welfare of a limited information policy. Before going on to that example, however, it is useful to interpret the results in the previous section to equilibrium and welfare under non-price competition.

In this model, non-price competition is simply the selection of some variable θ_i by brand - i at cost $C_i(\theta_i)$. Following the conventions established, we may treat lower θ's as more preferred settings of the non-price policy variable. Similarly, these more preferred settings may only be achieved at higher cost, so we adopt the convention that $C_i'(\theta_i) < 0$. Now the surplus function must be expanded to allow for different θ_i choices by competing firms and the cost of these θ_i choices

Denoting by $\theta = (\theta_1, \theta_2, \ldots, \theta_n)$ the vector of non-price variables at the n - firms, we have the expanded surplus function to be maximized in the first-best problem.

$$(4.1) \qquad S(n, \underline{\theta}) = W(n, \underline{\theta}) - mL - nF - \sum_{i=1}^{n} C_i(\theta_i)$$

The first-best policy problem is to choose $\hat{n}$ and $\hat{\underline{\theta}}$ to maximize surplus, or

$$(4.2) \qquad S_n = W_n(\hat{n}, \hat{\underline{\theta}}) - F = 0$$

$$(4.3) \qquad S_i = W_i(\hat{n}, \hat{\underline{\theta}}) - C'_i(\hat{\theta}_i) = 0$$

where S_i, W_i denote partial derivatives with respect to θ_i. Equation (4.2) is similar to the variety choice derived earlier. Equation (4.3) includes the possibility that lower θ_i entail higher costs so that θ_i must be chosen economically even in the first-best problem.

The conditions for a Nash price plus non-price competition monopolistically competitive equilibrium are the mark-up equation plus the efficient θ_i choice by each firm. If we denote the demand for brand - i as $D^i(\underline{p}, \underline{\theta})$, then we have the profit function

$$(4.4) \qquad \Pi^i = (p_i - m)D^i(\underline{p}, \underline{\theta}) - C_i(\theta_i)$$

where $(\underline{p}, \underline{\theta})$ denotes the vector of prices and θ_i's. Profit-maximization with respect to (p_i, θ_i) by each firm will yield an equilibrium $(p^*, \underline{\theta}^*)$. At a symmetric equilibrium $(p_i^* = p^*, \theta_i^* = \theta^*)$, we have the equilibrium conditions for each firm,

$$(4.5) \qquad p^*\{1 - \frac{1}{\varepsilon(\theta^*)}\} = m$$

$$(4.6) \qquad (p^* - m)D_\theta(p^*, \theta^*) = C'(\theta^*)$$

where D_θ denotes the effect on demand of a particular brand's change in its θ and $\varepsilon(\theta)$ denotes the price elasticity of demand. 1/

Even if we identify increases in demand from decreases in θ with the direct welfare increases from a decrease in θ, given by W_θ, then (4.6) expresses a qualitatively optimal choice of θ given the number of brands - $n(\theta^*)$ and the price p^*. However, the optimal second-best choice of θ from the point-of-view of society does not take the number of brands or the price as given. Instead, society will select a θ with an eye towards affecting the equilibrium number of brands, as set out in the Section III example of costless θ.

The lesson of this analysis is straightforward. The private incentives for non-price competition represents a tradeoff between revenue and costs of the non-price variable. The incentives of society in setting the non-price variable are a tradeoff between the welfare and costs of the policy with the additional twist that the effect of the non-price variable on the equilibrium number of brands is also taken into account. Thus, imperfect price and non-price competition contain two distortions in the non-price variable. First, revenue rather than welfare is traded-off against costs. In this arena, since revenue and welfare effects are in the same direction, the distortion

1/ The elasticity ε may also depend on the price vector p as well as θ. The price vector has been supressed for notational convenience.

is merely one of quantitative magnitudes. However, the second distortion is qualitative; the private incentives in the Nash game completely ignore the repercussions of the non-price variable on the ultimate equilibrium number of firms.

For example, suppose the non-price variable is advertising that improves consumer information regarding ones own brand. Beginning from an initial zero advertising equilibrium, each firm continues to advertise until the increased revenue no longer outweighs the additional advertising costs. In the shortrun, this advertising surely raises profits. However, these additional profits attract new entrants until profits equal zero in the longrun. If the advertising renders demand curves more elastic, the new advertising equilibrium will be at a lower price whereas if it renders demand curves less elastic, the final equilibrium will entail a higher price. Optimal second-best advertising policy takes this price effect into account, whereas Nash-competitors do not. In terms of the ice cream example, the perfectly competitive perfect ignorance equilibrium may be preferred to the perfect information monopolistically competitive equilibrium, eventhough the direct (shortrun) effect of improved information is to increase welfare.

V. Uncertainty and Brand Choice in Equilibrium

In this section, the effect of limited brand performance characteristics information on the consumer brand choice decision and the resulting monopolistically competitive equilibrium is studied.

In a Bayesian information-gathering framework, a consumer with a prior probability distribution function over a brand's performance characteristics - ℓ, summarized by the density $f(\ell)$, observes a set of n - signals $\sigma^n = (s_1, s_2, \ldots, s_n) \varepsilon S^n$ where each $s_j \varepsilon S$ varies according to the signal density $f(s|\ell)$, and then updates his prior by Bayes rule to form a posterior distribution of ℓ, summarized by the density $f(\ell|\sigma^n)$. If the consumer gathers information optimally, the optimal number - n of signals to observe has the property that the marginal benefit of further sampling is no greater than the marginal cost.

Armed with the posterior distribution density on each brand - i $f(\ell_i|\sigma_i^n)$, the consumer chooses the brand with the highest <u>ex ante</u> expected utility. If a consumer with a performance-characteristics preference parameter - x has an indirect utility function over performance characteristics - ℓ_i and known prices - p_i for each of the B - brands summarized by $U(x,\ell_i, p_i)$, then he chooses a brand according to

$$(5.1) \qquad \max_i \int U(x,\ell_i, p_i)\, f(\ell_i|\sigma_i^n)d\ell_i \qquad i = 1, 2, \ldots, B.$$

Rather than explicitly modeling the optimal learning process, we shall begin with a discussion of alternative unbiased posterior distributions. These distributions could have arisen from the interaction of optimizing behavior with particular signalling and performance technologies, or they could simply arise from satisfactory or arbitrary information-gathering decisions. By an <u>imprecise estimate</u>, we denote the case in which each consumer has an identical posterior density $f(\ell|\sigma)$

that yields a noisy, yet unbiased estimate of the true characteristics - ℓ, i.e $E(\ell|\sigma) = \ell$, var $(\ell|\sigma) > 0$. This case captures the intuitive notion of a consumer knowing only the "neighborhood" of a brand's performance characteristics. By a stochastic point-estimate, we denote the case in which the posterior distribution consists solely of a point-estimate of the performance characteristics, upon which the consumer acts. In this case, consumers who observe different signals $\sigma^n \in S^n$ form different point estimates, yet the point estimate distribution across consumers may still yield an unbiased estimate of the true parameter - ℓ. That is, if the point estimate distribution for brand - ℓ is given by $g(s|\ell)$, then it is unbiased if $\ell = \int s\, g(s|\ell)ds$. Thus, in the language of rational expectations, every consumer in the imprecise estimate case has noisy rational expectations, whereas in the stochastic point-estimate case, consumers as a group have rational expectations, though particular consumers' estimates are biased.

Either type of estimate can arise for rational consumers facing particular specifications of the performance characteristics, signalling technologies and information-gathering costs. For example, consider the example of wine-tasting prior to the purchase of a case of wine. Taste may in fact be fixed for every bottle in every case or may vary across bottles and/or cases. That is, the actual performance-characteristics may be fixed or stochastic. A particular glass tasted (the signal) may perfectly replicate the true taste of the particular bottle from which it was poured or be noisy and differ from the "true" taste according to the temperature of the glass or the interval between uncorking and tasting. From a single glass tasted, a consumer with a strong prior that every bottle and every glass is identical will form a posterior point-estimate; if in fact the signals are noisy or actual taste is stochastic, then the stochastic point-estimate case obtains. On the other hand if the consumer has prior beliefs that the signal or actual taste is noisy, then a single tasting yields an imprecise estimate. Multiple (infinite) tastings yield an imprecise estimate if the signal and actual taste are stochastic, though infinite tasting will yield perfect information if actual taste is fixed and each tasting is an unbiased estimate.

These various rationalizations of the derivation of the posterior distributions are of no concern in determining the consumer's ex ante brand choice. For that, the posterior can be taken as basic data. However, looking ahead to the welfare comparisons to be made, while the estimate should be used to evaluate consumers' ex ante welfare perceptions, the actual performance-characteristics (distributions) are more relevant for realized ex poste welfare comparisons. In the model to follow, both ex ante and ex poste comparisons will be made.

Now we derive the monopolistically competitive equilibrium when consumers make their brand choices on the basis of imprecise estimates. The model is the conventional spatial analogue to monopolistic competition. There are L - consumers with most-preferred characteristics - x distributed uniformly around the unit-circumference of a circle. Each consumer purchases one unit regardless of price.1/ We will solve for

1/ For the general model and the complications introduced by a maximum reservation price or elastic demands, see Salop (1976).

a symmetric-zero-profit-Nash equilibrium (SZPE) to be formally defined below. Basically, this equilibrium is a number of brands equally spaced in product space and a common price charged by each brand such that every brand earns zero profits.

Imprecise estimates are captured in the following way. Although the performance characteristic of a brand is actually located at some point - ℓ_i on the circle, each consumer treats this location as uncertain, specifically either $\ell_i + \theta$ or $\ell_i - \theta$, each with probability one-half.[1/] Thus, uncertainty is captured by use of a mean-preserving spread, as illustrated below, and increases in θ denote more consumer uncertainty.[2/]

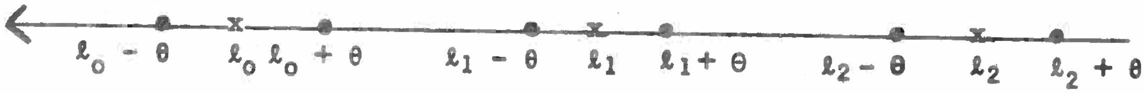


Figure 5.1: Actual and Perceived Locations

Each consumer chooses the brand that maximizes <u>ex ante</u> expected utility stated in (5.1). For example, consumer - x prefers brand - ℓ_1 to brand - ℓ_i if and only if

(5.2) $$\tfrac{1}{2}U(x, \ell_1 - \theta, p_1) + \tfrac{1}{2}U(x, \ell_1 + \theta, p_1) \geq \tfrac{1}{2}U(x, \ell_i - \theta, p_i) + \tfrac{1}{2}U(x, \ell_i + \theta, p_i)$$

where p_1 is the price of brand - i. If $p_1 = p_i$ and brands are equally spaced, then demand for brand - 1 consists of all those consumers in the interval $(\ell_1 - \hat{x}, \ell_1 + \hat{x})$ such that (5.2) is satisfied. Since consumers more preferred locations are distributed uniformly around the circle, the demand for firm - 1 is given by

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1/ Note that it is irrelevant for consumer choice whether $\ell_i \pm \theta$ represent imprecise estimates or true stochastic performance characteristics. However, the <u>ex</u> poste welfare judgements do depend on which assumption is adopted.

2/ See Rothschild-Stiglitz (1970) for the details of capturing uncertainty with a mean preserving spread.

(5.3) $$q_1 = 2L\hat{X}$$

The slope of demand is then given by

(5.4) $$\frac{\partial q_1}{\partial p_1} = 2L\left(\frac{\partial \hat{x}}{\partial p_1}\right)$$

where $\frac{\partial \hat{x}}{\partial p_1}$ is calculated from (5.2). The following "spatial" form of the utility function is convenient for deriving explicit results; therefore, let

(5.5) $$U(x, \ell, p) = u(|x - \ell|) - p, \quad u' < 0 \quad u'' \gtrless 0$$

where $u'' < 0$ denotes risk-averse and $u'' > 0$ risk-loving preferences in product space. Substituting (5.5) into (5.2) and defining $\hat{x}$ by equality in (5.2), we may calculate $\partial\hat{x}/\partial p_1$. At a symmetric equilibrium, we have

(5.6) $$\hat{x} = \frac{1}{2n}$$

where n denotes the number of brands and hence, the distance between brands on the unit-circumference circle is 1/n. At this symmetric equilibrium, we have

(5.7) $$\left.\frac{\partial \hat{x}}{\partial p_1}\right|_{\hat{x} = \frac{1}{2n}} = \frac{1}{u'(\frac{1}{2n} - \theta) + u'(\frac{1}{2n} + \theta)} < 0$$

Formally, a symmetric zero-profit equilibrium (SZPE) is a number of equally spaced brands - n and a price - p, such that the price - p is the best choice for each Nash price-setter, and profits at (p, n) equal zero for each brand. Thus, the maximum profit point for each brand earns zero profits; a necessary condition for this is that the demand curve is tangent to the average cost curve at the equilibrium. Since marginal costs are constant, the slope of the average cost curve at $\hat{x} = \frac{1}{2}\frac{1}{n}$, $q = L/n$ is given by

$$(5.8) \qquad \frac{\partial AC}{\partial q} = -\frac{F}{q^2} = -\frac{F}{L^2}n^2$$

Substituting (5.7) into (5.4), we have the slope of the demand curve,

$$(5.9) \qquad \frac{\partial p_1}{\partial q_1} = \frac{1}{2L}\{u'(\frac{1}{2n} - \theta) + u'(\frac{1}{2n} + \theta)\}$$

Setting (5.8) equal to (5.9), we have the equilibrium number of brands given by

$$(5.10) \qquad u'(\frac{1}{2n} - \theta) + u'(\frac{1}{2n} + \theta) = -\frac{2F}{L}n^2$$

Differentiating (5.10) totally, the effect of an increase in uncertainty - θ on the equilibrium number of brands $n \equiv n(\theta)$ may be calculated as follows.

$$(5.11) \qquad n'(\theta) = -\frac{1}{T}\{u''(\frac{1}{2n} + \theta) - u''(\frac{1}{2n} - \theta)\}$$

where

$$(5.12) \qquad T = 2n - \frac{1}{2n^2}\{u''(\frac{1}{2n} + \theta) + u''(\frac{1}{2n} - \theta)\}$$

Therefore, $T > 0$ for risk-averse consumers ($u'' < 0$), but

$$(5.13) \qquad n'(\theta) \underset{<}{\geq} 0 \iff u''' \underset{>}{\leq} 0$$

It is easy to check that $u''' \underset{<}{\overset{>}{=}} 0$ determines whether the price elasticity rises, stays constant, or falls as θ rises. Thus, (5.13) expresses the elasticity effect in (3.7) and (3.8).

One ex ante welfare measure $W(n(\theta), \theta)$ is simply the sum of consumer utilities. Since there are 2n intervals of length $\hat{x} = \frac{1}{2n}$ with L - consumers in each, we have

$$(5.14) \quad W(n,\theta) = 2nL \int_0^{\frac{1}{2n}} \{\tfrac{1}{2}u(|x - \theta|) + \tfrac{1}{2}u(|x + \theta|)\}\, dx$$

Eliminating absolute values and transforming the integrand and limit, we have

$$(5.15) \quad W(n,\theta) = nL \int_0^{\frac{1}{2n} - \theta} u(y)dy + nL \int_0^{\frac{1}{2n} + \theta} u(y)dy$$

Differentiating (5.15) with respect to n and θ, we have

$$(5.16) \quad W_n = n\{\frac{W(n\ \theta)}{L} - \{\tfrac{1}{2}u(\tfrac{1}{2n} - \theta) + \tfrac{1}{2}u(\tfrac{1}{2n} + \theta)\}\}$$

$$(5.17) \quad W_\theta = nL\{u'(\tfrac{1}{2n} + \theta) - u'(\tfrac{1}{2n} - \theta)\} < 0$$

The interpretation of these conditions is straightforward. W_n is proportional to the difference between the average utility per consumer and the utility of the marginal consumer $\frac{1}{2n}$. If an extra brand were added at the margin, marginal consumers become average consumers at each firm with the gain in welfare indicated by (5.16). $W_\theta < 0$ expresses the change in the deadweight loss as θ - rises. It is interesting that it does not depend on the risk-preference of consumers; this is because consumers in the neighborhood of a brand lose regardless of risk-preference, more than offsetting any risk-gains to risk-loving more distant consumers.[1/]

Finally, the first-best certainty number of brands - $\hat{n}$ may be calculated from (5.16), as the $\hat{n}$ such that

(5.18) $$W_n(\hat{n}, 0) = F$$

Now, these equations may be substituted into Condition I to calculate the possibility of improving welfare with second-best-policy - θ.

Examination of (5.17) and (5.13) reveals that the size of dead-weight loss depends on the magnitude of u' while the elasticity effect price and number of brands depends on the magnitude of u'''. Whether there are too many or too few brands depends on W_n which itself depends on u. Thus, even with the spatial, separable form of the utility function used, conclusions regarding the optimality of the second-best policy depend on the entire share of the utility function. All that can be concluded from these derivatives in conjunction with Condition I is that if the perfect information equilibrium - n(0) entails too many brands ($n(0) > \hat{n}$), then u''' must be positive and large relative

1/ That is consumer $x = \ell_1$ faces an $x - \ell_1 = 0$ when $\theta = 0$, but $x - \ell_1 = \theta$ for $\theta > 0$. Thus, "close" consumers lose regardless of risk-preference.

to u', whereas for too few brands at equilibrium, u''' must be negative and large relative to u'. 1/ Analysis of the stochastic point estimates case is similar, though equilibrium prices are not identical.

VIII. Conclusions

In this paper we analyzed the effect of uncertainty and imperfections in performance-characteristics information across differentiated brands in an imperfectly competitive industry. It was shown that although better information surely has private value for every risk-averse consumer, improvements in information also change the elasticity of the brands' demand curves. Comparing the price effects of the elasticity change to the direct private welfare effects, it was shown that aggregate welfare may rise or fall according to the shape of the consumers' preferences. In particular, if improved information creates more inelastic demand for each brand, then aggregate *ex ante* welfare may fall.

The driving force of the analysis is the fact that the allocation of resources under imperfect competition is not generally optimal. Therefore, as a second-best solution, adding a second distortion to the allocative process, imperfections of information or uncertainty here, may offset rather than magnify the first distortion. The notion also found application in an example of product selection in the misperceptions example and non-price competition in general.

1/ Actually, it is even more complicated than this. Since $n'(0) = W_\theta(n(0), 0) = 0$, then $\theta = 0$ is local maximum or a local minimum. The second-best policy is preferred at local minimum, or $W_n n''(\theta) + W_{\theta\theta} > 0$. $W_{\theta\theta} < 0$ with magnitude proportional to u'', whereas $n''(\theta)$ according to u''' as before.

The most serious shortcoming of the paper seems to be in the specification of preferences. A separable utility function in price and "distance" was used throughout the analysis, and even within that special case, the results depended on the relative magnitudes of the first and third derivations of the utility functions. This is surprising, for one might have expected that in the evaluation of welfare under uncertainty, the degree of risk aversion, that is the second derivative, would have played a crucial role. Instead the welfare loss was proportional to the first derivative and the price effect was proportional to the third derivative.

It is difficult to interpret these results because so little is known about the properties of indirect utility functions of price and "distance". This function should be derived directly from consumer preferences over commodity attributes, the technology of "home production" of these attributes from purchased commodities and the budget constraint. A priori, few conclusions may be drawn. For example, consider the preferences of a coffee drinker. Suppose he most prefers one lump of sugar in his coffee at current prices. Suppose he hates sugarless coffee but may be fairly content with two lumps. Yet, if extra sugar (or perhaps a sweet dessert) is also available, then even if he prefers two lumps to zero lumps, he may purchase sugarless coffee and add sugar at additional cost rather than consume overly sweet two-lump coffee. This makes risk-preferences very complex to derive. For example, faced with a lottery of sugarless coffee or four lumps versus two lumps with certainty, a risk-averse consumer may still prefer the lottery since sugar may be added but not subtracted. Moreover, free disposal of terrible coffee may lead to a non-concavity in a risk-averse consumer's indirect utility function. Clearly more work is needed on this aspect of the problem.

References

1. Butters, G. (1977), "Equilibrium Distributions of Sales and Advertising Prices," Review of Economic Studies, (October 1977)

2. Diamond, P. (1977), "Welfare Analyses of Imperfect Information Equilibrium," Bell Journal of Economics, forthcoming.

3. Dixit, A. K. and J. E. Stiglitz (1977), "Monopolistic Competition and Optimum Product Diversity," American Economic Review, 67 (June 1977), pp. 297-308.

4. Lancaster, K. (1975), "Socially Optimal Product Differentiation," American Economic Review, 65, (September 1975), pp. 567-585.

5. Rothschild, M. and J. E. Stiglitz (1970) "Increasing Risk: A Definition," Journal of Economic Theory, 2, (September 1970) pp. 225-43.

6. Salop, S. C. (1976), "Monopolistic Competition Reconstituted," Federal Reserve Board.

7. Salop, S. C. (1976a), "Information and Monopolistic Competition," American Economic Review, 66, (May 1976), pp. 240-245.

8. Salop, S. C. and J. E. Stiglitz (1977), "Bargains and Ripoffs: A Model of Monopolistically Competitive Price Dispersion," Review of Economic Studies, (October 1977)

9. Spence, A. M. (1976), "Product Selection, Fixed Costs and Monopolistic Competition," Review of Economic Studies, 43, (June 1976) pp. 217-36.

10. Von zur Muelen, P. (1976), "Sequential Search and Price Dispersion in Monopolistic Competition," Federal Reserve Board.

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