Product Diversity and Welfare:

Some Further Results

Alan Carruth

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Introduction*

The main purpose of this paper is to examine recent work in monopolistic competition theory concerned with the influence of fixed costs (nonconvexities) on Chamberlin's (1951) welfare 'ideal' which distinguishes a trade-off between allocative efficiency and product diversity. Inefficiency is shown to be no longer just a matter of non-marginal-cost pricing. The actual number of commodities and the product mix are important considerations for welfare analysis of product differentiation.

The first section provides a brief resumé of the contributions which have questioned contemporary wisdom on the alleged excessive diversity of monopolistic outcomes. The second section discusses the importance of fixed costs and increasing returns in exposing the weakness of profitability as a market signal determining product existence and survival. The third section sets out Spence's (1976a) analytical framework deriving his missing equations plus an extension to a 'true second-best' solution. The basis is a monopolistic model specified by linear demand and cost functions from which Spence derives a monopolistic equilibrium and two welfare outcomes (the optimum and a marginal cost pricing interpretation of market equilibrium) with respect to two control variables, the number of firms (n) and output per firm (x). My aim is to further this work by providing, first of all, the second-best welfare outcome which constrains the optimum by a zero profit condition and, secondly, a geometrical presentation of outcomes for

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* This paper is a revised version of part of a Master's degree dissertation. Its content has greatly benefitted from the suggestions and encouragement of Avinash Dixit and Jesus Seade. A personal copyright covers any remaining errors and misconceptions.

particular cases taken from the simulation results of Table A. The final section tabulates and compares the extended set of simulated numerical results based on an identical parameter set to that of Spence (1976a). The main points arising from the analysis are brought together in the form of a conclusion.

1. Diversity Versus Efficiency

This section outlines a number of contributions to monopolistic competition theory which have sought to clarify the importance of product diversity in relation to the well-known efficiency arguments of modern welfare economics. Work by Lancaster (1975), Dixit and Stiglitz (1975, 1977) and Meade (1974) will be briefly summarised in terms of basic approach and relevant findings. Subsequent sections will pursue the work of Spence (1976a, 1976b).

Lancaster's (1966) seminal work introduced the 'product characteristics' approach as a complete attempt to break with traditional modes of thought in consumer theory. Instead of commodities directly determining utility it is the 'intrinsic properties' or 'characteristics' of commodities which are to be the arguments of the utility function. Preference orderings will arrange collections of characteristics and not bundles of products. An important condition requires the independence of product characteristics with respect to household whims or tastes. In other words such intrinsic properties have a predetermined physical basis in characteristics space - a given parameter of the model. Lancaster demonstrated the many insights his framework could afford our understanding of individual behaviour. However information requirements appeared excessive for any potential applications.

Nevertheless Lancaster (1975) further utilised his characteristics methodology to examine the social optimality of different degrees of product differentiation. The main concern was with

> "How many different product variants should society provide?" (1975; pp. 756).

Much of the analysis is varied and complex, certainly outwith the scope of this paper. Suffice to report the important theorem for monopolistic competition (pp. 581), which states that under increasing returns and within the Chamberlin group restrictions,[1/] monopolistic competition will lead to a greater degree of product differentiation than is socially optimal. This indicates that the marginal welfare gain from an additional product is significantly outweighed by the marginal efficiency welfare loss. Support is evident for the proposition that welfare losses from monopolistic competition not only stem from inefficiency but also from having the wrong number of products - in this case from having too many products. Interestingly this result on deviations from optimal product numbers lies in one direction, whereas Dixit and Stiglitz (1975, 1977) and Spence (1976a, 1976b) derive results from their respective approaches which raise the possibility of welfare losses from deviations on both sides of the optimum number of products.

The contribution of Dixit and Stiglitz (1975, 1977) to the diversity issue embodies the most detailed approach to the debate through the formulation of a general equilibrium version of the Chamberlin group model. Their main purpose is to expose the firmly embeded belief that monopolistic competition involves excessive diversification of products. The diversity

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1/ notably the symmetry assumption which is required for Chamberlin's (Cournot's no reaction) perceived dd' demand curve.

issue is shown to be only part of the more general question involving a comparison between the market equilibrium and the optimum allocation under monopolistic conditions. Welfare losses from such a comparison can stem from having the wrong number of products (firms), an undesirable product mix and incorrect firm output levels. Moreover the optimum allocation involves a lump-sum mechanism to cover firm losses, so as to avoid introducing further distortion. Now lump-sum transfers are not a practicable instrument available to intervening bodies. Therefore a more appropriate notion of optimality may be constrained such that no firm operates at a loss. It is this constrained optimum, given a suitably defined objective function, which features in the extension to Spence's (1976a) simulation exercise, the main focus of this paper.

Unlike Lancaster (1975) Dixit and Stiglitz's analysis of variety is based on traditional consumer theory whereby variety is implicit in the convexity of preferences underlying a utility function defined over all possible products - actual and potential. The symmetry assumption removes the problem of the product mix. The number of commodities and firm output levels are the remaining concerns. Under this utilitarian framework the authors show that in one special case - constant elasticity of substitution - the monopolistic market equilibrium and the constrained optimum coincide. A similar result is obtained by Spence (1976b; pp. 232) using a partial equilibrium, consumer surplus model. In the more general variable elasticity of substitution case a number of possibilities arise in terms of output level or product variety inefficiency. The sections to follow will present these alternatives within the Spence (1976a) simulation framework.

Like Spence, Meade (1974) adopts a partial consumer surplus approach to the optimal variety question. He examines whether a specific town should be provided with either a gasworks or an electricity station or both together. The analysis brings out the importance of demand interdependence, that is, cross substitution effects, for the efficiency - diversity trade-offs. Such influences are also highlighted by the simulation exercise of this paper.

It seems clear from the above that the actual degree of variety provided by an uncontrolled market will not necessarily coincide with the socially optimal degree of variety. The welfare issues involved in this divergence are taken up in the rest of the paper.

2. Product Selection

According to Spence (1976a) an important function of the price mechanism or any comparative economic system is the choice of products to be placed on the market. This 'basic economic problem' is implicit to the product heterogeneity of the monopolistic model. But to quote Spence

> "The full range of products may be neither feasible nor desirable due to the presence of increasing returns to scale." (1976a; pp. 407)

Moreover most production units incur fixed costs which by definition are independent of output. It is the fixed costs which are instrumental in imperfectly competitive pricing and profitability ramifications. Thus with such non-convexities the efficiency/diversity trade-off has significant importance for a proper welfare analysis of monopolistic competition.

Yet early work neglected the diversity question, focussing on the welfare losses arising from non-marginal cost pricing. The previous section sketched how economic theory has sought to redress this imbalance with respect to diversity.

In a separate article Spence (1976b) provides the theoretical analysis which underlies his (1976a) paper. Attention is focussed on the effects of fixed costs and monopolistic competition[1/] on the selection of products within the Chamberlin group. Casual observation indicates that products which exist must be capable of extracting revenues sufficient to cover fixed and variable costs. However revenues do not provide an adequate measure of the social benefits derived from products, evidenced by the economics of consumer surplus. Only a perfectly price-discriminating monopolist can extract all consumer surplus.[2/] In this rather special case the welfare aspects of the product choice problem are eliminated simply by the rationale of revenues reflecting the true social benefits obtained from a product. This inability of firms to extract the true social benefits of their products is a market force working against product existence. In effect it provides a tendency to reduce variety, and is symptomatic of market failure.

Now product variety is desirable from the consumers point of view if the products are not very good substitutes for one another; variety is costly if the economies of large scale production are great. It is for these reasons that substitutability and scale economies are important elements in

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1/ No attempt will be made to question the existence of equilibrium in the face of economies of scale. The exposition will focus on market outcomes and welfare optima, as in Spence (1976a).

2/ A potential virtue in terms of no efficiency loss. Of course, lump-sum taxation would be needed to offset the ugly-looking distributive consideration.

determining variety of products. Several other market forces bring out the influences of these two factors.

Monopolistic market equilibrium coincides with a situation of falling average costs - the tangency solution. The presence of increasing returns results in price setting above marginal cost but equal to average cost assuming entry takes place. The outcome of this behaviour is not necessarily too many products (c.f. Lancaster (1975)); but certainly there will be more products than if non-negative profits were required and marginal cost pricing imposed - the well-known first-best solution. Evidence of this point can be found in the next section.

The actual degree to which firms can capture consumer surplus depends on the properties of individual demand functions. This introduces a bias in product selection. Spence (1976a) presents a restricted example to help clarify the issue. Suppose demand functions exist and have constant elasticities. Now let r be the ratio of total revenue to gross surplus. Therefore

$$r = \frac{\int MR(x)\ dx}{\int p(x)\ dx} \qquad (1)$$

where $p(x)$ is the inverse demand function, $MR(x)$ is the marginal revenue function and x is quantity. Now (1) can be manipulated 1/ to yield

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1/ $$\frac{\int MR(x)\ dx}{\int p(x)\ dx} = \frac{\int p(x)(1 - \frac{1}{e})dx}{\int p(x)\ dx} = (1 - \frac{1}{e}) \frac{\int p(x)\ dx}{\int p(x)\ dx}$$

$$r = (1 - \frac{1}{e}) \qquad (2)$$

where e is the constant own price elasticity of demand.

As e rises the ratio of total revenue to gross surplus, r, rises. The resulting implication is a product selection bias whereby it is not unreasonable for a product with a low price elasticity to have a higher net welfare surplus but lower profit than a product with a high own price elasticity. Hence there may be a greater tendency to lose low elasticity products, particularly in the light of fixed costs. Moreover an implicit welfare bias may also be distinguished as low elasticity products are often attached greater welfare weights. Subsequently the incentive for sellers to price discriminate will be greater for low elasticity products. It turns out, Spence (1976b), that it is not just elasticity that matters, but what fraction of net potential surplus for a product is capturable by a selling firm. This will involve both demand and costs. As a market force selection bias will also tend to eliminate products.

Finally market interdependence may lead to non-optimal degrees of variety. Consider the case where products are imperfect substitutes. When a new product is introduced it affects other firms' products by reducing their demand which leads to a contraction of output for the existing set of firms. Gains arise from the profit and consumer surplus of the new product but losses are incurred on the profit and surplus of the existing set of products. When products are fairly close substitutes losses can easily outweigh gains. However the entering firm does not take account of such interactions: it may enter when it is not generating a net social benefit. This is a market force tending to generate too many products. On the other hand if products are complements then the monopolistic equilibrium by reducing output and raising price above marginal cost lowers the demand for other complementary products. This induces further quantity cutbacks and possibly the exit of products from the market. The process reinforces itself and leads to an equilibrium where all outputs are below the optimum and some of the products in the optimal set are not produced at all.

Profitability, it has been argued, is to be considered a fairly weak criterion for product selection. However it is the only benchmark available and as Spence (1976a) points out

> "One can reasonably accept profitability as a constraint and pose the problem of product selection as that of determining the right set of products subject to that constraint. The solution to the problem includes specification of not only the products but also the prices. The prices will typically be above marginal cost, since that may be required to increase the profitability of products to permit the entry of products that are not profitable under marginal cost pricing. In short the solution to the second-best problem will include a trade-off between numbers of products on the one hand and the inefficiency due to non-marginal cost pricing on the other." (1976a; pp. 411).

Spence goes on to suggest that the monopolistic equilibrium has the qualitative features of the constrained optimum as both problems involve the trade-off between product variety and inefficiency through non-marginal cost pricing. However it is apparent that both outcomes have a different objective function and price-output configuration. Extension of Spence's (1976a) paper to include the 'true second-best' solution facilitates a comparison of the implications arising from these distinctions. The analytical framework employed by Spence (1976a) is set out in next section.

3. Analytical Framework for Simulated Welfare Analysis

The essence of the approach is a numerical comparison of the aforementioned market outcomes and welfare optima. Welfare is measured by the multiproduct net surplus which is simply the sum of producer and consumer surplus. Income effects are to be ignored. Recent work in this area by Dixit and Weller (1977), Seade (1978) and Willig (1976) indicates that this type of assumption is less restrictive for welfare analysis than was once thought. Nevertheless it should be borne in mind that it underlies all the subsequent analysis of this paper. A product's marginal contribution to total surplus is defined to be the area under the inverse demand function minus the costs of production for that particular product.[1/]

The main purpose of the numerical analysis is to tentatively illustrate the preceding market forces influencing product selection by showing that the fraction of the total welfare loss attributable to the non-marginal cost pricing of the equilibrium set of products varies considerably and is frequently less than half of the total welfare loss given specific demand and cost conditions.

The numerical results depend upon the following framework. The quantity of the i^{th} product is x_i. The inverse demand for the i^{th} product is

$$p_i = a - 2b\, x_i - 2d \sum_{i \neq j} x_j \quad (3)$$ [2/]

1/ Note that the total net surplus is not exactly equal to the sum of these marginal surpluses as account must be taken of entry repercussions on existing members of the 'group'.

2/ The analysis is restricted to linear functions.

The cost function of the i^{th} firm is

$$f + cx_i \tag{4}$$

In the calculations marginal cost, c, is fixed at unity in Table A; but will be allowed to change in Table B. The remaining parameters a, b, d, and f can vary: f is the fixed cost, d is the interaction effect, and a and b are the intercept and slope of the inverse demand for each product when there are no other products. Equations (3) and (4) plus the final form of the total surplus function are all the information provided by Spence (1976a) before he proceeds to his table of numerical results for the different market outcomes and welfare optima (1976a; pp. 412, Table I).

However I should like to bring out the underlying framework in greater detail to aid my extension to a "true second-best" solution. The remaining analysis of this section represents my own interpretation and extension of Spence's framework.

From (3) we know that the marginal revenue function has twice the slope of the inverse demand function, that is

$$MR_i = a - 4b\, x_i - 2d \sum_{i \neq j} x_j \tag{5}$$

Under the symmetry and uniformity assumptions of the Chamberlin 'group' the inverse demand function, equation (3), can be rewritten as

$$p_i = a - 2b\, x_i - 2d(n - 1)x_i \tag{6}$$

Now gross surplus (G.S.) is defined to be the area under the inverse demand function. Integrating[1/] (6) with respect to x_i gives

$$G.S. \quad = \quad a\,x_i - b\,x^2_i - d(n-1)x^2_i \tag{7}$$

Net surplus is simply G.S. minus production costs. Using equation (4) we obtain

$$N.S. \quad = \quad a\,x_i - b\,x^2_i - d(n-1)x^2_i - f - c\,x_i \tag{8}$$

Total net surplus can then be derived from (8) by summing over n firms and dropping labels (subscripts) due again to symmetry and uniformity. This gives[2/]

$$S(n, x) \quad = \quad n(ax - bx^2) - dn(n-1)x^2 - nf - ncx \tag{9}$$

which means that any outcome must be completely described by x, the output per firm and n, the number of firms. This is clearly restrictive, for in this type of problem we often have product ordering along a spectrum and two products closer together on this spectrum will be closer substitutes than two products further apart. With asymmetry the actual product labels will become important and (9) will not be valid. To aid tractable results symmetry will remain an integral assumption of this paper. Dixit and Stiglitz (1977) have provided some analysis of the asymmetry problem.

1/ Analytically symmetry means that $\int p_i\, dx_i$ is well defined with no 'path of integration' difficulties. This is an important property of compensated demand functions.

2/ Equation (9) corresponds to the surplus equation presented by Spence (1976a, in a footnote, pp. 412).

A profit function can also be derived from demand and cost conditions in a similar manner to the surplus function. Multiply (6) through by x_i to yield total revenue and then subtract the total cost function, equation (4). With the symmetry condition we can drop labels and so obtain

$$\pi(n, x) = ax - 2bx^2 - 2d(n - 1)x^2 - f - cx \tag{10}$$

With entry in monopolistic competition the zero profit condition is simply equivalent to average revenue equal to average cost, that is, AR = AC. Therefore, ignoring subscripts under symmetry, equation (6) and equation (4) can be used to obtain

$$a - 2bx - 2d(n - 1)x = c + \frac{f}{x} \tag{11}$$

This helps to simplify the derivation of the monopolistic equilibrium. Equations (3) to (11) provide all the necessary information for deriving the different outcomes in terms of our control variables n and x.

<u>The Optimum (O.)</u>

In this case we wish to

$$\text{Max. } S(n, x) = n(ax - bx^2) - dn(n - 1)x^2 - nf - ncx$$

First order conditions (F.O.C.) are

$$S_n = ax - bx^2 - (2n - 1)dx^2 - f - cx = 0 \tag{12}$$

$$S_x = n(a - 2bx) - dn(n - 1)2x - nc = 0 \tag{13}$$

where subscripts are used to denote partial derivatives. Rearranging (13) we can derive

$$x = \frac{a - c}{2b + 2d(n - 1)} \tag{14}$$

and by substitution for x in (12) a little manipulation yields

$$n = 1 + \left[\frac{(b - d)(a - c)^2}{4d^2 f}\right]^{\frac{1}{2}} - \frac{b}{d} \tag{15}$$

(14) and (15) enable the isolation of x and n for different initial values of our parameters which will consistently describe this welfare outcome and allow us to calculate total surplus, profit/loss, prices, revenues and costs.

<u>Market Equilibrium (E.)</u>

Within the monopolistic competition model this solution is described by the conditions that marginal revenue equals marginal cost and under free entry average revenue equals average cost. Now from (6), (5) and (4) $MR = MC$ gives

$$a - 4bx - 2d(n - 1)x = c \tag{16}$$

and from (11) $AR = AC$ [1/] implies

1/ I am grateful to Avinash Dixit for suggesting the simplification afforded by using equation (11) rather than (10).

$$a - 2bx - 2d(n - 1)x = c + \frac{f}{x} \tag{17}$$

(16) and (17) depict two simultaneous equations in n and x. Now by simply subtracting we obtain

$$x = \sqrt{\frac{f}{2b}} \tag{18}$$

Substitution for x yields

$$n = 1 + \left[\frac{2b(a - c)^2}{4d^2 f}\right]^{\frac{1}{2}} - \frac{2b}{d} \tag{19}$$

Hence equations (18) and (19) capture the monopolistic market equilibrium.

Equilibrium Number of Firms with MC Pricing (M.)

In this example we know that supply price has to equal marginal cost, c, and that n is constrained to equal (19). From equations (4) and (6), ignoring subscripts, we have

$$a - 2bx - 2d(n - 1)x = c$$

and

$$x = \frac{a - c}{2b + 2d(n - 1)} \tag{20}$$

Equations (19) and (20) model this outcome. Notice that equations (14) and (20) are identical which simply reflects first-best efficiency with no thought to loss-offset problems.

'Second-Best' Solution (S.B.) 1/

The analysis of this welfare outcome involves a fairly straightforward constrained optimisation problem. It can be expressed as

$$\text{Max } S(n, x) \qquad \text{S.t. } \pi(n, x) = 0 \tag{21}$$

In Lagrangean form (21) becomes

$$L = S(n, x) + \lambda \pi(n, x) \tag{22}$$

F.O.C.

$$L_x = S_x + \lambda\pi_x = 0 \tag{23}$$

$$L_n = S_n + \lambda\pi_n = 0 \tag{24}$$

$$L_\lambda = \pi(n, x) = 0 \tag{25}$$

From (9) and (10) we can substitute for the partials S_x, π_x, S_n, π_n to obtain a system of three nonlinear simultaneous equations in the three unknowns n, x and λ, the Lagrange multiplier. This gives

$$L_x = na - 2nbx - 2dn(n - 1)x - nc + \lambda(a - 4bx - 4d(n - 1)x - c) = 0 \tag{26}$$

1/ This welfare outcome was not considered by Spence (1976a), although it was discussed in (1976b). It can therefore be viewed as an extension of his (1976a) simulation exercise.

$$L_n \quad = \quad ax - bx^2 - (2n - 1)dx^2 - f - cx - \lambda 2dx^2 \quad = \quad 0 \tag{27}$$

$$L_\lambda \quad = \quad ax - 2bx^2 - 2d(n - 1)x^2 - f - cx \quad = \quad 0 \tag{28}$$

This system [(26) - (28)] did not manipulate into manageable formulas in x and n as in the previous solutions. To achieve some results a numerical approximation method was used with the aid of a computer algorithm[1/] developed by Powell (1968) for solving this type of system. Estimates of n, x and λ were obtained for different parameter sets of a, b, c, d and f. It is important to note that no global properties can be attributed to a numerical iteration approach of this nature owing to the arbitrary selection of initial starting values for n, x and λ.

Monopolistic Pricing and Surplus Maximisation (SM.)

This constrained outcome was examined by Spence (1976b) but not included in his simulation exercise (1976a). The idea is to

$$\begin{aligned} &\text{Max} \quad S(n, x) \\ &\text{S.t.} \quad MR \; = \; MC \end{aligned} \tag{29}$$

The problem is of a similar form to the second-best solution. However it was decided that the geometrical presentation of outcomes would be a sufficient illustration for this problem; although results can easily be derived from the numerical algorithm. The essential feature of (29) is that entry occurs

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1/ The algorithm was available as part of the Warwick University Computer Library facilities.

under monopolistic pricing until net surplus is maximised. For as Spence (1976a) suggests

> "Profits can be above or below zero when the marginal contribution of a product to the surplus falls to zero." (1976a; pp. 409).

Now simulated tables of numbers never appear very demonstrative of economic argument. For this reason an alternative procedure was sought to provide additional impact for the main contentions. It actually turned out to be relatively easy to portray all the outcomes on a contour diagram by plotting the requisite functions for particular parameters sets selected from Table A. The technique employed was 'Minighost', a library of computer subroutines which produces graphical output on a plotter. The best diagrams were achieved by plotting surplus contours for $S(n, x)$ along with the functions $\pi(n, x) = 0$ (or $AR = AC$), $MC = P$ and $MR = MC$. Four sets of parameters $(a, b, c, d, f,)$ 1/ were selected from Table A and with the aid of Minighost the different solutions were depicted - see Figures 1, 2, 3, 4.

From the first-order conditions of the second-best solution set out in equations (23), (24) and (25), the Lagrangean multiplier, λ , can be easily eliminated to yield the following condition

$$\frac{S_x}{S_n} = \frac{\pi_x}{\pi_n} \qquad (30)$$

1/ Parenthetical parameters represent a single set.

This equality simply states that the slope of the surplus contour curve must be the same as the slope of the profit constraint function at the optimal point - the familiar tangency of constrained optimisation. The point SB on the four diagrams attempts to approximate this tangency. Equilibrium, point E, on the figures is at the intersection of MR = MC with AR = AC ($\equiv \pi(n, x) = 0$). Marginal-cost pricing with the equilibrium set of firms, point M, lies on MC = P and at the same vertical height from the output axis as the equilibrium E.[1/] Surplus maximisation under monopolistic pricing is labelled SM. Here entry or exit occurs until the highest surplus contour is reached along the MC = MR function rather than the normal equilibrium case until profits are driven to zero. The diagrams clearly demonstrate that under monopolistic pricing the surplus will not necessarily be maximised at zero profits. Finally MSB depicts the outcome of marginal cost pricing with the second-best set of firms, which adopts equivalent reasoning to outcome M.

Figure 1 was selected from group I ($a = 10.0$, $b = 1.0$, $c = 1.0$, $d = 0.5$, $f = 6.0$) of Table A. It is easy to check that the rough visual outcomes for n and x gleaned from the diagram are consistent with the requisite row from Table A. A number of points for this particular case are illustrated on the figure. First the market equilibrium indicates a situation of too many products when compared to the optimum. Second marginal cost pricing under the equilibrium set of firms, M, yields a higher surplus than the second-best solution, SB. Note that this outcome does involve a loss and in the absence of a lump-sum solution must involve a trade-off between actual surplus and subsidy. Third the

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1/ This represents the restriction implied by equation (19).

solution, SM , suggests that the marginal contribution of a new product to the surplus reaches zero before profits are erased. Similar qualifications apply to Fig. 3; but with a substantially different parameter set (a = 10.0, b = 0.3, c = 1.0, d = 0.05, f = 10.0).

Fig. 2 provides an example where monopolistic competition leads to a situation of too few products (cf. Lancaster (1975)). Moreover the effects of entry lead to zero profit while the marginal contribution of a new product to surplus[1/] is still positive. (Compare E to O and SM to E). Marginal cost pricing again improves on the second best surplus with a similar proviso placed on the loss. Finally Fig. 4 depicts a situation where there are too many products in monopolistic equilibrium: the second-best solution yields higher welfare than marginal cost pricing with the equilibrium set of firms: positive profits result when the marginal contribution of a new product to the surplus falls to zero.

The next section examines the complete simulated numerical results set out in Table A. In particular the consequences of demand interdependence and fixed costs for welfare are highlighted.

4. Welfare Analysis of Numerical Examples

The initial numerical results are presented in Table A. Some columns were replications of Spence's Table 1 (pp. 412) as the chosen parameter sets are identical. It is pleasing that in these cases the results of

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1/ Remember that by surplus we mean "net surplus".

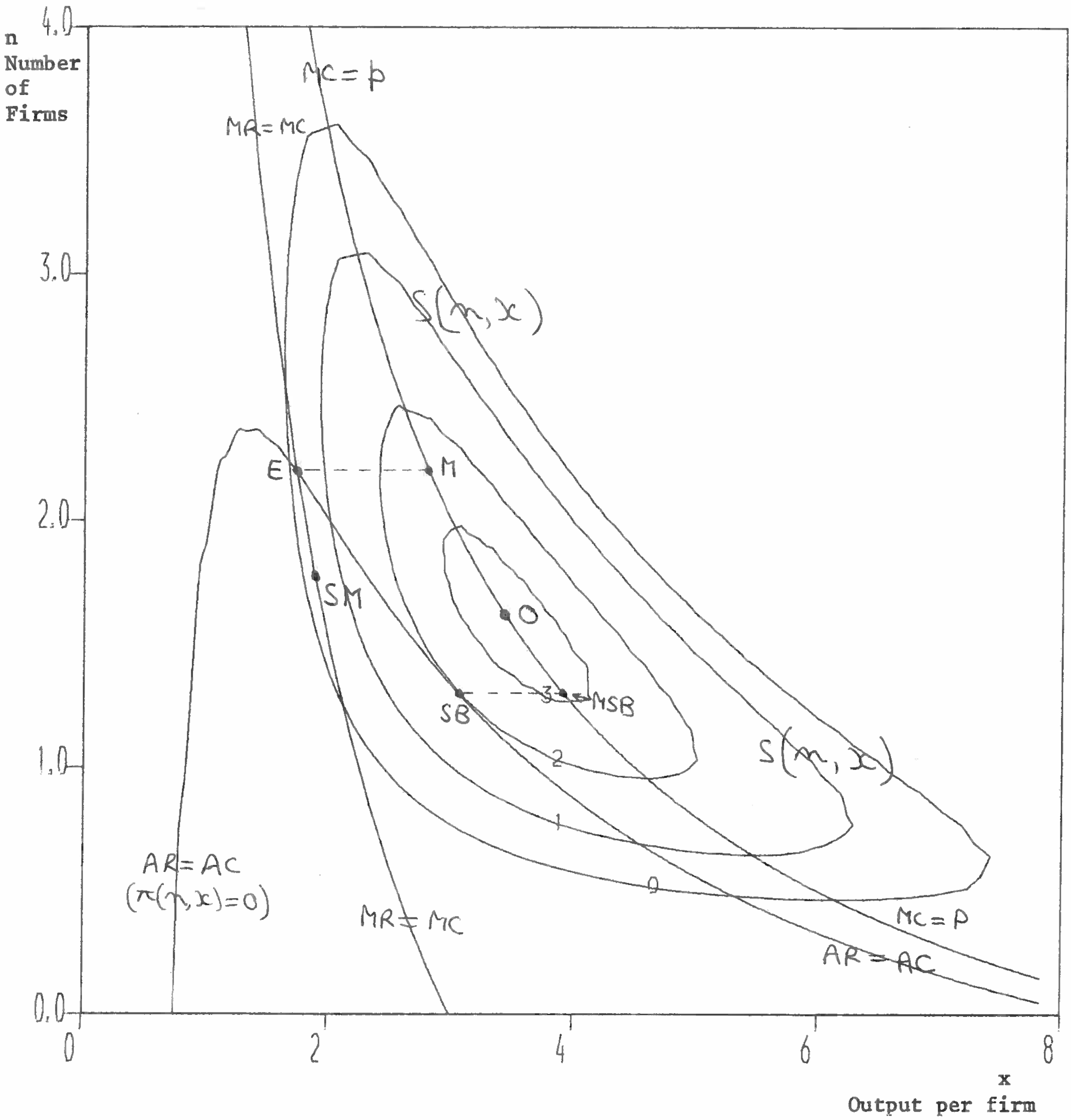

Fig. 1: Group I (a = 10.0; b = 1.0; c = 1.0; d = 0.5; f = 6.0)

Contour	Surplus
0	10
1	12
2	14
3	15

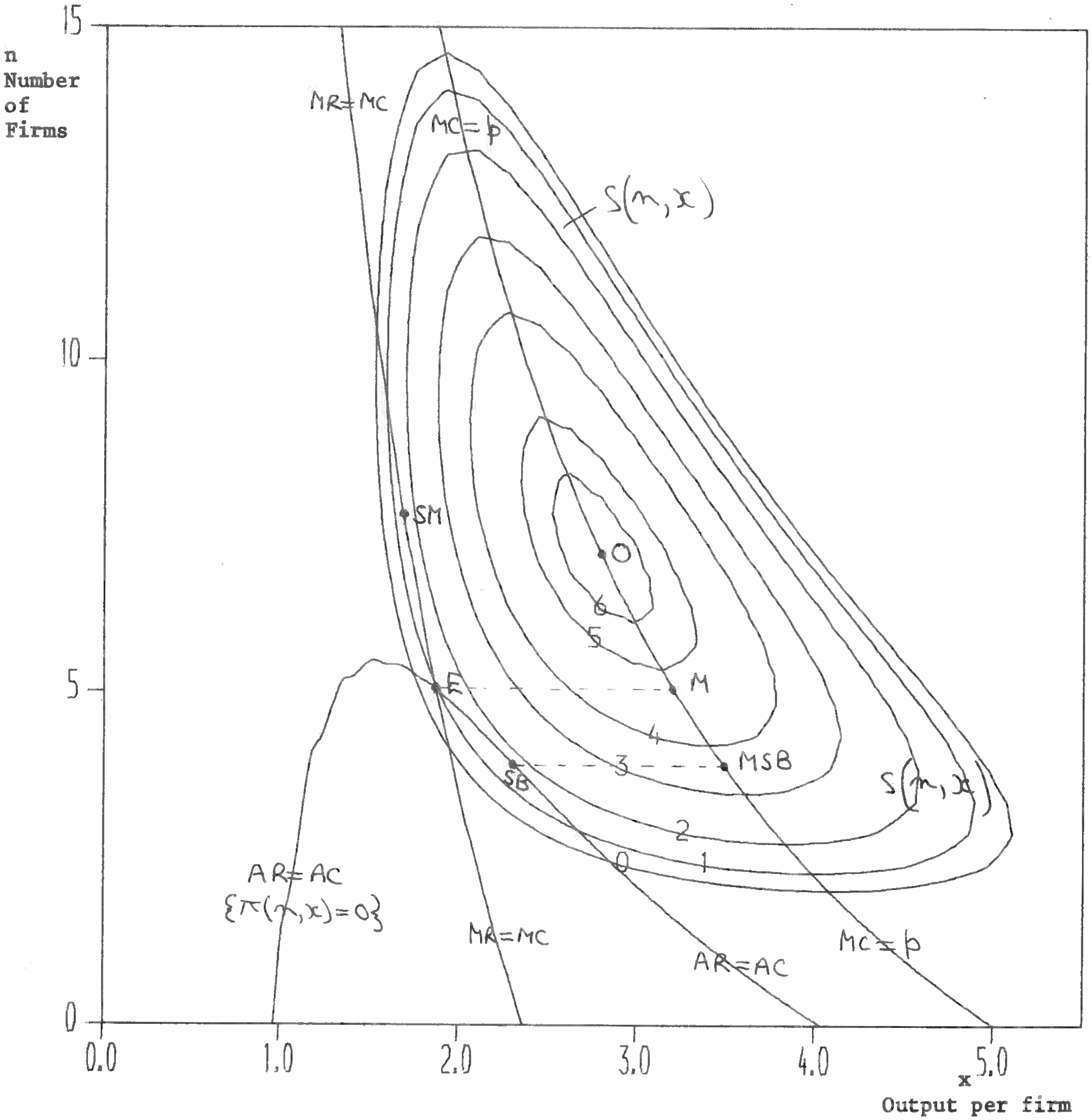

Fig. 2: Group III (a = 10.0; b = 1.0; c = 1.0; d = 0.1; f = 7.0)

Contour	Surplus
0	23
1	25
2	28
3	32
4	35
5	38
6	39

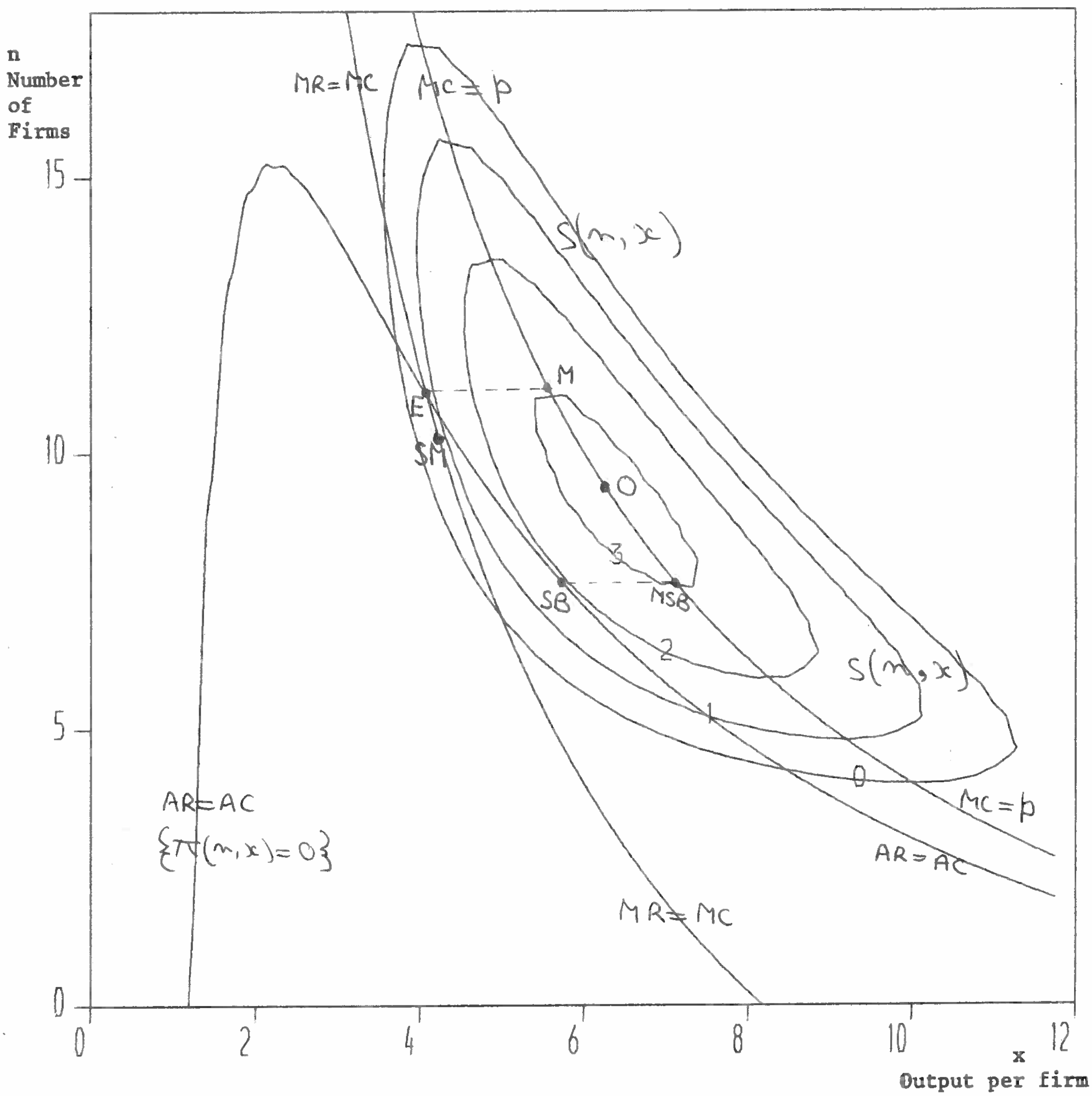

Fig. 3: Group VI (a = 10.0; b = 0.3; c = 1.0; d = 0.05; f = 10.0)

Contour	Surplus
0	140
1	150
2	160
3	168

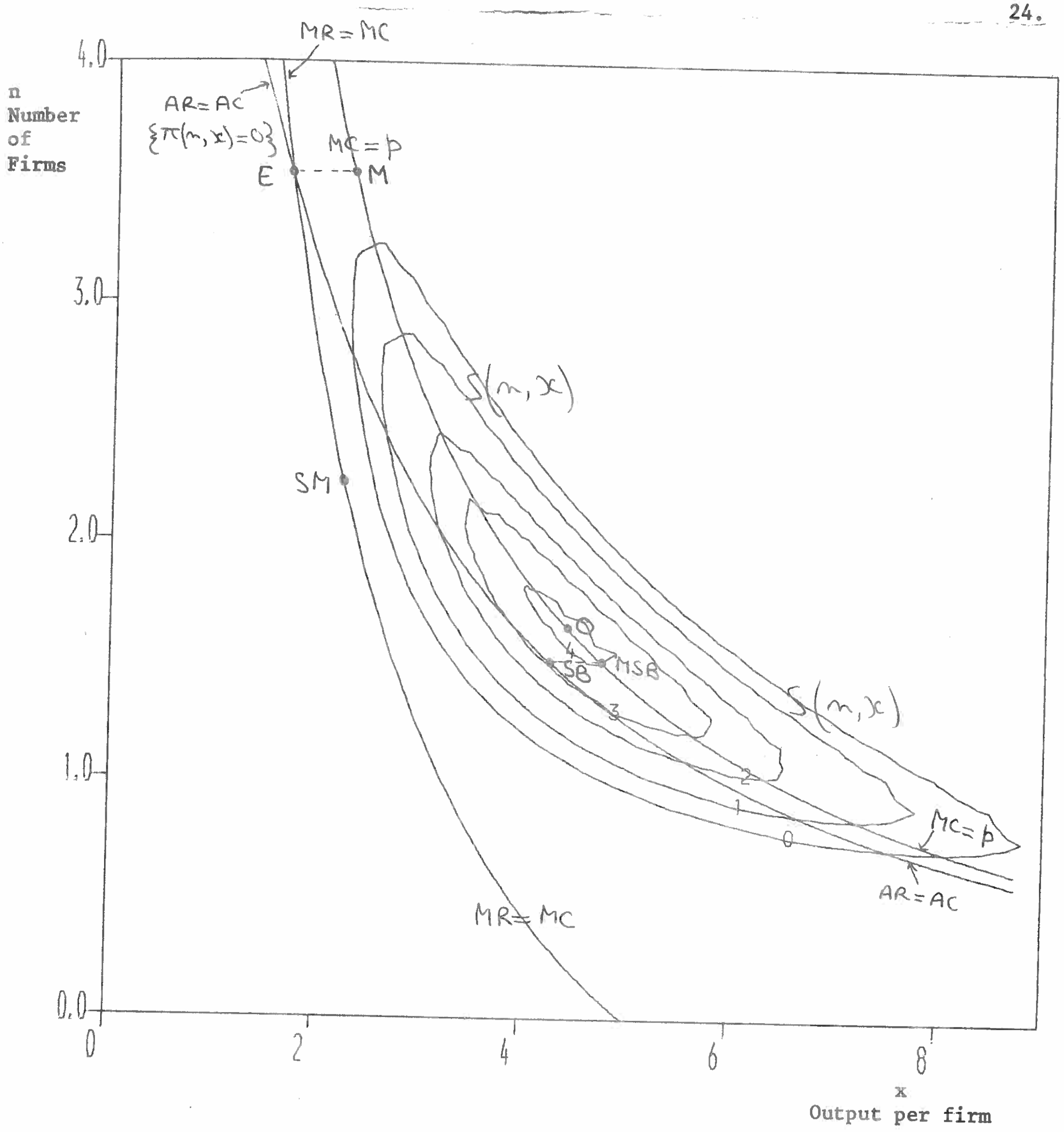

Fig. 4: Group IV (a = 10.0; b = 0.7; c = 1.0; d = 0.5; f = 4.0)

Contour	Surplus
0	23
1	24
2	25
3	25.5
4	25.9

both tables coincided. Table A is of course considerably extended compared to Spence's Table 1 to include the 'true' second-best solution, marginal cost pricing with the second-best set of firms, equilibrium and second-best prices and the Lagrange multiplier. T stands for total net surplus and subscripts O, E, M, SB, MSB are the optimum, market equilibrium, marginal cost pricing with equilibrium firms, second-best, and marginal cost pricing with second-best firms respectively. ΔT_i represents the difference between the optimum total net surplus and the surplus pertaining to the relevant subscript, i, i.e. welfare losses for (i = E, M, SB, MSB). N_j and X_j are the number of firms and output per firm respectively for the different outcomes, (j = 0, E, M, SB, MSB). P_E is the market equilibrium price and P_{SB} is the requisite price for the second-best solution. Finally λ_{SB} is the Lagrange multiplier implicit in the second-best solution.

Notice that the results of Table A imply treating n as a continuous variable a necessary condition for the calculus. This is a common practice and Seade (1978) has provided a justification for its adoption which depends upon the sign of the derivative function in n remaining unchanged as n goes from n to $(n + 1)$.[1/]

The Spence analysis focussed on columns ΔT_E and ΔT_M of Table A, which illustrates that in some cases the welfare loss from non-marginal cost pricing is sometimes less than half the total welfare loss. In other words any move from market equilibrium to marginal cost pricing with the equilibrium set of firms will only remove part of the welfare loss, that is, ΔT_M is never zero. This is in complete agreement with the proposition that in

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1/ This condition is fulfilled in this analysis.

TABLE A*

GROUP	CASE	a	b	d	f	T_O	T_{SB}	T_{MSB}	ΔT_E	ΔT_M	ΔT_{SB}	ΔT_{MSB}	N_O	N_E	N_{SB}	X_O	X_E	X_M	X_{SB}	X_{MSB}	P_E	P_{SB}	λ_{SB}
I	1	10	1	0.5	1	28.77	28.53	28.75	2.7	1.8	0.24	0.02	5.4	9.7	5.1	1.4	0.7	0.8	1.4	1.5	2.3	1.7	0.5
	2				2	24.50	24.04	24.46	3.5	1.8	0.46	0.04	3.5	6.0	3.2	2.0	1.0	1.3	1.9	2.1	3.0	2.1	0.5
	3				4	19.04	18.14	18.93	4.3	1.2	0.90	0.11	2.2	3.4	1.9	2.8	1.4	2.1	2.6	3.1	3.6	2.6	0.5
	4				6	15.32	14.00	15.09	4.8	0.6	1.32	0.23	1.6	2.2	1.3	3.5	1.7	2.8	3.1	3.9	4.5	3.0	0.5
	5				8	12.50	10.79	12.09	5.0	0.2	1.71	0.41	1.3	1.5	0.9	4.0	2.0	3.6	3.4	4.6	5.0	3.3	0.5
	6				10	10.25	8.18	9.62	5.1	0	2.07	0.63	1.0	1.0	0.7	4.5	2.2	4.4	3.7	5.3	5.5	3.7	0.5
II	7	10	2	1.5	2	8.20	8.01	8.15	2.6	1.4	0.19	0.05	1.2	2.6	1.1	2.0	0.7	1.0	1.9	2.1	3.8	2.1	0.17
	8				4	6.35	6.05	6.31	3.0	0.8	0.30	0.04	0.7	1.3	0.6	2.8	1.0	1.8	2.6	3.1	5.0	2.6	0.17
	9				6	5.11	4.67	5.04	3.1	0.3	0.44	0.07	0.5	0.8	0.4	3.4	1.2	2.7	3.1	3.9	5.9	3.0	0.17
III	10	10	1	0.1	2	99.75	99.65	99.20	8.7	1.3	4.10	0.55	21.2	26.0	18.6	1.5	1.0	1.3	1.4	1.6	3.0	2.5	4.5
	11				5	56.58	47.12	54.02	12.9	0.1	9.46	2.56	10.1	9.5	7.2	2.4	1.6	2.4	2.0	2.8	4.2	3.5	4.5
	12				7	39.60	27.07	34.53	14.7	2.2	12.53	5.07	7.1	5.0	4.1	2.8	1.9	3.2	2.3	3.4	4.7	4.1	4.5
	13				9	27.36	12.21	18.24	16.2	7.4	15.15	9.12	5.2	2.2	1.9	3.2	2.1	4.0	2.4	4.1	5.2	4.7	4.5
	14				10	22.50	6.26	10.41	16.8	11.3	16.24	12.09	4.5	1.1	1.0	3.3	2.2	4.4	2.5	4.5	5.5	5.0	4.5
IV	15	10	0.7	0.5	2	29.92	29.72	29.90	4.8	3.5	0.20	0.02	2.4	5.7	2.3	3.2	1.2	1.5	3.1	3.3	2.7	1.7	0.2
	16				4	26.00	25.62	25.97	6.2	3.7	0.38	0.03	1.6	3.5	1.5	4.5	1.7	2.3	4.2	4.7	3.4	1.9	0.2
	17				6	23.18	22.62	23.13	7.1	3.5	0.56	0.05	1.2	2.6	1.1	5.5	2.1	3.1	5.1	5.9	3.9	2.2	0.2
	18				8	20.93	20.19	20.85	7.6	3.0	0.74	0.08	1.0	2.0	0.9	6.3	2.4	3.8	5.8	6.9	4.3	2.4	0.2
V	19	10	0.3	0.2	2	82.13	81.88	82.13	7.6	6.2	0.25	0	4.5	10.3	4.4	4.8	1.8	2.1	4.4	4.6	2.1	1.5	0.25
	20				6	69.39	68.67	69.35	12.0	7.9	0.72	0.04	2.4	5.1	2.3	7.7	3.2	4.0	7.4	8.1	2.9	1.8	0.25
	21				10	61.25	60.08	61.16	14.3	7.7	1.17	0.09	1.8	3.5	1.6	10.0	4.1	5.6	9.4	10.6	3.4	2.1	0.25
	22				15	53.64	51.91	53.46	15.1	6.7	1.73	0.18	1.3	2.5	1.2	12.2	5.0	7.5	11.3	13.3	4.0	2.3	0.25
VI	23	10	0.3	0.05	10	170.39	159.24	168.69	22.7	2.1	11.15	1.70	9.2	11.0	7.7	6.3	4.1	5.6	5.7	7.1	3.4	2.8	2.5
	24				20	102.51	81.81	96.28	28.2	0.5	20.70	6.23	5.1	4.6	3.4	8.9	5.8	9.4	7.5	10.6	4.5	3.7	2.5
	25				30	62.05	33.34	46.97	32.9	9.8	28.71	15.08	3.2	1.7	1.4	10.9	7.1	13.6	8.6	14.0	5.2	4.5	2.5

* N.B. C ≡ Marginal Cost ≡ 1.0 in all 25 cases.

monopolistic competition it is not only optimal output levels which are important but also the optimal number of products, (firms).

The cases where welfare costs from having the wrong number of products are significant vary in respect of elasticities and fixed costs. When cross elasticities (d) are high they occur when fixed costs are low; and vice versa, for example, column ΔT_M groups III, V and VI. The table also illustrates that the equilibrium number of products can be above or below the optimum. Too many products tend to occur when cross elasticities (d) are high relative to own elasticities (b) and fixed costs are low, for example columns N_O, N_E groups I, II, IV, and V.

It is apparent in every group that equilibrium output is less than optimum output, a necessary condition for the consistency of the results. What is not apparent from Spence's table or Table A in this paper is that the losses which arise in the optimum and marginal cost pricing outcomes are quite substantial in relation to revenues and costs. To be fair to Spence he argued that the inability of firms to appropriate the true social benefit of their products implied that profitability was not the most desirable criterion for optimising an efficiency-variety trade-off. The losses may be offset by lump-sum transfers for the sake of exposition. In practice lump-sum transfers are not available to the authorities; hence the desirability of the second-best argument.

Now Spence's examination of the realities of profitability involved a zero profit market equilibrium as an approximation. This was certainly not unreasonable as regards the theory of the firm. However the results presented in Table A indicate that it was not such a close second-best approximation as

Spence may have imagined. In all groups the price-output configuration is substantially different between the market equilibrium and the numerically approximated second-best solution: the difference being that the second-best result has a lower price and higher output (c.f. columns X_E and X_{SB}, P_E and P_{SB}). In terms of variety the second-best solution has resulted in a situation of fewer products in every case. This reflects the welfare nature of the problem where the minimum amount to variety and output is being traded to cover the loss incurred at the optimum, rather than firms entering until profits are driven to zero.

Where the cross elasticities are high relative to the own elasticities (greater competition within the group), then the welfare loss arising from the second-best solution is fairly small (column ΔT_{SB}, groups I, II, IV and V). It is interesting to note that in some situations the second-best result would be considered an improvement on marginal cost pricing with the equilibrium set of firms (c.f. T_M and T_{SB}), e.g.: see Fig. 4. Also of relevance is the outcome of marginal cost pricing with the second best set of firms. Again Table A shows that not only do welfare losses occur from non-marginal cost pricing but also from having too few products (c.f. ΔT_{SB} and ΔT_{MSB}). Output under second best marginal cost pricing is higher than the optimal output (X_0 and X_{MSB}); so this may be involved in the welfare loss.

Finally the Lagrange multipliers (λ_{SB}) are constant within groups but vary between groups. Further experiments are presented in Table B, which shows that the Lagrange multiplier is unaffected by either changes in fixed cost (f) or marginal cost (c) or both together. It seems to be influenced solely by the demand parameters as evidenced in Table A.

Table B

Group	Case	a	b	c	d	f	T_{SB}	N_{SB}	X_{SB}	λ_{SB}
I	1	10.0	1.0	1.0	0.5	1.0	28.53	5.09	1.36	0.5
	2			2.0			21.45	4.39	1.35	0.5
	3			3.0			15.37	3.68	1.34	0.5
	4			4.0			10.28	2.97	1.32	0.5
	5			5.0			6.23	2.25	1.30	0.5
	6	10.0	1.0	2.0	0.5	2.0	17.54	2.72	1.86	0.5
	7			3.0		3.0	9.68	1.56	2.20	0.5
	8			4.0		4.0	4.19	0.80	2.40	0.5

This result suggests that a change in marginal cost and/or fixed cost will not influence the contribution of an additional unit of the profit constraint to the surplus (objective function). Intuitively it would appear that the linearity of the functional forms particularly the cost function and the similarity between criterion and constraint is imparting a certain degree of homogeneity on the second-best solution. This is indeed the case as the following argument demonstrates.1/

Now the basic second-best problem was

$$\begin{aligned} &\text{Max} \quad S(n, x) \\ &\text{S.t.} \quad \pi(n, x) \geq 0 \end{aligned} \tag{31}$$

with F.O.C.

1/ I am grateful to Avinash Dixit for pointing out this result to me.

$$S_n + \lambda\pi_n = 0 \tag{32}$$

$$S_x + \lambda\pi_x = 0 \tag{33}$$

From (9) and (10) we have

$$S(n, x) = n[ax - bx^2 - d(n - 1)x^2 - f - cx]$$

$$\pi(n, x) = ax - 2bx^2 - 2d(n - 1)x^2 - f - cx$$

Therefore

$$S_n = [ax - bx^2 - d(n - 1)x^2 - f - cx] - ndx^2$$

$$= [bx^2 + d(n - 1)x^2] - ndx^2 \quad \text{when} \quad \pi = 0$$

$$= (b - d)x^2$$

and

$$\pi_n = -2dx^2$$

From (32)

$$\lambda = -\frac{S_n}{\pi_n} \qquad \text{at} \quad \pi = 0$$

Hence

$$\lambda = \frac{b - d}{2d} \tag{34}$$

Equation (34) means that λ depends solely on b and d. Evaluating λ for the different values of b and d used in Table A gives coincidental results with the value of λ actually found by the numerical algorithm as we should expect for consistency. Thus λ is also independent of the demand intercept, a.

A final question of interest involves the implications for a regulatory body attempting to control for the different welfare outcomes examined in this paper. For simplicity take the market equilibrium as a benchmark where no intervention is apparent. The idea is that regulation can lead to welfare improving adjustments. The optimum will require regulation of n, the number of firms, and x, the output per firm (MC pricing). Moreover losses will have to be covered in some way. In the second-best outcome n and x need to be manipulated; but there is no necessity for loss-offset provision. Monopolistic pricing with the surplus criterion requires the regulator to set n, leaving firms to profit maximise. Losses may have to be taken into account. Imposing marginal cost pricing on the set of monopolistic products in existence again involves the regulatory body in loss-offset.

The suggestion that a substantial opportunity cost may be involved in any regulatory move from the market equilibrium to a welfare outcome adds a further dimension to the problem, where the final net welfare outcome may no longer be obvious. Examination of such a dimension is outwith the scope of this paper. Suffice that its existence is recognised.

Conclusion

The paper can be summarised by the following points.

First it is the non-convexities in the form of fixed costs and increasing returns which underlie the welfare issue of product diversity versus efficiency. Recent work has placed considerable emphasis on such phenomena.

Second inefficiency can arise from an undesired product mix, too little or too many products, as well as incorrect output levels. In particular the Spence analysis focussed on the trade-off between output levels and product numbers under a specified set of restrictions. It was evident that the degree of competition (demand interdependence, d) and the extent of fixed costs, f, had important bearings on where welfare losses could be attributed.

Finally the practical absence of a lump-sum tax mechanism and the obvious need to cover fixed costs suggested the appropriateness of analysing a constrained optimum or 'true' second-best solution to Spence's welfare approach. This provided an extension of Spence's (1976a) paper and afforded a useful comparison with the market equilibrium (an approximate second-best solution) and other welfare outcomes. Both graphical procedures and a numerical algorithm were used to further this work.

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