

Risk Taking and Taxation: An Alternative Perspective

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### Abstract

The analysis of risk taking and taxation has almost invariably been in a portfolio choice framework. This paper presents the alternative perspective of an occupational choice framework - where risk taking involves the additional element of discrete choice between safe and risky activities. It is shown that the specification of equilibrium must of necessity have a general equilibrium character. In this setting the paper develops rules for government intervention in the market equilibrium, and analyses the effects of taxation on risk taking.

1. Introduction

The study of risk taking and taxation has a long tradition in economic analysis. <sup>1/</sup> The "yield versus risk" analysis of Domar and Musgrave (1944) first drew attention to the effect of taxation on the choice between safe and risky assets. The "mean-variance" approach of Tobin (1958) was applied by Richter (1960), but the more general expected utility approach of Mossin (1968) and Stiglitz (1969) has now become standard. A distinctive feature of this whole literature is that the analysis is in a portfolio choice framework, where the agent controls continuous choice variables - the proportions of wealth invested in safe and risky assets - to maximise his objective function. The additional element of discrete choice between safe and risky activities (or occupations) is therefore not present in this tradition.

The object of this paper is to develop an alternative perspective on risk taking and taxation by modelling risk taking in an occupational choice framework, where the choice of a continuous control variable is combined with a discrete choice between occupations. It is shown that in this discrete choice setting the specification of equilibrium must of necessity have a general equilibrium character, and an analysis of the effects of taxation on risk taking and welfare must take these general equilibrium feedbacks into account. <sup>2/</sup> The plan of the paper is as follows. Section 2 presents the basic model of choice between entrepreneurship and wage labour, and specifies the equilibrium of the economy. Section 3 compares market equilibrium to that of a planned economy which controls access to the occupations. Section 4 develops rules for the introduction of an occupational tax subsidy scheme in an unplanned economy. Similarly, Section 5 develops rules for

the introduction of a progressive or regressive linear tax regime. Section 7 elaborates further on the effect of income taxation on risk taking in the model. Section 7 presents some concluding observations.

## 2. The Basic Model

The model of risk taking developed in this section is introduced in Kanbur (1977). It is a single period, general equilibrium model of occupational choice under risk. There are two occupations, centred on a risky production function which uses labour to produce a homogeneous output

$$F(L, \theta) \tag{1}$$

where  $L$  is the labour employed and  $\theta$  is a random variable defined on the range  $[\theta \text{ min}, \theta \text{ max}]$  with a probability density function  $g(\theta)$ . The production function satisfies the following properties

$$F_L > 0, F_{LL} < 0, F_\theta > 0, F_{\theta L} > 0, F(0, \theta) = 0 \tag{2}$$

where, as throughout the paper, subscripts denote partial derivatives.

The agent has two alternatives open to him. He can either become a "labourer", in which case he supplies a unit of labour<sup>3/</sup> of uniform quality and receives the safe competitive wage. Or he can become an "entrepreneur" - which in this model is simply the management of a production function (1) by employing labour at a guaranteed wage while bearing the production (and hence income) risk represented by  $\theta$ . One interpretation of  $\theta$  is in terms of ability risk.<sup>4/</sup> It is assumed that agents do not know their own entrepreneurial ability  $\theta$  but take  $g(\theta)$ , the density of  $\theta$  in the population, as the relevant risk. (The interpretation of  $\theta$  as an ability

index also makes intuitive sense of the assumptions  $F_{\theta} > 0$ ,  $F_{\theta L} > 0$  in (2).)

The risk faced by the prospective entrepreneur is that he has to make his labour hiring decision before he discovers his entrepreneurial ability  $\theta$ . The income of an entrepreneur who hires  $L$  units of labour and then discovers his ability to be  $\theta$  is given by

$$y = F(L, \theta) - wL \quad (3)$$

Agents are assumed to have a common von Neumann-Morgenstern utility function

$$U(y); U_y > 0, U_{yy} \leq 0 \quad (4)$$

and prospective entrepreneurs choose  $L$  to maximise expected utility of income, which gives

$$V(w) = \text{Max}_L E (U (F (L, \theta) - wL)) \quad (5)$$

where  $E$  is the expectation operator with respect to the density of  $\theta$ .

The first and second order conditions for the problem (5) are, respectively, <sup>5/</sup>

$$E(U_y [F_L - w]) = 0 \quad (6)$$

$$E(U_{yy} [F_L - w]^2) + E(U_y F_{LL}) < 0 \quad (7)$$

and it is seen from the properties of  $F$  and  $U$ , given in (3) and (5), that (8) is satisfied.

How is market equilibrium characterised in this model? Firstly, the labour market must clear - those who enter the entrepreneurial activity must demand just enough labour to employ those who enter wage labour. Secondly, to sustain equilibrium when all agents have a common utility function it must be the case that neither activity is preferred to the other. Along with the continuous variable choice problem (5) agents also face a discrete choice between the entrepreneurial activity, which gives an expected utility  $V(w)$ , and wage labour, which gives a utility  $U(w)$ . If either of these is strictly greater than the other then all agents will enter one activity or the other, and the labour market will not clear. Thus we require that

$$V(w) = U(w) \tag{8}$$

when this holds, the distribution of population between the two activities is determined by the full employment condition. Denoting  $x$  as the proportion of population (a representative sub-sample of the whole) that enters the risky occupation, and normalising population size at unity, full employment requires that

$$x\hat{L} = 1 - x$$

or

$$x = \frac{1}{1 + \hat{L}} \tag{9}$$

where  $\hat{L} = \hat{L}(w)$  is the optimised labour demand of a prospective entrepreneur, derived from (5).

Equilibrium is thus characterised by a wage which is the solution to (8). The solution exists because  $U(w)$  is a monotonic increasing function of  $w$  while from (5) and (6) it is clear that

$$V_w = -\hat{L} E(U_y) < 0 \quad (10)$$

so that  $V(w)$  is a monotonic decreasing function of  $w$ . The solution to (8) then determines a distribution of population via the full employment condition (9).

If the social welfare function is assumed to be the expost sum of utilities, then social welfare generated by the market equilibrium is given by

$$\begin{aligned} S &= x V(w) + [1 - x] U(w) \\ &= V(w) = U(w) \end{aligned} \quad (11)$$

Total output (or national income), on the other hand, is given by

$$Y = x E(F(\hat{L}, \theta)) = \frac{E(F(\hat{L}, \theta))}{1 + \hat{L}} \quad (12)$$

Does the market equilibrium have too few or too many entrepreneurs?

The answer to this question depends on the particular instruments the government has to effect control. The following sections consider three such instruments - control of access to occupations, occupational tax-subsidy schemes, and income taxation.



3. Control of Access to Occupations

In this section we will examine the response to market equilibrium of a government which has quantitative control on the occupational distribution of population (perhaps through a system of licences), but which does not control the labour hiring decision of entrepreneurs. The wage is then adjusted, for the chosen distribution of population, to clear the labour market. Thus the government chooses  $x$  and the wage  $w$  adjusts to satisfy

$$x \hat{L}(w) = 1 - x$$

$$\text{or } w = \hat{L}^{-1}\left(\frac{1-x}{x}\right) \quad (13)$$

Under these circumstances, would the government wish to direct more or less people into entrepreneurship? We assume that its objective function is the sum of ex post utilities:

$$\bar{S} = x E(U(F(\hat{L}, \theta) - w\hat{L})) + [1-x] U(w) \quad (14)$$

Then, differentiating (14) with respect to  $x$ ,

$$\frac{\partial \bar{S}}{\partial x} = [1-x] \frac{dw}{dx} [U_w - E(U_y)] + [E(U(y)) - U(w)] \quad (15)$$

where, from (13),

$$\frac{dw}{dx} = - \frac{1}{\hat{L}_w x^2} \quad (16)$$

We wish to analyse government policy when faced with market equilibrium. Thus we evaluate (15) at the market equilibrium (ME), and using the equilibrium condition (8) we get that

$$\left. \frac{\partial \bar{S}}{\partial x} \right|_{ME} = [1 - x] \frac{dw}{dx} [U_w - E(U_y)] \quad (17)$$

The expression (17) illuminates the possible non optimality of market equilibrium. Because of the discrete choice element in our modelling of risk taking, equilibrium requires that expected total utility in the two activities be equal. But from the policy point of view it is the difference between expected marginal utility which is important, and equality of total utility need not imply equality of marginal utility - hence the sub-optimality of market equilibrium.

Notice of course that when the utility function is linear,

$$\left. \frac{\partial \bar{S}}{\partial x} \right|_{ME} = 0$$

which leads us to our first Proposition.

Proposition 1. In a risk neutral society government control of access to occupations cannot improve upon market equilibrium

For a risk averse society, define the Arrow-Pratt measure of absolute risk aversion

$$A(y) = - \frac{U_{yy}}{U_y} \quad (18)$$

In the Appendix it is shown that

$$A_y < 0 \Rightarrow \hat{L}_w < 0 \quad (19)$$

so that, from (16),

$$A_y < 0 \Rightarrow \frac{dw}{dx} > 0 \quad (20)$$

when  $\frac{dw}{dx} > 0$ , (17) tells us that

$$\left. \frac{\partial \bar{S}}{\partial x} \right|_{ME} \begin{matrix} > \\ < \end{matrix} 0 \Leftrightarrow U_w \begin{matrix} > \\ < \end{matrix} E(U_y) \quad (21)$$

The expression (21) provides us with a general characterisation result which we state as a proposition.

Proposition 2. If there is non-increasing absolute risk aversion, then the government would wish to direct more people (fewer people) into the risky activity than present in the market equilibrium if the expected marginal utility in the risky activity is less than (greater than) the marginal utility in the safe activity.

The interpretation of this proposition is of interest, since it may seem surprising at first sight - the government is directing more people into the activity with lower expected marginal utility. But this result is a natural consequence of the general equilibrium nature of the problem. Under non-increasing absolute risk aversion  $\frac{dw}{dx} > 0$ , so that directing more people into an activity will lower its expected total utility (see (10)) because of these general equilibrium feedbacks, and precisely for this

reason raises expected marginal utility in that activity. Since increasing welfare requires a narrowing in the expected marginal utility differential between the two activities, we have the result that the government will direct more people into the activity with lower expected marginal utility.

The condition (21) can be related to certain restrictions on preferences and technology. Define the Arrow-Pratt measure of relative risk aversion.

$$R(y) = \frac{-yU_{yy}}{U_y} \quad (22)$$

Let us restrict the utility function to the constant relative risk aversion family

$$U(y) = \begin{cases} \frac{y^{1-\epsilon}}{1-\epsilon} & \text{for } 0 < \epsilon < 1 \text{ and } \epsilon > 1 \\ \log y & \text{for } \epsilon = 1 \end{cases} \quad (23)$$

It follows that

$$wU_w - E(yU_y) = U(w) - E(U(y)) = 0 \quad (24)$$

using the market equilibrium condition (8). If we further restrict the production function (1) to the separable form

$$F(L, \theta) = H(\theta) G(L) \quad (25)$$

then, using the first order condition (6) we get that

$$E(yU_y) = wL \left[ \frac{1}{\alpha} - 1 \right] E(U_y) \quad (26)$$

where

$$\alpha = \frac{LG_L}{G} \quad (27)$$

Substituting (26) in (24),

$$U_w - E(U_y) = -E(U_y) \left[ 1 - L \left[ \frac{1}{\alpha} - 1 \right] \right] \quad (28)$$

so that

$$\frac{\partial \bar{S}}{\partial x} \Bigg|_{ME} \begin{matrix} > \\ < \end{matrix} 0 \iff \alpha \begin{matrix} < \\ > \end{matrix} \frac{L}{1+L} \quad (29)$$

Thus when preferences satisfy (23) and technology is of the separable form (25), the characterisation result (21) can be reduced to a simple condition on technology, involving the labour elasticity of output.<sup>6/</sup>

The characterisation of the optimal distribution of population between safe and risky activities follows by setting the derivative (15) to zero, which gives us a condition on the differences between expected total utilities and expected marginal utilities:

$$\frac{w}{x\eta} = \frac{U(w) - E(U(y))}{U_w - E(U_y)} \quad (30)$$

where

$$\eta = - \frac{\hat{wL}_w}{\hat{L}} \quad (31)$$

We end this section with a characterisation of the effect of government control of access to occupations on national income. From (12),

$$\begin{aligned} \frac{dY}{dx} &= Y_L \hat{L}_w \frac{dw}{dx} \\ &= \frac{E(LF_L)}{E(F)} - \frac{L}{1+L} \\ &\quad - x^2 E(F) [1+L] L \end{aligned} \quad (32)$$

In the separable case (25), this gives us

$$\frac{dY}{dx} \begin{matrix} > \\ < \end{matrix} 0 \iff \alpha \begin{matrix} < \\ > \end{matrix} \frac{L}{1+L} \quad (33)$$

In other words, whether national income rises or falls when more people are directed into the risky activity depends on a condition on technology, as in (29). In fact, comparing (33) and (29) we can see that

$$\left. \frac{\partial \bar{S}}{\partial x} \right|_{ME} \begin{matrix} > \\ < \end{matrix} 0 \iff \frac{dY}{dx} \begin{matrix} > \\ < \end{matrix} 0 \quad (34)$$

Proposition 3. When preferences and technology are restricted to (23) and (25), then in the neighbourhood of market equilibrium the appropriate rule for restricting occupational entry is to intervene to increase national income.

This completes our analysis of the case where the government has direct control on the distribution of population between safe and risky activities - which we may think of as the "planned economy case". The next two sections will consider the case where the government is restricted to tax-subsidy schemes.

4. Occupational Taxation

In this section we assume that the government can identify the occupational origin of incomes, and can tax occupations differentially. Specifically, we restrict attention to a self-financing proportional tax-subsidy scheme across occupations. When faced with the market equilibrium, should the government tax or subsidise the risky activity? We develop rules which provide an answer to this question, at least in the neighbourhood of market equilibrium.

Consider a proportional tax rate  $t$  on incomes from the risky activity, and a proportional subsidy rate  $s$  on wage income (of course  $t$  and  $s$  can be negative, which is a tax on wage income to subsidise risky incomes). The post tax incomes in the risky and safe activities are given respectively by

$$y^* = [1 - t]y \quad (35)$$

$$w^* = [1 + s]w \quad (36)$$

The prospective entrepreneurs' problem is to maximise expected utility of post-tax income, which gives

$$V(t, w) = \underset{L}{\text{Max}} E U ([1 - t] [F(L, \theta) - wL]) \quad (37)$$

The first order condition is

$$E (U_y (y^*) [F_L - w]) = 0 \quad (38)$$

which defines labour demand

$$L(t, w) \tag{39}$$

Market equilibrium is specified, as before, by a labour market clearing occupational distribution of population

$$x = \frac{1}{1 + L(t, w)} \tag{40}$$

and the condition that agents be indifferent between the two activities:

$$V(t, w) = U([1 + s]w) \tag{41}$$

As previously, it can be shown that equilibrium exists and is unique.

The tax revenue raised and subsidy disbursed for given  $t$  and  $s$  depends on the market equilibrium which attains when this regime is imposed. Net tax revenue is given by

$$T = xtE(y) - [1 - x]sw \tag{42}$$

and the self-financing condition therefore requires that

$$s = \frac{t}{1-x} \cdot \frac{E(y)}{w} = \frac{t E(y)}{wL(t, w)} \tag{43}$$

We assume social welfare to be given by the expost sum of post tax utilities:



$$\begin{aligned}
S &= xV(t, w) + [1 - x] U([1 + s] w) \\
&= V(t, w) = U([1 + s] w)
\end{aligned} \tag{44}$$

using the market equilibrium condition (41). The effect on social welfare of a small change in the tax rate  $t$  is thus given by

$$\frac{dS}{dt} = U_w \left\{ [1 + s] \frac{dw}{dt} + w \frac{ds}{dt} \right\} \tag{45}$$

where  $\frac{dw}{dt}$  and  $\frac{ds}{dt}$  are jointly determined by the market equilibrium condition (41) and the self-financing condition (43). Differentiating (41) and (43) we get, respectively,

$$\frac{dw}{dt} \left\{ [1 - t] L(t, w) E(U_y) + U_w [1 + s] \right\} + E(yU_y) + wU_w \frac{ds}{dt} = 0 \tag{46}$$

$$\frac{ds}{dt} = \frac{E(y)}{wL(t, w)} + t \cdot d \left( \frac{E(y)}{wL(t, w)} \right) \tag{47}$$

A combination of (45), (46) and (47) will give us a general characterisation of the direction of welfare change consequent upon a tax change. As can be seen, this general case presents us with considerable intractability. However, we can restrict attention to government policy in the neighbourhood of market equilibrium. Mathematically this means that we evaluate the derivatives at  $t = 0$ ,  $s = 0$  :

$$\left. \frac{ds}{dt} \right|_{t=0, s=0} = \frac{E(y)}{wL} \tag{48}$$

$$\left. \frac{dw}{dt} \right|_{t=0, s=0} = - \frac{E(yU_y) + \frac{E(y)}{L} U_w}{LE(U_y) + U_w} \quad (49)$$

$$\begin{aligned} \left. \frac{dS}{dt} \right|_{t=0, s=0} &= U_w \left\{ \left. \frac{dw}{dt} \right|_{t=0, s=0} + w \left. \frac{ds}{dt} \right|_{t=0, s=0} \right\} \\ &= \frac{E(y) E(U_y) - E(yU_y)}{LE(U_y) + U_w} \cdot U_w \end{aligned} \quad (50)$$

Thus

$$\left. \frac{dS}{dt} \right|_{t=0, s=0} \begin{matrix} \geq \\ < \end{matrix} 0 \iff E(y) E(U_y) \begin{matrix} \geq \\ < \end{matrix} E(yU_y) \quad (51)$$

Ofcourse with decreasing marginal utility income and marginal utility are negatively correlated, so that

$$E(y) E(U_y) > E(yU_y)$$

and

$$\left. \frac{dS}{dt} \right|_{t=0, s=0} > 0$$

Proposition 4. Starting from a position of market equilibrium, the government should tax and subsidise the safe occupation.

What are the implications of the above policy for the distribution of population between safe and risky activities? Differentiating (40),

$$\begin{aligned} \frac{dx}{dt} &= x_L \frac{dL(t, w)}{dt} \\ &= - \frac{1}{[1 + L]^2} \left[ L_w \frac{dw}{dt} + L_t \right] \end{aligned} \quad (52)$$

In the Appendix it is shown that

$$R_y \begin{matrix} > \\ \cong \\ < \end{matrix} 0 \iff L_t \begin{matrix} > \\ \cong \\ < \end{matrix} 0 \quad (53)$$

Thus, since  $L_w < 0$  and  $\left. \frac{dw}{dt} \right|_{t=0, s=0} < 0$ , we have the result that

$$\left. \frac{dx}{dt} \right|_{t=0, s=0} < 0 \text{ if } R_y \geq 0 \quad (54)$$

Proposition 5. If there is non decreasing relative risk aversion, then a government imposing (marginally) a cross-occupational tax-subsidy scheme to increase welfare will do so to reduce entry into the risky activity.

Finally, let us consider the effect of such a cross-occupational tax-subsidy scheme on national income:

$$\frac{dY}{dt} = \frac{\frac{E(LF_L)}{E(F)} - \frac{L}{1+L}}{E(F)[1+L]L} \left[ L_w \frac{dw}{dt} + L_t \right] \quad (55)$$

In the separable technology case (25) we get that with non-decreasing relative risk aversion,

$$\left. \frac{dY}{dt} \right|_{t=0, s=0} \begin{matrix} > \\ = \\ < \end{matrix} 0 \iff \alpha \begin{matrix} > \\ = \\ < \end{matrix} \frac{L}{1+L} \quad (56)$$

Thus, unlike the conclusion of the previous section, in this taxation scheme there may be cases  $(\alpha < \frac{L}{1+L})$  when a government policy which increases welfare will lower national income.

## 5. Income Taxation

In this section we focus on the use of income taxation to modify market equilibrium in such a way as to increase welfare. In the context of risk taking, as modelled in the discrete choice framework of this paper, should self-financing income taxation be progressive or regressive? We will investigate this question in the framework of a linear tax regime.

$$y^* = a + by \quad (57)$$

where  $y$  is pretax and  $y^*$  is post tax income. The marginal tax rate is  $[1 - b]$  and the average tax rate is given by

$$\frac{-a}{y} + [1 - b]$$

Hence the regime is progressive (regressive), in the sense that the average tax rate rises (falls) with income, if  $a > 0$  ( $< 0$ ).

Prospective entrepreneurs will now choose their labour demand so as to maximise the expected utility of post tax income (57), which gives

$$V(a, b, w) = \underset{L}{\text{Max}} E (U (a + b[F(L, \theta) - wL])) \quad (58)$$

The first and second order conditions are given respectively by

$$E(U_y (y^*) [F_L - w]) = 0 \quad (59)$$

$$b^2 E (U_{yy} [F_L - w]^2) + b E (U_y F_{LL}) < 0 \quad (60)$$

and it is seen from the properties of the utility function and production function that (60) is satisfied. Labour demand is defined as a function of the wage and the tax parameters

$$L(a, b, w) \quad (61)$$

Equilibrium is specified as before by

$$x = \frac{1}{1 + L(a, b, w)} \quad (62)$$

from the labour market clearing condition, and

$$V(a, b, w) = U(w^*) \quad (63)$$

where

$$w^* = a + bw \quad (64)$$

is the post tax wage. As in the previous sections, for any given pair of tax parameters  $a$  and  $b$ , a wage exists which solves (63), and this with (62)

defines a unique equilibrium. If tax revenue is  $T$  then the self-financing requirement is that

$$\begin{aligned} T &= x E (y - y^*) + [1 - x] [w - w^*] \\ &= -a + [1 - b] Y (a, b, w) = 0 \end{aligned} \quad (65)$$

where

$$Y(a, b, w) = \frac{E(F(L(a, b, w), \theta))}{1 + L(a, b, w)} \quad (66)$$

is national income gross of tax.

Social welfare is again assumed to be the expost utilitarian sum:

$$\begin{aligned} S &= xV(a, b, w) + [1 - x] U(w^*) \\ &= V(a, b, w) = U(w^*) \end{aligned} \quad (67)$$

We parametrise tax regime changes by changes in  $b$ , and we wish to examine the effect on welfare of a change in the marginal tax rate subject to the self-financing condition (65), and the response of market equilibrium from (62) and (63). From (67) we see that social welfare is monotonically related to the post tax wage  $w^*$ . Differentiating this with respect to  $b$ ,

$$\frac{dw^*}{db} = \frac{da}{db} + b \frac{dw}{db} + w \quad (68)$$

The derivatives  $\frac{da}{db}$  and  $\frac{dw}{db}$  are jointly determined by the self-financing condition and the response of market equilibrium. Differentiating (63) and (65) we get, respectively,

$$\begin{aligned} \frac{da}{db} [E(U_y) - U_w(w^*)] - b \frac{dw}{db} [LE(U_y) + U_w(w^*)] \\ = wU_w(w^*) - E(yU_y) \end{aligned} \quad (69)$$

$$\frac{da}{db} = \frac{[1 - b] [Y_b + Y_w \frac{dw}{db}] - Y}{1 - [1 - b] Y_a} \quad (70)$$

Solving (69) and (70) simultaneously,

$$\frac{dw}{db} = \frac{wU_w(w^*) - E(yU_y) + \frac{[Y - [1 - b] Y_b] [E(U_y) - U_w(w^*)]}{1 - [1 - b] Y_a}}{\frac{[E(U_y) - U_w(w^*)] [1 - b] Y_w}{1 - [1 - b] Y_a} - b [LE(U_y) + U_w(w^*)]} \quad (71)$$

As can be seen, the effect on welfare of a revenue compensated change in the tax schedule is somewhat complicated in the general case. However, as in previous sections we propose to focus attention on a neighbourhood of market equilibrium. This means that we evaluate derivatives at  $b = 1$  ( $a = 0$ ):

$$\left. \frac{da}{db} \right|_{b=1} = -Y \quad (72)$$

$$\left. \frac{dw}{db} \right|_{b=1} = - \frac{wU_w - E(yU_y) + Y [E(U_y) - U_w]}{LE(U_y) + U_w} \quad (73)$$

In the case where the utility function is given by (23) and the production function is given by (25), then using (24) and (28) we can further simplify (73) to

$$\left. \frac{dw}{db} \right|_{b=1} = - \frac{\alpha Y}{L} [1 - L \left[ \frac{1}{\alpha} - 1 \right]] \quad (74)$$

so that

$$\left. \frac{dw^*}{db} \right|_{b=1} = \frac{wL - \alpha Y [1 + L]}{L} \quad (75)$$

In other words,

$$\left. \frac{dw^*}{db} \right|_{b=1} \begin{matrix} > \\ \equiv \\ < \end{matrix} 0 \iff \alpha \begin{matrix} \leq \\ > \end{matrix} \frac{w[1-x]}{Y} \quad (76)$$

Notice that under self-financing  $b > 1$  ( $\Rightarrow a < 0$ ) represents a regressive regime and  $b < 1$  ( $\Rightarrow a > 0$ ) represents a progressive regime. Further notice that when technology is of the Cobb-Douglas type  $\alpha$  is simply the share of labour in national income when there is no uncertainty i.e. when the distribution of  $\theta$  is degenerate. We thus have the following proposition:

Proposition 6. Under separable Cobb-Douglas technology and constant relative risk aversion, if marginal deviations from market equilibrium are considered, then progressive (regressive) taxation increases welfare if



the share of labour in national income under uncertainty is less than (greater than) its share under certainty.

This rule is interesting for two reasons. Firstly, the functional distribution of income is seen to provide information on the design of a policy whose chief focus (progressivity versus regressivity) is on the personal distribution of income. Secondly, the difference that uncertainty makes to the economy is identified as important in the design of tax policy. Thus if uncertainty lowers the share of labour, tax progressivity is desirable.

#### 6. Risk Taking

In the previous section we analysed the welfare effects of income taxation. However, the focus of the portfolio choice literature has been not so much on welfare as on the effects of taxation on "risk taking". Following the pioneering analysis of Domar and Musgrave (1944), risk taking in this literature has been defined simply as the demand for risky assets. The analysis is usually partial equilibrium in nature, and the effects of taxation on the supply side or on the pre-tax risk are not treated (an exception is Stiglitz (1970)). We have shown in this paper that a discrete choice or occupational choice framework for the analysis of risk taking necessarily requires a general equilibrium treatment of the problem. In this section we define "risk taking" in our occupational choice framework as the proportion of population engaged in the risky activity, analogously to investment in risky assets in the portfolio choice framework. Our object is to examine the effects of various tax changes on risk taking and compare the results with those obtained in the portfolio choice framework.

The equilibrium setting with linear income taxation has already been described in the previous section. A change in the tax parameters will lead to a change in the wage to satisfy the market equilibrium condition (63), which in turn will change the pre tax (and post tax) income risk faced by prospective entrepreneur's and hence their labour demand from (61), which is also affected directly by the tax parameters. This change in labour demand will lead to a change in our measure of risk taking  $x$ , as given in (62). Following the portfolio choice literature, our main interest will be in the effect on risk taking of a parametric change in the marginal tax rate  $[1 - b]$  accompanied by different types of "compensation" through a change in the lump sum tax  $a$ . Total differentiation of (62) with respect to  $b$  gives us

$$\frac{dx}{db} = x_L \left[ L_a \frac{da}{db} + L_b + L_w \frac{dw}{db} \right] \quad (77)$$

From (62) we know that

$$x_L = - \frac{1}{[1+L]^2} < 0 \quad (78)$$

But it is seen that in order to sign (77) we have five other terms to consider, each of which represents a separate aspect of the interrelated equilibrium system modelled in section 5.  $L_a$ ,  $L_b$  and  $L_w$  reflect the optimisation problem (58), while  $\frac{da}{db}$  and  $\frac{dw}{db}$  jointly depend on the equilibrium condition (63) and the specific compensation scheme being considered.

We can obtain general expressions for  $L_a$ ,  $L_b$  and  $L_w$  by differentiating the first order condition (59):

$$L_a = \frac{-E(U_{yy} [F_L - w])}{b E(U_{yy} [F_L - w]^2) + E(U_y F_{LL})} \quad (79)$$

$$L_b = \frac{-E(U_{yy} [F_L - w] [F - wL])}{b E(U_{yy} [F_L - w]^2) + E(U_y F_{LL})} \quad (80)$$

$$L_w = \frac{E(U_y) + b LE(U_{yy}) [F_L - w]}{b E(U_{yy} [F_L - w]^2) + E(U_y F_{LL})} \quad (81)$$

The signs of these expressions are investigated in the Appendix, where it is shown that conditions on the behaviour of the Arrow-Pratt measures of absolute and relative risk aversion, (18) and (22), will give us determinate signs on (79), (80) and (81).

In this section we will consider two types of compensation schemes which have received attention in the literature - welfare compensation and revenue compensation. However, before doing this we consider the case of no compensation - the simple imposition of a proportional income tax. A basic result in the literature on taxation and risk taking in a portfolio choice framework is that a proportional income tax necessarily increases risk taking given only that absolute risk aversion is decreasing (e.g. Mossin (1968), Stiglitz (1969)). Feldstein (1969) has provided a counter example to this in the occupational choice framework. He shows, as a partial equilibrium exercise, that with relative risk aversion constant a proportional income tax leaves the expected utility ordering of prospects unchanged, and thus has no effect on risk taking. It can be shown that this result carries over

to our general equilibrium specification because with constant relative risk aversion a proportional income tax leaves (i) the market equilibrium condition (63) unchanged and (ii) the labour demand of prospective entrepreneurs unchanged (see Appendix). Hence the equilibrium distribution of population between safe and risky activities is unaffected.

Moving to welfare compensation we notice from the market equilibrium condition (63) that the welfare of an individual, whether entrepreneur or labourer, is given by

$$V(a, b, w) = U(w^*)$$

so that a parametric change in  $b$  will require a corresponding compensation in  $a$  of

$$\frac{da}{db} = -w - b \frac{dw}{db} \quad (82)$$

to keep the individual at the same welfare level.<sup>7/</sup> A basic result in the portfolio choice literature is that increasing the marginal tax rate and compensating for loss of welfare must necessarily increase risk taking, given only decreasing absolute risk aversion (see Cowell (1975)). Does this result hold in the occupational choice framework? To see that it does not necessarily do so, substitute (82) into (69) to get

$$\frac{dw}{db} = - \frac{wE(U_y) - E(yU_y)}{[1 + L] E(U_y)} \quad (83)$$

At the no tax equilibrium, if technology is given by (25) then using (26) we get that

$$\left. \frac{dw}{db} \right|_{b=1} = -\frac{w}{1+L} \left[ 1 - L \left[ \frac{1}{\alpha} - 1 \right] \right] \begin{matrix} \geq 0 \\ < 0 \end{matrix} \Leftrightarrow \alpha \begin{matrix} \leq \\ > \end{matrix} \frac{L}{1+L} \quad (84)$$

Thus if  $\alpha \leq \frac{L}{1+L}$  then  $\left. \frac{dw}{db} \right|_{b=1} \geq 0$ . Moreover if there is increasing or constant relative risk aversion then it is shown in the Appendix that  $L_b \leq 0$ , and that non-increasing absolute risk aversion implies  $L_w < 0$  and  $L_a \geq 0$ . Using these signs in (77), we see that under these conditions

$$\left. \frac{dx}{db} \right|_{b=1} > 0.$$

Proposition 7. With separable technology, non-increasing absolute risk aversion and non-decreasing relative risk aversion, if  $\alpha \leq \frac{L}{1+L}$  then the introduction of a welfare compensated marginal tax rate will decrease or not affect risk taking.

The derivation of a general characterisation result in the occupational choice framework would require the evaluation and comparison of the absolute magnitudes of the expressions (79), (80), (81), (82) and (83). This presents intractabilities of some severity. What we have in Proposition 7, however, is a counterexample that warns against uncritical acceptance of the portfolio choice result.- welfare compensated taxation need not necessarily increase risk taking.

Let us finally consider the revenue compensation case, where as  $b$  is varied,  $a$  is also changed so as to balance the government's budget. A basic result in the portfolio choice literature is that revenue compensated

progressive taxation must necessarily increase risk taking, given only that absolute risk aversion be decreasing (e.g. Ahsan (1974)). Does this hold in the occupational choice framework? We can show that it does not necessarily do so. In the revenue compensated case we know from (72) that  $\left. \frac{da}{db} \right|_{b=1} < 0$  and from (74) that if  $\alpha < \frac{L}{1+L}$  then  $\left. \frac{dw}{db} \right|_{b=1} \geq 0$ . It is still the case that increasing or constant relative risk aversion implies  $L_b \leq 0$  and non-increasing absolute risk aversion implies  $L_w < 0$  and  $L_a \geq 0$ . Using these signs in (77), we see that  $\left. \frac{dx}{db} \right|_{b=1} \geq 0$ .

Proposition 8. With separable technology, non-increasing absolute risk aversion and non-decreasing relative risk aversion, if  $\alpha < \frac{L}{1+L}$  then the introduction of revenue compensated progressive taxation will decrease or not affect risk taking.

Again it should be clear that a general characterisation result presents severe intractabilities. The best that seems available is a counter-example giving conditions on the strength of diminishing returns ( $\alpha$ ) to labour in production which will ensure that progressive taxation will not increase risk taking. The results of the portfolio choice literature thus have to be treated with caution.

## 7. Conclusion

The first object of this paper has been to construct a model in which some old questions of taxation and risk taking can be posed in terms of occupational choice. The distinctive feature of the model is that risk taking combines an element of discrete choice with an element of continuous choice. The discrete choice element forces us to specify general equilibrium

in the model, with the result that in the analysis of government policy we have to take account of the general equilibrium feedbacks. In this framework, sections 3, 4 and 5 develop rules for government intervention in the market with three different types of instruments - control of access to occupations, occupational taxation and income taxation. It is shown that the market equilibrium is not necessarily optimal, since in each case we find conditions under which intervention would be desirable. Section 6 presents some counterexamples to existing results on taxation and risk taking in the portfolio choice framework.

The recognition of occupational choice as a major vehicle for risk taking goes at least as far back as Adam Smith. Yet the analysis of taxation and risk taking has almost invariably been in a portfolio choice framework. We hope this paper has shown that the alternative perspective of an occupational choice framework provides an interesting and open area for further research.

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Appendix

We will examine some properties of the solution to the optimisation problem (58),

$$\text{Max}_L E (U (a + b [F (L, \theta) - wL]))$$

with the first order condition (59). Differentiating (59) partially with respect to  $a$ ,  $b$  and  $w$  we get, respectively, (79), (80) and (81). Notice that the denominator of these three expressions is negative (see (60)). Hence the numerators have to be signed in order to sign the whole expression.

We start by considering the sign of  $E (U_{yy} |F_L - w|)$ . Define  $\hat{\theta}$  to be such that

$$F_L (L, \theta) - w \begin{matrix} > \\ = \\ < \end{matrix} 0 \iff \theta \begin{matrix} > \\ = \\ < \end{matrix} \hat{\theta} \quad (\text{A1})$$

From the properties of the production function given in (2),  $\hat{\theta}$  exists and is unique. Moreover notice that since  $F_\theta > 0$  from (2),  $y = F - wL$  is an increasing function of  $\theta$ . Thus, recalling that  $A(y)$  is the measure of absolute risk aversion,

$$A_y \leq 0 \implies \begin{cases} A(y(\theta)) \leq A(y(\hat{\theta})) & \text{if } \theta > \hat{\theta} \\ A(y(\theta)) = A(y(\hat{\theta})) & \text{if } \theta = \hat{\theta} \\ A(y(\theta)) \geq A(y(\hat{\theta})) & \text{if } \theta < \hat{\theta} \end{cases} \quad (\text{A2})$$

Using (A1) and (A2),



$$A_y \leq 0 \Rightarrow A(y(\theta)) [F_L - w] \leq A(y(\hat{\theta})) [F_L - w] \quad \forall \theta \quad (\text{A3})$$

From the definition of  $A(y)$ , in (18), it follows that

$$A_y \leq 0 \Rightarrow U_{yy} [F_L - w] \geq A(y(\hat{\theta})) U_y [F_L - w]$$

Taking expectations and using the first order condition (59),

$$A_y \leq 0 \Rightarrow E(U_{yy} [F_L - w]) \geq 0 \quad (\text{A4})$$

Hence

$$A_y \leq 0 \Rightarrow L_a \geq 0 \quad (\text{A5})$$

$$A_y \leq 0 \Rightarrow L_w < 0 \quad (\text{A6})$$

Turning now to the measure of relative risk aversion,

$$R_y \geq 0 \Rightarrow \begin{cases} R(y(\theta)) \geq R(y(\hat{\theta})) & \text{if } \theta > \hat{\theta} \\ R(y(\theta)) = R(y(\hat{\theta})) & \text{if } \theta = \hat{\theta} \\ R(y(\theta)) \leq R(y(\hat{\theta})) & \text{if } \theta < \hat{\theta} \end{cases} \quad (\text{A7})$$

From (A1) and (A7)

$$R_y \geq 0 \Rightarrow R(y(\theta)) [F_L - w] \geq R(y(\hat{\theta})) [F_L - w] \quad \forall \theta \quad (\text{A8})$$

Taking expectations in (A8) and using the definition of  $R(y)$  in (22) and the first order condition (59),

$$R_y \geq 0 \Rightarrow E(U_{yy} [F_L - w] [F - wL]) \leq -\frac{a}{b} E(U_{yy} [F_L - w]) \quad (A9)$$

Thus when  $a = 0$  and  $b = 1$ ,

$$R_y \geq 0 \Rightarrow L_b \leq 0 \quad (A10)$$

Footnotes

\* I am indebted to Joe Stiglitz, Kevin Roberts and participants at the SSRC Workshop on Public Economics, Warwick, 1978, for helpful discussions.

1/ For example, Domar and Musgrave (1944), Richter (1960), Mossin (1968), Stiglitz (1969), Allingham (1972), Ahsan (1974), (1975), Cowell (1975) and Sandmo (1977).

2/ Feldstein (1969) argues the need for analysis in an occupational choice framework, but fails to recognise the inherently general equilibrium nature of the problem.

3/ Problems of labour-leisure choice are abstracted from in order to focus on the essentials of risk taking behaviour.

4/ For a discussion of this interpretation, see Kanbur (1978).

5/ Throughout the paper, round brackets are used to enclose arguments of functions, whereas square brackets represent the usual bracket operation.

6/ Of course there are many families to which this condition can be related. For the Cobb-Douglas case, where  $\alpha$  is a constant between zero and one, the same inequality cannot be satisfied for all  $L$ . An example of a family of functions for which  $\alpha < \frac{L}{1+L}$  for all  $L$  is given by

$$G(L) = k_0(1 + L)L^{-k_1}; \quad k_0, k_1 > 0; \quad L > \frac{1 + k_1}{1 - k_1}$$

The other case of  $\alpha > \frac{L}{1+L}$  for all  $L$  is exemplified by

$$G(L) = k_0 \exp [1 + k_1 \log L]; \quad k_0, k_1, L > 0.$$

7/ Notice from (67) that if social welfare is assumed to be the Utilitarian sum, then this is also the condition for "social welfare compensation".

8/ See Wealth of Nations, Chapter 10.

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