POLICY DECENTRALIZATION AND EXCHANGE RATE MANAGEMENT IN INTERDEPENDENT ECONOMIES

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1. Introduction

The demise of Bretton Woods and of the short-lived Smithsonian agreement has raised questions about exchange rate management by monetary authorities acting in isolation from one another. For instance, will individual monetary authorities have an incentive to stabilise the exchange rate? To what extent will monetary actions abroad disrupt domestic monetary policy? What are the gains from co-ordinating monetary policy?

The problems that arise when different agents pursue independent policies in interdependent economies have been explored by a number of authors. Aoki (1976), Cooper (1969), Hamada (1976), Allen and Kenen (1980), McFadden (1967), Patrick (1973), Kydland (1976) and Pindyck (1976), among others, have made significant contributions. Different authors have focused on different aspects of decentralized policy formation. One purpose of this paper is to provide a general discussion of decentralization. In part 2 we provide a theoretical framework for analysing policy formation among independent authorities operating in an interdependent environment. We distinguish three dimensions of the problem and discuss, by way of example, the Mundell (1962) assignment problem in terms of our typology. We show that instability in Mundell's context does not arise because different authorities are assigned different and inappropriate targets, but because they fail to formulate strategies in a co-operative way.

A second purpose of this paper is to analyse the optimal design of monetary policy in interdependent economies. Previous analytic discussion of policy decentralization has been based on deterministic models. Furthermore, with the exception of Hamada (1976), who considers the problem in a classical, full-employment context, these models incorporate the traditional neo-Keynesian assumption of fixed prices. Studies incorporating stochastic elements have also used a neo-Keynesian framework and rely solely on simulation analysis (Pindyck (1976) and Kydland (1976)). None of the studies incorporates recent contributions to the theory of aggregate supply and expectations formation associated with the

"New Classical Macroeconomics".

We consider two monetary authorities pursuing domestic targets in two economies connected both by trade in real goods and in national monies. The model is presented in section 3. Each economy is characterised by a supply function of the Lucas type in which deviations of output from its natural level occur only because of deviations of the domestic price level from the value that was anticipated in the previous period. The natural level is itself stochastic. Agents in each economy hold domestic money for transactions purposes but may speculate on exchange rate movements by holding domestic or foreign money. We assume that money demands are also stochastic. The two economies are linked by a stochastic purchasing power parity relationship between the two price levels and the exchange rate.

Throughout, we assume that the only contemporaneous variable observed by the two monetary authorities and the private sectors is the exchange rate. Incomes and price levels are observed only with a one-period lag. Each monetary authority's problem, then, is to infer from the observed current exchange rate the type of shocks affecting the economy and to set the current money supply to offset these shocks, taking into account the response of the foreign monetary authority to its actions.

In section 4 we assume that each country's monetary authority pursue the objective of stabilizing output around the average or "*ex ante*", natural level of output. In section 5 we modify the objective to one of stabilizing income around the actual but unobserved natural rate. This second objective is equivalent to minimising price forecast errors and is more likely to lead to a policy of exchange rate stabilization. In section 6, we introduce exchange rate stabilization as an additional, independent goal.

Throughout, we derive the policy rules which obtain when the monetary authorities pursue distinct targets independently. In sections 4 and 5 the optimal policy rules are not affected if, instead, authorities pursue a common objective cooperatively. This result does not extend to more general models such as the one considered in section 6.

A main purpose of the model we develop in sections 3 through 6 is to provide insight into the design of optimal policies for dirty floating. Models in which current policy can only respond to past information must implicitly assume that either the exchange rate or the money supply is fixed within the period: exchange rates are either fixed or the float is clean within the period [see Buiter 1979a]. Our model, however, allows for a contemporaneous money supply response to the current exchange rate.[1/] Setting the money supply to fix the exchange rate or ignoring the exchange rate in setting the money supply constitute special cases of our model. Indeed, we find that optimal monetary policy can involve exacerbating exchange rate movements.[2/]

2. Alternative Definitions of Policy Decentralization and Coordination

The question of policy decentralization arises in interdependent systems in which distinct agents, whom we call *authorities*, have the ability to set instruments in the pursuit of possibly independent objectives. For simplicity, consider a system with two authorities. At time t the first authority seeks to maximize an objective function of the form

$$(2.1) \quad W_t^1 \equiv E[\sum_{\tau=t}^{\infty} (\delta^1)^{\tau-t} V_\tau^1(y_\tau, x_\tau^1, x_\tau^2) | \Omega_t^1] \qquad 0 \leq \delta^1 < 1$$

where y_τ^i represents a vector of state variables in period τ and x_τ^i a set of levels of the instrument variables under the control of authority i. Ω_t is the i^{th} authority's set of information about the state of the economy in period t.

A second authority may be maximizing an objective function of the form

$$(2.2) \quad W_t^2 \equiv E[\sum_{\tau=t}^{\infty} (\delta^2)^{\tau-t} V_\tau^2(y_\tau, x_\tau^1, x_\tau^2) | \Omega_t^2] \qquad 0 \leq \delta^2 < 1$$

Let us assume that the state variables y evolve according to

$$(2.3) \quad y_t = g(y_{t-1}; x_t^1, x_t^2; \ldots, \ldots, E(y_i|\Omega_j^1)\ldots,$$

$$\ldots E(y_i|\Omega_j^2)\ldots; u_t) \qquad j \leq t$$

If the authorities adopt time-consistent policies[3/] we can define their optimizing behaviour recursively as in (2.4) and (2.5)

$$(2.4) \quad W_t^1 \equiv \max_{x_t^1 \epsilon X_t^1} E[V_t^1(y_t, x_t^1, x_t^2) + \delta^1 W_{t+1}^1] | \Omega_t^1]$$

and

$$(2.5) \quad W_t^2 \equiv \max_{x_t^2 \epsilon X_t^2} E[V_t^2(y_t, x_t^1, x_t^2) + \delta^2 W_{t+1}^2] | \Omega_t^2]$$ [4/]

Here X_t^1 and X_t^2 represent the sets of feasible values of x_t^1 and x_t^2 respectively. If agents are pursuing memoryless Nash strategies (see Kydland (1976)) then x_t^1 is set taking agent 1's future behaviour and agent 2's current and future behaviour as given and similarly for agent 2. We thus define

$\hat{x}^1_t = \Phi^1_t\ (\Omega^1_t;\ \ldots,\ \Phi^1_\tau,\ldots;\ \ldots,\Phi^2_{\tau-1},\ \ldots)$ and $\hat{x}^2_t = \Phi^2_t\ (\Omega^2_t;\ \ldots,\ \Phi^2_\tau,\ \ldots;\ \ldots,\ \Phi^1_{\tau-1},\ \ldots)\ \tau \geq t+1$ as the values of x^1_t and x^2_t which attain W^1_t and W^2_t respectively. Within this framework we identify three types of decentralization and coordination.

(1) *Target* decentralization occurs when the two authorities have different objective functionals, i.e. when $w^1 \neq w^2$. Full coordination of targets requires that the authorities adopt a common objective functional w. This does not necessarily imply that authorities maximize w with respect to the same information, i.e. Ω^1_t may not equal Ω^2_t. Furthermore, even though authorities have a common set of objectives, they need not play a cooperative game in the formal sense; i.e. no binding pre-play agreements on the choice of Φ^1_t and Φ^2_t may have been established.

(2) Decentralization of *nonstrategic information* occurs when authorities have access to different information sets concerning the state of the economy $(\Omega^1_t \neq \Omega^2_t)$. Full coordination of such information requires that the authorities share this information. Each authority will then form expectations and policy in period t on the basis of $\Omega^1_t \cup \Omega^2_t$. This type of coordination need not imply that authorities adopt common objectives or that they formulate Φ^1_t and Φ^2_t cooperatively.

(3) Finally, decentralization of *strategic information* arises whenever Φ^1_t and Φ^2_t are chosen independently. This type of decentralization could even arise in situations where authorities shared objectives $(w^1 = w^2)$ *and* information about the state of the economy $(\Omega^1_t = \Omega^2_t)$ but the optimal cooperative strategy is non-unique. It is analogous to the decentralization problem faced by an American football team which has snapped the ball without having called a play in the huddle. Coordination of stragegic information means that authorities make binding, "pre-play" agreements on their choice of Φ^1_t and Φ^2_t, which they may do even though their targets and information about the state of the economy may differ.

Mundell's (1962) assignment problem seems to represent this last type of decentralization. Consider Mundell's two equation model of an open economy:

(2.6) $Y = \alpha_1 r + \alpha_2 G$; $\alpha_1 < 0, \alpha_2 > 0$

(2.7) $B = \beta_1 r + \beta_2 G$; $\beta_1 > 0, \beta_2 < 0$

where Y denotes income, r the interest rate, G government spending and B the balance of payments. The objective of policy is to stabilize Y and B; that is, to assure asymptotic convergents of Y and B to target levels which we set without loss of generality at $Y = B = 0$. Policy is restricted to finite instantaneous rates of change of G and r which are linear functions of authorities' information sets.[5/]

Mundell constrains his analysis to policies which assign one instrument, r, to one authority and G to another. The monetary authority responds only to B according to the rule.

(2.8) $\dot{r} = \delta B$

while the fiscal authority responds to Y according to

(2.9) $\dot{G} = \gamma Y$.

Mundell then shows that if the authorities choose $\delta < 0$ and $\gamma < 0$ - the natural "leaning against the wind" rules - then if $|-\alpha_1/\alpha_2|<|-\beta_1/\beta_2|$, i.e. instruments are not assigned according to "comparative advantage", the system represented by (2.6) - (2.9) is unstable.

Prima facie, this problem might be interpreted as one of either decentralized *non-strategic* information (each authority knows only one of the two state variables, Y and r, as well as the values of the structural parameters α_i and β_i, i=1, 2 or at least the relative strengths of the effects of the instruments on the targets) or one of decentralization of *strategic* information (each authority chooses its response function independently). In fact, it can only represent a failure to choose response functions jointly.

To show this, consider the two necessary and sufficient conditions for stability:

(2.10) $\delta\gamma(\beta_1\alpha_2 - \alpha_1\beta_2) > 0$

and

(2.11) $\delta\beta_1 + \gamma\alpha_2 < 0$.

For any values of the structural parameters there exist values of δ and γ (not necessarily the leaning against the wind values), which satisfy (2.10) and (2.11).[6/] Therefore, authorities can attain their objective without sharing information about Y and r. They need only cooperate by jointly choosing appropriate values of δ and γ.

Without such cooperation, sharing information on Y and r will not guarantee stability. Under full nonstrategic information sharing, the two authorities' response functions take the more general form:

(2.12) $\dot{G} = \gamma_1 B + \gamma_2 Y$

(2.13) $\dot{r} = \delta_1 B + \delta_2 Y$.

In this case stability obtains if and only if

(2.14) $\gamma_1\beta_1 + \gamma_2\alpha_1 + \delta_1\beta_2 + \delta_2\alpha_2 < 0$

and

(2.15) $(\gamma_2\delta_1 - \delta_2\gamma_1)(\alpha_1\beta_2 - \alpha_2\beta_1) > 0$.

Without knowing the parameters of the fiscal authority's response function, γ_1 and γ_2, there are no values of δ_1 and δ_2 which the monetary authority could select to ensure stability. An equivalent problem faces the fiscal authorities. Even though both authorities have full information about the state of the economy, they cannot be sure of attaining their objective if one authority does not know what the other is doing. Thus the assignment problem arises not because authorities have been arbitrarily assigned target variables but because they do not determine jointly how they will respond to these variables.

3. A Two-Country Model of Monetary Policy and Exchange Rate Determination

We now turn to a model in which *target* and *strategic information* decentralization both arise. Two national monetary authorities pursue separate objectives but have access to the same information about the state of the world. Each authority sets its money supply in response to the current exchange rate between the two monies, taking the other authority's response as given.

We derive optimal money supply rules in a model of considerable simplicity. Firstly, we consider two economies in which the deviation of actual output from its long-run normal level is proportional to the percentage deviation of the actual price level from the price level anticipated in the previous period. In the home country we have

$$(3.1) \qquad y_t = \Phi(p_t - p_{t|t-1}) + u_t^y \qquad\qquad \Phi \geq 0$$

while abroad

$$(3.2) \qquad y_t^* = \Phi(p_t^* - p_{t|t-1}^*) + u_t^{*y} \qquad\qquad \Phi \geq 0 .$$

Here y_t denotes the deviation of home-country output from the full employment level and p_t the logarithm of the home-country price level; $p_{t|t-1}$ denotes the expectation of p_t based on information available in period t-1 and u_t^y denotes a Gaussian white noise disturbance term with variance $\sigma_{u^y}^2$. Equivalent magnitudes in the foreign country are denoted with an asterisk. The actual or ex-post natural levels of outputs, y_t^n and y_t^{*n}, obtain when $p_t = p_{t|t-1}$ and $p_t^* = p_{t|t-1}^*$ respectively, that is $y_t^n = u_t^y$ and $y_t^{*n} = u_t^{*y}$. *Ex ante*, the expected natural levels are simply zero.

Justification for output supply equations of the form (3.1) and (3.2) is provided by Lucas (1972) and Sargent and Wallace (1975). Output equations of the form (3.1) and (3.2) would arise also if wage contracts are formed one period in advance. Wages in period t, it is assumed, cannot be modified by information that is not available before period t.[7/] Other motivations of

(3.1) and (3.2) involve imperfect observation of the contemporaneous aggregate price level.

Secondly, we assume that citizens in each country must hold domestic money for transactions purposes but may speculate by holding foreign money. Overall money demand is thus given by a set of currency substitution money demand functions:

$$(3.3) \quad m_t - p_t = \alpha y_t - \beta(e_{t+1|t} - e_t) + u_t^m \qquad \alpha, \beta \geq 0.$$

$$(3.4) \quad m_t^* - p_t^* = \alpha^* y_t^* + \beta^*(e_{t+1|t} - e_t) + u_t^{*m} \qquad \alpha^*, \beta^* \geq 0.$$

Here m_t denotes the logarithm of nominal home-country money balances in period t, e_t the logarithm of the spot price of foreign currency and $e_{t+1|t}$ the value of e_{t+1} expected to occur in period t. The terms u_t^m and u_t^{*m} represent Gaussian white noise disturbances with variances $\sigma^2_{u^m}$ and $\sigma^2_{u^{*m}}$, respectively. The parameter α denotes the income elasticity of demand for real money balances and β the expected exchange rate appreciation elasticity. When $e_{t+1|t}$ is high relative to e_t foreign money balances are more attractive.[8]

Finally, we assume that the domestic and foreign price levels are connected by a stochastic purchasing power parity relationship, i.e.,

$$(3.5) \quad p_t = e_t + p_t^* + u_t^e$$

where u_t^e represents a Gaussian white noise disturbance with variance $\sigma^2_{u^e}$. Also, for simplicity let contemporaneous values of u_t^y, u_t^{*y}, u_t^m, u_t^{*m} and u_t^e be uncorrelated.

We assume that e_t is the only endogenous or exoge nous variable observed contemporaneously. All past endogenous and exogenous variables are also part of the common private and public information set. The information set at time t when m_t is chosen, denoted by Ω_t, is therefore given by: [9]

(3.6) $$\Omega_t = \left[e_\tau,\ p_{\tau-1},\ p^*_{\tau-1},\ y_{\tau-1},\ y^*_{\tau-1} \right] \qquad \tau \leq t.$$

or equivalently by

$$\Omega_t = \left[e_\tau,\ u^y_{\tau-1},\ u^{*y}_{\tau-1},\ u^m_{\tau-1},\ u^{*m}_{\tau-1},\ u^e_{\tau-1} \right] \qquad \tau \leq t.$$

In addition both the authorities and the private sector know the true structure of the model, including the first and second moments of the distributions of the random disturbances. Expectations of p_t formed in period t-1 are conditional on Ω_{t-1}; and expectations of e_{t+1} formed in period t are conditional on Ω_t. The foreign country has the same information set as the home country.

In period t the monetary authorities in the home country choose m_t to minimise

(3.7a) $$E(V_t|\Omega_t) = \sum_{\tau=t}^{\infty} \delta^\tau E\left[(y_\tau - \bar{y}_\tau)^2 | \Omega_t \right] \qquad 0<\delta<1$$

The foreign monetary authorities choose m^*_t to minimise

(3.7b) $$E(V^*_t|\Omega_t) = \sum_{\tau=t}^{\infty} \delta^{*\tau} E\left[(y^*_\tau - \bar{y}^*_\tau)^2 | \Omega_t \right] \qquad 0<\delta^*<1$$

$\bar{y}_\tau$ and $\bar{y}^*_\tau$ are target real output at home and abroad, to be specified more precisely below. δ and δ^* are discount factors. Both m_t and m^*_t are chosen under the assumption that m_τ and m^*_τ minimise respectively $E(V_\tau|\Omega_\tau)$ and $E(V^*_\tau|\Omega_\tau)$ for all $\tau>t$. [10/]

By permitting money supplies to respond to the contemporaneous exchange rate but precluding the possibility of exchange-rate contingent money wage contracts, known monetary rules will affect real output. In this model policy makers set money supplies via transfers and taxes. Direct exchange market intervention provides another mode which is consistent with our specification if the other country sterilises the effect of exchange market intervention on its own money supply via transfers and taxes.

Equations (3.1) through (3.5) can be solved by substituting (3.1), (3.2) and (3.5) into (3.3) and (3.4) to obtain:

$$\begin{bmatrix} p_t \\ p_t^* \end{bmatrix} = B \begin{bmatrix} m_t + \alpha\phi p_{t|t-1} + \beta e_{t+1|t} + v_t + \beta u_t^e \\ m_t^* + \alpha^* \phi^* p^*_{t|t-1} - \beta^* e_{t+1|t} + v_t^* - \beta^* u_t^e \end{bmatrix} \Delta^{-1} \qquad (3.8a)$$

where

$$B \equiv \begin{bmatrix} \pi^* + \beta^* & \beta \\ \beta^* & \pi + \beta \end{bmatrix} \qquad (3.8b)$$

$$\pi \equiv 1 + \alpha\phi \qquad (3.8c)$$

$$\pi^* \equiv 1 + \alpha^* \phi^* \qquad (3.8d)$$

$$v_t \equiv -(\alpha u_t^y + u_t^m) \qquad (3.8e)$$

$$v_t^* \equiv -(\alpha^* u_t^{*y} + u_t^{*m}) \qquad (3.8f)$$

$$\Delta \equiv \pi\pi^* + \beta\pi^* + \beta^*\pi \qquad (3.8g)$$

Therefore, since $e_t = p_t - p_t^* - u_t^e$,

$$e_t = \left[\pi^*(m_t + \alpha\phi p_{t|t-1} + \beta e_{t+1|t} + v_t) - \pi(m_t^* + \alpha^*\phi^* p^*_{t|t-1} - \beta^* e_{t+1|t} + v_t^*) - \pi\pi^* u_t^e\right]\Delta^{-1} \qquad (3.9)$$

Equations (3.8) and (3.9) represent reduced form expressions for the price levels and the exchange rate given past expectations of the current price levels and current expectations of the future exchange rate. We now consider the design of monetary policy.

4. Optimal Monetary Policy: The Nash Solution

We first consider the optimal design of monetary policy when each monetary authority follows a memoryless Nash strategy; that is, each monetary authority sets its money supply as a function only of the contemporaneous value of the state variables, taking as given the other monetary authority's money supply rule.

In this section, target output is the *ex ante* natural level of output. We thus set $\bar{y}_\tau = \bar{y}^*_\tau = 0$ for all τ.

Since

$$y_t = \Phi(p_t - p_{t|t-1}) + u^y_t$$

$$y^*_t = \Phi^*(p^*_t - p^*_{t|t-1}) + u^{*y}_t$$

the objectives of minimizing

$$\sum_{\tau=t}^{\infty} \delta^\tau E(y^2_\tau|\Omega_t)$$

and

$$\sum_{\tau=t}^{\infty} \delta^{*\tau} E(y^{*2}_\tau|\Omega_\tau)$$

are equivalent to minimizing

$$(4.1) \qquad \sum_{\tau=t}^{\infty} \delta^\tau E[[\Phi(p_\tau - p_{\tau|\tau-1}) + u^y_\tau]^2|\Omega_t]$$

and

$$(4.2) \qquad \sum_{\tau=t}^{\infty} \delta^{*\tau} E[[\Phi^*(p^*_\tau - p^*_{\tau|\tau-1}) + u^{*y}_\tau]^2|\Omega_t]$$

respectively.

Note that for any variable q one has $q_{t|t-i} \equiv E(q_t|\Omega_{t-i})$ and that $E(E(q_t|\Omega_{t-i})|\Omega_{t-i-j}) = E(q_t|\Omega_{t-i-j})$, $i, j \geq 0$. We thus obtain, from (3.8a),

$$(4.3)\qquad \begin{bmatrix} p_{t|t-i} \\ p^*_{t|t-i} \end{bmatrix} = B \begin{bmatrix} m_{t|t-i} + \alpha\Phi p_{t|t-i} + \beta e_{t+1|t-i} \\ m^*_{t|t-i} + \alpha^* \Phi^* p^*_{t|t-i} - \beta^* e_{t+1|t-i} \end{bmatrix} \Delta^{-1}$$

Subtracting (4.1) for i=1 from (3.9) yields:

$$(4.4)\qquad \begin{bmatrix} p_t \\ p^*_t \end{bmatrix} - \begin{bmatrix} p_{t|t-1} \\ p^*_{t|t-1} \end{bmatrix} = B \begin{bmatrix} m_t - m_{t|t-1} + \beta(e_{t+1|t} - e_{t+1|t-1}) + v_t + \beta u^e_t \\ m^*_t - m^*_{t|t-1} - \beta^*(e_{t+1|t} - e_{t+1|t-1}) + v^*_t - \beta^* u^e_t \end{bmatrix} \Delta^{-1}$$

Using

$$(4.5)\qquad e_t = p_t - p^*_t - u^e_t$$

which, since $u^e_{t+1|t} = u^e_{t+1|t-1} = 0$, implies, for $i > 0$,

$$(4.6)\qquad e_{t+i|t} - e_{t+i|t-1} = p_{t+i|t} - p_{t+i|t-1} - (p^*_{t+i|t} - p^*_{t+i|t-1})$$

we may write (4.4) as

$$(4.7)\qquad \begin{bmatrix} p_t - p_{t|t-1} \\ p^*_t - p^*_{t|t-1} \end{bmatrix} = BD \begin{bmatrix} p_{t+1|t} - p_{t+1|t-1} \\ p^*_{t+1|t} - p^*_{t+1|t-1} \end{bmatrix} \Delta^{-1}$$

$$+ B \begin{bmatrix} m_t - m_{t|t-1} \\ m^*_t - m^*_{t|t-1} \end{bmatrix} \Delta^{-1} + B \begin{bmatrix} v_t + \beta u^e_t \\ v^*_t - \beta^* u^e_t \end{bmatrix} \Delta^{-1}$$

where $D = \begin{bmatrix} \beta & -\beta \\ -\beta^* & \beta^* \end{bmatrix}$

From (4.2) and (4.6), however, note that

$$(4.8)\quad \begin{bmatrix} p_{t+i|t} - p_{t+i|t-1} \\ p^*_{t+i|t} - p^*_{t+i|t-1} \end{bmatrix} = ABD \begin{bmatrix} p_{t+i+1|t} - p_{t+i+1|t-1} \\ p^*_{t+i+1|t} - p^*_{t+i+1|t-1} \end{bmatrix} \Delta^{-1}$$

$$+ AB \begin{bmatrix} m_{t+i|t} - m_{t+i|t-1} \\ m^*_{t+i|t} - m^*_{t+i|t-1} \end{bmatrix} \Delta^{-1}$$

where

$$A \equiv \left\{ I - B \begin{bmatrix} \alpha\Phi & 0 \\ 0 & \alpha^*\Phi^* \end{bmatrix} \Delta^{-1} \right\}^{-1}$$

By repeated forward substitution and assuming stability we may write (4.8) as

$$(4.9)\quad \begin{bmatrix} p_{t+i|t} - p_{t+i|t-1} \\ p^*_{t+i|t} - p^*_{t+i|t-1} \end{bmatrix} = \sum_{j=i}^{\infty} C^{j-i} AB \begin{bmatrix} m_{t+j|t} - m_{t+j|t-1} \\ m^*_{t+j|t} - m^*_{t+j|t-1} \end{bmatrix}$$

where

$$C \equiv ABD$$

Substituting (4.9) into (4.7) implies that

$$(4.10)\quad \begin{bmatrix} p_t - p_{t|t-1} \\ p^*_t - p^*_{t|t-1} \end{bmatrix} = BD \sum_{j=1}^{\infty} C^j AB \begin{bmatrix} m_{t+j|t} - m_{t+j|t-1} \\ m^*_{t+j|t} - m^*_{t+j|t-1} \end{bmatrix} \Delta^{-1}$$

$$+ B \begin{bmatrix} m_t - m_{t|t-1} \\ m^*_t - m^*_{t|t-1} \end{bmatrix} \Delta^{-1} + B \begin{bmatrix} v_t + \beta u^e_t \\ v^*_t - \beta^* u^e_t \end{bmatrix} \Delta^{-1}$$

We define the following

$$u_t \equiv (u^y_t, u^{*y}_t, u^m_t, u^{*m}_t, u^e_t)$$

$$\tilde{u}_t \equiv u_t - E(u_t|\Omega_t)$$

$$\tilde{e}_t \equiv e_t - E(e_t|\Omega_{t-1})$$

We may thus define

$$\tilde{\Omega}_t \equiv (\tilde{e}_t, \tilde{u}_{t-1})$$

as the new information available in period t. Since u_{t-i}, $i \geq 2$ is known at period t-1, $\tilde{e}_t$ and thus $\tilde{\Omega}_t$ can depend only on $\tilde{u}_{t-1}$ and u_t unless monetary policy is itself random. Revisions of expectations in period t about monetary policy can only depend on information newly available in period t, i.e. $\tilde{\Omega}_t$. Restricting ourselves to linear time-invariant nonstochastic policies, we may write

$$(4.11)\quad m_{t+j|t} - m_{t+j|t-1} = \gamma_j u_t + \gamma' \tilde{u}_{t-1} \qquad j \geq 0.$$

and

$$(4.12)\quad m^*_{t+j|t} - m^*_{t+j|t-1} = \gamma^*_j u_t + \gamma^{*'}_j \tilde{u}_{t-1}$$

Since $\tilde{u}_{t-1}$ is known at time t, γ_j' and $\gamma_j^{*'}$ are purely determined by policy. Since u_t is only observed imperfectly in period t via e_t, γ_j and γ_j^* depend both on the policy rules and the structure of the model.

Substituting (4.11) and (4.12) into (4.10) we obtain

$$(4.13)\quad \begin{bmatrix} p_t - p_{t|t-1} \\ p_t^* - p_{t|t-1}^* \end{bmatrix} = BD \sum_{j=1}^{\infty} C^j AB \begin{bmatrix} \gamma_j u_t + \gamma_j' \tilde{u}_{t-1} \\ \gamma_j^* u_t + \gamma_j^{*'} \tilde{u}_{t-1} \end{bmatrix} \Delta^{-1}$$

$$+ B \begin{bmatrix} \gamma_0 u_t + \gamma_0' \tilde{u}_{t-1} \\ \gamma_0^* u_t + \gamma_0^{*'} \tilde{u}_{t-1} \end{bmatrix} \Delta^{-1} + B \begin{bmatrix} v_t + \beta u_t^e \\ v_t^* + \beta^* u_t^e \end{bmatrix} \Delta^{-1}$$

Substituting (4.13) into (4.1) and (4.2) it is clear, since u_t and u_{t-1}, and therefore u_t and $\tilde{u}_{t-1}$, are orthogonal, that policies for which γ_j' and $\gamma_j^{*'}$ are non-zero increase the minimum expected loss. Such policies introduce additional randomness, in the form of the unobserved (as of last period) component of last period's disturbance, into the current period price forecast error. We thus restrict ourselves to monetary policies which do not respond to $\tilde{u}_{t-1}$.

If, in fact, $\gamma_j' = \gamma_j^{*'} = 0$, then policy responds only to currently observed components of the current disturbances. Since these can only be observed via e_t, policy can only respond to e_t. We thus restrict ourselves to policies of the form

$$(4.14)\quad m_t = \sum_{\tau=-\infty}^{t} a_{t-\tau} e_\tau$$

$$(4.15)\quad m_t^* = \sum_{\tau=-\infty}^{t} a_{t-\tau}^* e_\tau$$

Substituting (4.14) and (4.15) into (4.10) we obtain

$$(4.16) \quad \begin{bmatrix} p_t - p_{t|t-1} \\ p^*_t - p^*_{t|t-1} \end{bmatrix} = \begin{bmatrix} \Psi \\ \Psi^* \end{bmatrix} e_t + B \begin{bmatrix} v_t + \beta u^e_t \\ v^*_t - \beta^* u^e_t \end{bmatrix} \Delta^{-1}$$

where

$$\Psi \equiv BD \sum_{j=1}^{\infty} C^j ABa_j + Ba_0.$$

$$\Psi^* \equiv BD \sum_{j=1}^{\infty} C^j ABa^*_j + Ba^*_0.$$

Observe that any given values of Ψ and Ψ^* can be achieved via linear combinations of an infinite number of variations of the underlying policy parameters a_j and a^*_j. For example, a policy rule which sets $a_j = 0, j \neq 0$, and $a_0 = \bar{a}_0$ will have the same effect on the objective functional as one which sets $a_j = 0$, $j \neq 1$ and $a_1 = (BDCAB)^{-1}B\bar{a}_0$. In general, the government can achieve the same objective by responding only currently to current information (e_t) that it can achieve by responding to this information at later dates. It is interesting to note that even if the government were to have inferior information to the private sector in the sense that they learn e_t at a later date, they can achieve their objectives equally well. Turnovsky (1980) provides another example of this phenomenon. (See also Buiter, 1980c). For convenience, we restrict ourselves to current response only. We thus assume $a_j = a^*_j = 0 \quad j \neq 0$. This restriction uniquely has the virtue of yielding time consistent policies. A monetary policy which responds in period t to e_τ, $\tau < t$, does not affect y_t by this response, but only y_τ. Since y_τ is at period t a bygone, time-consistent monetary authorities will not stabilize output via expectations of future policy. We thus consider policies of the form

$$(4.17) \quad m_t = ae_t$$

$$(4.18) \quad m^*_t = a^* e_t .$$

Since m_t and m_t^* respond only to e_t, all endogenous variables in our model depend, given expectations, only on current disturbances. Taylor (1977) has shown that while models such as ours, which incorporate current or past expectations of *future* endogenous variables, have an infinite number of solutions in which current endogenous variables depend on lagged exogenous variables, in the minimum variance solution such lagged variables do not enter. We restrict our analysis to this minimum variance solution. Thus e_t, y_t, y_t^*, p_t and p_t^* depend linearly only on u_t so that

(4.19) $$p_{t|t-1} = p^*_{t|t-1} = e_{t+1|t} = 0.$$

Since y_t and y_t^* do not depend on m_τ and m_τ^* for $t \neq \tau$, the authorities' problem reduces to one of choosing a to minimize $E(y_t^2|e_t)$ for the home country and a^* to minimize $E(y_t^{*2}|e_t)$ for the foreign country in each period t.

Each country will optimally choose its monetary policy rule, taking as given the rule of the other country. Considering the home country first, minimisation of $E(y_t^2|e_t)$ is equivalent to choosing a money supply rule such that $E(y_t|e_t) = 0$,[11] given the rule followed by the foreign authority.

(4.20) $$E(y_t|e_t) = \Delta^{-1}\phi\Big[(\pi^*+\beta^*)ae_t + \beta a^* e_t + (\pi^*+\beta^*)E(v_t|e_t) + \beta E(v_t^*|e_t) + \pi^*\beta E(u_t^e|e_t)\Big] + E(u_t^y|e_t) = 0$$

The foreign country chooses its money supply rule such that $E(y_t^*|e_t) = 0$, given the rule followed by the domestic authority.

$$(4.21) \qquad E(y_t^*|e_t) = \Delta^{-1}\Phi^* \Big[\beta^* ae_t + (\pi+\beta)a^* e_t + \beta^* E(v_t|e_t)$$

$$+(\pi+\beta)E(v_t^*|e_t) - \pi\beta^* E(u_t^e|e_t)\Big] + E(u_t^{*y}|e_t) = 0$$

(4.20) and (4.21) can be rewritten as reaction functions as in (4.22) and (4.23).

$$(4.22) \qquad ae_t = -\Big[\frac{\beta}{\pi^*+\beta^*} a^* e_t + E(v_t|e_t) + \frac{\beta}{\pi^*+\beta^*} E(v_t^*|e_t) + \frac{\pi^*\beta}{\pi^*+\beta^*} E(u_t^e|e_t)$$

$$+ \frac{\Delta\Phi^{-1}}{\pi^*+\beta^*} E(u_t^y|e_t)\Big]$$

$$(4.23) \qquad a^* e_t = -\Big[\frac{\beta^*}{\pi+\beta} ae_t + \frac{\beta^*}{\pi+\beta} E(v_t|e_t) + E(v_t^*|e_t) - \frac{\pi\beta^*}{\pi+\beta} E(u_t^e|e_t)$$

$$+ \frac{\Delta\Phi^{*-1}}{\pi+\beta} E(u_t^{*y}|e_t)\Big]$$

Note from (4.22) and (4.23) that the domestic money supply responds negatively to the money supply abroad. An increase in the foreign money supply causes an appreciation of the exchange rate, creating expectations of depreciation which reduce the demand for domestic currency. To prevent the reduction in demand for domestic currency from raising the domestic price level, an accommodating reduction in domestic money supply must occur. Note also that *given* the money supply in the foreign country the optimal domestic money supply in general responds to expectations of all types of shocks, both domestic and foreign and both monetary and real. Using (3.8e), (3.8f) and (4.5) and noting that the optimal (least squares) predictor of some variable z_t given e_t is given by

$$(4.24) \qquad E(z_t|e_t) = E(e_t)^{2^{-1}} E(z_t e_t) \cdot e_t$$

we obtain

(4.25a) $$E(v_t | e_t) = \Delta\Lambda\pi^{*}\sigma_v^2\Sigma^{-1}e_t$$

(4.25b) $$E(v^{*}_t | e_t) = -\Delta\Lambda\pi\sigma^2_{v^{*}}\Sigma^{-1}e_t$$

(4.25c) $$E(u^{e}_t | e_t) = -\Delta\Lambda\pi\pi^{*}\sigma^2_{u^e}\Sigma^{-1}e_t$$

(4.25d) $$E(u^{y}_t | e_t) = -\Delta\Lambda\alpha\pi^{*}\sigma^2_{u^y}\Sigma^{-1}e_t$$

(4.25e) $$E(u^{*y}_t | e_t) = \Delta\Lambda\alpha^{*}\pi\sigma^2_{u^{*y}}\Sigma^{-1}e_t$$

(4.25f) $$\Sigma \equiv \pi^{*2}\sigma_v^2 + \pi^2\sigma^2_{v^{*}} + (\pi\pi^{*})^2\sigma^2_{u^e}$$

(4.25g) $$\Lambda \equiv 1 - \Delta^{-1}(\pi^{*}a - \pi a^{*})$$

(4.26) $$\sigma_v^2 \equiv E(v_t^2);\ \sigma^2_{v^{*}} \equiv E(v_t^{*2});\ \sigma^2_{u^e} \equiv E(u_t^{e2});\ \sigma^2_{u^y} \equiv E((u_t^y)^2);\ \sigma^2_{u^{*y}} \equiv E((u_t^{*y})^2)$$

Assuming the system given by (4.22) and (4.23) to be of full rank, the Nash equilibrium solution for a and a^{*} is given by:

(4.27) $$a = \beta + \frac{\phi^{-1}\alpha\pi\pi^{*}\sigma^2_{u^y} - \pi^{*}\sigma^2_v}{\pi\pi^{*}\sigma^2_{u^e} + \phi^{-1}\alpha\pi^{*}\sigma^2_{u^y} + \phi^{*-1}\alpha^{*}\pi\sigma^2_{u^{*y}}}$$

(4.28) $$a^{*} = -\beta^{*} + \frac{-\phi^{*-1}\alpha^{*}\pi\pi^{*}\sigma^2_{u^{*y}} + \pi\sigma^2_{v^{*}}}{\pi\pi^{*}\sigma^2_{u^e} + \phi^{-1}\alpha\pi^{*}\sigma^2_{u^y} + \phi^{*-1}\alpha^{*}\pi\sigma^2_{u^{y*}}}$$

or, noting that $\sigma_v^2 = \alpha^2\sigma^2_{u^y} + \sigma^2_{u^m}$ and $\sigma^2_{v^{*}} = \alpha^{*2}\sigma^2_{u^{*y}} + \sigma^2_{u^{*m}}$

(4.27') $$a = \beta + \frac{\phi^{-1}\alpha\pi^{*}\sigma^2_{u^y} - \pi^{*}\sigma^2_{u^m}}{\pi\pi^{*}\sigma^2_{u^e} + \phi^{-1}\alpha\pi^{*}\sigma^2_{u^y} + \phi^{*-1}\alpha^{*}\pi\sigma^2_{u^{*y}}}$$

(4.28') $$a^{*} = -\beta^{*} + \frac{-\phi^{*-1}\alpha^{*}\pi\sigma^2_{u^{*y}} + \pi\sigma^2_{u^{*m}}}{\pi\pi^{*}\sigma^2_{u^e} + \phi^{-1}\alpha\pi^{*}\sigma^2_{u^y} + \phi^{*-1}\alpha^{*}\pi\sigma^2_{u^{*y}}}$$

β is the (absolute value of the) elasticity of demand for domestic money with respect to the expected proportional rate of depreciation of the domestic currency and $-\beta^*$ the corresponding elasticity for the foreign currency. In a currency substitution framework they can be viewed as the exchange rate speculation elasticities of home and foreign currency respectively. The first terms of (4.27) or (4.27') and (4.28) or (4.28') therefore suggest that monetary policy accommodates changes in the demand for money due to unanticipated changes in the exchange rate, thereby neutralising the effect of unanticipated exchange rate changes on the price level. This policy insulates the economy from real effects of unanticipated exchange rate changes. Remember that since $e_{t|t-1} = 0$, a monetary rule contingent on e_t is a monetary rule contingent on the deviation of the actual exchange rate in period t from the exchange rate for period t anticipated in period t-1. As all disturbances are i.i.d. and there are no other sources of inertia in the model (specifically $m_{t|t-1} = 0$), rational expectations are regressive.[12/]

To the extent that monetary policy does accommodate swings in speculative demand, monetary authorities "lean with the wind", in the foreign exchange market, i.e. expand the money supply when the price of domestic currency is lower than had been expected and conversely, i.e. $a > 0$ and $a^* < 0$. Such a policy will exacerbate movements in the exchange rate, as can be seen from equation (4.29), the reduced form expression for the exchange rate.

$$(4.29) \qquad e_t = (\pi^* v_t - \pi v_t^* - \pi\pi^* u_t^e)(\Delta - \pi^* a + \pi a^*)^{-1} .$$

The denominator of the second term on the right-hand side of (4.27') and (4.28') is positive. Thus an increase in the variability of the demand for *domestic* money ($\sigma^2_{u^m}$) will reduce the degree to which the authorities lean with the wind and may even reverse this policy. An expected increase in the demand for domestic money will be associated with an unanticipated appreciation of the home currency. Rather than contracting the money supply as would be optimal if the main sources of uncertainty were foreign, optimal monetary policy will at least in part accommodate the unexpected increase in the demand for money by expanding the money supply.

The variability of *foreign* money demand has no effect on optimal domestic monetary policy, however. This result may seem surprising since, from (4.7) and (4.8), foreign monetary shocks do affect domestic income and, from (4.9) and (4.10), domestic monetary policy, *given* foreign monetary policy, does respond to perceived shocks in the demand for foreign money. If *foreign* monetary authorities pursue an optimal monetary policy, however, they minimize the effect of their own monetary disturbances. Domestic monetary policy can then ignore such disturbances.

An increase in the variance of domestic income shocks raises the optimal degree to which monetary authorities should lean with the wind. A positive income shock raises the demand for money and appreciates the exchange rate. To offset the effect of a positive income shock authorities should contract the money supply. Hence when exchange rate variation is caused in large part by instability in the supply of domestic output, monetary authorities should act to augment exchange rate changes.

The variability of foreign output shocks, unlike the variability of foreign monetary shocks, does affect the optimal intervention policy. A positive foreign output shock will tend to depreciate the exchange rate and engender a foreign monetary action which further depreciates the exchange rate. (In contrast, a foreign monetary disturbance engenders an offsetting foreign monetary action). Foreign output shocks thus create exchange rate variability which is unrelated to domestic disturbances. Any response designed to offset the effects of domestic shocks, as perceived through exchange rate variation, on domestic targets will be diminished. As $\sigma^2_{y^*}$ rises, optimal domestic policy is aimed increasingly at offsetting speculative behaviour.

For the same reason increased variability in shocks to the purchasing power parity relationship also reduce the extent to which monetary policy can offset the effects of domestic shocks on income. As $\sigma^2_{u^e}$ rises, then policy should increasingly isolate the domestic price level from the effects of exchange rate speculation.

It is interesting to consider monetary policy in four special cases of the model.

(a) <u>No domestic shocks</u>

When $\sigma^2_{u^y} = \sigma^2_{u^m} = 0$, there are no domestic sources of disturbances in the home country. The only shocks it faces are exchange rate disturbances resulting either from the stochastic nature of the purchasing power parity relationship ($\sigma^2_{u^e} > 0$) or from uncertainty in the rest of the world ($\sigma^2_{u^{*m}}$, $\sigma^2_{u^{*y}} > 0$). In this case (4.27') reduces to $a = \beta$. If there were no sources of disturbances internal to the foreign country ($\sigma^2_{u^{*y}} = \sigma^2_{u^{*m}} = 0$) (4.28') reduces to $a^* = -\beta^*$. The money supply rule is entirely accommodating. When the exchange rate depreciates unexpectedly, the money supply expands. In the absence of changes in the money supply a depreciation of the exchange rate creates expectations of appreciation (since $e_{t+1/t} = 0$). These expectations increase the speculative demand for home country money which would lower the home country price level and therefore income. To offset this, the monetary authority acts so as to accommodate exactly the higher money demand with a higher supply. Therefore, a country facing shocks largely from abroad through the exchange rate will adopt a monetary rule that exacerbates the exchange rate changes in order to stabilise real income.

(b) No domestic shocks and no currency substitution

If there are no domestic shocks *and* if the demand for domestic currency is inelastic with respect to exchange rate changes (i.e. if $\sigma_{u^y}^2 = \sigma_{u^m}^2 = \beta = 0$) then the optimal money supply is independent of the exchange rate (a = 0). Thus, except in the improbable event that the various components of (4.27') cancel exactly, a policy of free floating is optimal if and only if (1) the demand for money is interest inelastic *and* (2) there are no domestic disturbances. Even if the demand for money does not depend on the expected change of the exchange rate (i.e. if $\beta = 0$) exchange rate changes signal in part domestic shocks to which the money supply should respond. This result is analogous to Poole's (1970) finding that in the closed economy IS-LM model the optimal money supply is invariant to the interest rate if and only if (1) the demand for money is interest inelastic and (2) the economy is not subject to a variable demand for money .

(c) No real or purchasing power parity shocks

When the only source of uncertainty is in the demand for either currency (i.e. when $\sigma_{u^y}^2 = \sigma_{u^{*y}}^2 = \sigma_{u^e}^2 = 0$) then policy makes the supply of money perfectly elastic. The exchange rate is pegged. This result is analogous to Poole's finding that for a closed economy a policy of fixing the interest rate is optimal when the only source of disturbances is in the demand for money. Note that if pegging the exchange rate is the optimal policy for one country, it is so for both. Unless the two countries peg at the same level, however, the model become inconsistent.

(d) Infinitely elastic currency substitution

If individuals view domestic and foreign currency as perfect substitutes then $\beta = \beta^* = \infty$ and a policy of pegging minimizes income variability even if the economy is subject to real disturbances. If the authorities fail to peg the exchange rate, exchange rate changes will subject both economies to wide swings in the demand for money. These will create large price changes which will in turn destabilize income. Again, consistency requires that monetary authorities peg to the same exchange rate.

5. Optimal Exchange Rate Management when Minimising the Price Forecast Error is the Objective

So far we have assumed that the policy makers' objectives are to minimise output variation around the ex ante expected natural rates $(\bar{y}_\tau = \bar{y}^*_\tau = 0)$. One might assume, instead, that policy makers are concerned with the deviation of income around the ex post actual natural rates $(\bar{y}_\tau = u^y_\tau,\ \bar{y}^*_\tau = u^{y*}_t)$ which are unobserved contemporaneously. Such an objective is equivalent to minimising price forecast errors since

(5.1) $$y_t - u^y_t = \phi(p_t - p_{t|t-1})$$

and

(5.2) $$y^*_t - u^{y*}_t = \phi^*(p^*_t - p^*_{t|t-1})$$

The alternative specification of objective functions as

(5.3) $$E[(y_t - u_t)^2|\Omega_t]$$

and

(5.4) $$E\left[(y^*_t - u^*_t)^2|\Omega_t)\right]$$

is plausible if one believes that price forecast errors themselves, rather than output fluctuations, are a primary source of inefficiency. If such a specification is adopted, optimal policy rules are derived for the home country by choosing a such that $E(y_t - u_t|\Omega_t) = 0$, given a^* and for the foreign country by choosing a^* such that $E(y^*_t - u^*_t|\Omega_t) = 0$, given a. This yields

(5.5) $$a = \beta - (\sigma^2_{u^m} + \alpha^2\sigma^2_{u^y})/[(1 + \alpha\phi)\sigma^2_{u^e}]$$

(5.6) $$a^* = -\beta^* + (\sigma^2_{u^{*m}} + \alpha^{*2}\sigma^2_{u^{*y}})/[(1 + \alpha^*\phi^*)\sigma^2_{u^e}]$$

Supply uncertainty now contributes toward the optimality of a policy of exchange rate stabilisation (leaning against the wind in the exchange market) rather than the opposite. The reason is that an unanticipated increase in output, *ceteris paribus*, will increase money demand, lower the price level and cause the currency to appreciate. To eliminate the unanticipated price decline money *expansion* is now appropriate, dampening the exchange rate change.

The variability of foreign shocks, regardless of whether they are monetary or real in origin, has no effect on optimal domestic monetary policy. The domestic effects of foreign shocks of either type are minimized by optimal foreign monetary policy. Each monetary authority, in other words, acts to offset the effects of local shocks on both itself and the other country.

Regardless of the variability of money demand or output supply shocks in either economy, if the purchasing power relationship is non-stochastic a policy of pegging the exchange rate is optimal. In this case if the exchange rate is fixed then so are prices.

6. The Cooperative Pareto-Optimal Solution

So far we have assumed that each monetary authority acts independently to attain a domestic policy objective, taking the monetary policy of the other country as given. In this section we compare such policies with those that would arise if the two monetary authorities were to cooperate in setting monetary policy to attain a mutual objective. To derive the set of Pareto-optimal policies we assume that policy makers jointly set monetary policy in period t to minimise an objective of the form

$$(6.1) \quad w \sum_{\tau=t}^{\infty} E[(y_\tau^2|\tilde{\Omega}_\tau)|\tilde{\Omega}_t] + w^* \sum_{\tau=t}^{\infty} E[(y_\tau^{*2}|\tilde{\Omega}_\tau)|\tilde{\Omega}_t] \qquad w, w^* > 0$$

in period t.

As section 4 demonstrated, however, current values of m_t and m_t^* do not affect values of y_τ and y_τ^* for $\tau > t$. Choosing m_t^* to minimise (6.1) is equivalent to choosing m_t and m_t^* to minimise

(6.2) $$wE(y_t^2|\tilde{\Omega}_t) + w^*E(y_t^{*2}|\tilde{\Omega}_t)$$
$$= wE[(y_t - E(y_t|\tilde{\Omega}_t))^2|\tilde{\Omega}_t] + w^*E\left[(y_t^* - E(y_t^*|\tilde{\Omega}_t))^2|\tilde{\Omega}_t\right]$$
$$+ w[E(y_t|\tilde{\Omega}_t)]^2 + w^*[E(y_t^*|\tilde{\Omega}_t)]^2.$$

From footnote 11 it follows that the first two terms of the right-hand side of expression (6.2) are independent of m_t and m_t^*. Minimising (6.2) with respect to m_t and m_t^*, then, is equivalent to minimising

(6.3) $$wE[(y_t|\tilde{\Omega}_t)^2] + w^*E[(y_t^*|\tilde{\Omega}_t)^2].$$

First order conditions for a minimum are

(6.4) $$wE(y_t\frac{dy_t}{dm_t}|\tilde{\Omega}_t) + w^*E(y_t^*\frac{dy_t^*}{dm_t}|\tilde{\Omega}_t) = 0$$

(6.5) $$wE(y_t\frac{dy_t}{dm_t^*}|\tilde{\Omega}_t) + w^*E(y_t^*\frac{dy_t^*}{dm_t^*}|\tilde{\Omega}_t) = 0$$

$\frac{dy_t}{dm_t}$, $\frac{dy_t^*}{dm_t}$, $\frac{dy_t}{dm_t^*}$, and $\frac{dy_t^*}{dm_t^*}$ are constants. These first-order conditions therefore obtain when m_t and m_t^* satisfy:

(6.6) $$E(y_t|\tilde{\Omega}_t) = E(y_t^*|\tilde{\Omega}_t) = 0 .$$

Since (6.4) and (6.5) are linear functions of m_t and m_t^*, the values of m_t and m_t^* which satisfy (6.6) constitute a unique solution. These are exactly the same values of m_t and m_t^* which satisfy the Nash equilibrium. In our model, then, the Nash solution is also the unique Pareto-optimal solution. This result is not surprising since, in our model, each country has one independent instrument, its money supply, and one independent target, the level of its income. In such a context there are no gains from policy coordination.

To show that the equivalence of the Nash and Pareto-optimal solutions does not generalise to systems in which there are more targets than instruments consider a system in which one or both countries also have exchange rate stabilisation as another goal, i.e. in period t the home country seeks to minimise.

$$(6.7) \qquad E\sum_{\tau=t}^{\infty}[E(y_\tau^2 + \omega\, e_\tau^2|\tilde{\Omega}_\tau)|\tilde{\Omega}_t] \qquad \omega > 0$$

while the foreign country minimises

$$(6.8) \qquad E\sum_{\tau=t}^{\infty}[E(y_\tau^{*2} + \omega^* e_\tau^2|\tilde{\Omega}_\tau)|\tilde{\Omega}_t] \qquad \omega^* > 0$$

First-order conditions for Nash equilibrium values of m_t and m_t^* are given by

$$(6.9) \qquad E(y_t|\tilde{\Omega}_t)\frac{dy_t}{dm_t} + \omega\, e_t \frac{de_t}{dm_t} = 0$$

$$(6.10) \qquad E(y_t^*|\tilde{\Omega}_t)\frac{dy_t^*}{dm_t^*} + \omega^* e_t \frac{de_t}{dm_t^*} = 0.$$

With weights of w and w^* placed on the home and foreign countries' objective functions, however, first-order conditions for Pareto optimal values of m_t and m_t^* are

$$(6.11) \qquad wE(y_t|\tilde{\Omega}_t)\frac{dy_t}{dm_t} + w^*E(y_t^*|\tilde{\Omega}_t)\frac{dy_t^*}{dm_t} + (w\omega + w^*\omega^*)\frac{de_t}{dm_t} = 0$$

$$(6.12) \qquad wE(y_t|\tilde{\Omega}_t)\frac{dy_t}{dm_t^*} + w^*E(y_t^*|\tilde{\Omega}_t)\frac{dy_t^*}{dm_t^*} + (w\omega + w^*\omega^*)\frac{de_t}{dm_t^*} = 0$$

These are not equivalent to (6.9) and (6.10) except when $\omega = \omega^* = 0$. The Nash solution is not, in general, Pareto optimal.

7. Conclusion

This study of optimal monetary policy or exchange rate management in interdependent economies has abstracted from many real-world complications to obtain a transparent structure. Nevertheless, a number of results are likely to be robust under further generalisations of the model:

1). Neither a fixed nor a freely-floating exchange rate is likely to be optimal. Optimal monetary policy in general requires a finite response of the money supply to the exchange rate.

2). Output stabilising monetary policy may well require "leaning with the wind" in the foreign exchange markets, expanding the money supply when the home currency depreciates, thus increasing the volatility of the exchange rate.

3). Monetary authorities can stabilise real variables when private and public opportunity sets differ. In our model the monetary authorities are able and willing to establish (one period ahead) contingent forward contracts making the money supply in period t a known function of the contemporaneously-observed exchange rate. The private sector is assumed not to make exchange-rate contingent forward contracts. This asymmetry creates an opportunity for output stabilising (or destabilising) current exchange rate-contingent monetary policy.

4). There are likely to be gains from policy coordination.

Footnotes

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1/ In this respect our model resembles that of Boyer (1978) and Roper and Turnovsky (1980). They, however, consider a single open economy characterized by Keynesian unemployment. In a closed economy setting the current response issue has been studied by G. Woglom (1979) and McCallum and Whitaker (1979).

2/ In a very different model Canzoneri (1979) obtains a similar result.

3/ A time-consistent policy or plan is a sequence of rules, one for each period, which specifies policy actions contingent on the state of the world in that period. Each rule has the property of being optimal given the subsequent elements in the sequence. When the current state depends on anticipations of future states, such "optimal" time-consistent policies may fail to take account of the impact of future policy measures on the current state through the changes in current behaviour induced by anticipation of these future policy measures. In such models, the optimal plan in subsequent periods may therefore not be the continuation of the first-period optimal plan over the remainder of the planning period, i.e. the optimal plan will not be time-consistent. See Kydland and Prescott (1977), Fischer (1980), Buiter (1980 a, b).

4/ If $i > t$ for some i, that is if the current state is a function of current or past anticipations of a future state then the time inconsistency problems referred to in the previous footnote may arise. In the model developed in sections 3 through 6, the time-consistent policy is also the optimal policy even though expectations of the future exchange rate influence the current exchange rate.

5/ In the modern theory of optimal policy design in stochastic models, achievement of convergence has been replaced by the more general objective of minimizing deviations of target variables from their desired values over some finite or infinite time horizon. Nevertheless, the distinctions we have made about different forms of decentralization extend to the Mundellian problem.

6/ Trivial exceptions arise when $\beta_1 = \alpha_2 = 0$ or when $\alpha_2 \beta_1 = \alpha_1 \beta_2$. In this case the model is in neutral equilibrium irrespective of policy.

7/ Contracts which do not allow wages to respond to contemporaneous data might arise because such data are not available symmetrically to workers and employers, leading to problems of moral hazard. See Eaton and Quandt (1979) for a discussion.

8/ Barro (1978) also assumes that the demand for money responds to expected exchange rate changes in his model of monetary policy in a small open economy.

9/ We assume away all problems of non-uniqueness through extraneous information. The information sets of all agents are therefore limited to variables which appear in the structural model, given expectations, that is to market fundamentals in the sense of Flood and Garber (1979). See also Taylor (1977).

10/ Kydland (1976) provides a discussion of optimal stabilisation policies in a two-country, multi-period context.

11/ First note that $E\{y_t^2 | e_t\}$ =

$$E\left\{\left(y_t - E\{y_t | e_t\}\right)^2 | e_t\right\} + \left[E\{y_t | e_t\}\right]^2$$

In our model y_t and e_t are jointly normally distributed. The conditional variance of y_t is independent of e_t (see e.g. Hogg and Craig (1965), pp. 63-65 and pp. 102-104. and Buiter (1979b)). It is therefore also independent of any known linear function of e_t such as m_t or m_t^*. Minimising $E\{y_t^2 | e_t\}$ is therefore equivalent to minimising $\left[E\{y_t | e_t\}\right]^2$. This is achieved by choosing a such that $E\{y_t | e_t\} = 0$.

12/ See Harris and Purvis (1979) for a single economy model of exchange rate determination including permanent as well as transitary disturbances.

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