THE GENERAL CONJECTURAL MODEL OF OLIGOPOLY -

SOME CLASSICAL POINTS REVISITED

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The general conjectural model of oligopoly - some classical points revisited

My excuse for going into this topic is that I - as many of the readers - from time to time have to teach oligopoly theory. Although not specialists in the field, we have to take up problems on how to present the theory, pick out what we consider to be the important points and criticise popular misunderstandings and misinterpretations. Some of these points are more relevant when teaching on an elementary or intermediate level. When teaching at advanced level we can often use recent research - material more directly.

The way of teaching oligopoly at an elementary level is however not only a question of how to make life easier for young students, but also and much more interesting, two other aspects. First and most important is the connection between elementary teaching and economic policy. If there is any connection today between elementary oligopoly theory and economic policy, the connection is probably based upon Cournot, and this could be a bad state of affairs. Second, and a bit more ambiguous, is the connection between elementary oligopoly theory and new approaches in economic research. Cournot and his reaction functions appear rather frequently in economic research papers in spite of the fact that more direct investigations upon this theory have a tendency to conclude that any expansion of his theory is a waste of time.[1/]

In recent years I have used "The general conjectural model" as a base for oligoply theory. This model is based upon works by

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[1/] See e.g. Simon, Puig and Aschoff: "A Duopoly Simulation and Richer Theory: An End to Cournot", The Review of Economic Studies, 1973, or more indirectly but very clear: William Fellner: "Competition Among the Few", New York, 1960.

Ragnar Frisch , 1/partly/published in the mid-thirties, a work not very well known.

It was impressive, I think, in the thirties, but today you might have

problems in understanding the difference between this model and a standard

textbook approach. But anyhow, given demand 2/

$$p_1 = p_1(x_1, x_2) \qquad\qquad x_1 = x_1(p_1, p_2)$$
$$\text{or}$$
$$p_2 = p_2(x_1, x_2) \qquad\qquad x_2 = x_2(p_1, p_2)$$

and production costs

$$b_1 = b_1(x_1)$$
$$(2)$$
$$b_2 = b_2(x_2)$$

or more directly, given profit functions

$$\Pi_1 = \Pi_1(x_1, x_2) = \Pi_1^*(p_1, p_2) = x_1 \cdot p_1(x_1, x_2) - b_1(x_1)$$
$$(3)$$
$$\Pi_2 = \Pi_2(x_1, x_2) = \Pi_2^*(p_1, p_2) = x_2 \cdot p_2(x_1, x_2) - b_2(x_2)$$

where we have the usual assumptions (concavity, continuity) to secure a

unique interior maximum (leaving out entry and falling out of the market, and

thereby the theory of monopolistic competition). Then the central building

1/ Ragnar Frisch: Lecture notes (Norwegian) mimeographed Oslo 1940
    "Polypolteori". Partly published in "NationalØkonomisk Tidsskrift,
    Copenhagen 1933 pp.271-259: "Polypole La notion de force dans l'economie".
    Translated to English in: International Economic Papers. Vol.1 (1951)
    p.23-36.

2/ The model is not necessarily duopoly. The generalisation to n
    producers is straightforward, but will be left out here.

stones, the conjectural derivatives

$$\left(\frac{dx_2}{dx_1}\right)^* = a_1(x_1, x_2)$$

(4)

$$\left(\frac{dx_1}{dx_2}\right)^* = a_2(x_1, x_2)$$

As usual we assume profit maximisation behaviour and we have

$$(5) \quad \frac{d\Pi_1}{dx_1} = \frac{\partial\Pi_1}{\partial x_1} + \frac{\partial\Pi_1}{\partial x_2}\left(\frac{dx_2}{dx_1}\right)^* = \Pi_{11} + \Pi_{12}a_1 = p_1 + x_1 p_{11} + x_1 p_{12} a_1 - b_{11} = 0$$

and similar for producer 2.

The fulfillment of (5) will define a locus in the quantity or in the price space to be labelled <u>the reaction function</u> which represent our theory about producer behaviour. Different assumptions about the conjectural derivatives will now generate all possible (under our assumptions) oligopoly theories. To familiarise ourselves with the model, let us recognise a few of the most well-known.

First

$$(6) \quad a_i = 0 \qquad i = 1, 2 \qquad \text{(Cournot)}$$

is of course Cournot where the producer thinks that his opponents will not react (in quantity) at all.[1] Similar assumptions about the price

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[1] See fig.B in the appendix for illustration.

$$\left(\frac{dp_2}{dp_1}\right)^* = 0 \quad \rightarrow \quad a_1 = -\frac{p_{21}}{p_{22}}$$

(7)   (Nash)

$$\left(\frac{dp_1}{dp_2}\right)^* = 0 \quad \rightarrow \quad a_2 = -\frac{p_{12}}{p_{11}}$$

where the opponents price is assumed constant is another Nash-equilibrium different from Cournot, and we might label it "Nash" for short.

Before leaving Frisch, let me select a few points from his work. First about the choice between price or quantity as "action parameter". Frisch calls this a "typographical choice". He recognises of course the difference between Cournot and Nash, as we have labelled them, but as long as we assume a one-to-one relationship between price and quantity (known to the producer) the choice between them is trivial. I like this presentation, and react negatively towards presentations "assuming" price or quantity as action parameter.

Another of Frisch' points I like to stress is the necessity of the assumptions concerning conjectural derivatives (4). Before we are willing to assume something about the conjectural derivatives, the polists will have no behaviour at all. Remember for a moment the traditional textbook present-ations of oligopoly theory, leading up to the Cournot solution. There usually follows a criticism of the Cournot-assumption (6) saying that it is unrealistic or even wrong. This might be, but the logic in it is in any case rather difficult and it is important to remember that before some alternative assumptions about the conjectural derivative (4) are chosen, nothing is "right", because the polists have no defined behaviour at all.

Let us now depart from Frisch and let us use his model to develop other possible oligopoly theories.   If we assume

(8)
$$a_1 = \frac{x_2}{x_1} \qquad \text{(the proportional quantity solution)}$$

$$a_2 = \frac{x_1}{x_2}$$

or analogous for price[1]

(9)
$$\left(\frac{dp_2}{dp_1}\right)^* = \frac{p_2}{p_1} \;\rightarrow\; a_1 = \frac{x_2}{x_1} \cdot \frac{\hat{p}_{11}-\hat{p}_{21}}{\hat{p}_{22}-\hat{p}_{12}} \qquad \begin{array}{l}\text{(the proportional}\\ \text{price solution)}\end{array}$$

$$\left(\frac{dp_1}{dp_2}\right)^* = \frac{p_1}{p_2} \;\rightarrow\; a_2 = \frac{x_1}{x_2} \cdot \frac{\hat{p}_{22}-\hat{p}_{12}}{\hat{p}_{11}-\hat{p}_{21}}$$

or a combination where the polist believes that his opponent will keep his market-share of total revenue constant[2]

(10)
$$a_1 = \frac{x_2}{x_1} \cdot \frac{1+\hat{p}_{11} - \hat{p}_{21}}{1+\hat{p}_{22} - \hat{p}_{12}} \qquad \begin{array}{l}\text{(the proportional revenue}\\ \text{solution)}\end{array}$$

$$a_2 = \frac{x_1}{x_2} \cdot \frac{1+\hat{p}_{22} - \hat{p}_{12}}{1+\hat{p}_{11} - \hat{p}_{21}}$$

---

[1] $\hat{p}_{ij}$ stands for elasticities instead of p_{ij} derivatives.

[2] It is   tempting but not necessary in a static model to introduce what we might call "the conjectural functions", the functions behind the conjectural drivatives.   In Cournot they are straight horizontal lines,in (8) the genera market share solution they are rays through the origin etc. (in the quantity space).   Here in (10) they might be written

$$p_2 x_2 = a_1(p_1 x_1 + p_2 x_2)$$

but this could be misleading.   In general they might be written

$$x^*_2 = f^*_1(x_1^*, x_1, x_2)$$

where * indicates the conjectural parts.   This might give an indication about the difficulties we are avoiding, as Frisch, is not introducing the conjectural functions explicitly.

These three solutions are all Pareto-optimal in profits [1]. If we add a few assumptions about some symmetry in demand, they will all be the same and equal to the monopoly solution [2]. Under this assumption let us label all of them "The general market share solution" since "The market share solution" in the literature means something different, namely the asymetric solution where one polist keeps his market share constant and the other, knowing this, maximises his profit.

In saying that the general market share solution is equal to the monopoly solution, it is important to stress that the market share solution is a theory of decentralised decisions in contrast to monopoly, collusion solution or other more or less formal centralised decisions or agreements.

Before going further about the general market share solution let us use the general conjectural model to develop a few more oligopoly theories. From the literature we remember "The kinked demand curve solution" assuming the polist to believe his opponent to react as in Nash to a price increase and as in general market share to a price decrease, we have

(11)
$$- \frac{p_{21}}{p_{22}} \leq a_1 \leq \frac{x_2}{x_1}$$

$$- \frac{p_{12}}{p_{11}} \leq a_2 \leq \frac{x_1}{x_2}$$

(the kinked demand curve solution)

1/ Intuitively this should be rather obvious since the conjectural functions for the polists are identical. In the solution the iso-profit-curves for these polists will have a common tangent and thus, Pareto-optimality.

2/ (8) gives monopoly solution if $p_{12} = p_{21}$, that is if the real income effect is the same e.g. negligible. (9) gives monopoly solution if $(p_2 - b_{22}) / p_2 = p_{12}(p_1 - b_{11})/p_1$ and (10) if $p_{22}b_{22}/p_2 = p_{12}b_{11}/p_1$. One of the three might give monopoly alone but two cannot without the third. (10) performs better (closer) to monopoly on the average than the others.

when we select (8)[1]/ to represent the general market share. The reaction

functions here are not (in duopoly) one-dimensional curves but cover the

whole area between the reaction functions in Nash and the general market share.

The solution covers in fact all other solutions specified in this article.[3]/

As we know, the kined demand curve solution is a theory trying to explain the

non-existence of price competition in oligopoly. The large solution area

describes that, meaning that for market points or changes of market points

within this area, the polists will do nothing, will not react. This "no-

reaction" as a property of the points located on a polists reaction locus (or

curve) could be an argument for labelling the reaction curve the "no-reaction"

curve, but I will not propose to do so.

This "no-reaction" has two aspects I will stress. First a

discussion about what the polist will do facing a market point outside his

reaction locus demands a different kind of model than our static one[2]/. Second,

it is rather common to mix up "no-reaction" in the above sense with the

"no-reaction" we assume as a basis for the Nash solution (when no reaction

means no change in price as a response). My feeling is that it is rather

common to confuse these two logically quite different concepts, not only in

elementary textbooks but also in more advanced literature. It is rather

common to restrain the discussion of realism to the assumptions about the

conjectural derivatives and almost forget about the realism in the total

theory, to discuss how realistic the prescribed behaviour of the polists are

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1/ Some presentations of the kinked demand curve solution will use (9).

2/ It is fairly straightforward to build a corresponding dynamic model.
   We have to add a few assumptions about action sequences etc.
   and we have to introduce the conjectural functions explicitly.
   My experience is that stability is no problem for a rather large
   variety of "normal" numerical examples.

3/ The possible exceptions are "mixed solutions" where one of the polists
   follows (10) and the other something else.

as given by the reaction functions. This might be illustrated by the fact that if we demand consistency here – that the polists should <u>know</u> and not only <u>believe</u> (conjecture) – there is no room for further discussion. We have then reached a new oligopoly theory without further assumptions – a theory we will label "the full information solution".

The theory goes like this. Let us write the reaction functions (5) as

(12)
$$\Pi_{11} + \Pi_{12}a_1 = g_1(x_1, x_2, a_1) = 0$$

$$\Pi_{22} + \Pi_{21}a_2 = g_2(x_1, x_2, a_2) = 0$$

since we have full information, these reaction functions are now also regarded as the base for the conjectural derivatives

(13)
$$a_1 = \frac{g_{21}}{g_{22}} \qquad \text{(The full information solution)}$$

$$a_2 = \frac{g_{12}}{g_{11}}$$

where (12) and (13) form four relations to determine the quantities and the values of the conjectural derivatives in equilibrium.[1]

To grasp the uniqueness of the full information solution, consider the following reasoning. We define an isocline (same inclination, slope) as a locus in the x_1-x_2 space where the iso-profit-loci have the same slope a. The isocline-functions can be written as (12) where a_i is the constant numerical value of the slope. Select one isocline for 1. and locate

....

[1] We have here assumed the reaction function to be linear (locally). I am not afraid of this assumption since it seems rather unlikely in this kind of theory for curvatures to enter in any serious manner. For further discussion – see appendix.

maximum profit for No.2 along this isocline. Do this for all possible isoclines for No.1 and these maximum profit points for 2 will satisfy

$$
\begin{vmatrix}
0 & \Pi_{21} & \Pi_{22} \\
\Pi_{11} & \Pi_{111} & \Pi_{112} \\
\Pi_{112} & \Pi_{121} & \Pi_{122}
\end{vmatrix} = 0
$$

having some similarities with a reaction function (for 2). The same reasoning for the other polist gives us

$$
\begin{vmatrix}
0 & \Pi_{11} & \Pi_{12} \\
\Pi_{21} & \Pi_{211} & \Pi_{212} \\
\Pi_{22} & \Pi_{221} & \Pi_{222}
\end{vmatrix} = 0
$$

to determine the full information solution. This does not prove the uniqueness of this solution but I see no reason why the uniqueness of this solution should be questioned more than the uniqueness of the other solutions presented above. Another argument in this direction is that it is not difficult to prove that the full information solution always will be located between the Cournot and the Nash solution, although I will not give the proof here.

I find the full information solution interesting enough to be mentioned among the oligopoly theories known from the literature. Not because I think it is very realistic but because pointing out its existence might shed

some light upon the understanding of the content of traditional oligopoly

theories. I have for example always been a little uneasy about the logic

involved in the Stackelberg-solutions coming to the phrase that the

duopolist observes his own and his opponent's reaction functions (Cournot) in

order to decide to desire to become a follower or a leader [1] The question

"What have we now assumed about the conjectural derivatives?" is in any case

a difficult one. This is logically improved when we as Fellner [2] extend the

theory to interpret a Stackelberg leadership as a theory where the leader has

the power to force the opponent into a followership. But, as Fellner points

out, if it has that power, why would it not prefer to force its rival into

something much more favourable (for both) than anything on a Cournot reaction

function.

If we should take up a discussion similar to Stackelberg, following

the logic in the general conjectural model, it might go as follows: starting

with Cournot (or any other assumption about the conjectural derivatives) the

polist, if given that opportunity, will inspect his rival's reaction curve.

He will always choose to be a leader (if the other is a follower) (the only

exception is in full information equilibrium where the choice is trivial).

Thereby, his reaction function will alter because the assumption about his

conjectural derivative has altered. If now in turn the other polist is able

to inspect _that_ reaction function - and so on - this "Ustinov"-process [3]

will lead to full information solution regardless of the initial assumption

about the conjectural derivatives.

1/ For example Henderson & Quandt: "Microeconomic theory", London 1971, p.229
but also many other places and of course H.von Stackelberg: "The theory of
the market economy (English translation of the 1948 German) London, 1952.

2/ William Fellner: "Competition among the few", New York, 1960, p.118.

3/ This should be called something and I remembered the 1958 Peter Ustinov
movie "Romanov and Juliette" where he, as president in a mini-state, is
running back and forth between the American and the Russian ambassador
telling: "Do you know that they know that you know that they know ...."

Some conclusions

If I should suggest some conclusions from what I have presented here I should like to underline the pedagogical merits in the general conjectural model in teaching oligopoly theory. In a systematic and simple way both analytically and graphically, the general conjectural model gives a way to present many classical and some new oligopoly theories, superior I think, to what we find in textbooks on elementary or intermediate levels.

In my own teaching therefore I will go on using the general conjectural model and within this, I will pay my respect to Courtnot, Nash, Stackelberg and so forth as historical landmarks in the development of oligopoly theory.

What about the full information solution? I think it is an interesting idea, but regarded as an oligopoly theory I find it not very realistic. The full information solution is more competitive than Cournot and almost as competitive as Nash, and I think it is correct to say that there is a clear tendency in more advanced oligopoly literature to regard as realistic much less competitive solutions.[1]

To bridge this gap, the general conjectural model offers what I have labelled the general market share solutions. This produces oligopoly theories with much more realistic solutions than the classical ones, having the same point of gravity as more advanced theories. In general market share solutions is however the assumption about the conjectural derivatives as far

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[1] The ranking of oligopoly solutions has a distinct pattern. For complements we have (where X is total quantity and $|\overset{v}{p}_{ij}| \leq \overset{v}{p}_{ij}$) X free comp. $\geq X$ monopoly $\geq X$ Nash $\geq X$ full inf. $\geq X$ Cournot and the same for alternatives except that X monopoly moves to the extreme right.

from the actual behaviour (as given by the reaction functions) as can be. In order to defend the realism in these assumptions we must accept a distinction between "tactics" (short time behaviour in order to obtain a favourable position - as an argument for realism in assuming constant market share) and "strategy" (a more ultimate aim - as an argument for maximizing behaviour). If we refuse this distinction - we are back in full information solution.

APPENDIX

A. Graphic treatment - a numerical example.

B. Further remarks about the full information solution.

A. I have chosen linear demand functions, proportional marginal costs
and heterogenous alternative symmetric products for this example.
Thereby we have a linear transformation from price to quantity space
as shown from Fig. A to B.   Only reaction curves and iso-profit-curves
for polist no. one are drawn.   Similar concepts for no.two are
symmetric around the $X_1 = X_2$ line.

The table and figures can be read straightforward with possible
exceptions for some theories not mentioned above.   The "Free
Competition" solution where the polist believes his own price to be
kept constant by his opponent might have some merits.   A similar
assumption about the quantity ($\alpha_i = 2$) gives of course zero quantity.
The "Equal response" solution gives the same as general market share
when polists are equal.   When they differ considerably in size it
gives something else but then the assumption is less tempting.
The "Stackelberg disequilibrium" solutions labelled "Price leaders"
and "Price leaders[2]" follows the reasoning from p.10 above giving
two steps (or four half-steps) in the "Ustinov" process starting
with Nash.

Numerical example:

$$P_i = 1 - 2/3 x_i - 1/3 x_j \qquad b_i = 1/2 x_i^2$$

$$x_i = 1 - 2p_i + p_j$$

Conjectural derivatives		Name of Theory	Reaction functions		Solution Values			
$a_i = dx_j/dx_i$	$a_i = dp_j/dp_i$		Quantity-form	Price-form	x_i	p_i	$-g_{ij}/g_{ii}$	Π_i
0	0.5	Cournot	$3-7x_i-x_j = 0$	$5-13p_i+5p_j = 0$	3/8	5/8	$-1/7 = -0.143$	0.1641
-0.5	0	Nash (price)	$6-13x_i-2x_j = 0$	$3-8p_i-3p_j = 0$	2/5	3/5	$-2/13 = -0.154$	0.1600
x_j/x_i	$(1-p_j)/(1-p_i)$	Proportional Quantity	$3-7x_i-2x_j = 0$	$2-4p_i+p_j = 0$	1/3	2/3	$2/7 = -0.286$	0.1667
p_j/p_i	$(1-x_j)/(1-x_i)$	Proportional price	$3-11x_i+7x_i^2+2x_ix_j-x_j = 0$	$3p_i-8p_i^2+6p_ip_j-p_j-p_j^2 = 0$	1/3	2/3	$-1/17 = -0.059$	0.1667
$\dfrac{3/x_i-2/p_i+1/p_i}{3/x_j-2/p_j+1/p_i}$	$\dfrac{1/p_i-2/x_i-1/x_j}{1/p_j-2/x_j-1/x_i}$	Proportional revenue	$\dfrac{3p_i}{x_j} - \dfrac{2p_i}{p_j} - \dfrac{5x_i}{x_j} + \dfrac{3x_i}{p_j}$	$\dfrac{x_i}{p_j} - \dfrac{x_i}{p_i} = 0$	1/3	2/3	$-\dfrac{23}{67} = -0.373$	0.1667
$\dfrac{-1}{7+a_j} = \dfrac{3\sqrt5-7}{2}$	$\dfrac{3-a_j}{8-3a_j} = \tfrac{1}{4}(3-\sqrt5)$	Full Information	$3 \cdot (3.5+\tfrac32\sqrt5)x_i-x_j = 0$	$1- \dfrac{7+3\sqrt5}{3+\sqrt5}\,p_i + p_j = 0$	0.3820	0.618034	-0.145898	0.163119
-2	$\infty,\ dp_i = 0$	"Free Competition"	$3 - 5x_1 - x_2 = 0$		0.5	0.5	-0.2	0.1250
1	1	Equal response (quantity or price)	$3 - 8x_1 - x_2 = 0$	$1 - 5p_1 + 2p_2$	1/3	2/3	-0.125	0.1667
$\geqq -0.5$ $\leqq x_j/x_i$	$\leqq (1-p_j)/(1-p_i)$	Kinked Demand Curve	$\dfrac{3}{7} - \dfrac{2}{7}x_j \leqq x_i \leqq \dfrac{6}{13} - \dfrac{2}{13}x_j$	$\dfrac{3}{8} + \dfrac{3}{8}p_j \leqq p_i \leqq \tfrac14 + \tfrac14 p_j$	-	-	-	-
$-\dfrac{2}{13} = -0.154$	$\dfrac{3}{8}$	Both price leaders	$3 - (7 - \tfrac{2}{13})x_i - x_j = 0$	$1 - 55/21\, p_i + p_j = 0$	13/34	0.6176	-0.1460	0.163062
$-\dfrac{13}{89} = -0.146$	$\dfrac{21}{55}$	Both price leaders$_2$	$3 - (7 - \tfrac{13}{89})x_i - x_j = 0$	$1 - 377/144\, p_i + p_j = 0$	89/233	0.6182035	-0.145902	0.163118

General market share

Fig. A

Fig. B

NASH

FULL INFORMATION

COURNOT

PROP. PRICE

PROP. QUANTITY

PROP. REVENUE

Kinked-
demand-
curve-
solution

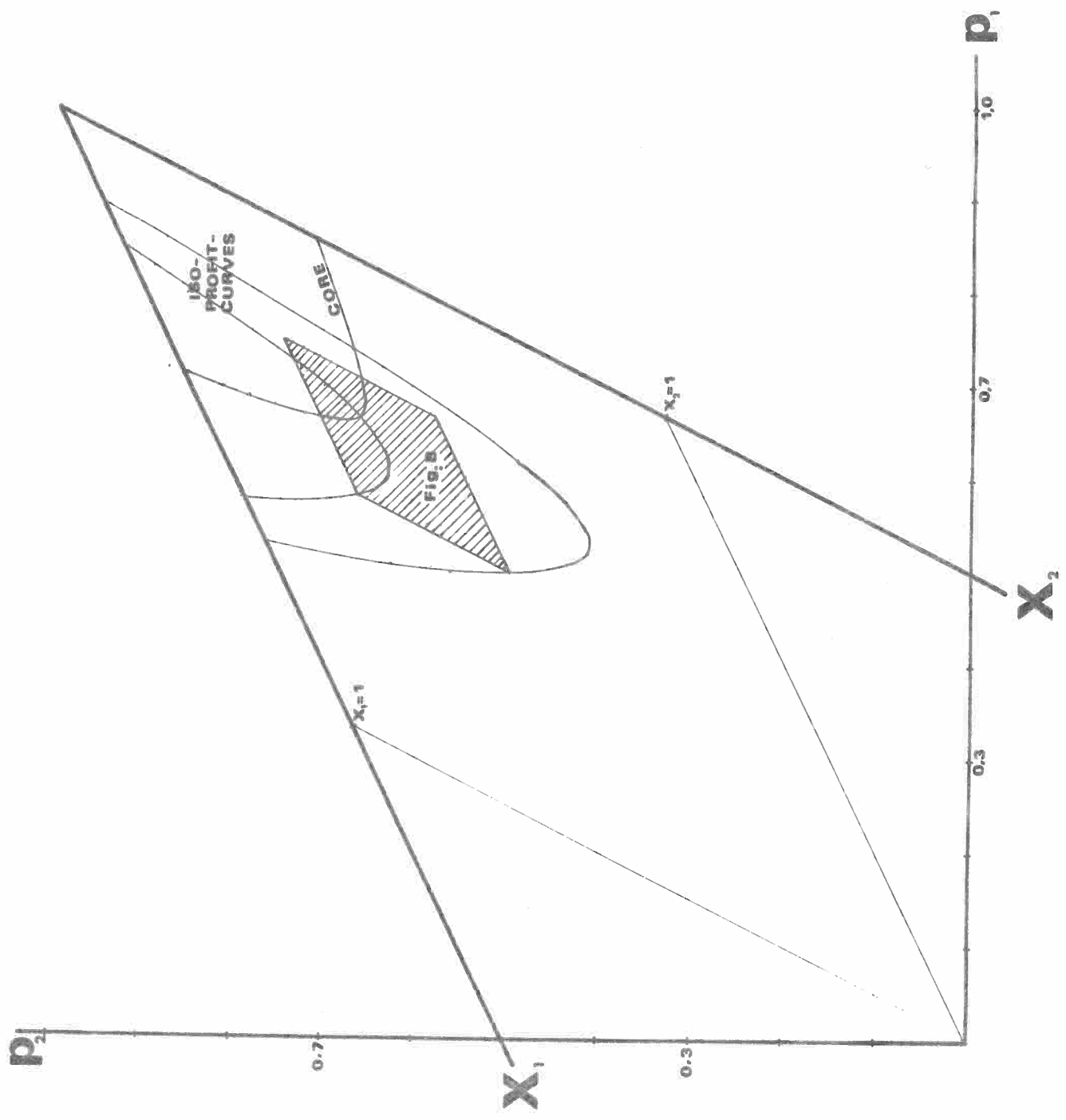ISO-PROFIT-
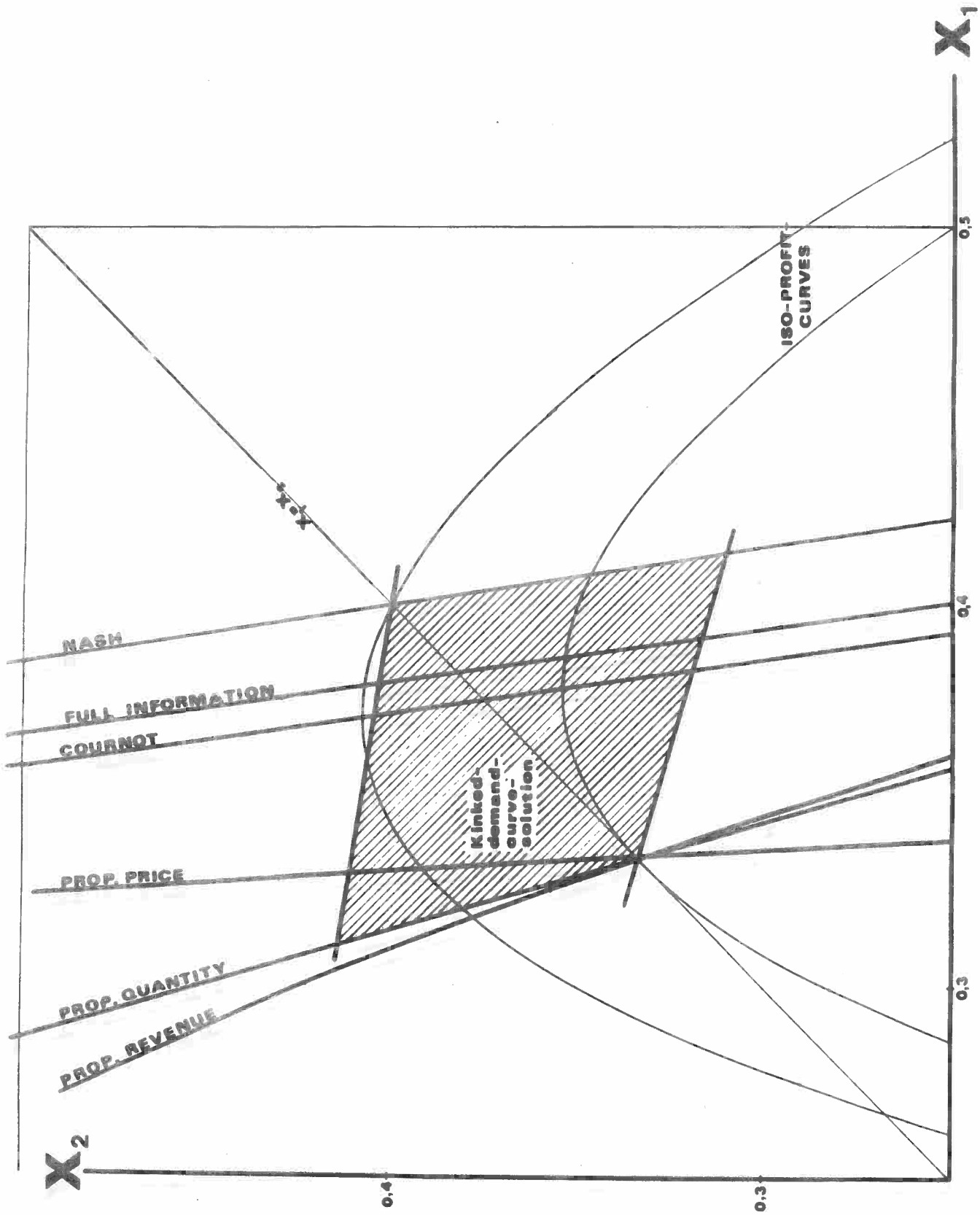CURVES

$X_1 = X_2$

X_1

X_2

0.5

0.4

0.3

0.4

0.3

B.

Further remarks about the full information solution.

In the introduction of the "full information solution" above it is a built-in assumption that the slope of the conjectural function (the opponents reaction function) is a constant, at least locally. This "linearity" assumption is in my opinion rather acceptable but let us look into the matter and see what happens if we abandon this assumption.

Let us rewrite (5) as

i) $\quad \Pi_{11} + \Pi_{12}f_1'(X_1) = 0 \qquad X_1 = f_2(X_2)$

ii) $\quad \Pi_{22} + \Pi_{21}f_2'(X_2) = 0 \qquad X_2 = f_1(X_1)$

where $f_i(x_i)$ is defined as i) and ii) on explicit form. If we now maximise profit using i) directly, we have

iii) $\quad \dfrac{\Pi_{221} + f_2'\Pi_{211}}{\Pi_{222} + f_2'\Pi_{212} + \Pi_{21}f_2''} = -f_1'$

iv) $\quad \dfrac{\Pi_{112} + f_1'\Pi_{122}}{\Pi_{111} + f_1'\Pi_{121} + \Pi_{12}f_1''} = -f_2'$

If we assume $f_i'' = 0$ we have as above four equations to determine f_i' and x_i and full information solution is determined. To abandon the linearity assumption means to discuss what can happen to the full information solution when the last term in the denominator in iii) and iv) is different from zero.

Having alternatives we know under our assumptions that all isoclines will be negatively inclined in the quantity-space, thus

$$- \frac{dx_1}{dx_2} = \frac{\Pi_{112} + f_1' \Pi_{122}}{\Pi_{111} + f_1' \Pi_{121}} > 0 \quad \text{for all } f_1'$$

Since $\Pi_{111} < 0$ we have that both numerator and denominator will be negative, and similar for the other polist. It follows that since $\Pi_{ij} < 0 \ i \neq j$ the full information solution will be located between "linear" full information on Cournot as long as

$$0 \leq f_i'' < \infty$$

We can of course have $f_i'' < 0$ but not to a large degree in order to have a well defined profit maximum. It can be shown that for $f_i'' < 0$ the full information equilibrium must stay between "linear" full information and Nash (a similar discussion as above using $\Pi_i(p_1 p_2)$).

Although I am not inclined to do so, we can conclude that taking conjectural curvatives into consideration this means only minor adjustments to the full information equilibrium. Full information equilibrium will still be less competitive than Nash and more competitive than Cournot.

But of course, we need the linear assumption to secure the uniqueness of full information solution. One obvious example where this is not fulfilled is when demand functions are non-linear. Then "linear" assumption means something different in price and quantity. But we still are talking about much smaller adjustments than Nash solutions giving Cournot in the quantity space and Nash (price) in the price space. To illustrate this, our numerical

example defines an area for possible full information solutions when

$$-1 \leq f_i^{''} \frac{x_i}{f_i^{'}} \leq 1$$

being only 0.014% of the area limited by the Cournot-Nash reaction functions. Even if we choose practically extreme boundaries for the second order derivatives (both price and quantities)

$$-10 \leq f_i^{''} \frac{x_i}{f_i^{'}} \leq + 10$$

and

$$-10 \leq \frac{d\alpha_i}{dp_i} \frac{p_i}{\alpha_i} \leq + 10$$

the area for possible full information solutions will only be 2.03% of the Cournot-Nash area.