On the Size of a Controlling Shareholding

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1. Introduction

A large number of empirical studies have examined the question of the divorce between ownership and control as to either its extent (following Berle and Means (1932)) or its implications for behaviour (following Marris (1964)). These studies have used different samples, different variables and have employed very different criteria to decide on the location of control within each corporation (see Table 1). In addition there appears to be considerable confusion over nomenclature, in particular the meaning of the phrase "owner-controlled". It is therefore not surprising that they come to different conclusions on both questions.

As Table 1 shows the main criterion used in determining the location of control is the size of the biggest block of voting shares and who owns that block. Implicitly, what is being envisaged is a voting model in which there may or may not be a group which by virtue of its control of a significant block of votes and the dispersion of the remainder, is able to secure a majority regularly at company meetings. It is with such a group that control is deemed to lie. One purpose of this paper is to replace this implicit voting model with an explicit one and thereby derive better criteria for determining the location of corporate control.

In most of the studies in Table 1 great importance attaches to the precise proportion of shares used as the criterion of control type. Allowing the possibility of minority control is a recognition of the reality of dispersion of ownership and the concomitant low rates of participation by shareholders in major decision making. But that these can vary between firms is not usually explicitly recognised in the choice of criterion to be applied in any particular case. For firms whose

TABLE 1 - Main criteria used to categorise control type

Unless otherwise specified figures represent percentages of voting shares required by controlling group

	Total no. of categories	Category 1 e.g. "managerially controlled"	Category 2 e.g. "owner controlled"	Other categories
Berle & Means (1932)	5	< 20%	Approaching 100%	1. Legal device. 2. Majority control (50%) 3. Minority control (> 20% usually) a). remaining stock dispersed b). joint minority control
Bothwell (1980)	See Palmer			
Boudreaux (1973)	2	< 5%	> 20% or > 10% + board representation	
Burch (1972)	3	No sizeable ownership by directors	"Probably family" (>~ 4-5% held by family or cohesive group of families)	Marked family representation on board
Grabowski & Mueller (1972)	2	Categorisation according to % of directors holding executive posts – as in Williamson		
Hindley (1970)	3	Largest 20 holders own < 20%	Closely held "Largest 20 holders own > 40%"	Intermediate
Kamerschen (1968)	See Larner			
Kania & Mckean (1976)	2	Residual	> 10% (500 largest corporations) > 15% (second 500 corporations) > 20% (next 800 corporations)	
Larner (1970)	5	Residual	Privately owned block > 80%	Majority ownership as Berle & Means. Minority control (50% > biggest block > 20% or 10% depending on dispersion)
McEachern & Romeo (1978)	2	< 4%	> 4%	
Monsen, Chiu & Cooley (1968)	2	No block > 5%	Board member > 10%, otherwise > 20%	
Nyman & Silberston (1978)	3-4	< 5%	> 5% or chairman member of founder family	Industrial Capital Ownership Finance Capital Ownership
Palmer (1973)	3	< 10%	Strong owner (> 30%)	Weak owner < 30% but > 10%
Pederson & Tabb (1976)	5	Residual	> 5%	
Qualls (1972)	See Palmer			
Radice (1971)	2	< 5%	> 15%	
Stano (1976)	3	See Palmer	See Palmer	
Sorenson (1974)	2	< 5%	> 20%	
Thonet & Poensgen (1979)	4	Residual	> 25% by one family	1) > 25% by other corporation government or institution 2) unknown
Williamson (1964)		Categorisation according to % of directors holding executive posts		

shareholding is highly dispersed it is possible that only a very small proportion of voting shares is sufficient to give effective control. Similarly, if a firm's shareholding has become more dispersed over time through new issues, this proportion is likely to have declined. Moreover criteria based on a fixed and predetermined figure are inevitably arbitrary and it is not clear how such a rule of thumb has been arrived at except as a figure which appears reasonable when considered on a percentage scale. For example Larner classifies a firm as owner controlled "if 10 per cent or more of its voting stock is held by an individual, family, corporation or group of business associates" on the grounds that "In view of the greater size of the 200 largest nonfinancial corporations in 1963 and the wider dispersion of their stock, this lower limit (20% used by Berle and Means) seems too high". From Table 1 it appears that the most popular criterion for internal control is ownership by the board of 5% of the voting shares with figures usually ranging between 10% and 20% for external control. Only to a very limited extent[1/] does the dispersion of the remaining shareholdings influence the categorisation of control types. One possible reason for the failure to pay more than lip service to the distribution of non-controlling shareholdings is the lack of a theoretical model giving any indication of how this can be dealt with. The model presented in this paper deals explicitly with this question.

It is therefore proposed that the criterion used to determine control type should reflect the particular circumstances of firms in terms of the number of shareholdings and their distribution. This is not to argue that this should be the only criterion. On the contrary, other factors may be equally important. Nyman and Silberston (1978) mention the following in addition to those relating to dispersion of shareholdings:

whether the board of directors includes the founder of the company, a member of his family or descendents; other directorships in relation to major institutional shareholders; and the identities of the chairman and managing director, their career histories and the manner in which they came to be appointed.

In Section 2 we set up a probabilistic voting model on certain stylised assumptions about shareholders' behaviour in relation to company decision making and derive a measure of the size of a controlling holding. In Section 3 we show that this measure can be quite small when applied to observed dispersions of shareholdings and conclude that some firms might reasonably be regarded as controlled by a shareholder with much less than 5% of the voting stock.

At this point it is worth mentioning a problem of nomenclature. In the original study by Berle and Means owner control was synonymous with "control through almost complete ownership" and was contrasted with majority and minority control as well as managerial control. Most of the more recent authors have not kept to this convention and regard owner control as control by a group with a substantial ownership interest. Whilst there may be grounds for defending this change in nomenclature it does have the effect of distracting attention away from Berle and Means' concern for those shareholders not in a position to control, however misplaced this concern may or may not have been.[2/] Given the likelihood of confusion arising from the use of the term "owner-control" we shall avoid it as far as possible.

2. A Probabilistic voting model [3/]

In giving a precise definition of a controlling shareholding there is obviously no difficulty in the case of majority control since there can be no threat either from existing outside shareholders or from takeovers. With minority control, however, there is always a possibility that a new coalition of outside shareholders could emerge and transfer control to itself. This could happen either through disaffected shareholders forming themselves into a pressure group or, more likely, through takeover by another firm.

A corollary, therefore, of minority control is the existence of some degree of uncertainty. The definition of a controlling shareholding must reflect this and therefore be framed in probabilistic terms. If it is assumed that shareholders exercise their right to participate in decision making (e.g. by attendance at the company meeting or by appointing proxies) according to some probability then the largest shareholding (or group of shareholdings) is said to be a controlling holding if it commands enough votes to be almost certain of majority support. By "almost certain" is meant a preassigned probability such as 95%, 99% or 99.9%. This is merely a formalisation of the notion of minority control since this probability can be 100% only in the case of majority control.

In setting up the probabilistic voting model it is assumed that: (1) the key parameter is the probability of participating (i.e. not abstaining); (2) that shareholders who exercise their votes are equally likely to vote either way; (3) shareholders vote independently. The uncertainty facing the controlling group is then the residual uncertainty arising from its ignorance of the opinions and intentions of other shareholders combined with the formal, legal reality that crucial decisions (such as the appointment of directors) are made by majority vote.[4/]

Suppose shareholder (or identifiable group) i holds S_i shares (and therefore commands S_i votes) and the largest block of shares is S_o. If S_o is greater than the criterion derived below it is defined as a controlling group. This criterion is the number of shares required by this group in order to secure majority support with probability α. There are $N + 1$ shareholdings and the total number of shares is $T + S_o = \sum_{i=1}^{N} S_i + S_o$. The probability that a shareholder exercises his right to vote is Π and $\Pi/2$ is the probability of a vote for (or against). The probability of abstention is $1 - \Pi$.

Let X_i be the number of votes for (supporting the largest group) cast by i (votes against are negative). Then on these assumptions X_i is a random variable with probability distribution given in Table 2.

Table 2

X_i	Probability
S_i	$\Pi/2$
0	$1 - \Pi$
$-S_i$	$\Pi/2$

The majority is then $M = \sum_{i=1}^{N} X_i + S_o = Y + S_o$. Since the number of holdings, $N + 1$, is large and shareholders are assumed to vote independently with constant probability, the distribution of Y can be approximated with negligible error by a normal distribution. Its mean is zero since the mean of X_i is zero for all i. It variance is $\sigma_Y^2 = \sum_{i=1}^{N} \sigma_i^2$ where σ_i^2 is the variance of X_i. From Table 2, $\sigma_i^2 = \Pi S_i^2$ and therefore $\sigma_Y^2 = \Pi \sum_{i=1}^{N} S_i^2$.

A controlling shareholding is defined as S* such that

$$\alpha = Pr[M > 0] = Pr[S^* + Y > 0] = Pr[Y > -S^*].$$

Let Z be the standard normal deviate and Z_α such that $Pr[Z > -Z_\alpha] = \alpha$.

Then $Z = Y/\sigma_Y$ and hence $S^* = Z_\alpha \sigma_Y = Z_\alpha \sqrt{\Pi \sum_{i=1}^{N} S_i^2}$.

The proportion of shares needed for control is

$$P^* = \frac{S^*}{T + S^*} = Z_\alpha \sqrt{\Pi \sum_{i=1}^{N} \left(\frac{Si}{T+S^*}\right)^2}$$

$$= Z_\alpha \sqrt{\Pi \sum_{i=1}^{N} P_i^2}$$

where $P_i = S_i/(T + S^*)$ is the proportional holding of shareholder i. This can be written in terms of the Herfindahl index of concentration,

$$H = P^{*2} + \sum_{i=1}^{N} P_i^2, \text{ as}$$

(1) $$P^* = Z_\alpha \sqrt{\Pi (H - P^{*2})}$$

and therefore

$$(2) \qquad P^* = Z_\alpha \sqrt{\frac{\Pi H}{1 + Z_\alpha^2 \Pi}} \quad .$$

It is useful to derive an alternative form of this condition in terms of the Herfindahl index for shareholders other than the largest.

Defining $H_T = \sum_{i=1}^{N} (\frac{S_i}{T})^2$

we can write

$$H = P^{*2} + H_T(1 - P^*)^2$$

and therefore substituting into (1),

$$P^* = Z_\alpha (1 - P^*) \sqrt{\Pi H_T}$$

which gives

$$(3) \qquad P^* = \frac{Z_\alpha \sqrt{\Pi H_T}}{1 + Z_\alpha \sqrt{\Pi H_T}}$$

Expressions (2) and (3) are alternative forms of the condition for the proportion of shares required for control as a function of three parameters: (1) the probability of majority support, α, specified in the definition of minority control; (2) the probability of shareholder participation, Π; (3) the Herfindahl index of concentration.[5] This last parameter reflects both the number and dispersion of shareholdings.

Table 3 gives values of 100P* from condition (3) for various values of α, Π and H_T and the figures are generally very small except for large values of H_T. The question of what constitutes a realistic value for the Herfindahl is deal with in the next section.

TABLE 3

100P* : Percentage shareholding for minority control

	H_T			
Π	0.0001	0.001	0.01	0.1
α = 0.95				
1.0	1.62	4.94	14.13	34.22
0.5	1.15	3.55	10.42	26.89
0.1	0.52	1.62	4.94	14.13
0.01	0.16	0.52	1.62	4.94
α = 0.99				
1.0	2.27	6.85	18.87	42.38
0.5	1.62	4.94	14.12	34.22
0.1	0.73	2.27	6.85	18.87
0.01	0.23	0.73	2.27	6.85
α = 0.999				
1.0	2.98	8.86	23.52	49.30
0.5	2.13	6.43	17.86	40.74
0.1	0.96	2.98	8.86	23.52
0.01	0.31	0.96	2.98	8.86

The model gives a measure of the proportion of shares needed for control. This is evaluated on assumptions which are unfavourable to the existence of a controlling group. The measure is therefore biased but the assumptions are such that this bias is positive. The determination of control type can be made in a specific instance by a comparison of this with the largest actual proportion, together with other relevant criteria referred to above. Thus the assumption that votes are equally likely to be cast either way is not meant to be a faithful reflection of real behaviour in company meetings. Typically votes against resolutions proposed by the controlling group are a much smaller proportion than votes for, itself usually a very small proportion of the total number of shares (see Midgley (1974)), and if this were built into the model the typical firm would almost always appear to be controlled by a consensus of shareholders. A justification for this assumption is that only controversial issues (those with a significant probability of a vote against the board) are relevant to the issue of control within this model. The probability of any shareholder voting against the controlling group could be made to depend on managerial behaviour but this would unnecessarily complicate the model. Making no allowance for this will mean that the measure will be further biased upwards. Similarly the probability of participation is not independent of holding. Assuming it to be constant gives more weight to smaller shareholders and therefore increases the necessary size of a controlling block in the model. The assumption of independence of voting behaviour is also likely to bias the measure upwards to the extent that information about collusive arrangements is unavailable. Institutions with large holdings may take an active part in decision making through private, informal contact with management. Such holdings should properly be considered part of the controlling group and not doing so makes the necessary size of a controlling shareholding appear larger.

3. Estimates of Orders of Magnitude for UK Companies

The measure (2) or (3) above gives the size of a controlling shareholding in terms of the Herfindahl index of concentration of shareholdings. In order to give the analysis an empirical dimension it is necessary to calculate Herfindahl indices for actual companies. Two sets of estimates are presented. The first set are derived from analyses of shareholdings which some companies provide in their annual company reports. Despite the limitations of this as a data source it was felt that it might provide enough information to obtain orders of magnitude of the indices and therefore an impression of the likely values of P^*. The second set of estimates are based on concentration measures for large shareholdings by Collett and Yarrow (1976).

Crude estimates based on analyses of shareholdings in company reports for a sample of companies are presented in Table 4. Most of these companies give analyses of shareholdings in two ways: by size of holding and by type. We distinguish only two types, individuals and institutions. Estimates are obtained by fitting an assumed size distribution to this data (a mixture of two lognormals). The use of a mixture distribution was motivated by the fact that a single lognormal distribution appeared to fit the entire distribution badly although seeming to provide a reasonable fit to either tail separately in all cases considered. Moreover the upper tail is dominated by institutional shareholders and the lower tail by individuals. It therefore seems reasonable to divide shareholders into two populations and fit a separate lognormal distribution to each. The Herfindahl index is correspondingly decomposed and separate values for the two populations are combined into a single estimate.

Generally, suppose shareholders are divided into g groups, labelled $G_1, G_2, \ldots G_g$ and the total shareholding is $T = \sum_{j=1}^{g} T_j$. The Herfindahl index can be written as

$$H_T = \frac{1}{T^2} \sum_{i=1}^{N} S_i^2 = \sum_{j=1}^{g} \left(\frac{T_j}{T}\right)^2 \sum_{i \varepsilon G_j} \left(\frac{S_i}{T_j}\right)^2 ,$$

hence

(4) $$H_T = \sum_{j=1}^{g} \left(\frac{T_j}{T}\right)^2 H_j .$$

H_j is the Herfindahl index for group j and T_j/T is the proportion of shares held by the group.

Suppose further that within group j holdings are distributed with mean $M_j = T_j/N_j$ and variance V_j. Write

(5) $$H_j = \sum_{i \varepsilon G_j} \left(\frac{S_i}{T_j}\right)^2 = \frac{1}{N_j} \left(\frac{N_j}{T_j}\right)^2 \sum_{i \varepsilon G_j} \frac{S_i^2}{N_j} .$$

The summation term in (5) is the second moment about the origin of the distribution of S_i and therefore

$$\frac{1}{N_j} \sum_{i \varepsilon G_j} S_i^2 = V_j + M_j^2 .$$

Hence,

(6) $$H_j = \frac{1}{N_j} \left(\frac{1}{M_j}\right)^2 (V_j + M_j^2) = \frac{1}{N_j} \left(\frac{V_j}{M_i^2} + 1\right).$$

Assuming that S_i has a lognormal distribution for $i \epsilon G_j$ with parameters μ_j and σ_j^2 implies that

$$M_j = e^{\mu_j + \frac{1}{2}\sigma_j^2}$$

and

$$V_j = e^{2\mu_j + \sigma_j^2}(e^{\sigma_j^2} - 1) .$$

Substituting these into (6) gives

$$(7) \qquad H_j = \frac{e^{\sigma_j^2}}{N_j} .$$

In the calculations reported in Table 4 it has been assumed that $g = 2$ and that the size distribution has distribution function (the proportion of holdings smaller than S) $F(S) = \lambda F_1(S) + (1-\lambda)F_2(S)$ where $F_1(\cdot)$ and $F_2(\cdot)$ are separate lognormal distribution functions, $F_i(S) = \Lambda(S|\mu_i, \sigma_i^2)$. The weight λ has been taken as the proportion of shareholders who are individuals, usually between 0.90 and 0.95. The parameter σ_1 was estimated by fitting a lognormal distribution graphically using logarithmic probability graph paper (Aitchison and Brown (1957) p.31) to the lower tail of $F(S)/\lambda = F_1(S) + F_2(S)(1-\lambda)/\lambda$. If $F_2(S) \doteq 0$ for small values of S this will approximate $F_1(S)$. The distribution for institutional shareholders was obtained by fitting a lognormal to $(F(S)-\lambda)/(1-\lambda) = \lambda(F_1(S) - 1)/(1-\lambda) + F_2(S)$. If $F_1(S) \doteq 1$ for large values of S, this gives an approximation to $F_2(S)$ and hence an estimate of σ_2 on the lognormality assumption. With such large values for λ

it is expected that this procedure will give reasonably accurate estimates of σ_1 but relatively imprecise estimates of σ_2, particularly in view of the fact that some large shareholders are individuals.

The values of H are all of the order of 0.0001. If they are taken as values of H_T (the difference is small except where there is a very large shareholding such as Unilever and Muirhead - in the latter case the value reported is of H_T) then the corresponding values for P* are remarkably small, even assuming high values for Π and α. The largest value for H_T, for example, in Table 3 is 2.41×10^{-3}. Assuming $\Pi = 1$ and $100\alpha = 99.9\%$ this gives (expression (3)) 100P* = 13.2%. Assuming a more realistic (although still high) value for Π, $\Pi = 0.1$ and $100\alpha =$ 99.9%, gives 100P* = 4.6%. It is clear that, if these figures can be taken as realistic, the probabilistic voting model predicts that control can be exercised with what appears to be a very small shareholding indeed.

This procedure for estimating the Herfindahl indices is expected to give reasonably precise estimates of σ_1 but much less precise estimates of σ_2 which can at best be regarded as rough orders of magnitude. A more serious source of error in estimating H, however, is the assumption about the number in the two separate populations in the two tails of the distribution. The indices H_1 and H_2 reflect both the concentration and number of shareholdings in each group and, even though the above method might be capable of giving orders of magnitude for σ_1 and σ_2, the resulting estimates of H_1 and H_2 are highly sensitive to assumptions about N_1 and N_2. The assumption made above was that the two groups were individuals and institutions and that all individuals holdings belong to one population and all institutional holdings

TABLE 4 - Herfindahl indices (1)

Company	T_1/T	σ_1^2	H_1	σ_2^2	H_2	$(T_1/T)^2H_1$	$(T_2/T)^2H_2$	H
Tate & Lyle	0.3482	0.053	2.8E-5	1.000	7.6E-4	3.64E-6	3.24E-4	3.28E-4
Reed International	0.3235	0.168	1.7E-5	1.051	1.1E-3	1.78E-6	4.82E-4	4.84E-4
GKN	0.4370	0.168	1.6E-5	0.397	5.5E-4	3.06E-6	1.74E-4	1.77E-4
Unilever(a)	0.2938	0.204	1.7E-5			3.88E-5		
Boots	0.4760	0.185	1.1E-5	1.113	3.9E-4	2.49E-6	1.07E-4	1.10E-4
Unigate	0.3436	0.181	2.5E-5	1.440	1.3E-3	2.95E-6	5.60E-4	5.63E-4
J.Sainsbury	0.8028	0.308	6.1E-5	1.596	7.2E-3	3.93E-5	2.80E-4	3.19E-4
House of Fraser	0.5070	0.181	3.2E-5	0.781	1.4E-3	8.22E-6	3.40E-4	3.48E-4
Muirhead(b)	0.4850	0.271	4.5E-4	o.686	7.9E-3	3.13E-4	2.10E-3	2.41E-3
Barclays Bank	0.3939	0.324	1.1E-5	1.171	4.8E-4	1.55E-6	1.76E-4	1.78E-4
National Westminster Bank	0.3167	0.200	1.3E-5	2.045	5.5E-4	1.00E-6	2.57E-4	2.58E-4

(a) Unilever Trust control 18% of shares

(b) Figures given obtained after removing substantial shareholdings of 5.4% and 8.4%

to the other. In all cases considered the number of institutional holdings was quite large and hence the estimates of H_2 have been generally low. If some smaller institutional holdings have been wrongly assigned to the second group this will have seriously biased the estimate of H_2. One alternative to the above procedure is to treat λ as an additional parameter to be estimated along with μ_1, μ_2, σ_1 and σ_2 but the data contained in company reports is too sketchy to be the basis for this. A second alternative, adopted below, is to look at the largest shareholdings and assume that the Herfindahl index for this group dominates that for the whole distribution.

The second set of estimates are presented in Table 5 and are based on measures of shareholder concentration given in Collett and Yarrow (1976). Their sample consisted of 85 firms in the engineering, electrical engineering/electronics, food and textile industries and they fitted a Pareto distribution to the largest 50 shareholdings of each and a lognormal to the full sample of at least 100 shareholdings of each firm. A chi-square goodness of fit test rejected the Pareto distribution in only 8 cases but the lognormal, fitted to the full sample, was rejected in 71 out of 93 cases. The estimates presented provide a basis for calculating approximate orders of magnitude for Herfindahls in the upper tail of the distribution and, assuming this dominates the Herfindahl index for the whole distribution, as in Table 4, obtaining likely values for H.

Unfortunately the results for the Pareto distribution are unsuitable to this purpose, despite their better fit than the lognormal, because the Herfindahl index depends on the second moment of the distribution which only exists for a Pareto population for values of the parameter $\alpha > 2$. All the estimates presented by Collett and Yarrow have $\alpha < 2$ and therefore the Herfindahl for the top fifty shareholdings is undefined. This is a serious limitation of the Pareto distribution as a theoretical model in this context. For the lognormal distribution, however, we can obtain estimates for firms for which this distribution is not rejected.

Dividing shareholders into the largest 100 and the remainder the Herfindahl index is written as

$$H = \left(\frac{T_3}{T}\right)^2 H_3 + \left(\frac{T_4}{T}\right)^2 H_4$$

where T_4/T is the proportion of shares held by the largest 100 shareholders, H_4 is the corresponding Herfindahl index for this group, H_3 is the Herfindahl index for the remaining shareholders and $T_3/T = 1 - T_4/T$. Assuming, on the basis of the estimates given in Table 4 that H_3 is much less than H_4, H can be crudely approximated by $(T_4/T)^2 H_4$. The lognormality assumption means that $H_4 = e^{\sigma_4^2}/100$ where σ_4^2 is the parameter of the lognormal distribution, tabulated by Collett and Yarrow. These latter estimates are based on the full sample of shareholdings for each firm which in some cases is greater than 100, but the actual number is not given. The value of T_4/T (C(100) in terms of Collett and Yarrow's notation) is not given but can be found from the value of σ_4^2 and the figure for C(50), the proportion of shares held by the largest 50 shareholders, which is reported for each firm.

If S is the size of shareholding and is distributed as lognormal with parameters μ and σ^2, the proportion of shareholders with holdings greater than x is

$$Pr[S > x] = 1 - \Lambda(x|\mu,\sigma^2).$$

The proportion of shares in holdings no bigger than x is the first moment distribution and is also lognormally distributed (Aitchison and Brown (1957) p.15). The first moment distribution function is

$$\Lambda_1(x|\mu,\sigma^2) = \Lambda(x|\mu + \sigma^2, \sigma^2).$$

Thus, if C(50) is the proportion of shares held by the top 50 shareholders, $1 - C(50) = \Lambda_1(x|\mu,\sigma^2) = \Lambda(x|\mu + \sigma^2, \sigma^2) = N(Z_1|0,1)$. Letting P(50) be the proportion of shareholders represented by the top 50, $P(50) = 1 - \Lambda(x|\mu,\sigma^2) = 1 - N(Z_2|0,1)$ where $Z_2 = Z_1 + \sigma$. Hence $P(100) = 2P(50)$ and $1-P(100) = \Lambda(y|\mu,\sigma^2) = N(Z_3|0,1)$ and therefore $C(100) = 1-\Lambda_1(y|\mu,\sigma^2) = 1-\Lambda(y|\mu + \sigma^2, \sigma^2) = 1-N(Z_4|0,1)$ where $Z_4 = Z_3 - \sigma$. Thus $T_4/T(= C(100))$ can be obtained by a simple calculation from C(50) and σ for a lognormal distribution.

The estimates given in Table 5 are generally much larger than those in Table 4 although they are not directly comparable since, apart from one, they refer to different companies. The one company for which there is an estimate in both Table 4 and Table 5 is GKN and the values of H are respectively 0.0002 and 0.003. This difference is far greater than that due to differences in the estimated variance which suggests that too many shareholdings were assigned to the institutional group in the first set of calculations and therefore the Herfindahl indices in Table 4 should be regarded as underestimates.

TABLE 5 - Herfindahl indices (2)

	$C(50)^{(a)}$	T_4/T [$\doteq C(100)$]	$\sigma_4^{2(a)}$	$(T_4/T)^2 H_4$ [$\doteq H$]
Northern Dairies	0.4971	0.6879	1.15	0.0178
Melbray	0.4143	0.5199	4.33	0.2063
Baker Perkins	0.5915	0.7140	4.43	0.4279
Thomas Ward	0.4086	0.5596	1.76	0.0182
GKN	0.2529	0.3897	0.68	0.0030
English Calico	0.3887	0.5596	1.11	0.0095
George Cohen	0.4948	0.7190	0.68	0.0102
John Brown	0.4754	0.6664	1.07	0.0129
Chubb	0.3550	0.5398	0.66	0.0056
Associated Biscuits(B)	0.5403	0.7673	0.72	0.0121
Mann Egerton	0.5294	0.6443	5.09	- (b)
Simon Engineering	0.4572	0.5832	3.37	0.0989
Firth Cleveland	0.8990	0.9463	9.75	- (b)
Tilling	0.2733	0.3632	3.81	0.0596
Burton(A)	0.4357	0.6064	1.35	0.0142
Bovril	0.3437	0.4641	2.66	0.0308
Westinghouse	0.5557	0.7257	1.73	0.0297
Pegler Hattersley	0.4952	0.6026	5.56	- (b)
Fitch Lovell	0.3663	0.5438	0.85	0.0069
Cammel Laird	0.3779	0.5398	1.21	0.0098
Selincourt	0.3737	0.4840	4.91	- (b)
Morgan Crucible	0.3814	0.5596	0.86	0.0074

(a) Taken from Tables 1 and 2 of Collett and Yarrow (1976)

(b) Estimated $H_4 > 1$, i.e. $e^{\sigma_4^2}/100 > 1$. The assumption that $N_4 = 100$ is presumably invalid in these cases.

Many of the estimates in Table 5 are of the order of 0.01 and for this value of H_T in Table 3, P* depends strongly in Π. If we take a figure of $\Pi = 0.1$ which is probably high for many firms, this gives a controlling shareholding at $\alpha = 0.99$ of 6.85% and at $\alpha = 0.999$ of 8.86%. A value of $\Pi = 0.01$ gives a controlling shareholding of well under 5% for all values of α in Table 3. These figures are consistent with cases reported by Burch (1972), Nyman and Silberston (1978) and Beed (1966) suggesting that many firms may be controlled through substantial ownership interests even though the largest shareholder owns apparently an extremely small proportion of the voting shares. The analysis further suggests that present disclosure rules which require disclosure of substantial holdings of 5% or more, may be too loose to provide real information about possible control in many cases.

Separate Herfindahl indices are reported in Table 4 for individuals and institutions. It is apparent that the index is dominated by institutional holdings and that the distribution among individuals has little influence. This points to institutional shareholders as the key group within the firm. If the assumption of random voting by institutions is relaxed then it is clear that given the typical concentration of institutional shareholdings in the firms listed in Table 4 (and in Table 5 if the largest shareholders are institutions) and if these firms are regarded as a typical sample, then minority control by institutions must be regarded as the norm among British companies.

4. Conclusions

This paper has derived a measure of the size of a controlling shareholding as a proportion of the total shares, based on a probabilistic voting model. A definition of control is given, not as an absolute concept, but in terms of a preassigned probability of the controlling group winning majority support on clearly stated assumptions about voting behaviour. These assumptions are such as to make the measure biased upwards.

The measure turns out to be proportional to the square root of the Herfindahl index of concentration of shareholdings in the firm. An important conclusion is that the measure varies between firms according to the number and dispersion of shareholdings. Specifically, it depends on three parameters, (1) the Herfindahl index, (2) the probability of winning majority support specified in the definition of control, (3) the level of participation by shareholders in the firm's decision making.

Two sets of estimates of Herfindahl indices for individual companies have been given based on different assumptions about the size distribution of shareholdings. These estimates are such as to suggest that, even assuming an unrealistically high level of participation by shareholders and requiring a high probability of winning a vote, the proportion of shares needed for control is often much lower than that frequently adopted in empirical studies.

The present paper does not explore the question of the categorisation of firms into "owner-controlled", "management-controlled", etc. The results allow the possibility or likelihood of control being exercised by a small minority of shareholders by virtue of their large (although small as a proportion of the total) holdings but not by other shareholders with smaller holdings. It is therefore inappropriate to attempt to dichotomise firms into "management-controlled" and "owner-controlled" but instead there should be a more neutral set of nomenclature for different control categories.

Footnotes:

1. Berle and Means (1932) do go to some trouble to deal with the question of dispersion and similar considerations underly Kania and McKean's (1976) method of categorisation. It was also discussed by Florence (1961) who hinted informally at some of the results of the present paper in a speculative section (Florence (1961), pp.193-195) drawing on earlier results of Penrose (1946).

2. For a criticism of this concern for shareholders see Alchian (1969).

3. An alternative approach to the question of the distribution of power among shareholders could be made through game theory by computing the Shapley value for each holding (see e.g. Owen (1975)). This approach seems difficult to apply in the present context without resorting to probabilistic assumptions and the method adopted below is preferable since it leads to a measure with a direct interpretation and which incorporates explicitly an index of dispersion.

4. The model assumes that the controlling shareholder or group is completely ignorant of the intentions of all other shareholders. Where there are two groups competing for control this is obviously not the case but the analysis can be applied to the difference in shareholdings between the two groups. That is the question is whether the largest group has enough votes more than the competing group to secure majority support with the preassigned probability. In this case we can also extend the notion of participation by shareholders in decision making to include selling their shares to either group as well as voting or appointing proxies. The assumption that shareholders vote randomly is not meant to be taken literally but is a formalisation of the ignorance of the controlling group about the opinions of the remaining shareholders.

5. It is easily seen that in the case where shareholdings other than the largest are equally distributed this model specialises to that considered by Penrose (1946) whose main result was that the number of votes needed to achieve a given degree of control is proportional to the square root of the electorate. In this case $H_T = 1/N$ and therefore $S^* = Z_\alpha q\sqrt{\Pi N}$ where q is the n mber of shares held by each shareholder other than the largest, and therefore S^* is proportional to $\sqrt{N}$.

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