INTRA-FIRM DIFFUSION, LEARNING AND PROFITABILITY

P. Stoneman

NUMBER 135

WARWICK ECONOMIC RESEARCH PAPERS

DEPARTMENT OF ECONOMICS

UNIVERSITY OF WARWICK
COVENTRY

INTRA-FIRM DIFFUSION, LEARNING AND PROFITABILITY

P. Stoneman

NUMBER 135

September 1978

This paper is circulated for discussion purposes only and its contents should be considered preliminary

Intra-Firm Diffusion, Learning and Profitability

Summary

In this paper we construct a model of the diffusion process based on a means-variance approach to the choice of techniques. We show under what conditions S shaped diffusion curves can be predicted and consider the resultant relationship between diffusion speed and profitability. In a world of constant profitability we show that the shape of the diffusion curve and the determinants of the diffusion speed depend crucially on the nature of the mechanism by which the entrepreneur learns. If profitability is changing we show that diffusion speed depends on the speed at which profitability increases and its determinants.

P. Stoneman
University of Warwick

September, 1978

Intra-Firm Diffusion, Learning and Profitability

I. Introduction

The core of micro-economic studies of technological change is the theory of diffusion - the theory of the determinants of the spread of a new technology over all its possible uses. Although numerous empirical studies have been undertaken over the years (see Kennedy and Thirlwall (1972)), the theory itself is still in its infancy. The central approach is attributed to Mansfield (1968) but his theory involves little more than a Taylor's expansion of an assumed general form (the work of Davies (1978), represents something of an advance on this). However, one of the big problems associated with a new technology is its riskiness, and the existing literature makes little allowance for this in the modelling process. In this paper[1/] we face the problem squarely by the use of a means-variance approach to technique choice. We realise that this approach has its problems, but as Green (1971) states this model has the great advantage of simplicity. The primary objective is to see under what conditions the model will predict the central result of diffusion studies - the existence of an S shaped diffusion curve, and what this implies about the determinants of the diffusion speed.

II. The Model

The environment we had in mind when building this model was one of the spread of "new wheat" across all possible uses. Thus we consider there to be a fixed acreage to be planted in the proportions $\alpha : 1 - \alpha$, new to old wheat. The general framework may also cover a firm with a fixed capital stock

---

1/ I wish to thank Avinash Dixit, Nick Stern and Keith Cowling and an anonymous referee for comments on earlier drafts of this paper, and am grateful for views expressed by members of the SSRC Industrial Economic Study Group. Of course all errors that remain are the sole responsibility of the author.

to be distributed in the proportions $\alpha : 1 - \alpha$, new to old technology. The new and old technologies are assumed to have perceived returns normally distributed according to (1) and (2).

$$\text{New} : N(\mu_N, \gamma_N^2) \tag{1}$$

$$\text{Old} : N(\mu_O, \gamma_O^2) \tag{2}$$

An individual entrepreneur will face the additive distribution with mean and variance shown by (3) and (4),

$$\mu = \alpha\mu_N + (1 - \alpha)\mu_O \tag{3}$$

$$\gamma^2 = \alpha^2\gamma_N^2 + (1 - \alpha)^2\gamma_O^2 + 2(\alpha)(1 - \alpha)\gamma_{NO} \tag{4}$$

where γ_{NO} is the covariance of the returns to the old and new technologies. The entrepreneur is assumed to have a utility function (5).

$$U = U(\mu, \gamma^2) \quad \frac{dU}{d\mu} > 0 , \frac{dU}{d\gamma^2} < 0 \tag{5}$$

For simplicity sake we assume that (5) can be written as (6), which is a convenient form with some pedigree (see Chipman (1973)).

$$U = a\mu - \tfrac{1}{2}b\gamma^2 \quad b > 0 \quad a > 0 \tag{6}$$

From (3) and (4) we obtain the slope of the transformation curve between μ and γ^2:

$$\frac{d\mu}{d\gamma^2} = \frac{\mu_N - \mu_O}{2(\alpha\gamma_N^2 - (1 - \alpha)\gamma_O^2 + \gamma_{NO}(1 - 2\alpha))} \tag{7}$$

and from (6) the slope of the indifference curve is

$$\frac{d\mu}{d\gamma^2} = \frac{b}{2a} \tag{8}$$

At the optimum the two will be equal and thus from (7) and (8) we obtain (9).

$$\alpha = \frac{\frac{a}{b}\left[\mu_N - \mu_O\right] + \gamma_O^2 - \gamma_{NO}}{\gamma_N^2 + \gamma_O^2 - 2\gamma_{NO}} \tag{9}$$

As can be seen the proportion of acreage planted with new wheat is positively related to the difference in mean returns and negatively related to the sum of the variance of the returns. We note first that if the new technology is to be used at all $(\alpha > 0)$ then μ_N must be greater than $\mu_O - \frac{b}{a}(\gamma_O^2 - \gamma_{NO})$ and $\gamma_N^2 + \gamma_O^2 - 2\gamma_{NO} > 0$. We feel that the typical case will be one where the new technology, especially in the early stages of the diffusion process will have a higher expected return than the old technology $(\mu_N > \mu_O)$ but because of its novelty will be considered to carry high risk i.e. to have a high subjective variance of returns $(\gamma_N^2 > \gamma_O^2)$. As the diffusion proceeds α will increase but there is an upper limit of unity. We restrict our analysis therefore to values of α such that $0 \leq \alpha \leq 1$.

There are a number of ways to proceed from this point in order to produce a theory of diffusion. One approach is to consider a number of different entrepreneurs with different attitudes to risk (different a's and b's) and then view the diffusion process as the spread of the new technology across all entrepreneurs. Alternatively one can consider the model as referring to a representative entrepreneur, assuming a and b are constant over time, and then argue that the parameters of the distribution of returns change over time generating a time path for α. In this context, however, the device of the representative entrepreneur may well be an unreasonable one. Thus we prefer to consider the model as being concerned with an individual entrepreneur (for whom a and b remain constant over time) rather than a representative entrepreneur and our results then provide an intra-firm theory of diffusion rather than a generalised conception of an industry or economy wide diffusion process.

We proceed by arguing that over time either the mean of the distribution of returns and, or, the subjective variance of the returns for the new technology will change, these changes generating a time path for α that is the diffusion path. We assume μ_0 and γ_0^2 remain constant over time. The realism of this assumption may be open to argument. For example, Harley(1973) argues that a new technology can further stimulate the development of the old technology and thus μ_0 and γ_0^2 may change over time. By the same token, but in the reverse direction, Schumpeter (1912) can argue that the appearance of a new technology may lead to a greater demand for primary inputs and a greater supply of outputs. The resultant price changes would obviously lead to changes in the returns to the old technology. Our analysis below may thus be criticised on the grounds that these effects are ignored. However at a number of stages it is clear that this framework is capable of development in many directions, and the varying of this assumption is one of them. We have attempted to take the simplest form of the model to illustrate its implications in the clearest way, and such modifications, although feasible, would cloud the main issues under discussion. Thus,similarly,we will also assume that γ_{NO} remains constant over time, essentially on the grounds that this seems to be a "neutral" assumption and there seems little a priori reason for preferring any other assumption over this. We can then derive from (9) that the change in usage over time is given by (10).

$$\frac{d\alpha_t}{dt} = \frac{-\alpha_t^2}{\frac{a}{b}(\mu_N-\mu_O) + \gamma_O^2 - \gamma_{NO}} \cdot \frac{d\gamma_{Nt}^2}{dt} + \frac{a}{b(\gamma_N^2+\gamma_O^2-2\gamma_{NO})} \cdot \frac{d\mu_{Nt}}{dt} \qquad (10)$$

where the first term shows the effect of reducing risk and the second the effect of increasing (expected) profitability.

III. Predicting the S shaped diffusion curve

The common denominator of all diffusion studies is the finding that the plot of the percentage of output produced (or acreage planted) with the new technology against time is S shaped. Three S shaped curves have merited most attention in the diffusion literature - the logistic, the Gompertz, and the Log Normal.

The logistic may be expressed as equation (11)

$$\log_e \frac{\alpha_t}{\bar{\alpha} - \alpha_t} = a_1 + b_1 t \quad , \quad b_1 > 0 \qquad (11)$$

where $\bar{\alpha}$ is the limiting value of α_t , $\bar{\alpha} \leq 1$. This may also be written as (12)

$$\frac{d\alpha_t}{dt} \frac{1}{\alpha_t} = b_1 \cdot \left(\frac{\bar{\alpha} - \alpha_t}{\bar{\alpha}} \right) \qquad (12)$$

The equivalent expression for the Gompertz cuve is (13) and a linear approximation to the Log-Normal is given by (14)

$$\frac{d\alpha_t}{dt} \cdot \frac{1}{\alpha_t} = -b_2 (\log \alpha_t - \log \bar{\alpha}) \quad b_2 > 0 \tag{13}$$

$$\frac{d\alpha_t}{dt} \cdot \frac{1}{\alpha_t} = \frac{b_3}{t}\left(\frac{\bar{\alpha} - \alpha_t}{\bar{\alpha}}\right) \qquad b_3 > 0 \tag{14}$$

For the three curves b_1, b_2 and b_3 respectively are independent of time and yield estimates of "the diffusion speed".

From equation (10) we see that changes over time in α_t are generated by changes in γ^2_{Nt} (i.e. the risk associated with the new technology) and or changes in μ_{Nt} (the mean return to the new technology) whereas at a moment in time the levels of γ^2_{Nt} and μ_{Nt} determine the level of α_t . We can contrast this with the approach of Mansfield (1968 pp. 180-181). Using the current terminology as far as possible, Mansfield defines $W(t)$ for a given firm as

$$W(t) = \frac{\alpha_{t+1}}{\bar{\alpha} - \alpha_t} - \frac{\alpha_t}{\bar{\alpha} - \alpha_t} \tag{15}$$

and then "supposes that" $W(t) = f(\pi, U(t) \ldots .)$ where π is the profitability of the new technology (note it is considered independent of time) and $U(t)$ the risk associated with the technology at time t. He then allows that $U(t)$ is a function of $\alpha_t/\bar{\alpha}$ (although arguing that α_t would be just as good) and after substitution generates a logistic diffusion curve (via a Taylor's expansion) in which $b_1 = c_1 + c_2 \pi$ i.e. in which the speed of diffusion is a linear positive function of the level of profitability.

Mansfield's "supposition" is equivalent to an assumption that the change in α_t is dependent on the levels of profit and uncertainty. This is in direct contrast to the predictions of the means variance framework where the levels of profit and uncertainty determine the level of α_t, and it is the changes in profit and uncertainty that determine the changes in α_t. It should be no surprise therefore that the results generated below differ from those generated by Mansfield.

Our approach is to argue that as S shaped curves are found so frequently in diffusion studies we should proceed by seeing under what conditions the means-variance framework will predict the existence of such curves and whether these conditions are reasonable. Then, assuming that these conditions do exist, we ask what does the model predict to be the determinants of the diffusion speed? We have argued that the time path for α_t is generated by changes in γ^2_{Nt} and, or μ_{Nt}. We concentrate below on the case where $\frac{d\mu_{Nt}}{dt} = 0$, for all t, for this is the scenario analysed by Mansfield and it is on his theory that most empirical work is based. We shall thus be stressing the situation most commonly analysed. We shall however also have some comments to make on diffusion paths in a world with changing profitability.

(a) Constant mean profitability

Consider, then, a world in which μ_{Nt} is fixed, but γ^2_{Nt} will vary over time. We consider that γ^2_{Nt} will vary from $\hat{\gamma}^2_N$ at time zero to $\bar{\gamma}^2_N$ for $t = \infty$. If we define Z_1 as in (16)

$$Z_1 \equiv \frac{a}{b}\left[\mu_N - \mu_O\right] + \gamma^2_O - \gamma_{NO} \qquad (16)$$

then α_t will vary from $\hat{\alpha} = \dfrac{Z_1}{\hat{\gamma}_N^2 + \gamma_O^2 - 2\gamma_{NO}}$ in time zero to

$\bar{\alpha} = \dfrac{Z_1}{\bar{\gamma}_N^2 + \gamma_O^2 - 2\gamma_{NO}}$ at its limit.1/ From equation (10) we know that

if $\dfrac{d\mu_{Nt}}{dt} = 0$ then $\dfrac{d\alpha_t}{dt} \cdot \dfrac{1}{\alpha_t}$ is given by (17)

$$\frac{d\alpha_t}{dt} \cdot \frac{1}{\alpha_t} = -\frac{\alpha_t}{Z_1} \cdot \frac{d\gamma_{Nt}^2}{dt} \tag{17}$$

Thus if the time path from $\hat{\alpha}$ to $\bar{\alpha}$ is to be logistic $\dfrac{d\gamma_{Nt}^2}{dt}$ must be such that (18) holds.

$$b_1\left(\frac{\bar{\alpha} - \alpha_t}{\bar{\alpha}}\right) = -\frac{\alpha_t}{Z_1} \frac{d\gamma_{Nt}^2}{dt} \tag{18}$$

which implies that the diffusion speed b_1 must satisfy (19)

$$b_1 = \frac{-d\gamma_{Nt}^2/dt}{Z_1\left(\dfrac{\bar{\alpha} - \alpha_t}{\bar{\alpha}\,\alpha_t}\right)} \tag{19}$$

As b_1 must be independent of time we are led to the conclusion that the variance must move over time according to some function such as (20) where v is a vector of variables independent of time

$$\frac{d\gamma_{Nt}^2}{dt} = -f\left(v, \frac{\bar{\alpha} - \alpha_t}{\bar{\alpha}\,\alpha_t}\right) \tag{20}$$

1/ We will not introduce the complication of the case where $\bar{\gamma}_N^2$ is such that it implies $\bar{\alpha} > 1$.

Equation (20) relates the change in the subjective value of the variance to the level of current use of the new technology α_t. Such a relationship is commonly considered to be at work in the diffusion process, for example, as we discussed above, Mansfield considers that the risk attached to a technology is related to the level of use. Chow (1967) argues that a prime reason for the existence of the S shaped curve is that as the diffusion proceeds more information is gained about the new technology and this leads to more decisions to accept. In fact these concepts of learning are the basis of the "epidemic" theories of diffusion.

By a procedure similar to the above we derive that if $d\gamma^2_{Nt}/dt$ follows a function such as (21) then a Gompertz diffusion curve will exist, and if some learning mechanism such as (22) operates then the log normal curve will result.

$$\frac{d\gamma^2_{Nt}}{dt} = -g\left(v, \frac{1}{\alpha_t} \log \frac{\bar{\alpha}_t}{\alpha_t}\right) \tag{21}$$

$$\frac{d\gamma^2_{Nt}}{dt} = -h\left(v, \frac{1}{t}\left(\frac{\bar{\alpha} - \alpha_t}{\bar{\alpha}\,\alpha_t}\right)\right) \tag{22}$$

We can restrict the range of acceptable learning concepts further. We know that $\bar{\alpha}$ is the limiting value of α_t, thus $d\gamma^2_{Nt}/dt$ must equal zero when $\alpha_t = \bar{\alpha}$ or α_t will continue to change. Thus the functional forms of the learning mechanism must be multiplicative between v and the appropriate expressions containing α_t. Thus we may write (20) as (23).

$$\frac{dY^2_{Nt}}{dt} = - F(v) . \frac{\bar{\alpha} - \alpha_t}{\bar{\alpha}\, \alpha_t} \tag{23}$$

where F is some function of v, and then the diffusion speed, b_1, is given by $\frac{F(v)}{Z_1}$ (from 19). As should be clear without further discussion, the diffusion speed is obviously going to be determined by the arguments of F(v).

The fact that this concept of diffusion relies upon learning as a function of use gives it a certain reliability. However if one is to proceed any further than this one needs to know what is an appropriate theory of learning. Unfortunately Economics gives us very little guide here. The well known learning-by-doing hypothesis of Arrow (1962) relating experience or knowledge to cumulative use will not give us S shaped diffusion curves, and despite a search of the psychology and sociological literature we have found little to guide us on this question. In fact Economics seems to have proceeded as far as the simple stock adjustment and distributed lag expectations models for empirical purposes without really giving any insight into this basic problem of learning.

What we have done therefore is to consider specific forms of (23) (and the equivalent expressions for the Gompertz and log normal curves) and investigate their implications for the determinants of the speed of diffusion without making any choice of one over any other. The motive for this is to show that if in empirical work any specific shape for the S shaped curve is chosen or any determinants of the diffusion speed detailed it is incumbant upon the researcher to justify exactly why the learning mechanism that implies these relationships is appropriate in the case under consideration.

Case 1

The simplest case is where $F(v)$ equals an arbitrary constant S. Then $b_1 = \frac{S}{Z_1}$. As Z_1 includes μ_N, the profitability of the technique, this case implies that the diffusion speed is inversely related to the level of profitability, quite the reverse of the Mansfield result. Similarly if we rewrite (21) and (22) as multiplicative forms (24) and (25).

$$\frac{d\gamma_N^2}{dt} = - G(v) . \frac{1}{\alpha_t} \log \frac{\bar{\alpha}}{\alpha_t} \tag{24}$$

$$\frac{d\gamma_N^2}{dt} = - H(v) . \frac{1}{t}\left(\frac{\bar{\alpha} - \alpha_t}{\bar{\alpha}\, \alpha_t}\right) \tag{25}$$

and let $G(v) = T$ and $H(v) = Q$ where Q and T are arbitrary constants, then we obtain $b_2 = {}^T/Z_1$ and $b_3 = {}^Q/Z_1$, and again diffusion speed is inversely related to profitability. Thus the functional form of the relationship between $\frac{d\gamma_{Nt}^2}{dt}$ and α_t will determine the shape of the diffusion curve, and if $F(v)$, $G(v)$ and $H(v)$ are arbitrary constants diffusion speed will be inversely related to profitability.

Case 2

The next obvious case is to consider $F(v)$, $G(v)$ or $H(v)$ as functions of the initial or final values for α_t. Thus if we allow e.g. $F(v) = S_1 \cdot \bar{\alpha}$ we can rewrite (23) as (26)

$$\frac{d\gamma_{Nt}^2}{dt} = - S_1 \cdot \frac{\bar{\alpha} - \alpha_t}{\alpha_t} \tag{26}$$

in which case the variance changes in proportion to the distance still to travel along the diffusion curve relative to the distance actually travelled. In this case $b_1 = \frac{S_1 \bar{\alpha}}{Z_1} = \frac{S_1}{\bar{\gamma}_N^2 + \gamma_0^2 - 2\gamma_{NO}}$ and the diffusion speed is now independent of the level of profitability. Similarly, one could generate corresponding results for the log normal and Gompertz curve and achieve this same independence.

Case 3

The third case is one that will generate a diffusion speed positively related to profitability. One can argue that γ_{Nt}^2 is not a good measure of risk, but that γ_{Nt}^2/μ_N is better. The reasoning is that a large variance of returns is not too important if the mean return is also high. We then allow that the entrepreneur varies γ_{Nt}^2/μ_N as α_t changes in a manner similar to Case 2. Thus as $\frac{d(\gamma_{Nt}^2/\mu_N)}{dt} = \frac{d\gamma_{Nt}^2}{dt} \cdot \frac{1}{\mu_N}$ we have that (27) holds

$$\frac{d\gamma_{Nt}^2}{dt} = -\mu_N \cdot S_1 \cdot \frac{\bar{\alpha} - \alpha_t}{\alpha_t} . \tag{27}$$

Then $b_1 = \frac{S\,\mu_N}{\bar{\gamma}_N^2 + \gamma_0^2 - 2\gamma_{NO}}$ and the diffusion speed is positively related to the level of profitability. Similar arguments can be used for the Gompertz and Log-Normal curves.

These three cases are obviously not exhaustive, but they show what is required to relate diffusion speed to profitability. However, we still consider it an open question whether experience gained over time should lead to changes in γ^2_{Nt} or γ^2_{Nt/μ_N}, or whether the learning mechanism is a function of $\frac{\bar{\alpha} - \alpha_t}{\bar{\alpha}\ \ \alpha_t}$, $\frac{\bar{\alpha} - \alpha_t}{\alpha_t}$, $\log \frac{\bar{\alpha}}{\alpha_t}$, $\bar{\alpha} \log \frac{\bar{\alpha}}{\alpha_t}$, $\frac{1}{t}\left(\frac{\bar{\alpha} - \alpha_t}{\bar{\alpha}\ \ \alpha_t}\right)$ or $\frac{1}{t}\left(\frac{\bar{\alpha} - \alpha_t}{\alpha_t}\right)$, each combination of which produces a different diffusion curve with its own determinants of the diffusion speed. We have not found any good a priori reasoning for the choice of any particular combination. However these mechanisms depend essentially on relations between $\bar{\alpha}_t$ and α_t. If we consider $\bar{\alpha}_t$ as implying full information and α_t as measuring current information, then we have e.g. learning as a function of $\bar{\alpha}_t - \alpha_t$, which we could consider to be a measure of information yet to be received, or $\frac{\bar{\alpha} - \alpha_t}{\bar{\alpha}}$, the proportion of total information yet to be learnt, or $\frac{\bar{\alpha}}{\alpha_t}$, the log of the inverse of the proportion of total information held. Then the amount of the current lack of information being made up and influencing the variance is, for example, a function of $\frac{1}{\alpha_t}$, or $1/t$, i.e. related to the distance travelled along the diffusion curve. The general principle being implied is that learning in a period is related to the amount yet to be learnt and the probability of learning anything new in the current period. Thus although we make no specific choice, the principle behind these mechanisms seems sufficiently realistic to make us reasonably satisfied that this is a viable theory of diffusion.

(b) <u>Variable mean profitability</u>

Consider that $d\gamma^2_{Nt}/dt = 0$, for all t, then from equation (10) we may derive that

$$\frac{d\alpha_t}{dt} = Z_2 \frac{d\mu_{Nt}}{dt} \qquad (28)$$

where $Z_2 = \dfrac{a}{b(\gamma_N^2 + \gamma_O^2 - 2\gamma_{NO})}$. Davies (1978) has argued that as time proceeds further technological advances increase the profitability of the new technology relative to the old. Although this may be in contradiction to the Schumpeter (1912) view of declining entrepreneurial profit with increasing usage we may model this process by finding the time paths of μ_{Nt} which when substituted into (15) will produce the S shaped diffusion curves. An alternative approach is to argue that the entrepreneur's subjective evaluation of expected profitability will change with usage, as did γ_{Nt}^2 in the learning mechanisms detailed above. The problem with this approach is that if α_t is to increase over time μ_{Nt} must rise monotonically and there seems no good reason for arguing that the entrepreneur will always underestimate the mean of the distribution of returns in the early years [1/] although it was reasonable to consider him overestimating risk in the above framework. However to illustrate for the logistic case, if μ_{Nt} is determined by the learning mechanism (29), where J(v) is some function of variables independent of time.

$$\frac{d\mu_{Nt}}{dt} = J(v) \cdot \alpha_t \cdot \frac{\bar{\alpha} - \alpha_t}{\bar{\alpha}} \qquad (29)$$

then a logistic curve results and $b_1 = Z_2.J(v)$. The determinants of J(v) thus determine the speed of diffusion.

Consider then the preferred approach where μ_{Nt} rises through further technological progress. Letting $\hat{\mu}_N = \mu_{Nt}$ for $t = 0$ and $\bar{\mu}_N = \mu_{Nt}$

1/ Unless one considers a relationship of μ_{Nt} to α_t to reflect scale economies, except it then becomes difficult to justify the appearance of $\ddot{\alpha}$ in (29).

at the limit (which in turn determine $\hat{\alpha}_t$ and $\bar{\alpha}_t$) to generate a logistic diffusion curve we require that μ_{Nt} followsa time path given by (30)

$$\mu_{Nt} = \bar{\mu}_N - \frac{(\bar{\mu}_N - \mu_0 + \frac{b}{a}(\gamma_0^2 - \gamma_{NO})}{\frac{(\hat{\mu}_N - \mu_0) + \frac{b}{a}(\gamma_0^2 - \gamma_{NO})}{\bar{\mu}_N - \mu_N} \exp Yt + 1} \tag{30}$$

where Y is the parameter determining the speed of transition of μ_{Nt} from $\hat{\mu}_N$ to $\bar{\mu}_N$. If μ_{Nt} follows a time path of the form of (30) then $b_1 = Y$. Similar results can be derived for the Gompertz and Log-Normal curves.

With this scenario therefore, the factors determining the diffusion speed are those factors that determine the rate of technological improvement of a new process. One might turn here to Schmookler (1966) or Rosenberg (1974) arguing perhaps that the total potential market and technological opportunity are the key factors.

As we require $b_1 > 0$, this model predicts that the diffusion will occur if technology improves over time. The main objection to this approach is that we have no good reason for arguing that μ_{Nt} follows any particular time path. The fact that S shaped curves do so often result in practice would, under this approach, imply that such special time paths for μ_{Nt} are common, but there seems no good reason why they should be. The important point, however, is that this approach does produce a predictable result, that the speed of diffusion is determined by the speed at which expected profitability increases over time. It would in fact seem realistic to argue that for a given level of risk extensions of usage of the technology require greater profit to compensate.

The final version of the model is one in which both γ^2_{Nt} and μ_{Nt} are changing over time. The path of α_t is then given by (10) which may be written as (31).

$$\frac{d\alpha_t}{dt} \cdot \frac{1}{\alpha_t} = \frac{a/b}{Z_1} \frac{d\mu_{Nt}}{dt} - \frac{\alpha_t}{Z_1} \frac{d\gamma^2_{Nt}}{dt} \tag{31}$$

Considering the logistic case, if a logistic diffusion curve is to result from movements in μ_{Nt} and γ^2_{Nt} (32) must hold.

$$b_1\left(\frac{\bar{\alpha} - \alpha_t}{\bar{\alpha}}\right) = \frac{a/b}{Z_1} \frac{d\mu_{Nt}}{dt} - \frac{\alpha_t}{Z_1} \frac{d\gamma^2_{Nt}}{dt} \tag{32}$$

from which we can see that if $d\gamma^2_{Nt/dt} > 0$ then $\frac{d\mu_{Nt}}{dt}$ must be greater than zero. We would expect however that $\frac{d\gamma^2_{Nt}}{dt} < 0$, in which case $\frac{d\mu_{Nt}}{dt}$ may be greater or less than zero. As no further general results seem forthcoming, we will assume that $d\gamma^2_{Nt/dt}$ falls over time in accordance with the learning mechanism (20) as discussed in Case 1 above. We then find that if a logistic curve is to result $d\mu_{Nt/dt}$ must satisfy (33)

$$\frac{d\mu_{Nt}}{dt} = \left(\frac{\bar{\alpha} - \alpha_t}{\bar{\alpha}}\right)\left(b_1\mu_{Nt} - b_1\mu_0 + b_1 \frac{b}{a} (\gamma^2_0 - \gamma_{NO}) - \frac{b}{a} S\right) \tag{33}$$

which implies (34).

$$\frac{d\mu_{Nt}}{dt}\left(1 + \exp (a_1 + b_1 t)\right) - b_1\mu_{Nt} = - b_1\mu_0 + b_1 \frac{b}{a}(\gamma^2_0 - \gamma_{NO}) - \frac{b}{a} S \tag{34}$$

Equation (34) has the solution (35),

$$\mu_{Nt} = \frac{\exp(a_1 + b_1 t)}{1 + \exp(a_1 + b_1 t)} + \mu_0 - \frac{b}{a}(\gamma_0^2 - \gamma_{N0}) + \frac{b}{a}\frac{S}{b_1} . \qquad (35)$$

Thus, to generate a logistic diffusion curve μ_{Nt} must grow along a curve such as (35) and the rate at which it grows determines the speed of diffusion, i.e. the speed of diffusion is determined by the rate of growth of profitability. If we wish to generate Gompertz or log normal curves we would replace learning mechanism (20) by (21) or (22) and solve for the path of μ_{Nt} that generates the appropriate curves. In each case the diffusion speed is determined by the rate at which profitability grows.

To summarise, we have constructed a model of the diffusion process where changes over time in the usage of a new technology are determined by changes in the subjective distribution of its returns. We have shown that if profitability is constant then S shaped diffusion curves can be generated by relating reductions in risk to a learning mechanism and the shape of the diffusion curve and the determinants of the diffusion speed depend on the learning mechanism adopted. If profitability is changing, then we have shown that if either γ_{Nt}^2 is constant or is related via a learning mechanism to the usage of the new technology, then the diffusion curve will be generated by the time path of μ_{Nt}, and the rate at which profitability grows determines the speed of diffusion.

Conclusions

We have constructed a theory of intra-firm diffusion based on a means-variance approach to the technique choice decision. If one were willing to accept the device of the representative entrepreneur this would yield either an industry or economy wide theory of diffusion. Even if one does reject this device the basis of the model should however be amenable to development into an alternative industry or economy wide theory of diffusion based on differences in individual entrepreneurs (following a line similar to Davies (1978)). We suggest that the approach yields a reliable theory of, at least, intra-firm diffusion.

We have shown that the model can be used to predict the existence of S shaped diffusion curves by using either a learning mechanism to reduce risk, holding profitability constant or by allowing profitability to increase over time with risk constant or being determined by a learning mechanism. This is in conflict with existing theory as presented by Mansfield (1968) in which he "supposes" that the change in usage is related to the levels of risk and return rather than the changes in risk and return.

Assuming, perhaps unrealistically, that the returns to the old technology have remained constant over time, we concentrated on a scenario where the mean return to the new technique remains constant over time but the risk attached to the new technology is allowed to change over time. We have shown that S shaped diffusion curves will be generated if risk reduces over time according to a learning mechanism relating the change in risk to the level of usage of the new technology. The form of this learning mechanism determines not only the shape of the diffusion curve but also specifies the determinants of diffusion speed. We have shown the implications that arise from choosing different learning mechanisms, concentrating on the empirically much studied question (Kennedy and Thirlwall (1972)) of the effect of profitability on the diffusion speed.

We have also investigated a world in which profitability changes over time, and although showing that this requires special time paths for profitability to produce S shaped diffusion curves, we have argued that if such time paths do exist the rate at which profitability increases will determine the diffusion speed. We suggest that this rate will depend on the determinants of the rate of technological change experienced by the new technology.

This framework has been used to illustrate these two main scenarios, but it is capable of extension to either an industry or economy wide theory of diffusion. Moreover the variation of the assumptions on the constancy of the returns to the old technology represents another avenue of development. But it would seem clear that in whatever direction one develops one will meet the need to specify learning mechanisms, and it is precisely in this field that we seem to be short of adequate theory.

P. Stoneman
September 1978

References

K.J. Arrow (1962) "The Economic Implications of Learning by Doing", Review of Economic Studies, pp. 155-173.

J.S. Chipman (1973) "The Ordering of Portfolios in Terms of Mean and Variance", Review of Economic Studies, pp. 167-190.

G.C. Chow (1967) "Technical Change and the Demand for Computers", American Economic Review, pp.1117-1130.

S. Davies (1978) The Diffusion of Process Innovations, Cambridge University Press (forthcoming).

Z. Griliches (1957) "Hybrid Corn: An Exploration in the Economics of Technical Change", Econometrica, pp.501-522.

H.A.J. Green (1971) Consumer Theory, Penguin, Harmondsworth.

C.K. Harley (1973) "On the Persistence of Old Techniques: the Case of North American Wooden Shipbuilding", Journal of Economic History, pp.372-398.

C. Kennedy and A. Thirlwall (1972) "Technical Progress - A Survey", Economic Journal, pp. 11-72.

E. Mansfield (1968) Industrial Research and Technological Innovation, New York, Norton.

N. Rosenberg (1974) "Science, Invention and Economic Growth," Economic Journal pp. 90-108.

J. Schmookler (1966) Invention and Economic Growth, Harvard University Press.

J.A. Schumpeter (1912) The Theory of Economic Development, Oxford University Press.