KEYNESIAN EQUILIBRIUM AND FIX-PRICE EQUILIBRIA

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The recent development of macroeconomic models with fixed prices has shown the importance of considering the type of unemployment in the economy, since this determines the effects of economic policy decisions. These models differ from the Keynesian approach by the hypothesis of rigidity of all prices, by the fact that equilibrium results from a tatonnement on quantities, and by the behaviour of firms whose role is as passive as that of households: firms and households are symmetrically treated. The most active role is that of the auctioneer who instead of adjusting prices, adjusts quantities.

Keynesian equilibrium is defined by "Aggregated Demand and Aggregate Supply Functions" and given an aggregate production function, it corresponds to a determined price level. But given fixed prices, there is generally no equality between production and aggregate supply and demand.

I try to restore the non-symmetrical Keynesian view and I study the formation and the type of equilibrium expected by firms facing constraints which arise from aggregate demand and from aggregate supply when prices are fixed. The existence and uniqueness of such an equilibrium is guaranteed without convexity. And when the expectations of firms are exact (when they coincide with the effective demand for goods and the effective supply of labour by households), the expected equilibrium is a fixed price equilibrium in the usual sense, i.e. the asymmetrical approach gives the same result as the symmetrical one.

Introduction : A Simple Model of Keynesian Equilibrium

The equilibrium level of employment, "the effective demand", is defined by Keynes [7] as the intersection of two curves:

the Aggregate Supply Function which expresses sales required $Z(N)$ as a function of employment N.

the Aggregate Demand Function which expresses expected sales $D(N)$ as a function of employment N.

Because goods are heterogeneous, Keynes chose as a unit of measure homogeneous quantity of labour, which permitted him, given a nominal wage rate w, to obtain monetary quantities $wZ(N)$ and $wD(N)$. Consider the following simple expression for the Aggregate Supply and Demand functions:

$$wZ(N) = wN + \lambda VK \ , \quad \text{and} \quad wD(N) = V\tilde{C} + VA$$

VK is the monetary value of capital which depreciates at rate μ and yields the profit at rate ρ; $\lambda = \mu + \rho$ is the sum of the rate of depreciation and the rate of profit. VA is the monetary value of autonomous demand which includes investments (linked to long run expectations) and public consumption. $V\tilde{C}$ is the expected monetary value of all the other demands directly linked to employment N.

Indeterminacy of the general price level, emphasised by Keynes, comes from its relation to effective production. But at the Keynesian equilibrium N^* defined by $Z(N^*) = D(N^*)$, this indeterminacy can be lifted by considering production. With an aggregate production function $F(N)$ taking into account the expected rate of capacity utilisation, the equilibrium price level p^* is defined by : $p^* F(N^*) = wZ(N^*) = wD(N^*)$. And the aggregate supply and demand functions:

$$wZ(N) = wN + \lambda pK \quad \text{and} \quad wD(N) = p\tilde{C} + pA$$

determine the same equilibrium that obtains with $VK = pK$, $V\tilde{C} = p\tilde{C}$ and $VA = pA$, provided one imposes the consistency condition on p with aggregate supply $wZ(N) = p\,F(N)$[1/].

1/ With the demand expectation $\tilde{C} = \alpha\, C_o + \beta(w/p)N$, the Keynesian equilibrium is defined by $F(N^*) = (\alpha C_o + A - \beta\lambda K)/(1-\beta)$ and does not depend on the absolute level of prices. But with the demand expectation $\tilde{C} = \alpha C_o + \beta(w/p)N + \gamma(M_o/p + K)$, there is a real balance effect of the real money stock M_o/p and the Keynesian equilibrium does depend on the absolute level of prices. [11].

Keynes did not consider the problem of differences which could exist between expected demand $\tilde{C}^*$ at equilibrium N^* and the demand $\hat{C}$ expressed on the goods market. It is often assumed that implicit in Keynes is an ex-post price mechanism which equilibrates demand and supply on the goods market. In this model, this ex-post price $\bar{p}$ is determined by relation : $\hat{C}(N^*, w, \bar{p}) = F(N^*) - A$. Then there is no change in the wage rate, nor in the expected equilibrium employment N^* which coincides in Keynes Model with realized employment.

Another possibility is that the expected equilibrium (p^*, N^*) is fixed by firms and that the realised equilibrium results from adjustments of quantities : realized sales correspond to the minimum of supply (equal to expected demand $F(N^*) = \tilde{C}^* + A$) and effective demand at price p^* : $\hat{C}(N^*, w, p^*) + A$. Then there are three possible cases : equality of demand and supply (when firms expectations are exact), excess supply, and excess demand for goods.

The usual approach of this problem, namely the fix-price equilibrium ([1], [2], [4], [6] . . .), is based on a symmetrical formulation of the demands of households and of firms. One find the same symmetry in the studies of the effects of expectations ([5], [10]). My approach is different, trying to restore the Keynesian asymmetry between firms and households : I only consider the expectations of the firms and I show that this asymmetrical approach gives the same result as the symmetrical one assuming that the firms' expectations are exact.

I. Fixed-price equilibrium : Equilibrium expected by firms

Firms form their expectations on given price level p and wage rate w . We consider the following functions of N :

- a production function $F(N)$ which takes into account the rate of capacity utilisation corresponding to N ;

- aggregate supply $wZ(N)$;
- aggregate demand $wD(N) = p(\tilde{C} + A)$ determined by expected demand $\tilde{C}(N)$ and autonomous demand A.

At fixed prices there is in general no equality between production and aggregate demand and supply. So aggregate demand and supply play the role of constraints. The supply constraint expresses the fact that the sales must be at least equal to the "Supply Price" $wZ(N)$:

(C1) $pF(N) \geq wZ(N)$

The demand constraint expresses the fact that sales cannot exceed the "Demand Price" :

(C2) $pF(N) \leq p(\tilde{C}(N) + A)$

In addition we consider the labour supply constraint : employment cannot exceed the expected supply of labour $\tilde{N}$

(C3) $N \leq \tilde{N}$

$\tilde{N}$ may be related to N through households' consumption which is $F(N) - A$ in the case where all production is bought and autonomous demand is satisfied. The domain Δ of employment levels $N \geq 0$ which satisfy the three constraints C1, C2, C3, is assumed to be non empty. According to Keynes, as long as there exist additional employment possibilities allowing sales at least equal to the Supply Price and which are feasible because they do not exceed the Demand Price, there will be firms which want to hire the additional labour.

Such behaviour based on profitability constraint C1,production outlets constraint C2 and employment possibility constraint C3, seems more adequate to describe reality than maximization of global profit. In the case of monotonic returns, the maximum of global profit $pF(N) - wN$ is equivalent to the maximization of employment subject to the supply constraint defined by $Z(N) = F(N)/F'(N)$. So, profit maximization is a particular case of our formulation. But this particular case does not seem to me any more Keynesian than any other.

The expected fixprice equilibrium is the maximum employment N^* which satisfies the three constraints C1, C2 and C3.

There always exists a unique expected equilibrium provided that the feasible set Δ is bounded, not empty and the functions are continuous.

To obtain the Keynesian equilibrium where the wage rate alone is fixed, I consider the case where there is no constraint $\tilde{N}$ on available employment; and I assume that 1°) the excess of sales relative to profitable supply : $pF(N) - wZ(N, p)$ is an increasing function of p , with N and w fixed; and 2°) expected demand $\tilde{C}(N, p)$ is a decreasing function of p , with N and w fixed.

Proposition. Under the above assumptions, the expected equilibrium with a fixed wage rate and a flexible price level, is a Keynesian equilibrium.

Proof. Let $\bar{N}$ be the employment level which maximizes N subject to the condition :

there exists p such that N satisfies C1 and C2.

Thus $\bar{N}$ satisfies C1 and C2 at a price level $\bar{p}$:

$$\bar{p}F(\bar{N}) \geq wZ(\bar{N}, \bar{p}) \ , \quad \text{and} \quad F(\bar{N}) \leq \tilde{C}(\bar{N}, \bar{p}) + A \ .$$

If neither of these two constraints is binding, an increase in $\bar{N}$ holding $\bar{p}$ fixed would be possible. If the first constraint alone is not binding, then there exists a price $p < \bar{p}$ at which : $pF(\bar{N}) - wZ(\bar{N}, p) > 0$ and $F(\bar{N}) < \tilde{C}(\bar{N}, p) + A$. Therefore an increase of $\bar{N}$ at the price p would be possible.

If the second constraint alone is not binding, then there exists a price $p' > \bar{p}$ at which :

$$F(\bar{N}) < \tilde{C}(\bar{N}, p') + A \quad \text{and} \quad p'F(\bar{N}) - wZ(\bar{N}, p') > 0 \ ;$$

again an increase of $\bar{N}$ at the price p' would be possible.

So the only remaining possibility is that both constraints are binding :

$$\bar{p}F(\bar{N}) = wZ(\bar{N}, \bar{p}) = \bar{p}(\tilde{C}(\bar{N}, \bar{p}) + A)$$

which shows that $\bar{N}$ is a Keynesian equilibrium.

Types of expected equilibria

When the two constraints C1 and C2 are binding at the expected equilibrium N^*, this is a Keynesian equilibrium, namely, the intersection of aggregate supply and demand functions. For $N^* < \tilde{N}$, this is an unemployment equilibrium. For $N^* = \tilde{N}$, this is a full employment equilibrium : the firms expect a Walrasian equilibrium by which I mean that expected supply and demand are equal both on the labour market and the goods market.

Like in the classical fix-price models ([8]), we will distinguish three major expected equilibrium regimes :

- Classical unemployment when only the constraint C1 is binding : $pF(N^*) = wZ(N^*)$. In this case, there is an expected excess demand for goods and it is also an expected excess supply of labour.
- Keynesian unemployment where only the constraint C2 is binding : $F(N^*) = \tilde{C}(N^*) + A$. In this case, there is an expected excess supply of labour and a potential excess supply of good in the sense that the only thing that prevents firms from increasing production is the demand constraint.

- Repressed inflation where only the constraint C3 is binding : $N^* = \tilde{N}$. In this case, there is an expected excess demand for good and a potential excess demand for labour.

Economic policy effects on the equilibrium expected by firms

If firms expectations did not respond to changes in economic policies, then the expected equilibrium would not change either. We will assume, as in rational expectation models, that there is a rather direct link between firms expectations and households' effective demands. In addition we will assume that the Aggregate Supply function takes the following simple form (considered in the Introduction) : $wZ(N) = wN + \lambda pK$.

1°) In the case of Classical Unemployment, the equation $F(N^*) = (w/p)N^* + \lambda K$ determines equilibrium employment level N^* , and constraints C2 and C3 are not binding. Therefore there exists a neighbourhood of N^* where constraints C2 and C3 are not binding, and in which, for $N > N^*$, $F(N) < (w/p)N + \lambda K$, because N^* is an expected fixprice equilibrium. The constraint C1 can only be satisfied by decreasing w/p or λ ; a decrease in either implies that the constraint C1 is no longer binding in N^* , so there would be an increase in equilibrium employment. Thus to increase employment in the classical unemployment regime requires a decrease in the real rate at which one of the two factors of production, capital or labour, is remunerated.

2°) In the Keynesian unemployment case, equilibrium employment is determined by $F(N^*) = \tilde{C}(N^*) + A$, and the two constraints C1 and C3 are not binding. An increase in equilibrium employment results from an increase in autonomous demand A which has a multiplier effect corresponding to the relationship between expected demand and employment. In the differentiable case, the multiplier is:

$$\frac{dF}{dN} / (\frac{dF}{dN} - \frac{d\tilde{C}}{dN}) .$$

An increase in employment is also positively related to the expected demand $\tilde{C}$. Demand for goods is positively related to the wage rate w and negatively related to the price level p ; then the same effects apply to the expected equilibrium, to the extent that $\tilde{C}$ is influenced by current demand. The effects of prices and wages can validly be considered separately as long as they vary independently. With a constant real wage, the real balance effect will predominate and the expected demand $\tilde{C}$ will increase with the real value of the money stock as at the Keynesian equilibrium. With a variable real wage, an increase in real wage will increase demand and hence employment and production.

3°) <u>In Repressed Inflation</u>, the equilibrium employment level is determined by expected labour supply $\tilde{N}$. The price effects are complex. For example, if these expectations are determined by current supply, given an increase in the wage rate, there will be both an income effect and a substitution effect; and the overall effect is ambiguous. A decrease in the general price level has the same effects, but the income effect in reinforced by the real balance effect; and supply of labour decreases. Thus an increase in prices will lead to an increase of employment level and production.

II. Expected equilibrium compared with the usual fix-price equilibrium

Fixed price equilibria in Macroeconomics models are very differently defined. In these models, the description of the behaviour of firms is based not on expected curves, but on constant constraints which are linked to the agents' supplies and demands.

So in our model, constraint curves C2 and C3 are replaced by constant constraints $\bar{Y}$ on sales and $\bar{N}$ on employment. The set of possibilities corresponding to constraints $\bar{Y}$ and $\bar{N}$ is :

$$G(\bar{Y}, \bar{N}) = \{(Y, N); Y = F(N), pF(N) \geq wZ(N), Y \leq \bar{Y} \text{ and } N \leq \bar{N}\} ;$$

and firms' constrained plan is the possibility $(\hat{Y}, \hat{N})$ which maximizes the employment level in $G(\bar{Y}, \bar{N})$.

The constrained demands. Even in the convex case, the effective demands or constrained demands are not uniquely well defined ([3]) . We shall assume the following relations (CD) between constraints $\bar{N}$, $\bar{Y}$, constrained plan $(\hat{Y}, \hat{N})$ and labour demand N_d and goods supply Y_s of firms :

(CD) $\quad \text{Min}\,\{N_d, \bar{N}\} = \hat{N}$ and $\text{Min}\,\{Y_s, \bar{Y}\} = \hat{Y}$.

This condition expresses that a constrained demand or supply 1°) must allow constrained plan's realization ($N_d \geq \hat{N}$ and $Y_s \geq \hat{Y}$) and 2°) must coincide with the constrained plan in the case where this plan is smaller than the constraint ($\hat{N} < \bar{N} \Rightarrow N_d = \hat{N}$ and $\hat{Y} < \bar{Y} \Rightarrow Y_s = \hat{Y}$) .

The condition that the two sides of a market (supply and demand) are not both constrained ([4]) has to be modified in a nonconvex case : the constrained plan may be smaller than the constraint and at the same time the agent can prefer some level greater than the constraint. It is thus necessary to make explicitly assumption (CD) for the nonconvex case; under convexity it is a direct implication of the definition of equilibrium.

Households Behaviour is defined by the maximization of a continuous utility function $u(C, L, M)$ depending on consumption C , labour L and final money stock M . Given an initial money stock M_o and non-labour income R , households budgetary constraint is $pC + M \leq wL + R + M_o$. Their possibilities set is, with constraints $\bar{C}$ and $\bar{L}$.

$$H(\bar{C}, \bar{L}; R) = \{(C, L, M); pC + M \leq wL + R + M_o , M \geq 0 , 0 \leq C \leq \bar{C} , \text{ and } 0 \leq L \leq \bar{L}\}$$

and a corresponding constrained plan $(\hat{C}, \hat{L}, \hat{M})$ or simply $(\hat{C}, \hat{L})$, maximizes the utility function u on $H(\bar{C}, \bar{L}; R)$. The constrained goods demand C_d and labour supply L_s are assumed to satisfy:

(CD') $\quad \mathrm{Min}\,\{L_s, \bar{L}\} = \hat{L}$, and $\mathrm{Min}\,\{C_d, \bar{C}\} = \hat{C}$.

We can now define a <u>fixed price equilibrium</u> as constraints $\bar{Y}, \bar{N}, \bar{C}, \bar{L}$, corresponding constrained plans $\hat{Y}, \hat{N}, \hat{C}, \hat{L}$, and constrained supplies and demands Y_s, N_d, L_s and C_d which satisfy the three following conditions :

1°) an agent perceives the constraints which are supplies and demands of the others : $\bar{Y} = C_d + A$, $\bar{N} = L_s$, $\bar{L} = N_d$ and $\bar{C} = Y_s - A$ (assuming the autonomous demand to be satisfied);

2°) constrained plans are compatible : $\hat{Y} = \hat{C} + A$ and $\hat{N} = \hat{L}$;

3°) constrained plans are on both markets (goods and labour) equal to the minimum of supply and demand : $\hat{Y} = \mathrm{Min}\,\{Y_s, C_d + A\}$ and $\hat{N} = \mathrm{Min}\,\{L_s, N_d\}$.

This definition is more explicit than in [1] and [2] ; it is equivalent in the convex case; but the complete formulation is necessary in the general case without convexity.

<u>Theorem</u>. <u>The equilibrium expected by firms, when firms expectations are exact, is a fixed price equilibrium, assuming that expected curves</u> $\tilde{C}(N)$ <u>and</u> $\tilde{N}(N)$ <u>are nondecreasing</u>.

<u>Proof</u>. Let N^* be a fixed price equilibrium expected by firms. We consider the constraints

$$\bar{N} = \tilde{N}(N^*) \quad \text{and} \quad \bar{Y} = \tilde{C}(N^*) + A .$$

a) We shall prove first that $\hat{N} = N^*$, $\hat{Y} = F(N^*)$ is the constrained firms' plan, with constraints $\bar{N}, \bar{Y}$. It is easy to see that $(\hat{Y}, \hat{N})$ belongs to $G(\bar{Y}, \bar{N})$, because N^* verifies the three conditions C1, C2 and C3. If $(\hat{Y}, \hat{N})$ would not be the constrained plan, it would exist (Y, N) belonging to $G(\bar{Y}, \bar{N})$ and such that $N > \hat{N}$. (Y, N) verifies C1 because it belongs to $G(\bar{Y}, \bar{N})$; it verifies also

$$F(N) \leqslant \bar{Y} = \tilde{C}(\hat{N}) + A \leqslant \tilde{C}(N) + A$$

$$N \leqslant \bar{N} = \tilde{N}(\hat{N}) \leqslant \tilde{N}(N)$$

because expected curves $\tilde{C}$ and $\tilde{N}$ are nondecreasing. Consequently, N verifies the three constraints C1, C2 and C3, and $N > N^* = \hat{N}$; then N^* is not an expected equilibrium which gives a contradiction.

(b) Constrained supplies and demands and exact expectations. To the constraints $\bar{N}$, $\bar{Y}$ correspond the constrained plan $(\hat{Y}, \hat{N})$ and constrained supply Y_s and demand N_d which verify (CD). We set

$$\bar{C} = Y_s - A \quad \text{and} \quad \bar{L} = N_d$$

Let R^* be the non-labour income of Households when expected equilibrium is N^* ; their constrained plan, subject to $\bar{C}$ and $\bar{L}$, with income R^*, is $(\hat{C}, \hat{L})$; labour supply is L_s and goods demand is C_d. The expectations of firms are <u>exact</u> at N^* iff $\tilde{C}(N^*) = C_d$ and $\tilde{N}(N^*) = L_s$. Then it results from definition of $\bar{N}$ and $\bar{Y}$, that : $\bar{N} = L_s$ and $\bar{Y} = C_d + A$; and the condition 1^o of fixed price equilibrium definition is satisfied.

(c) In the case $Y_s > \hat{Y}$, we shall prove conditions 2^o and 3^o on the goods market to be satisfied. Equality $\hat{Y} = \text{Min}\,\{Y_s, \bar{Y}\}$ gives then : $\bar{Y} = \hat{Y}$, and so $C_d = \hat{Y} - A$. We have also : $\bar{C} = Y_s - A > C_d$ and we obtain with assumption (CD')

$$\hat{C} = \text{Min}\,\{C_d, \bar{C}\} = C_d = \hat{Y} - A$$

$$\hat{Y} = C_d + A = \text{Min}\,\{Y_s, C_d + A\}$$

Conditions 2^o and 3^o are verified on the goods market.

(d) They are also verified in the case $Y_s = \hat{Y}$. In this case, we have :

$$\bar{C} = Y_s - A = \hat{Y} - A$$

$$C_d = \bar{Y} - A \geqslant \hat{Y} - A = \bar{C}$$

$$\hat{C} = \text{Min}\,\{C_d, \bar{C}\} = \bar{C} = \hat{Y} - A$$

$$\hat{Y} = Y_s = \text{Min}\,\{Y_s, C_d + A\}.$$

(e) The study of conditions 2^{o} and 3^{o} on the labour market is quite similar, distinguishing between the two cases $N_d > \hat{N}$ and $N_d = \hat{N}$. The proof is complete.

Remark. The reverse of the theorem is not true : the expected equilibrium is unique and that is not necessarily the case for the fixed price equilibrium. But, it is shown in [9] that the stable fix-price equilibria are locally optimal relative to the set of the constrained plans of the agents which are compatible together. Consequently, they are also locally expected equilibria (for exact expectations), i.e. they are local maxima of N in the set Δ defined by constraints C1, C2 and C3.

Conclusion

The approach of fixed price equilibrium expected by firms gives a more natural formulation of fix-price equilibrium : it results from the firms' expectations and not from some tatonnement on quantities. It also makes it possible to select a particular equilibrium in the case where uniqueness does not obtain.

This approach restores the Keynesian asymmetry between firms and households : firms play an active role deciding the level of equilibrium activity.

On the other hand, the assumption of fixed prices seems to be much more realistic applied to expectations than for the tatonnement on quantities.

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