ON THE CAPITALIZATION HYPOTHESIS FOR LOCAL PUBLIC FINANCE*

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I. Introduction.

The idea that the value of local public goods projects will be capitalized into land rents has a long tradition in the public finance literature, dating back at least as far as Tiebout (1956). Empirical evidence has been mixed, but some positive results have been reported by Oates (1969) among others. This simple idea is potentially very powerful. It provides a simple measure of project benefits which reveals (if perhaps after the fact) individual preferences for public goods. And if rent changes can be predicted in advance, it may provide an objective function for community decision making.

Furthermore, the presence of capitalization forces has important equity implications. To the extent that capitalization occurs, benefits tend to accrue to landlords at the expense of renters. We will show that capitalization reflects a real welfare increase only if these two groups are considered equally deserving. If, on the other hand, renters are considered more deserving, then the presence of capitalization clearly will worsen the distribution of benefits within society.

The 'Capitalization Hypothesis' is interesting from a purely theoretical point of view as well. It is clear that increases in land rent values are not themselves a net social benefit; owners of the land will benefit, but renters will suffer. Indeed, in the standard surplus theory of welfare measurement, changes in land prices (or prices in any other competitive market) never appear as net benefits (in aggregate, the gains to winners just offset losses to losers). Thus, if capitalization occurs, it must happen because the true benefits *induce* a corresponding change in rents.

We will argue that there are two separate and independent forces which can induce capitalization, one involving competitive forces *between* communities, and one involving such forces *within* a particular community. Furthermore, these two forces have quite different implications for (1) what benefits are capitalized and (2) where those benefits are capitalized. Thus, we will derive several different capitalization theorems and explore the conditions under which each holds. In the process, we will find some conditions under which capitalization does not occur at all or occurs only partially.

Let us refer to capitalization which derives from forces between communities, *external capitalization*, and capitalization which derives from forces within communities, *internal capitalization*. The intuitive argument for external capitalization is quite simple. Suppose that a project is built in one community designed to make people better off there. Now if people in all communities have similar tastes and are free to move among the communities, outsiders will necessarily be attracted to the project-building community. And they will continue to move until the welfare incentive disappears. The only factor which can stop this movement is a differential location cost, that is, an increase in land rents in the project-building community. This type of argument is the one used by Polinski and Rubenfeld (1976) to justify a form of capitalization. However, the Polinski/Rubenfeld model is not closed since the benefits do not accrue to land owners. Increases in land values will generally overstate true net *social* benefit, although they may be a good approximation to *community* net benefit.

Naturally, when we drop the assumption of free mobility between communities or the assumption of similar tastes between communities, the intuition for external capitalization is weaker. Indeed, we will show that when either assumption is relaxed slightly, external capitalization (for small projects) may disappear completely.

The argument for internal capitalization is quite different in that it is based on specific properties of local public goods.

Presumably public goods are local in character when proximity matters. For example, one has to make trips in order to appreciate such public goods as parks, museums, civic centers, sports complexes, and even roads and highways. Now, if there is a positive correlation between the amounts of public goods and the differential desirability of various locations, some degree of capitalization will occur as rents adjust. We will show later exactly what conditions are necessary for full capitalization. This type of argument has been used in the literature to justify the use of rent gradients for measuring the cost of pollution near a source (such as an airport or factory).[1] It is also in the spirit of arguments given for capitalization given by a number of authors, among them Strotz (1968), Lind (1974) and Pines and Weiss (1976). These authors essentially start with the assumption that a project will differentially improve land quality and explore whether or not those improvements will translate into rent increases. The results are instructive, but the underlying approach seems to beg the question somewhat. The primary question is: Will a local public project change the differential quality of land? For the case of irrigation projects discussed by Lind, the answer is clearly yes, but in other cases it is less obvious. Indeed, we will show that for some types of public goods, no capitalization occurs at all.

We will want to pay attention to the system of taxation in discussing both types of capitalization. One might guess that the form and nature of capitalization will depend on whether or not the public goods are financed out of a property tax. We will show that this is sometimes, but not always, the case.

The remainder of the paper will be divided into two parts. In the next section, we present a 'bare-bones' model designed to capture the essence of the local public goods situation. For this model we derive a reasonably exhaustive set of capitalization (and

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1. For an example of this argument in the literature, see Freeman (1971).

non-capitalization) results and explore the relationships among them. Then, in the last section we introduce a reasonably general model and prove two theorems, one on external capitalization and one on internal capitalization.

II. A Prototype Model.

Throughout this section we deal with a model in which there is a single local public good (q) being produced independently by a number of communities. Further, we will suppose that projects initiated in a community will not affect private goods prices other than land rents. This may seem a serious restriction, but it is not, really, since changes in other prices will have the usual cancelling effects on the two sides of the market. With non-land private goods prices fixed, we can aggregate all these goods into a single commodity and let this commodity be the numeraire in what follows.

Agents must engage in effort (take trips) to enjoy the public good. Naturally, such trips are costly; here we will assume that the expense can be represented by a simple cost function in terms of the numeraire: $f(g,s)$, where g is the number of trips taken, and s stands for location within the community. Location will be specified by dividing the community's land up into discrete zones and assuming that location can adequately be specified by designating the appropriate zone.

Each consumer gets to choose a zone and an amount of land within the zone (ℓ). The market for land will be assumed competitive and r_s will stand for the rental rate in zone s.

Consumers earn income in terms of the numeraire. We will want to separate this income up into three terms: profit shares (Π), rental income on land (R) and other income (mainly labor): (Y). Thus, an individual in income class i has income $I^i = \Pi^i + Y^i + R^i$. Such an individual will be required to pay taxes T^i. T^i may be a function of other parameters depending on the nature of taxation.

A consumer of preference type a has a preference function of the form:

$$U^a(q, g, \ell, I^i - T^i - f(g,s) - r_s \ell) .$$

The arguments of this function are the level of public goods provision, the number of trips taken, the amount of land consumed, and the amount of other private goods consumed. It seems appropriate that both g and q should affect utility. For example, if the public good is a museum, agents will care about the size of the museum as well as the number of visits made.

Each consumer in a particular community gets to choose $g, \ell,$ and s. (At a later stage such a consumer may be able to choose a community of residence as well.) We will assume throughout that there is a free mobility *within* the community, so that s is a free choice. It is analytically convenient to think of each resident as making a conditional choice of g and ℓ for each s. This process generates a first stage optimization problem:

$$(2.1) \qquad \max_{g\ell} U^a(q, g, \ell, I^i - T^i - f(g,s) - r_s \ell).$$

A solution to problem (2.1) defines demand functions:

$$g_s^{ai} \equiv g^a(q, I^i - T^i, r_s, s)$$

$$\ell_s^{ai} \equiv \ell^a(q, I^i - T^i, r_s, s)$$

and an indirect utility function

$$V_s^{ai} \equiv V^a(q, r_s, I^i - T^i, s) \ .$$

At a later 'stage', the consumer makes a discrete location choice, seeking to

$$\underset{s}{\text{Max}} \ V_s^{ai}$$

We will use the notation V_s^i to denote the final indirect utility function.

The remainder of the community economy consists of firms, about which we need to say little except that they behave competitively and are not affected by the public goods, and the government which produces the public good from private goods, paying for those goods with revenues from the taxes levied on households. Letting $\Gamma(q)$ stand for the public goods cost function, the government must satisfy a balanced budget condition:

$$T = \Gamma(q)$$

where T stands for total tax revenue.

We assume that the private sector is always in market equilibrium. For non-land commodities, this simply means that markets clear at fixed (unchanging) prices. For land, it means that rental rates must adjust until the market for land clears in each zone.

We will formulate questions about capitalization as follows. We measure welfare using a standard Bergson-Samuelson formulation. We ignore distributional considerations by assuming that the distribution of income is initially optimal (such an assumption is clearly necessary for pure forms of capitalization). Next, we consider an initial situation and a proposed new project. Some form of capitalization occurs when there is a correlation between the resulting welfare change and the associated rental change. We will see that this correlation can take many forms.

All precise statements about internal capitalization requires some boundary condition on community rents. This boundary condition will take different forms depending on the structure of the town and the organization of taxation in the town. We will discuss two different types of communities in this regard. In the first type of community the boundaries will be considered potentially variable, with 'outside' land always available to the community at an exogenously given opportunity cost; the boundary rent is thus exogenously given in such a community. The real prototype for such a community would be a town or metropolitan area located in the middle of farmland. We will refer to such communities as *isolated*.

In the second type of community, the boundaries are predetermined (presumably by legal arrangement). The effect of a project on boundary rent will be determined by changes (if any) in the aggregate demand for land in the community. The real prototype here would be a sub-community within a larger metropolitan area. We will refer to such communities as *adjacent*.

The local public goods model being proposed here seems most applicable to the case of isolated communities since it is difficult to ignore the importance of direct spillovers among adjacent communities. However, it still seems useful to treat the adjacent case.

To demonstrate that there are separate forces generating external and internal capitalization, we will need to show that it is possible to have each without the other. We will begin with a discussion in which we assume that no external forces are present; that is, we assume that community policy will not lead to any net migration to or from the community. Within this context, we explore the conditions for internal capitalization. We derive alternative sets of assumptions under which there is no capitalization or full capitalization.

Then, we will return to the issue of external forces. We will give these forces their best chance to work by assuming *free* mobility *between* communities. To isolate the effect of these forces, we will adopt a model of the community which is consistent with no-internal-capitalization. Within this context, we again exhibit conditions under which there is either no capitalization or full capitalization.

Later, in the section on general models, we will argue that when both types of forces are present, those of external capitalization tend to dominate; that is, the type of capitalization consistent with external forces will prevail in those circumstances.

A. Internal Capitalization.

Assuming away migration effects, there are essentially two ways in which project benefits can get translated into increased land values: through a change in the *extensive margin* or the *intensive margin*. If a project increases the aggregate demand for land, it may lead to an increase in the general level of rents (by operating on the extensive margin for land). Intuitively, this force could go either way. When taxes are raised (to pay for a new project) there is likely to be a substitution away from goods (including land). On the other hand, when public goods provision is increased, there is likely to be a complementary increase in the demand for land. Since the net effect on rents is clearly ambiguous, there is no reason to expect systematic capitalization from impact on the extensive margin. In fact, we will argue that the extensive margin is likely to be of no importance especially in isolated communities.

However, the project may affect the differential value of locations and thereby influence the rent structure through the internal margin. We will show that this force may lead to systematic capitalization in relatively large communities, within which there is a wide range of choice regarding use of the public goods. Restricting this range of choice can lead to any intermediate case from full capitalization to no capitalization.

B. Sufficient Conditions for No-Internal-Capitalization.

The essential restriction which is needed to rule out internal capitalization is that there be *no* effective differential choice in the use of public goods within the community. A number of sets of assumptions will do for this purpose.

The easiest case to explain is one in which there is no differential choice within the community at all. By this, we mean that $g = \overline{g}$ and $\ell = \overline{\ell}$ for all agents in all zones; these conditions would hold if the plot size is institutionally given and the public good has the characteristics of national defense so that residents get the benefits regardless of their actions.

A resident of preference type a and income class i now chooses s (the only remaining choice variable) to

$$\max_{s} U^a(q,\overline{g},\overline{\ell},I^i - T^i - f(\overline{g},s) - r_s\overline{\ell})$$

Clearly, this problem reduces for every agent to one of minimizing costs:

$$\min_{s}[f(\overline{g},s) - r_s\overline{\ell}] \ .$$

Since the costs to be minimized are the same for all agents, the only possible equilibrium rent structure must be one which makes those costs independent of s. (At such a cost structure, everyone is obviously indifferent as to where they live.). Now this independence condition must be true both before and after the project is initiated. It follows that the derivative of costs with respect to q must be independent of s. But with g and ℓ fixed this means that $\frac{d}{dq}r_s$ must be independent of s; that is, if rents change at all they must change in a uniform way. However, rents at the boundary cannot change: there is no change in the aggregate demand for community land, so the boundary rent will not change in either isolated or adjacent communities. Hence there is no capitalization.

We can introduce further choice into the community and obtain the same results as long as we are willing to restrict the degree of diversity and the degree of complementarity of tastes. For example, suppose that tastes are separable and similar in that preferences of all agents can be represented in the form:

$$U = \hat{U}^1(q,g) + \hat{U}^2(\ell) + I^i - T^i - f(g,s) - r_s\ell ,$$

where i indexes incomes class, as before. Now, if it should turn out that the optimal choice of g is independent of s (so that there is no effective differential choice within the town) we can again argue that there will be no capitalization. Clearly agents still make the same choices regardless of income class. Thus, rents must still adjust so that everyone is indifferent as to where they live (V_s^i is independent of s). This condition must be true both before and after the project so $\frac{d}{dq} V_s^i$ must be independent of s. Differentiating and using the envelope theorem, this condition becomes that

$$(2.2) \qquad \frac{d}{dq} \hat{U}(q,g_s) + \ell_s \frac{dr_s}{dq}$$

is independent of s . But since we assumed that g_s was independent of s , the first term of (2.2) in independent of s and we are back to the statement that the rent structure must change uniformly if at all. In fact, it cannot change at all, because there is no change in the extensive demand for land (given separable preferences).

C. Sufficient Conditions for 'Full' Internal Capitalization.

Internal capitalization can take a number of different forms depending on specific assumptions made concerning the boundary conditions, the type of taxes imposed, and the nature of property ownership. We will delineate these cases as we go along. As will be seen, all of the propositions reported here rely exclusively on the intensive land margin to 'enforce' capitalization. There may be some very special functional forms which will generate systematic capitalization from the extensive margin, but these would seem too special to be of much interest.

There are two principle assumptions which taken together guarantee that project benefits will be translated to land rents through the intensive margin. The first is that all the benefits of the project must be 'intramarginal' in that boundary residents are marginally unaffected. We would expect this condition to hold if marginal residents choose $g = 0$ (so that there is a complete revealed range of choice within the community). But the condition they may hold more generally; for example in the case of a museum or park, 'boundary' residents may make so few trips that, at their current level of activity, they could make no better use of a larger facility. We formalize this condition as follows: Let σ stand for some boundary region. Then *marginal indifference* will mean that

$$(2.3) \qquad \frac{\partial}{\partial q} U(q, g_\sigma^i, \ell_\sigma^i, I^i - T^i - f(g,\sigma) - r_\sigma \ell_\sigma^i) = 0 \quad .$$

The second necessary assumption is that residents do not tend to sort themselves out within the town according to their relative preference for the public good. Since systematic differences will generally lead to sorting, this means that the community must be reasonably homogeneous in attitudes toward public goods. We will see later that when sorting occurs, it will tend to mitigate capitalization.

The required homogeneity will result whenever agents are additively separable in their preferences on any component of consumption concerning which they systematically differ. Here we will treat the case where individuals differ in incomes and have preferences which are additively separable in the associated numeraire consumption good.

Consequently, we now assume that a resident (i) of the community has preferences which can be represented in the form

$$U^i = \hat{U}(q,g,\ell) + I^i - T^i - f(g,s) - r_s\ell \quad .$$

Now, it is obvious that residents will not sort themselves out by income class. Conditional on living at s , all residents would make the same choices of g and ℓ $(g^i_s = g_s, \ell^i_s = \ell_s$, all s) so at any specified rent structure, they would all agree on the best location; hence, as before, the rent structure must adjust until all locations are equally desirable for all residents.

We now evaluate the first order welfare effect of a new project:

$$dW = \sum_i \omega_i dV^i \quad . \tag{2.4}$$

Since we are ignoring distributional factors, we must assume that whatever income differences prevail are 'optimal', implying that the welfare weights (ω_i) are equal (otherwise, pure transfers could be made in such a way as to improve net welfare). We normalize units by setting the common weight equal to one. In evaluating (2.4) we treat everyone as if they were living at the boundary region σ and take into account all potential effects which q could have on the parameters faced by consumers.

Performing the differentiation, applying the envelope theorem where appropriate and aggregating where possible, we have

$$\frac{dW}{dq} = N\frac{\partial}{\partial q}\hat{U}(q, g_\sigma, \ell_\sigma) - N\ell_\sigma \frac{dr_\sigma}{dq} + \frac{dY}{dq} + \frac{d\Pi}{dq} + \frac{dR}{dq} - \frac{dT}{dq}$$

where N stands for the total population of the community, and the income variables without superscripts stand for aggregates over all residents. Now, marginal indifference (2.3) means that the first term is zero and $\frac{dY}{dq}$ must be zero as well since Y cannot change under the assumption that non-land private goods prices are fixed. Furthermore, government budget balance implies that $\frac{dT}{dq} = \frac{d\Gamma}{dq}$. Making these substitutions, we have finally

$$\text{(2.5)} \qquad \frac{dW}{dq} = \frac{dR}{dq} - \frac{d\Gamma}{dq} + \frac{d\Pi}{dq} - N\ell_\sigma \frac{dr_\sigma}{dq} \quad .$$

The formula (2.5) can be thought of as a generic capitalization result from which a variety of specific relationships can be derived.

1. Local Ownership.

Let us first look at the case where local firms (as well as local land) are all owned by local residents. In that case Π should be thought of as total profits generated in the region. Assuming that local firms behave competitively, we know that $d\Pi$ can be written in the form $d\Pi = X\cdot\delta p$, where x is the vector of net outputs and p the private goods price vector. But the only prices which change are land prices, so we have $\frac{d\Pi}{dq} = -\frac{dR_f}{dq}$ where R_f is the value of land used by firms. Substituting this relationship into (2.5) we have

$$(2.6) \qquad \frac{dW}{dq} = \frac{dR_r}{dq} - \frac{d\Gamma}{dq} - N\ell_\sigma \frac{dr_\sigma}{dq}$$

where R_r is the value of residential land.

The final form capitalization takes depends on the boundary conditions. In the isolated case, it seems reasonable to suppose that $\frac{dr_\sigma}{dq} = 0$ even though the boundary of the town may shift. Conceptually, we can think of there being many boundary regions which are shared by residents and farmers. As long as some of these regions are still shared after the project is initiated, boundary rent cannot change. Thus for the case of isolated communities, we assert

$$\frac{dW}{dq} = \frac{dR_r}{dq} - \frac{d\Gamma}{dq}$$

Verbally, the *gross* benefits of the project are capitalized into *residential* land values. Net benefits $(\frac{dW}{dq})$ are gross benefits minus costs $(\frac{d\Gamma}{dq})$.

A more general statement can be made which will apply to adjacent communities as well. If the aggregate demand for land increases so that $\frac{dr_\sigma}{dq} > 0$ then the value of residential land overcapitalizes gross benefits $\left(\frac{dR_r}{dq} - N\ell_\sigma \frac{dr_\sigma}{dq}\right.$ exactly capitalizes gross benefits) while if $\frac{dr_\sigma}{dq} < 0$, then the value of residential land undercapitalizes gross benefits.

2. National Ownership.

The majority of firms are not locally owned. Indeed, most firms are national in the scope of their operations and local ownership has no meaning for such firms. Suppose we take the position that all ownership is in national firms. What happens to the analysis? Actually, not very much. Indeed, the formula (2.6) is still correct for society as a whole, but now some of the costs of the project are paid by 'foreigners' who own shares in the firms which are operating locally. Indeed, if we take the position that each community is small relative to the country, then it is a good approximation to assume that local branches of national firms are owned entirely by outsiders. In that case, it follows that gross benefits *to residents of the community* are capitalized into *total* land value, while gross benefits to *society as a whole* are capitalized into *residential* land values. The difference represents an externality. We will discuss other examples of such externalities later.

3. Property Taxes.

Up until now we have dealt with only the case in which taxes were treated as lump sum taxes by consumers. Since the major locally imposed tax is a property tax, it is important to see how the analysis needs to be modified in that case. If an ad valorem tax at rate t is imposed, then an agent using an amount of land ℓ in zone s will pay taxes $tr_s\ell$, where r_s must now be thought of as the rent net of tax payments.[2]

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2. We will have more to say about this formulation of property taxation in the section on general models.

Now, the problem for a typical individual can be stated as

$$\underset{g,\ell,s}{\text{Max}} \quad \hat{U}(q,g,\ell) + I^{i} - f(g,s) - r_s(1+t)\ell \quad .$$

Clearly, it is still true that all agents will make the same choices and indeed, the method of analysis is exactly as before. Performing the calculations (and taking the point of view of society in order to avoid any ambiguity concerning the ownership of firms) we derive

$$\text{(2.7)} \qquad \frac{dW}{dq} = \frac{dR_r}{dq} - Nr_\sigma \ell_\sigma \frac{dt}{dq} - N\ell_\sigma (1+t)\frac{dr_\sigma}{dq} \quad ,$$

where R_r now stands for total residential rent net of taxes.

The exact form of capitalization again depends on boundary conditions. And now there is some ambiguity concerning these even in the case of isolated communities. The issue revolves around whether or not farmers in the boundary regions do or do not pay the taxes. If they do pay the taxes, then the free boundary condition is as before. However, if they don't pay the taxes (but would have to if they acquired property 'within' the community), then the appropriate condition is $r_\sigma(1+t) = \bar{r}$ where $\bar{r}$ is the opportunity cost of land to a farmer. We will refer to this arrangement as *agricultural zoning*, and the other case as *blind zoning*.

The case of agricultural zoning is easiest to analyze. Differentiating the boundary condition with respect to q we have

$$(1+t) \frac{dr_\sigma}{dq} + r_\sigma \frac{dt}{dq} = 0 \quad .$$

Therefore, referring back to (2.7),

$$\frac{dW}{dq} = \frac{dR_r}{dq} \,, \tag{2.8}$$

Here *net* project benefits are capitalized into *net-of-tax* residential property values, while gross project benefits are capitalized into before tax property values.

For the case of blind zoning, the term involving $\frac{dr_\sigma}{dq}$ vanishes in (2.7) and we are left to evaluate $Nr_\sigma \ell_\sigma \frac{dt}{dq}$. Suppose that only residential land is taxed, so that the government budget balance condition is $tR_r = \Gamma$. Differentiating this equation with respect to q and substituting for $\frac{dt}{dq}$ in (2.7) yields

$$\frac{dW}{dq} = \frac{dR_r}{dq} - \alpha\left(\frac{d\Gamma}{dq} - t\frac{dR_r}{dq}\right) = (1+\alpha t)\frac{dR_r}{dq} - \alpha\,\frac{d\Gamma}{dq} \,, \tag{2.9}$$

where

$$\alpha = \frac{N\bar{r}\ell_\sigma}{R_r} \,.$$

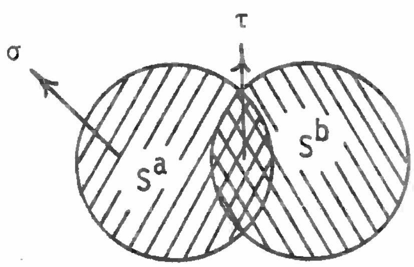
The net benefit is now a weighted difference between increases in after tax residential property values and increases in cost. Note that the smaller is α, the closer we approximate the results for the previous case. α stands for the ratio of the value of land residents would use if they all lived at the boundary to the actual value of residential land.

More generally, however, (2.9) tells us that increases in residential property value over-capitalize net benefits by an amount related positively to the gap between extra costs and the extra revenue that would be generated without changing tax rates. Clearly, this result is intermediate between pure gross capitalization and pure net capitalization.

D. Preference Differences Within the Community.

Once systemmatic preference differences are introduced into the communities, the exact capitalization results tend to break down. Here, we demonstrate this and indicate the type of modification which is required. For this purpose, let us consider the simplest extension possible, one with two different types of residents in the town (indexed a and b). Let there be $N_a(N_b)$ a-type (b-type) residents. It is convenient to assume that all a-type residents have the same preferences and income (although different incomes could be incorporated as before). Also, we find it necessary to suppose that the plot size within any given zone must be uniform, and we may as well assume that this plot size is predetermined.

Now we let S^a stand for the set of zones occupied by a-type residents and S^b for the corresponding set of zones for b-type residents. Obviously if σ is included in both sets, the analysis is exactly as in previous sections; indeed, all previous theorems go through with preference differences as long as those differences do not lead to systemmatic sorting. Therefore, without loss of generality, we can assume that the zoning arrangements are as in the following Venn diagram:



Since the zoning is discrete, there is little loss of generality in assuming that at least one zone is occupied by both types. We let

τ stand for such a zone.

In what follows, it is convenient to use the shorthand notation

$$F_s^x(q) = \hat{U}^x(q, g_s^x, \ell_s) - f(g_s^x, s) \ , \quad \text{all } s; \quad x = a,b \ .$$

Thus, V_s^a can be written in the form:

$$V_s^a \equiv F_s^a(q) - r_s \ell_s + I^a \ .$$

Next we evaluate welfare as before except that now we cannot evaluate everyone's welfare at σ ; instead, we evaluate a-type welfare at σ and b-type welfare at τ .:

$$\text{(2.10)} \qquad W = N_a V_\sigma^a + N_b V_\tau^b$$

$$= N_a F_\sigma^a + N_b F_\tau^b + I - N_a r_\sigma \ell_\sigma - N_b r_\tau \ell_\tau \ .$$

Finally, we make use of the fact that a-type agents are indifferent between location τ and σ . This indifference implies that

$$\text{(2.11)} \qquad r_\tau \ell_\tau = r_\sigma \ell_\sigma + F_\tau^a(q) - F_\sigma^b(q) \ .$$

Substituting (2.11) into (2.10) yields

$$W = NF_\sigma^a(q) + N_b[F_\tau^b(q) - F_\tau^a(q)] + I - Nr_\sigma \ell_\sigma \quad ,$$

and differentiating with respect to q , we obtain:

$$\frac{dW}{dq} = N \frac{d\hat{U}^a}{dq}(q, g_\sigma^a, \ell_\sigma) - N\ell_\sigma \frac{dr_\sigma}{dq} + \frac{d\Pi}{dq} + \frac{dR}{dq} - \frac{dT}{dq}$$

(2.12)

$$+ N_b[\frac{d\hat{U}^b}{dq}(q, g_\tau^b, \ell_\tau) - \frac{d\hat{U}^a}{dq}(q, g_\tau^a, \ell_\tau)]$$

Examining (2.12) we see that the terms are just as before except for the intramarginal benefit term:

(2.13) $$Nb[\frac{d\hat{U}^a}{dq}(q, g_\tau^b, \ell_\tau) - \frac{d\hat{U}^a}{dq}(q, g_\tau^a, \ell_\tau)] \quad .$$

We will argue that there is a strong presumption that this term is positive. To begin with, the condition $g_\tau^b > g_\tau^a$ is a stability condition for the town structure we have specified. It guarantees that there is a differential benefit to an a-type agent relative to a b-type agent as one moves toward the boundary of the town (b-type agents gain relatively more from proximity to the center since they make more trips). Therefore, as long as the marginal benefit from more public goods increases with the number of trips, the intramarginal term (2.13) must be positive.

Thus, when systematic preference differences are introduced into any particular variant of the model studied above, land rents will tend to undercapitalize the associated benefit. Operationally, this means that if land rents were used as a decision criteria, we might end up rejecting projects which ought to be accepted.

D. Sufficient Conditions for No-External-Capitalization.

Having explored the conditions for internal capitalization in a community that was insulated from immigration forces, we turn now to a study of those external forces. We will show first that any systemmatic differences between towns can serve to neutralize the external forces at least with respect to their effect on small projects. There is an important caveat here, however. What we show is that communities *could* arrange things so as to insulate themselves; we cannot show that they have an incentive to follow such policies. Indeed, we have shown in some related work that communities do not have a myopic incentive to insulate in many instances. We will ignore this issue here.

We employ the simplest model of community here, in which there are no choices with regard to public goods use. The reader might usefully recall that this is one of the cases in which there is no internal capitalization. All agents within the town are assumed alike. To make the point about potential insulation in the strongest possible way, we will also assume that preferences of all individuals in all towns are alike and that towns differ only in income levels; that difference alone is enough to insulate towns.

Referring back to the first model in Section IIA, we can specify the characteristics of community k completely by a level of public goods (q^k), an income level of residents $(\bar{I}^k)$,[3] and a cost level $(h^k = T^k + f^k(\bar{g}^k, s) + r_s^k \bar{\ell}_s)$ which must, at equilibrium, be independent of location (s). It is convenient to assume that the $\bar{g}$ and $\bar{\ell}$ levels are the same everywhere so that they can be deleted from the analysis, although this is not really necessary.

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3. $\bar{I}^k$ is clearly the average income in the community.

Now, the welfare (U^{kk}) of an individual with income I^k living in his own community can be expressed as

$$U^{kk} \equiv U(q^k, I^k - h^k)$$

We now ask the following question. Suppose that a set of q's, I's and h's is specified. Will anyone have an incentive to move immediately, and if not would anyone see such an incentive if one of the q's were changed marginally? We will show that an appropriate initial choice will make the answer to both these questions 'no' as long as income effects are not negligible. Suppose a k-agent considers moving to j. We assume that if he does this, he must pay the taxes (and other costs) appropriate to j. (If instead he expects to pay the taxes appropriate to k, as he surely would if the tax were a property tax, the results are still the same, but we would have to incorporate the taxes in the I-term rather than the h-term.) On the other hand, an agents income is determined by his place of initial ownership. This must be true of property income (as long as there are no unanticipated capital gains), and labor income is the same everywhere, by assumption. Hence, he would expect to get utility

$$U^{kj} \equiv U(q^j, I^k - h^j)$$

The move is not desirable, and would not be desirable for any sufficiently small variation in q^j, as long as

$$U^{kk} > U^{kj} ,$$

Similarly, a move from j to k is not desirable as long as

$$U^{jk} \equiv U(q^k, \bar{I}^j - h^k) < U(q^j, \bar{I}^j - h^j) \equiv U^{jj} \quad .$$

Are these two conditions consistent? Let us order j and k in such a way that $q^k > q^j$. Then stability certainly requires $h^k > h^j$. To see what else is required, it is convenient to change variables slightly. If we define

$$\Delta q = q^j - q^k \ ; \quad z^k = \bar{I}^k - h^k \ ; \quad z^j = \bar{I}^j - h^k \ ; \quad \Delta z = h^k - h^j \ ,$$

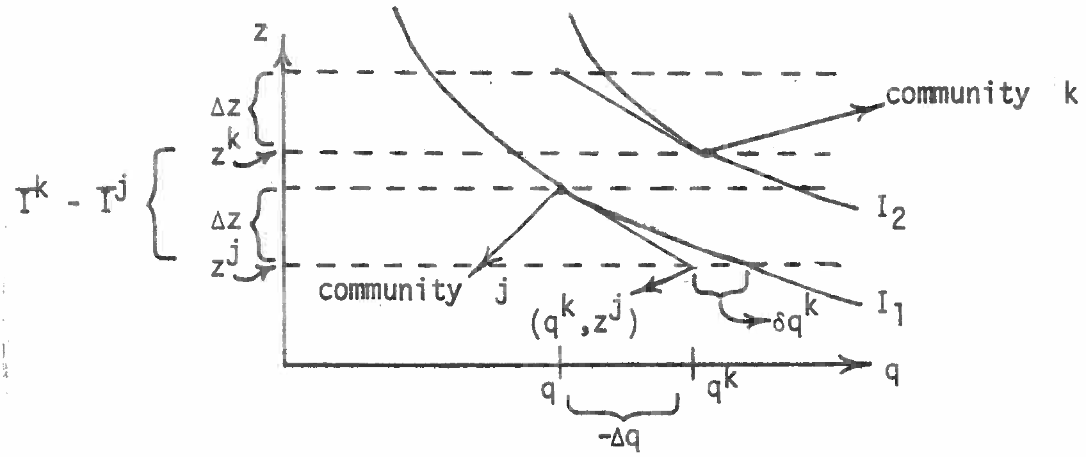
then the two stability conditions may be written as

$$U(q^k, z^k) > U(q^k + \Delta q, z^k + \Delta z)$$

$$U(q^k, z^j) < U(q^k + \Delta q, z^j + \Delta z)$$

With $\Delta q < 0$ and $\Delta z > 0$.

These conditions are consistent as long as $\bar{I}^k > \bar{I}^j$ as can be seen in the following diagram:



Of course, we have assumed in the construction that the income effect is normal and significant, so that increases in income increase the Marginal rate of substitution of z for q. Clearly, community k could engage in any project of size less than or equal to δq^k without inducing entry, while a similar statement holds for community j.

Naturally if projects undertaken are too large, entry may be induced. However, we assert that any systematic differences between towns (direct preference differences would have done as well) can imply no-external-capitalization of marginal projects.

It is interesting to note that if we now reintroduce the possibility of internal capitalization, its presence may be enough to netrualize external forces, even if there are no intrinsic differences at all among people. That is, if $q^k > q^j$ implies $R^k > R^j$, we will automatically have $I^k > I^j$ assuming that there are no other income differences.

We chose income differences to illustrate the possibility of insulation because they were the most innocuous sounding difference we could think of. Obviously, preference differences are more likely to serve this purpose. Indeed, some might argue that income effects are unimportant in the context discussed here. Suppose that we reexamine our problem in the absence of income effects. Interestingly, the whole character of the analysis changes.

F. Sufficient Conditions for Full External Capitalization.

Without income effects or preference differences it is impossible to insulate. After writing the preference function as additively separable in income, the two conditions for (weak) insulation become:

$$\hat{U}(q^k) + \bar{I}^k - h^k \geq \hat{U}(q^j) + \bar{I}^k - h^j \tag{2.14}$$

$$\hat{U}(q^k) + \bar{I}^j - h^k \leq \hat{U}(q^j) + \bar{I}^j - h^j \quad . \tag{2.15}$$

Clearly strict inequality in either direction is impossible, and both relations must hold as equalities in any equilibrium situation; all residents must be indifferent at the margin as to where they live. Any welfare improving project anywhere must induce entry.

We can now use equations (2.14), (2.15) to generate a simple external capitalization result. These equations must hold both before and after a project (in some particular town z), so we can differentiate them with respect to q^z . In doing so we must allow for the possibility that a change in q^z could affect I^j and $h^j (j \neq z)$. We have already seen one way in which this could happen through external ownership of local firms; now that migration is certain, there are many other ways as well. Postponing a detailed discussion of these effects until later, let us simply compute total differentials

$$\text{(2.16)} \qquad dV^z \equiv d\hat{U}(q^z) + dI^z - dh^z = dI^z - dh^j$$

$$\text{(2.17)} \qquad dV^j = d\hat{U}(q^z) + dI^j - dh^z = dI^j - dh^j \ ,$$

where dV^i is shorthand notation for the change in welfare of a representative resident of community i .

Now, if we knew that dV^j were zero for all $j \neq i$, equations (2.16) and (2.17) would imply a very simple capitalization relationship. But we are after a more general result, since we know that in most cases, the project will have external effects. Let us define $dW^{\neq}$ to be the total external effect; that is,

$$dW^{\neq} = \sum_j N^j dV^j = dI^{\neq} - \sum_{j \neq s} N^j dh^j \ .$$ [4]

And, $dW^z = N^z dV^z$ will stand for the welfare change in community z . Clearly, $dW = dW^{\neq} + dW^z$. Simple substitutions yield the following two equations for the change in community z welfare and the change in social welfare:

$$dW^z = \frac{N^z}{N^{\neq}} dW^{\neq} - \frac{N^z}{N^{\neq}} (dR^{\neq} + d\Pi^{\neq}) + dR^z + d\Pi^z \ .$$

---

4. I^i stands for total income in community i , and the symbol $\neq$ means the associated variable is aggregated over all communities except z .

$$dW = \frac{N}{N^{\#}} dW^{\#} - \frac{N^z}{N^{\#}} (dR^{\#} + d\Pi^{\#}) + dR^z + d\Pi^z \ .$$

Finally, we must distinguish cases again, according to the nature of firm ownership.

1. Neutral Ownership.

Suppose that each community owns its share of national profits. Then, $N^z\Pi^{\#} - N^{\#}\Pi^z = 0$ and we have

$$(2.18) \qquad dW^z = \frac{N^z}{N^{\#}} dW^{\#} + dR^z - \frac{N^z}{N^{\#}} dR^{\#}$$

$$(2.19) \qquad dW = \frac{N}{N^{\#}} dW^{\#} + dR^z - \frac{N}{N^{\#}} dR^{\#} \ .$$

Clearly, *total* rent change in z is a good approximation to *net* welfare benefits *in z* as long as community z is small relative to the nation (so that $N^z/N^{\#}$) is small). However, net *social* benefits will be misrepresented by rent increases to the extent that the external effect is significant.

We will have more to say about the size of the external effect in the next section. But before moving on it is worthwhile to discuss the other major difference from the internal-capitalization results: *net* benefits are externally capitalized while *gross* benefits were internally capitalized. The economic reason for this discrepancy is not difficult to find. The marginal potential resident *pays* local taxes in the internal model (he is the boundary resident) while the marginal potential resident *does not pay* local taxes in the external model. Naturally, given this discrepancy it is important to know which set of forces will dominate when both are present. We argue in the next section that external forces will tend to dominate.

2. Local Ownership.

If each community owns its own firms, then $d\Pi^{\#} = 0$ and we have

$$(2.20) \qquad dW^z = \frac{N^z}{N^{\#}}\, dW^{\#} + dR_r^z - \frac{N^z}{N^{\#}}\, dR^{\#}$$

$$(2.21) \qquad dW = \frac{N}{N^{\#}}\, dW^{\#} + dR_r^z - \frac{N^z}{N^{\#}}\, dR^{\#} \;.$$

The form of the results is the same except that capitalization is into *residential* rather than *total* land value. The reader might think at first that there is a contradiction between equations (2.19) and (2.21). After all, it cannot matter from a social point of view where the profits are owned, so why do these equations look different? The answer is that any increased cost of local industrial land will be reflected in the term $dW^{\#}$ in equation (2.19) but not in (2.21). Thus, even in the case of national ownership, only increases in residential land values represent real social benefit, once adverse land-value-externalities are cancelled out.

III. A General Model.

Our general model of local communities can be characterized as follows. The community provides a vector of local public goods. This vector may be disaggregated over locations within the community as well as over different types of public goods; q will now stand for this vector. (Thus, our approach will be general enough to encompass road systems and schools as well as the museum-type local public goods discussed earlier.) As before, we will assume that access to the public goods is 'free' in that no price is charged for entry. However, we now allow for the possibility that such goods will become congested with use. We assume that all important economic aspects of congestion can be summarized in a vector of congestion levels (Q) ; it is convenient to think of these congestion levels as measuring total use of the associated facility, although this interpretation is not really necessary in what follows. Public goods will be produced from private goods which are paid for out of tax revenues (again, we will not allow borrowing by the government).

All 'non-public' goods will be allocated through a market system in the private sector. The behavior of this sector will be assumed standard in every way except that we will allow consumers to engage in some 'production' activities in addition to their market transactions. We now allow essentially arbitrary diversity of tastes and incomes among agents. However, we will treat these differences in a discrete way by assuming that there are a finite number of different preference/labor resource types and property income classes.

It is convenient to think of the problem faced by a representative individual in a series of stages. At stage one, the individual will take as given his community of residence, zone within community (s) and market transaction vector (x) and seek to maximize with respect to his choice of non-market activities. At stage two, he will pick an optimal market activity vector, still holding fixed his location choices. At stage three he will optimize on choice of zone within community (s) and finally, at stage four, he will optimize on the choice of community (k) .

Let us focus on an individual of preference type a and income class i .[5] At the first stage, he will choose vectors of consumption of non-land private goods (c) , and public services

5. For the moment, we will suppress the community index, which otherwise should be attached to all variables.

(g), his choice being subject to a generalized activity technology relating feasible consumption to associated market transactions, public goods availability, and congestion levels. We formalize this problem as

$$\max_{c,g} \overline{U}^a(c,\ell,g,s) \quad \text{subject to}$$

(3.1)

$$(c,g,x+h^a,q,\overline{Q}) \in \Omega^s \quad ,$$

where $\overline{U}^a$ is the utility function of an a-type individual, h^a is the vector of non-land exogenous resources held by an a-type agent and Ω^s stands for the technologically feasible set for an individual in zone s. The formulation (3.1) is general enough to cover virtually any type of nonmarket activity contemplated by the consumer. In the most natural case of transportation activities, one should think of $c-x-h^a$ as representing private goods used up in the process of travel.

The outcome of our first stage problem defines an indirect utility function of the form:

$$U^{as} \equiv U^a(x+h^a,\ell,q,Q,s) \quad .$$

Frequently, we will identify this function using the shorthand notation U^{as}. Next, we formulate the second stage (market choice) problem. Initially we treat direct taxation, which we postulate formally as

P. 1 (Direct taxation).

Tax payments are treated as parameters by all individuals. We let T^{ai} stand for taxes paid by an (a,i)-type individual.

Now the second stage problem may be formulated as

$$\max_{x,\ell} U^a(x+h^a,\ell,q,Q,s) \quad \text{subject to}$$

(3.2)

$$p\cdot x + r_s\cdot\ell = R^i + \Pi^i - T^{ai} \quad ,$$

where p is a vector of non-land private good prices, r_s is the rental rate in zone s , and the income terms have the same meaning as in Section II. The indirect utility function from this problem will be written as

$$V^{ais} \equiv V^a(q,Q,p,r_s,R^i + \Pi^i - T^{ai},s) \ ,$$

and we will frequently index the associated choice variables as x^{ais} and ℓ^{ais} . Similarly, we let λ^{ais} stand for the Lagrange Multiplier (marginal utility of income) associated with problem (3.2).

At stage three, the zone (s) is chosen so as to

$$\max_{s} V^{ais}$$

The outcome of this choice determines a common utility level (V^{ai}) and a set of zones which will be occupied by a resident of preference type a and income class i ; we label this set: $s(a,i)$. For any zone in the set, let n^{ais} stand for the number of a,i-type residents in that zone.

The description of the government is as before except that now the cost function for public goods production must depend on the (potentially varying) price vector; that is $\Gamma = \Gamma(q,p)$. Government budget balance requires

$$T \equiv \sum_{ais} n^{ais} T^{ai} = \Gamma(q,p) \quad .$$

Throughout this section we assume the existence of a market clearing private sector equilibrium for any fixed levels of public goods provision. For some discussion and support for this assumption, see Starrett (1978).

A. Internal Capitalization.

We ignore the last (fourth stage) problem in this subsection. That is, we consider a community in isolation, implicitly assuming that no migration (in or out) will occur in response to projects initiated in the community. Such insulation can be justified either by an assumption that moving costs between communities are prohibitive, or by some systematic differences among communities of the type explicated in the previous section.

We now present some general conditions for internal capitalization. The most important of these are a 'no sorting' condition and a 'marginal indifference' condition. Both of these conditions refer to behavior of agents with respect to some 'boundary' zone or zones. As before, we will give the label σ to such a zone.

Formally, we postulate

<u>P. 2 (No sorting)</u>

For all a and i , $\sigma \varepsilon S(a,i)$.

<u>P. 3 (Marginal Indifference)</u>

For all a and i ,

$$\nabla_q V^{ai\sigma} = 0 \ , \quad \nabla_Q V^{ai\sigma} = 0 \ .$$

Clearly, P.2 may hold even when agents differ in some respects, although the analysis of the previous section should convince readers that the allowable differences are severely restricted. Since the model now includes congestion, postulate P.3 now takes a somewhat different form than before; the boundary resident must be indifferent to increases in both the public goods supply and the attendant level of congestion. The reader might object that (in the presence of congestion) the marginal resident should be expected to be worse off from certain kinds of marginal expansions. We agree, and will consider a modification of P.3 later.

We must be a little more careful with the welfare formulation now that general preference and income differences are being allowed. In particular we must deal with a problem first raised by Mirrlees. Suppose that we start with a general Bergson-Samuelson welfare formulation, but with the proviso that the government cannot interfere with private markets. Operationally, this assumption implies that the government's social welfare function must take Indirect utility levels for its arguments. Thus, in differential form, we can write:

$dW = \sum_{ais} n^{ais} \omega_{ais} dV^{ais}$ where the ω's are marginal welfare weights.[6]

Now, if we wanted to make a neutrality assumption concerning the income distribution we would ordinarily assume that the weights were inversely related to the marginal utility of a dollar for each agent. But we would also want to assume that two agents who are identical in all characteristics and who end up with the same utility levels should have the same welfare weight. (That is $\omega_{ais} = \omega_{ait} \equiv \omega_{ai}$, all $s,t \in s(a,i)$). However, these two conditions are inconsistent since similar agents living at different places within the town will have different marginal utility of a dollar.[7]

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6. We are implicitly assuming that agents who are identical in terms of both characteristics *and* behavior, get the same weight.
7. Residents near the boundary will spend more of the extra dollar (either directly or indirectly) on transportation.

Here we will assume that no transfer of income among *boundary* residents can be made which will improve social welfare. Formally, we postulate

P.4 (Neutral Income Distribution).

The initial distribution of income is such that

$$\omega^{ai} = \frac{1}{\lambda^{ai\sigma}} \, , \quad \text{all} \quad a,i \quad .$$

Obviously, there is some arbitrariness in the assumption P.4, but the potential distortion from using some other normalization will be small unless the differences in λ^{ais} as one changes position in the town vary significantly for different types of individuals.

Finally, we need (for exact capitalization) some condition that will fix boundary rents. And now we will also need a similar condition on other boundary prices. For this purpose, we postulate

P.5 (External Boundary Markets)

Let δp and δr be equilibrium market responses to an arbitrary public project. Then,

$$\delta r_\sigma = 0 \quad \text{and} \quad \sum_{ais} n^{ais} x^{ai\sigma} \cdot \delta p = 0$$

If boundary residents purchase only at the boundary, these conditions will be satisfied if the boundary prices are always determined by external forces (so that they do not change as a result of the project).

Theorem 1. (Internal Capitalization)

Given P.1-P.5, the welfare change (to residents) from a small project (δq) can be written in the form:

$$dW = \nabla_q R \cdot \delta q - d\Gamma + \nabla_q \Pi \cdot \delta q \quad , \tag{3.3}$$

where $d\Gamma$ stands for the total change in government cost as a result of the project.

Proof: Using P.2 and P.4, we can write

$$dW = \sum_{ais} n^{ais} \omega^{ai} dV^{ai} = \sum_{ai} (dV^{ai\sigma}/\lambda^{ai\sigma}) \cdot (\sum_{s} n^{ais}) \quad .$$

Differentiating the indirect utility functions and aggregating where possible yields

$$\begin{aligned} dW = & \sum_{ais} \left(n^{ais} \frac{\nabla_q U^{ai\sigma}}{\lambda^{ai\sigma}}\right) \delta q + \sum_{ais} \left(n^{ais} \frac{\nabla_Q U^{ai\sigma}}{\lambda^{ai\sigma}}\right) \delta_q - \sum_{ais} n^{ais} x^{ai\sigma} \cdot \delta p \\ & - \sum_{ais} n^{ais} \ell^{ai\sigma} dr_\sigma + \nabla_q R \cdot \delta q + \nabla_g \Pi \cdot \delta q - \nabla_q T \cdot \delta q \quad , \end{aligned} \tag{3.4}$$

where income variables without superscripts stand for aggregates over the whole community. Now P.3 and P.5 imply that the first four terms in (3.4) are zero, while government budget balance implies $\nabla_q T \cdot \delta q = [\nabla_q \Gamma + \nabla_p \Gamma \cdot \nabla_q p] \cdot \delta q \equiv d\Gamma$. Making these substitutions, we arrive at (3.3).

As in the earlier examples, internal capitalization tends to be *gross* capitalization; and we can again argue that residential land value is the appropriate base to use, at least from the point of view of society as a whole. Indeed, if ownership is strictly local then the term $-\nabla_q R_f$ will always be represented as part of $\nabla_q \Pi$. Of course, $\nabla_q \Pi$ may now contain other terms as well if congestion or price changes affect local profits.

In the case of national ownership, the profit change (together with any other changes in benefits to 'foreigners') must be included in total social welfare so it would still be correct in that case to think of residential land values as capitalizing true benefits.[8]

Since marginal indifference is unlikely to hold exactly, particularly in the presence of congestion, it is useful to state the following trivial corollary to Theorem 1:

Corollary 1.

Given P.1, P.2, P.4 and P.5, the welfare change (to residents) from a small project (δq) can be written as

$$dW = \alpha_q^\sigma \delta q - \alpha_Q^\sigma \delta Q + \nabla_q R \cdot \delta q - d\Gamma + \nabla_q \Pi \cdot \delta q$$

where α_q^σ and α_Q^σ are the vectors of marginal public good benefits and congestion costs to boundary residents, respectively.

Interestingly, if congestion is more important to marginal residents than are public goods levels, the term $\alpha_q^\sigma \delta q - \alpha_Q^\sigma \cdot \delta Q$ may be negative, in which case we actually get *over*capitalization of project benefits.

Finally, we state corresponding results for the case of ad valorem property taxes. Recall that in this case, P.4 corresponds to a situation of blind zoning in which farmers as well as residents

8. Now that all prices are variable there are other potential externalities associated with the terms of trade (even if there is no potential migration). For a discussion of these externalities, see Starrett (1978). They are ignored in the sequel.

pay the taxes. We replace P.1 by

P.1' (Ad Valorem Property Taxation)

Government revenue is generated by an ad valorem property tax. Let t stand for the tax rate. All rental values are now net-of-tax.

Corollary 2.

Given P.1' and P.2-P.5, we have

$$(3.5) \qquad \phi dW = [1+t(\frac{L_\sigma r_\sigma}{R})]\nabla_q R\cdot\delta q - \frac{L_\sigma r_\sigma}{R}\, d\Gamma - \nabla_q \Pi\cdot\delta q$$

where L_σ stands for the amount of land which would be demanded in region σ if everyone in fact lived there.

Proof: Setting taxes for a boundary agent of type (a,i) equal to $tr_\sigma \ell^{ai\sigma}$, it is easy to see that the tax term in (3.4) is replaced by

$$(3.6) \qquad [L_\sigma r_\sigma \nabla_q t + L_\sigma (1+t)\nabla_g r_\sigma]\delta q = L_\sigma r_\sigma \nabla_q t\cdot\delta q ,$$

since P.4 implies that $\nabla_q r_\sigma = 0$. Now the government budget balance condition must be written as $tR = \Gamma$. Differentiating with respect to q and using the result to substitute for $\nabla_q t$ in (3.6), we obtain (3.5).

If P.4 is replaced by the agricultural zoning condition:

P.4' (External Boundary Markets with Agricultural Zoning)

$$\delta[r_\sigma(1+t)] = 0 \quad \text{and} \quad \sum_{ais} n^{ais} x^{ai\sigma}\cdot\delta p = 0 ,$$

we have

<u>Corollary 3</u>

Let P.4 be replaced by P.4' in Corollary 2. Then

$$(3.7) \qquad dW = \nabla_q R \cdot \delta q + \nabla_q \Pi \cdot \delta q \ .$$

Proof: Differentiating the agricultural zoning condition with respect to q, we see that

$$(1+t)\nabla_q r_\sigma + r_\sigma \nabla_q t = 0 \ .$$

Inserting this condition at the first step in the proof of Corollary 2, we see that the tax term disappears completely. Hence (3.4) reduces to (3.7).

The interpretations of Corollaries 2 and 3 are exactly as in the model of Section II.

B. External Capitalization.

We now reintroduce the fourth stage problem (choice of community) and consider the possibility that migration may enforce 'external capitalization'. To avoid unnecessary confusion, we will use prescripts to denote community labels. A prescript in the upper position will refer to community of residence while a prescript in the lower position will refer to community of *initial* ownership. Thus, ${}^k q$ and ${}^k Q$ will stand for the public goods and congestion vectors in community k; ${}_j R^i$ will stand for the rental income of a person of property income class i who owns land in community j; and ${}^k_j V^{ai}$ will stand for the utility level of an individual with a-type preferences, from property income class i, who lives in community k and initially owned property in community j . For convenience, we will suppose that individuals own initially in only one community.

Our treatment of the tax system can be simplified here. We no longer have to treat several special cases depending on the nature of taxation. The only type of tax which is difficult to include in a general treatment is a tax imposed on residents of (say) k , on profit income earned outside of k . To rule this out, we replace P.1 by

P.1" (No taxation of profit income).

The tax system faced by a resident of community k does not include taxes on profit income.

We let ${}^{k}t^{a}$ stand for the set of parameters which characterize the tax system for an a-type individual in k ; for example, if the tax system consists of lump sum taxes then ${}^{k}t^{a}$ would stand for the amount of such a tax, while if the tax is on labor income, ${}^{k}t^{a}$ would be a vector of marginal tax rates on labor income brackets.

Property taxes paid to the community of ownership also can be parametrized in this way as long as all property values and rental rates are thought of as net-of-tax. Adopting that convention, taxes always appear to be paid to the government by land users and ${}^{k}t^{a} = {}^{k}t$ is simply the ad valorem tax rate (or rate schedule).[9] Of course, the real incidence of the tax may well be partly on owners, but the nature of incidence does not affect the form of results as long as all income variables are net-of-tax.

We saw in the previous section that exact external capitalization will not occur whenever there are income effects on the demand for public goods; indeed, if communities are sufficiently different

9. Most property taxes are 'collected' from owners, but it is well known that the physical point of collection is irrelevant to incidence in competitive markets.

in this respect, there will be no external capitalization. Hence, the major restrictive assumption of this section will be that there are no such effects over the 'relevant range' of incomes. Formally, we postulate

P.6 (Absence of Income Effects).

Preferences of an (a,i)-type individual living in zone s of community k, and owning in j can be represented in the form

$$(3.8) \qquad {}^k_jv^{ais} \equiv \hat{v}^{ask}({}^kq, {}^kQ, {}^kp, {}^kr_s, {}^kt^a) + {}_jR^i + \pi^i$$

Observe that the Mirrlees problem disappears once P.5 is assumed, since the marginal utility of a dollar can no longer vary within the town for a particular type of individual. Therefore, P.4 can be replaced by

P.4' (Neutral Income Distribution).

The initial distribution of income is such that

$${}_j\omega^{ai} = 1 , \quad \text{all } a,i,j .$$

P.6 also implies that the third and fourth stage problems look the same to all agents of property income class i who own in j. That is, if we let ${}_jV^{ai}$ stand for the indirect utility function derived from a combined third and fourth stage problem:

$${}_jV^{ai} \equiv \underset{sk}{\text{Max}}\ {}^k_jv^{ais} ,$$

then this function can be written in the form

$$(3.9) \qquad {}_jV^{ai} = \hat{V}^a(q,Q,p,r,t) + {}_jR^i + \Pi^i \ .$$

Our external capitalization results take the following form. A small project is initiated in community z $(\delta^z q)$. We know that such a project generally will induce welfare changes in other communities as well as community z . In what follows, we will not be concerned with which particular communities receive the external effects but only with the total external effect aggregated over all communities except z . As before the symbol $\#$ will be used to denote such aggregation; thus, $d^{\#}W$ will stand for the total welfare effect on property owners outside z from a project in z . Similarly, the symbol $*$ will be used to denote aggregation over all communities. We now show that the welfare change to owners in z (d_zW) and to society as a whole (d^*W) can be expressed as functions of the external effect and changes in certain land and property value aggregates. The aggregates which appear in this regard are: total value of land in community j $({}_j\tilde{R})$, total profit income to land owners in j $({}_j\tilde{\Pi})$, total population of type a owning in j $({}_jN^a)$ and total population of all types in j $({}_jN)$.[10]

10.

$${}_j\tilde{R} = \sum_{ai} {}_jn^{ai} {}_jR^i$$

$${}_j\tilde{\Pi} = \sum_{ai} {}_jn^{ai} \Pi^i$$

$${}_jN^a = \sum_{i} {}_jn^{ai}$$

$${}_jN = \sum_{a} {}_jN^a$$

Theorem 2 (External Capitalization).

Given P.1", P.4' and P.6, welfare changes from a project $(\delta^z q)$ may be expressed in the form

$$(3.10)\quad d_\star W = d_z\tilde{R} - \frac{{}_zN}{{}_{\neq}N}\, d_{\neq}\tilde{R} + d_z\tilde{\Pi} - \frac{{}_zN}{{}_{\neq}N}\, d_{\neq}\tilde{\Pi} + \frac{{}_\star N}{{}_{\neq}N}\, d_{\neq}W + {}_\star N \sum_a \left(\frac{{}_\star N^a}{{}_\star N} - \frac{{}_{\neq}N^a}{{}_{\neq}N}\right) d_\ell V^{ah}$$

$$(3.11)\quad d_z W = d_z\tilde{R} - \frac{{}_zN}{{}_{\neq}N}\, d_{\neq}\tilde{R} + d_z\tilde{\Pi} - \frac{{}_zN}{{}_{\neq}N}\, d_{\neq}\tilde{\Pi} + \frac{{}_zN}{{}_{\neq}N}\, d_{\neq}W + N_\star \sum_a \left(\frac{{}_\star N^a}{{}_\star N} - \frac{{}_{\neq}N^a}{{}_{\neq}N}\right) d_\ell V^{ah} ,$$

where ℓ and h index an arbitrarily chosen community and property income class, respectively.

Proof:

Using (3.9) we can write

$$(3.12)\quad {}_jV^{ai} = {}_\ell V^{ah} + {}_jR^i - {}_\ell R^h + \Pi^i - \Pi^h , \quad \text{all } i,j ,$$

where ℓ is a particular community and h a particular property income class. Taking total differentials in (3.12) and summing over appropriate collections of individuals, we can derive the following expressions for average welfare change outside z, and in society as a whole:

$$(3.13) \qquad \frac{d_{*}W}{{}_{*}N} = \sum_{jai} {}_{j}n^{ai} d_{j}V^{ai} = \sum_{a} \frac{{}_{*}N^{a}}{{}_{*}N} d_{\ell}V^{ah}$$

$$+ \frac{d_{*}\tilde{R}}{{}_{*}N} - d_{\ell}\tilde{R}^{h} + \frac{d_{*}\tilde{\Pi}}{{}_{*}N} - d\tilde{\Pi}^{h}$$

$$(3.14) \qquad \frac{d_{\neq}W}{{}_{\neq}N} = \sum_{\substack{ai \\ j\neq z}} {}_{j}n^{ai} d_{j}V^{ai} = \sum_{a} \frac{{}_{\neq}N^{a}}{{}_{\neq}N} d_{\ell}V^{ah}$$

$$+ \frac{d_{\neq}\tilde{R}}{{}_{\neq}N} - d_{\ell}\tilde{R}^{h} + \frac{d_{\neq}\tilde{\Pi}}{{}_{\neq}N} - d\tilde{\Pi}^{h}$$

Subtracting (3.14) from (3.13), multiplying by ${}_{*}N$ and rearranging terms, we obtain (3.10). (3.11) is then obtained by subtracting $d_{\neq}W$ from (3.10).

The first thing to notice about formulas (3.10) and (3.11) is that the representations of welfare change are incomplete in two important respects. We have not evaluated the externality $(d_{\neq}W)$ or the last term, as functions of the parameters; indeed, these evaluations cannot be done without more information on the structure of the problem.

Consider the last term in each equation. It is a term which reflects deviations from 'pure capitalization' which will result if the composition of community z by type (a) differs from that of the general society. Indeed, if the composition is the same everywhere, or if the distribution of benefits is neutral across types, the last term disappears. We formalize this result as follows: Consider the postulates

P.7 (Homogeneous Distribution by Type).

The distribution of owners in z by type is the same as in the general population.

P.8 (Neutral Distribution of Benefits).

For a given project $(\delta^z q)$ there exists an ℓ and h such that $d_\ell V^{ah}$ is independent of a.

Now, the following corollary is a trivial consequence of Theorem 2:

Corollary 2.1.

If P.7 or P.8 is added to the postulates of Theorem 2, then

$$(3.15) \qquad d_* W = d_z \tilde{R} - \frac{{}_z N}{{}_{\neq} N} d_{\neq} \tilde{R} + d_z \tilde{\Pi} - \frac{{}_z N}{{}_{\neq} N} d_{\neq} \tilde{\Pi} + \frac{{}_* N}{{}_{\neq} N} d_{\neq} W$$

$$(3.16) \qquad d_z W = d_z \tilde{R} - \frac{{}_z N}{{}_{\neq} N} d_{\neq} \tilde{R} + d_z \tilde{\Pi} - \frac{{}_z N}{{}_{\neq} N} d_{\neq} \tilde{\Pi} + \frac{{}_* N}{{}_{\neq} N} d_{\neq} W \; .$$

Of course, the distribution term may be very important if communities specialize in the type of owners they attract and succeed in confining the benefits to this group (so that neither P.7 or P.8 hold). For example, consider the extreme case in which only a-type agents own in z and the differences among communities are sufficient to provide insulation from migration as in Section II. Then, (taking $\ell = z$),

$$d_z V^{bh} = 0 \;\; b \neq z \quad \text{and} \quad {}_{\neq} N^a = 0 \; ,$$

so the last term in (3.11) reduces to $N^a d_z V^a$. Since this term is equal to $d_z W$ all the other terms in (3.11) or (3.10) must reduce to zero; that is, there is no capitalization as we would expect.

Clearly, external capitalization is going to occur only to the extent that communities *do not* succeed in specializing; a Tiebout-like world is inconsistent with external capitalization. On the other hand, specialization fosters a similarity of type within the community which is conducive to internal capitalization.

Before discussing the general nature of external capitalization, we note one further simplifying assumption which can be made. The profit terms in (3.15) and (3.16) disappear if we impose neutral ownership as introduced in the previous section:

P.9 (Neutral Ownership).

Each community claims its share of national profits, regardless of how these profits are generated.

Corollary 2.2

If P.9 is added to the postulates of Corollary 2.1, then

$$(3.17) \qquad d_{*}W = d_{z}\tilde{R} - \frac{{}_{z}N}{{}_{\#}N}\, d_{\#}\tilde{R} + \frac{{}_{*}N}{{}_{\#}N}\, d_{\#}W$$

$$(3.18) \qquad d_{z}W = d_{z}\tilde{R} - \frac{{}_{z}N}{{}_{\#}N}\, d_{\#}\tilde{R} + \frac{{}_{z}N}{{}_{\#}N}\, d_{\#}W \; .$$

The formulas of Corollary 2.2 are very similar in interpretation to the external capitalization results of the previous section. The only real difference is that all constituencies referred to in Corollary 2.2 are constituencies of *owners* as distinct from *residents* whereas there could be no distinction between these two groups in the previous section.

Thus, whenever community z is small relative to the whole (so that ${}_{z}N/{}_{\#}N$ is small), the change in land rents in z will be a good approximation to *net* benefits *for owners in* z . However, if externalities are important, the rent change will tend to misrepresent net social benefit by an amount, and in a direction, which depends on the size and sign of the externality. As before, one element of the externality will be the increase in land costs to national firms. However, there are generally other terms as well. We have studied the measurement of these externalities elsewhere and merely report the main result here.

When community z engages in a project in the context of assumptions in Corollary 2.2, that project is certain to induce some entry. Other communities suffer real welfare losses which are equal to lost tax revenue (from the migrators) minus any attendant improvement in congestion 'costs'. Unless communities have already seriously overexpanded, the net benefit to outsiders is negative $(d_{\neq}W < 0)$. Hence, even changes in *residential* land values will overstate *net* social benefit, quite generally.

Of course, if community z is not small as a fraction of the country, the theory takes a somewhat more complicated general form. Aside from the external terms, net benefits are capitalized into the *difference* between the change in land value in z and the *'average'* change in land values elsewhere.

Now that we have propositions about external and internal capitalization in models which are compatible with each other, an interesting question arises: Which theorems apply when both types of forces are present? The answer is actually very simple if we note that while internal capitalization results were derived under the explicit assumption that no external capitalization forces were present, the external capitalization results were derived without making any assumptionsabout internal capitalization forces. Obviously, then, the external forces dominate when they are present. In particular, any tendency to gross capitalization will be blocked in these circumstances (people would move out if it started to happen) and we expect net capitalization rather than gross.

IV. Summary.

We have shown in the previous sections that there is considerable diversity in the form and extent of 'capitalization' depending on the nature of the assumptions made. However, some fairly general principles emerge, and we summarize these in this section.

First, all of the models studied lead to the conclusion that from the point of view of society as a whole, it is *residential* property values that capitalize project benefits (rather than total land value). The theories differ on the appropriate land base only from the perspective of the project-generating community; if lost profits from increased land values are exported, then increases in *total land value* measure benefits to the community but if not, then it is *residential land values.*

The internal and external capitalization models generally disagree on whether gross or net benefits are capitalized. However, it is worth pointing out one important case in which the two theories agree on this issue. This is the case of property taxation with agricultural zoning, for which both theories suggest that increases in *net-of-tax* land values capitalize *net* welfare benefits (and, naturally, increases in gross-of-tax land values capitalize *gross* benefits).

The main differences between internal and external capitalization results can be traced to the boundary conditions imposed on a representative community. Notice that such boundary conditions play no role at all in external capitalization; indeed, the 'external' results hold regardless of what happens to prices or benefits at the boundary of the communities.

However, internal capitalization results vary considerably depending on the assumptions governing behavior of land rent and the impact of incremental projects, at the boundary. In the extreme case in which the impact is nil and the (net-of-tax) rent goes down by the 'rate of taxation', internal capitalization agrees with external capitalization. But if boundary rent is unchanged (as it will under many plausible conditions) then *gross* project benefits are internally capitalized. And if marginal projects actually make boundary residents worse off (because of a dominating congestion effect) then land values will overcapitalize even gross benefits.

We should recognize that in all cases considered, land values will fail to capture some of the 'external' benefits (or more likely, costs) of a project. When the only external effect is through nationally owned profits as was the case in the pure internal capitalization model, we could correct for the 'externality' by using residential land value rather than total land value. However, in the free-trade, free-migration model of external capitalization, the external costs of a project are generally much larger and we conclude that increases in residential land value will *overstate* true net social benefit by an amount closely related to the size of the externality.

We close with some remarks on the basic assumptions underlying capitalization results. The two types of capitalization rest on different, and to some extent complementary, assumptions. External capitalization requires that the distribution of agents by economic characteristics be the same in one community as in all others, while internal capitalization requires that agents within a particular community must have similar economic characteristics (though these characteristics may differ from community to community . Thus, the internal capitalization results are most applicable in a 'pure Tiebout' world where each community specializes and attracts a single type of agent. The external capitalization model is most applicable in the opposite extreme world of 'pure scrambling' in which all communities look alike in terms of the characteristics of agents. Naturally, there are many intermediate cases between these extremes (including the real world, presumably!). To the extent that neither extreme is a good approximation, capitalization will not hold in either form, or will hold only partially. On the other hand, in the unlikely event that both types of assumptions hold simultaneously, we showed that the external capitalization results win out.

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