THE LONG RUN EVOLUTION OF A RATIONED EQUILIBRIUM MODEL

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Introduction

In his recent book "The Theory of Unemployment Reconsidered" Malinvaud {1977} presents a model in which prices and wages are fixed and equilibria are achieved by quantity adjustments in the goods and labour markets. These adjustments are effected by rationing schemes. This model, although it provides a framework for macroeconomic analysis, has the drawback that it is wholly static. The notion that prices and wages are completely rigid in the short run is acceptable as an idealisation of the real situation. However, in the longer term market forces must influence prices and wages, and the latter will respond to excess demands and supplies in different markets. Our aim is to specify such an adjustment process, which will be slow relative to the quantity adjustments in each market, and then to examine the evolution and stability of prices and wages. Although it is possible with suitable assumptions to obtain stability results, the framework inherited from Malinvaud and developed by Hildenbrand & Hildenbrand {1976} is not a wholly satisfactory one in which to study the dynamics of prices and wages. In particular the simple imposition of a plausible adjustment process on the Malinvaud model does not produce the sort of phenomena one might naturally expect to observe. This raises some interesting, and we think, fundamental problems, which will not be described here, but will be the subject of a later paper by one of us.

The Model

The model we shall use in this paper is a "generalized" Maulinvaud model presented by K.Hildenbrand and W. Hildenbrand in their discussion paper "On Malinvaud's "Reconsideration of the Theory of Unemployment". For a general presentation of this model the reader is referred to that paper. Here we shall state only the main features of the model.

There are three goods in the economy: a consumption good, labour, and money. The price of the good (labour) is denoted by p,(w), while the price of money is 1. There are N consumers. Consumer a is endowed with some initial moneyholdings e_a. Considering a continuous time model the consumer must then at each point in time decide, how much he wants to supply of work (l_a), how much to consume (q_a), and how much money (m_a) to carry forward to the next period. This decision is derived from a multiperiod utility maximization (given the budget constraint), which is based on the consumer's expectations on quantity rationing and prices on the two markets. We shall here adopt the Hildenbrand notation:

Let (p,w) be given. If the consumer expects no rationing at all in the current or future periods, his decisions (i.e. his notional demand (supply) functions) are denoted $q_a^*(p,w), l_a^*(p,w)$, and $m_a^*(p,w)$.

If the consumer expects no rationing on the good market, and he is in the current period unrationed on the labour market, we denote his demand (supply) functions by

$$q_a(p,w),\ l_a(p,w),\ \text{and}\ m_a(p,w).$$

If he is rationed on the labour market in the current period, which means that he is completely unemployed, the functions are denoted

$$\bar{q}_a(p,w), \bar{l}_a(p,w), \text{ and } \bar{m}_a(p,w).$$

Rationing on the good market is assumed to be described by an upper bound x on the amount of the good available to the consumer. So corresponding to the two cases above, if the consumer is rationed on the good market in the current period (i.e. $x < +\infty$), we get the functions

$$q_a(p,w,x), l_a(p,w,x) \text{ and } m_a(p,w,x)$$

and

$$\bar{q}_a(p,w,x), \bar{l}_a(p,w,x) \text{ and } \bar{m}_a(p,w,x).$$

The production side of the economy is described by an aggregated production function F, relating labour input (z) to output of good. The assumptions on F are:

$$F(0) = 0, F'(z) > 0, F''(z) < 0$$

Let $y^*(p,w)$ denote the profit maximising output, given the price and wage rate. Finally we denote the exogenous (i.e. government) demand in the economy by g.[1/]

1/ For a discussion of the way this is financed, see Malinvaud {1977}, p.40.

Equilibrium concepts

With prices and wages fixed the Walrasian equilibrium, where excess demand equals zero on all markets, becomes a very special situation. On the other hand a model where adjustments take place only in quantities seems to call for a new equilibrium concept, which specifically takes into account the possibility of rationing on one or both markets. Three types of equilibria are considered here:

A: The Keynesian equilibrium, where consumers are rationed on the labour market, and producers are rationed on the good market.

B: The Classical equilibrium, where consumers are rationed on both markets.

C: The Repressed inflation equilibrium, where consumers are rationed on the good market, and producers are rationed on the labour market.

(For precise definitions see below).

By u we denote the unemployment rate, i.e. $(1-u)$ is the ratio of actual employment to effective labour supply.

The rationing scheme on the labour market is assumed to be such that a consumer is either fully employed or totally unemployed. By allowing consumers to differ in characteristics, we shall have to assume that unemployment is uniformly distributed over that part of the population, who wants to work. For N large (i.e. many consumers of each type of consumption characteristics) the mean effective labour supply is then independent of which particular individuals, who are rationed.

Let μ denote the distribution of consumer characteristics. Corresponding to the demand (supply) functions defined above, we define the mean demand (supply) functions by

$$\bar{q} = \int_N \bar{q}_a d\mu, q = \int_N q_a d\mu, l = \int_N l_a d\mu, \text{ and } l^* = \int_N l^*_a d\mu.$$

We then have the following definition.

Definition:

Let the price level p, the wage rate w, and the exogenous demand g be given.

A. $(p,w,g,y,u,x), x = \infty$ (i.e. consumers not rationed on goods market) is called a Keynesian equilibrium, if

1^o $y = g + u\bar{q}(p,w) + (1-u)q(p,w)$

2^o $F^{-1}(y) = (1-u)l(p,w)$

where y is production and u the corresponding unemployment rate.

B. $(p,w,g,y,u,x), y = y^*$ is called a <u>Classical</u> equilibrium, if

3^o $y^*(p,w) = g + u\bar{q}(p,w,x) + (1-u)q(p,w,x)$

4^o $F^{-1}(y^*(p,w)) = (1-u)l(p,w,x)$

where x is the rationing level and u the unemployment rate.

C. $(p,w,g,y,u,x), u = 0$ is called a <u>Repressed inflation</u> equilibrium, if

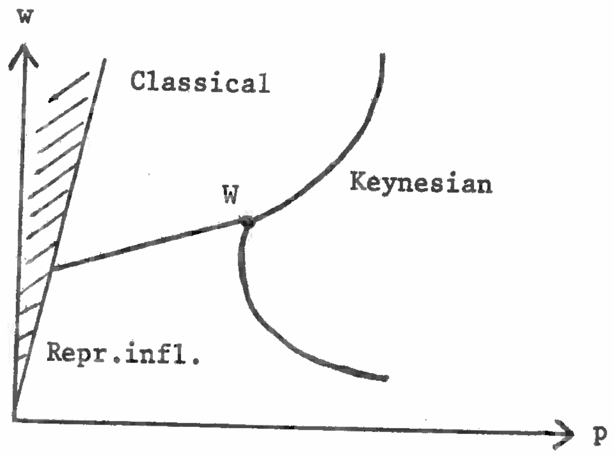
5^o $y = g + q(p,w,x)$

6^o $F^{-1}(y) = l(p,w,x)$

where y is production and x the rationing level.

<u>A long run Adjustment Process</u>

For fixed g and μ we get the (p,w) plane divided into regions of either Keynesian, Classical, or Repressed inflation equilibrium. The following picture is presented in the Hildenbrand paper. (W denotes the Walrasian equilibrium).



Fixing (p,w,g), adjustments in only (y,u,x) are possible. In this section we assume that adjustments in (y,u,x) do take place. In fact we shall assume that these adjustments are infinitely fast, i.e. we can concentrate attention on the resulting equilibrium values. So for given (p,w) the state of the economy will be an equilibrium point of one of the three types mentioned above, depending on the region to which (p,w) belongs. This sort of assumption is frequently implicitly made in economics. As we have said, the assumption made in all the literature on rationed equilibrium theory that prices and wage rate remain fixed, is only reasonable in the short run, where it might be argued that quantities are more likely to perform the adjustments. But in the long run excess demands and excess supplies will tend to force changes in p and w. As we in this section are looking for a description of the evolution of the economy in the long run, we shall now have to introduce a precise formulation of the variation in prices and wages through time.

Before we proceed, a remark on μ is necessary. We have assumed the distribution of consumer characteristics to be fixed, which is an acceptable idealisation, except for the consumers' moneyholdings. As soon as the consumers engage in trades, the distribution of money will change. The fixed - μ assumption therefore represents a rather peculiar taxation system: at the end of each period the government redistributes the money such that the original money distribution is obtained. Of course this is not a satisfactory description, and it will be omitted in a forthcoming paper. Here we shall keep it for simplicity.

As the economic conditions of the agents vary with the regions, it is reasonable to expect different descriptions of the adjustments in the three regions. It should be stressed again that these adjustment processes are considered to be slow compared to the above adjustments in the quantities. First we now state the three processes, and then we discuss their contents and plausibility.

Keynesian region :

$$\dot{p} = K_p((1-u)q(p,w) + u\bar{q}(p,w) + g - y^*(p,w)) \qquad K_p \in \mathbb{R}_+$$

$$\dot{w} = K_w(F^{-1}(y(p,w)) - 1^*(p,w)) \qquad K_w \in \mathbb{R}_+$$

Classical region :

$$\dot{p} = C_p((1-u)\tilde{q}(p,w) + u\tilde{\bar{q}}(p,w) + g - y(p,w)) \qquad C_p \in \mathbb{R}_+$$

$$\dot{w} = C_w(F^{-1}(y(p,w)) - 1^*(p,w)) \qquad C_w \in \mathbb{R}_+$$

Repressed inflation region :

$$\dot{p} = R_p(\tilde{q}(p,w) + g - y(p,w)) \qquad R_p \in \mathbb{R}_+$$

$$\dot{w} = R_w(F^{-1}(y^*(p,w)) - 1(p,w)) \qquad R_w \in \mathbb{R}_+$$

where $\tilde{q}$ $(\tilde{\bar{q}})$ denotes the mean demand function from the employed (unemployed) consumers, which will result, when the consumers ignore the actual rationing on the good market.

The first thing to note is that we consider the adjustment processes to take place in continuous time, so they may be formulated by differential equations. As p and w vary, so do y and u as described by the equilibrium equations in the section above. Let us first consider the process in the Keynesian region.. $\dot{p}$ is defined to depend on the difference between the total actual demand from consumers plus government and the optimal production, while $\dot{w}$ depends on the difference between actual employment and optimal supply of labour from the consumers. These equations express the following viewpoint: Given p and w, the producers want to sell $y^*(p,w)$. When the effective demand is less than y^*, they will decrease the price according to an excess supply of the good. On the other hand, the wage rate is set by negotiations between workers and producers, and the workers will tend to accept increases or decreases in the wage rate depending on the excess demand on the labour market.

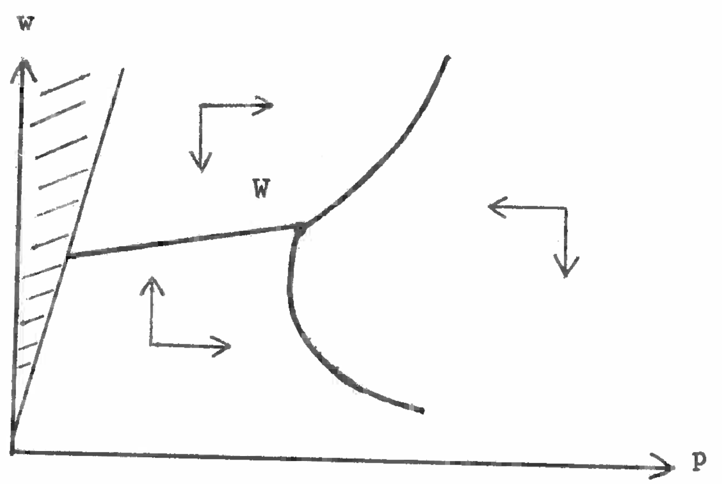
Next we look at the process in the Classical region. In this case $\dot{p}$ depends on the difference between the demand the consumers would have, if they ignored any rationing on the good market, and the actual production (which is equal to the optimal production, as the producers are not rationed at all). The $\dot{w}$ process is as in the Keynesian region. The idea behind this formulation of the $\dot{p}$ process is as follows: when consumers are rationed on the good market, it implies that total demand cannot be fulfilled. Even though consumers, due to this fact, have to take this restriction into account, when making their plans, it is the unrestricted demand which in the long run makes the pressure on the market.

Finally, consider the process in the Repressed inflation region. The $\dot{p}$ process is as in the Classical region with $u = 0$, as the consumers are unrationed on the labour market, but still rationed on the good market. The $\dot{w}$ process depends on the difference between the optimal employment by producers and the actual supply of labour by the consumers. This formulation reflects the viewpoint that when producers are rationed in the labour market that is to say, when they are unable to produce the optimal output, they will try to persuade the consumers to offer further work by increasing the wage rate.

Long run Stability

In this section we shall discuss the kind of stability result which may be obtained from the above adjustment process.

It follows immediately from the definition of the three types of disequilibrium that the sign of the changes $(\dot{p},\dot{w})$ can be represented in the following diagram:



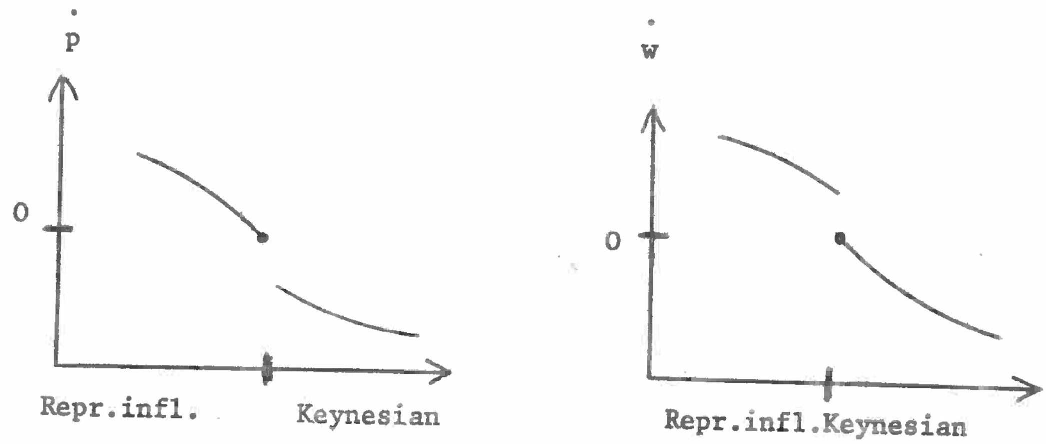
We shall now have to discuss what happens on the boundaries between the three regions:

On the boundary between the Keynesian and the Classical region y is equal to y^*, and $\tilde{q} = q$ $(\tilde{\bar{q}} = \bar{q})$, as the rationing on the good market disappears on this boundary. So this implies that the direction of the vector field describing the adjustment process is continuous on this boundary.

Similarly, as $y = y^*$ and $l = l^*$ on the boundary between the Classical and the Repressed inflation region, the direction of the vector field describing the adjustment process is continuous on this boundary.

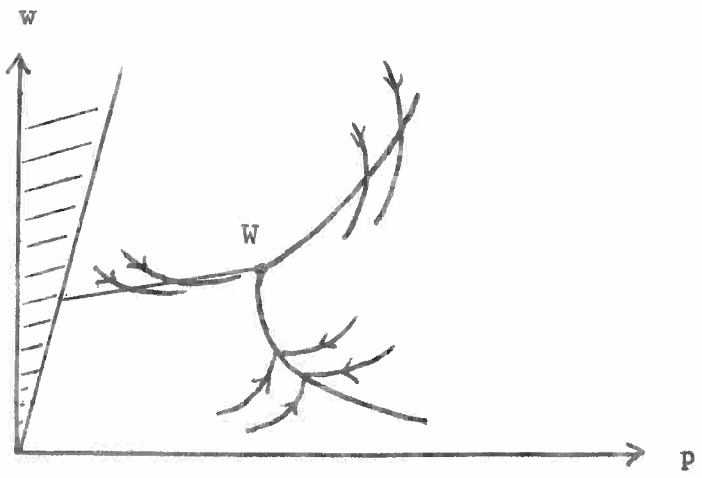
On the other hand we have the following picture for the boundary between Repressed inflation and Keynesian equilibrium:

2/



2/ Behaviour on this boundary is discussed in detail by Malinvaud {1977} pp.103-104.

which altogether leaves us with the following qualitative picture:



We shall therefore have to define the adjustment process on this boundary. On the boundary the consumers are not rationed at all, while producers are rationed on both markets, as they produce less than they want to. If we look at a sequence of points on this boundary, converging to the Walrasian equilibrium, we notice that consumers stay unrationed all the time, while the producers get still "closer" to the optimal production. If then we were to make the inviting assumption that the movement on this boundary was toward the Walrasian equilibrium, we would be able to proceed as follows. Moving from the Keynesian region and supposing it were possible to cross the boundary, a point would immediately get caught by the vector field in the Repressed inflation region, and this would take the point upwards towards the boundary. An analogue argument is available, when crossing the boundary from the Repressed inflation region. So we would have the following result:

Theorem

Suppose the long run adjustment processes in p and w are described by the differential equations above. If the price-wage combinations on the boundary between Repressed inflation and Keynesian equilibrium move towards the Walrasian equilibrium, the Walrasian equilibrium is globally stable.

However, we are then led to ask, is the qualitative picture obtained by introduction of this assumption in conformity with what one might expect the evolution of the model to be? The answer must, as but, be only a qualified yes. An obvious and important feature that would be lacking in the model would be cycles from the Repressed inflation to the Keynesian region, indeed it has been argued that this is the basic feature of the evolution of modern economies. One possibility would be to suggest that such cycles are generated by changes in autonomous or exogenous variables. Changes in g, for example, move the boundaries between the regions and would produce such cycles. This would, however, suggest that the economy left to its own devices would not exhibit such cycles and would move to the Walrasian equilibrium.

Although the reader will draw his own conclusions, we would suggest that the assumption on the behaviour of price and wage changes on the Keynesian - Repressed inflation boundary is one of convenience and avoids the real question raised by the apparent discontinuity on this line. We would further suggest that far from being of a technical nature, the difficulties presented in defining suitable adjustment processes on either side of this boundary suggest that the model as it stands, is not a satisfactory vehicle for a true dynamic analysis and needs fundamentally reformulating for this purpose.

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