'DIRECT' VS. 'INDIRECT' TAXATION OF EXTERNALITIES: A GENERAL TREATMENT

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1. Introduction

It is well-known that it may be possible to attain a Pareto-efficient allocation in an economy with consumption externalities by the imposition of suitable excise taxes and subsidies, although the imposition of such taxes may not be sufficient.[1/]

In addition, the structure of these Pigovian excise taxes is familiar; they are levied only on the externality-causing goods, and in general will differ across individuals. In practice, however, it is usually prohibitively costly to attempt to distinguish perfectly between individual externality creators, whereas it is often feasible to impose ordinary commodity taxes on externality-creating goods as a 'second-best' corrective measure. An example that is often cited is the case of pollution from the internal combustion engine; it is infeasible to monitor the pollution emission of each separate car owner and tax him accordingly, but it is quite possible to tax petrol at a rate which is uniform across consumers.

This fact has an important economic implication, which is that if corrective excise taxes are constrained to be uniform across individuals, 'first-best' allocations are usually not attainable,[2/] and therefore the choice of optimal tax structure is then a 'second-best' problem, even if lump-sum redistribution is possible.

1/ This is due to the fact that if there are multiple equilibria with taxation, one equilibrium may be Pareto-dominated by another, and thus be inefficient, even though the first order conditions for an optimum may be satisfied. (For a discussion of the non-convexities giving rise to this, see Scheffman (1975).)

2/ There are special cases where uniform taxes are desired at a 'first-best' allocation. One of these is the 'symmetric' case, where individuals have identical preferences and incomes. Another case is where externalities are of the Meade 'atmosphere' type.

Starting with Diamond (1973) there have been several attempts to investigate the optimum uniform corrective (OUC) tax structure. One of the notable difference between this structure and that of the Pigovian 'first-best' taxes is that in the OUC case, it is usually optimal to tax not only the externality-creating good itself, but also goods that are complements or substitutes to it.

In a general equilibrium context, this is not greatly surprising: if, for example, there is a good that is highly complementary with a negative externality, while all other compensated cross-elasticities, and its own compensated elasticity, are small, then imposing a positive tax on this good will entail a relatively small dead-weight loss while inducing a relatively large reduction in the undesirable externality.

A major policy question then arises; given that the government is constrained to OUC taxes, should it simply tax the externality-causing good itself ('direct' taxation), or impose taxes and subsidies on related goods ('indirect' taxation) ? 3/

Both Green and Sheshinski (1976) and Balcer (1980) have investigated this question in the context of a simple three-good model with utility functions linear in income, and find that there are three conditions each of which is sufficient for indirect taxes to be zero at a 'second-best' optimum.

3/ The terms 'direct' and 'indirect' are used in this way in what follows, although they are in parentheses to distinguish them from their more common usage, which distinguishes between income and commodity taxation.

The first two are; (i) that all individuals have identical preferences and incomes, and (ii) that the externality be of the 'atmosphere' type, where the externality level depends on the simple sum of the consumption levels of the externality-creating good, and the individual ignores his contribution to the externality. These conditions, as well as being restrictive, are not particularly interesting, as they are precisely the conditions for which 'first-best' Pigovian taxes are uniform, and as mentioned above, the Pigovian solution requires 'direct' taxes only.

Green and Sheshinski (1976) also noted a rather more interesting condition, which was that preferences be separable in one of the non-externality-causing goods; this turns out to be consistent with, and in part a special case of, the following analysis.

The purpose of this paper, then, is to examine the 'direct/indirect' problem in the the OUC tax structure in the context of a rather more general model than the one previously used, which allows for income effects and an arbitrary number of goods. We find that, for the case where preferences are separable in the externality, general conditions, on preferences can be found which are sufficient for 'indirect' taxes to be zero at the optimum and interestingly, that these conditions are precisely those which are sufficient for uniform commodity taxation (where 'commodities' are goods $1,..,N + 1$) in the standard many-person Ramsey optimal taxation problem. It is also found that these conditions are not sufficient to guarantee zero indirect taxation for the non-separable case; this is shown by counterexample.

2. The Model

For simplicity we assume that there are two individuals. The results obtained are easily generalised to an arbitrary number of individuals.

We also assume N + 2 goods (i = 0,..,N + 1) and we assume good 0 is externality-causing so that the utility functions are defined as

$$U^1 = U^1(x_o^1, \underset{\sim}{x}^1, e^1) \quad (1)$$

$$U^2 = U^2(x_o^2, \underset{\sim}{x}^2, e^2)$$

where $\underset{\sim}{x}^h = (x_1^h \ldots x_{N+1}^h) \qquad h = 1,2,$

and $e^1 = x_o^2$, $e^2 = x_o^1$. Also, we make the assumption of separability in e^1, e^2 so that

$$U^h = U^h(f^h(x_o^h, \underset{\sim}{x}^h), e^h) \qquad h = 1,2 \quad (2)$$

This assumption greatly simplifies the analysis by removing the dependence of demand of the freely chosen goods on externality levels.

In the subsequent analysis we will need to consider the possibility of taxing the externality-causing good 'directly' and for this reason must adopt some price normalisation that allows us to do this. For convenience, we adopt good N + 1 as the untaxable numeraire, and suppose that each individual has an endowment of the numeraire $(\bar{x}^h_{N+1})$ plus a lump-sum tax or subsidy of the same which is controlled by the government (T^h). It should be stressed that the results obtained below are entirely independent of the choice of normalisation.

The budget constraints are

$$P_o x_o^h + \sum_{i=1}^{N} P_i x_i^h + x_{N+1}^h \leqq \bar{x}_{N+1}^h + T^h \quad h = 1,2 \tag{3}$$

Each consumer maximises (2) subject to (3). The indirect utility functions are therefore of the form

$$V^h = V^h (\hat{f} (P_o , \underset{\sim}{p} , T^h) , e^h) \quad h = 1, 2 \tag{4}$$

and we have

$$\frac{\partial V^h}{\partial e^h} = \frac{\partial U^h}{\partial e^h} = r^h \qquad h = 1, 2 \tag{5}$$

We assume a constant returns to scale production function so that the aggregate production feasibility constraint can be replaced by a government revenue constraint in the usual way.

The production constraint can be written

$$C_o X_o + \sum_{i=1}^{N} C_i X_i + X_{N+1} - \bar{X}_{N+1} \leqq 0 \tag{6}$$

where $X_o = \sum_h x_o^h$, $X_i = \sum_h x_i^h$, and

$X_{N+1} = \sum_h x_{N+1}^h$, $\bar{X}_{N+1} = \sum_h \bar{x}_{N+1}$

Non-satiation will ensure that (3) will hold with equality, so that summing (3) over h and subtracting from (6), we obtain

$$t_o X_o + \sum_{i=1}^{N} X_i \, t_i - T^1 - T^2 \geq 0 \tag{7}$$

The optimal tax problem is now to maximise the sum of the V^h subject to (6), the control variables being t_n, t_i $(i = 1 \ldots N)$ and T_1, T_2 .

The first-order conditions are :

$$- \sum^{h} \gamma^h x_o^h + r_1 \frac{\partial \; {}_o^2}{\partial P_o} + r^2 \frac{\partial x_o^1}{\partial P_o} + \tag{8}$$

$$\mu \left[X_o + t_n \frac{\partial X_o}{\partial P_o} + \sum_{i=1}^{N} t_i \cdot \frac{\partial X_i}{\partial P_o} \right] = 0$$

$$- \sum^{h} \lambda^h x_j^h + r_1 \frac{\partial x_o^2}{\partial P_j} + r_2 \frac{\partial x_o^1}{\partial P_j} + \mu \left[X_j + \frac{\partial X_o}{\partial P_j} + \sum t_i \frac{\partial X_i}{\partial P_j} \right]$$

$$= 0 \quad \forall_j \tag{9}$$

for the choice of t_o, t_j $(j = 1 \ldots . N)$.

For the choice of T_1 , T_2 we obtain

$$\lambda^1 + r^2 . \frac{\partial x_o^1}{\partial M} + \mu \left[t_o \frac{\partial x_o^1}{\partial M} + \sum_{i=1}^{N} t_i \cdot \frac{\partial x_i^1}{\partial M} - 1 \right] \tag{10}$$

and

$$\gamma^2 + r^1 \cdot \frac{\partial x_o^2}{\partial M} + \mu \left[t_o \cdot \frac{\partial x_o^2}{\partial M} + \sum_{i=1}^{N} t_i \cdot \frac{\partial x_i^2}{\partial M} - 1 \right] = 0 \tag{11}$$

where μ is the multiplier on the government revenue constraint, and λ^h are marginal utilities of income for $h = 1, 2$.

Multiplying (10) and (11) by x_o^1 and x_o^2 respectively and adding to (8) we obtain

$$r^1 \cdot S_{oo}^2 + r^2 S_{oo}^1 + \mu \left[t_o (S_{oo}^1 + S_{oo}^2) + \sum_{i=1}^{N} t_i \cdot (S_{oi}^1 + S_{oi}^2) \right] = 0 \tag{12}$$

and similarly from (9) we can obtain

$$r^1 \cdot S_{oj}^2 + r^2 S_{oj}^1 + \mu \left[t_o (S_{oj}^1 + S_{oj}^2) + \sum_{i=1}^{N} t_i (S_{ji}^1 + S_{ji}^2) \right] = 0 , \forall j \tag{13}$$

Here S_{ij}^h $(i,j = 0....N)$ is the ij^{th} element of the h^{th} consumer's Slutsky matrix.

Equations (12) and (13) are very similar to optimal tax formulae for the many-person Ramsey tax problem and can be interpreted in a similar way.

First, the r^h are a measure of the distortion due to the externality, as r^h/λ^h $(h = 1, 2)$ represent the difference between the h^{th} person's marginal willingness to pay for the externality level and the amount (zero) that he actually does pay.

Therefore $(-r_1 S^2_{oj} + r_2 S^1_{oj})$ may be interpreted as the amount, to the first order and at constant levels of utility [4/], by which a unit change in the j^{th} tax reduces the distortion due to the externality. In the special case where $S_{ij} = 0 \quad i \neq j$, then

$$-(r_1 S^2_{oj} + r_2 S^1_{oj}) = \mu t_j (S^1_{jj} + S^2_{jj}) \qquad (14)$$

so that the marginal welfare gain in terms of "reduced distortion" induced by the j^{th} tax equals the deadweight loss to first order, imposed by that level of t.

In the general case, the interpretation is slightly modified as the first order conditions are interdependent. Now the marginal welfare gain from the j^{th} tax is proportional to the distortion in consumption of the j^{th} commodity induced by the tax system. For example, suppose X_j is highly complementary to a positive externality $(S_{oj} < 0)$; then one would be willing to bear a tax system which induced a high distortion in the consumption of X_j compared to some good k for which $S_{ok} \approx 0$, because increasing consumption of X_j is a more 'efficient' way of indirectly affecting X_o than is changing X_k.

From this discussion it is clear that the j^{th} tax will be non-

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4/ This is as it should be, as the ability to effect lump-sum transfers is equivalent to the ability to manipulate utility levels independently of commodity taxes.

zero at the optimum only if it provides an indirect way, via relationships of substitutability, of influencing consumption levels of good O. Therefore we would expect (and it will be proved below) that if $S_{oj} = 0$ (j = 1....N) then optimal 'indirect' taxes will all be zero.

A weaker condition for zero 'indirect' taxes can,however, be derived. Suppose that relationships of complemntarity and substitutability are such that <u>if the externality causing good were untaxable</u>, then the optimum 'indirect' tax structure would be uniform. In this case, nothing would be gained in welfare terms by allowing 'indirect' taxation as an additional policy instrument, supplementing 'direct' taxation. Therefore, at the 'second-best' optimum with <u>both</u> types of taxation as available instruments, 'indirect' taxes would all be zero. Also, because of the similarity of this problem to the many-person Ramsey tax problem, it is perhaps not surprising that one can provide sufficient conditions for uniformity of the 'indirect' tax structure which are identical to those for the Ramsey many-person optimal tax problem (the latter conditions in their most general form are given in Deaton (1979)).

This argument can easily be formalised, and leads to the following result :

<u>Proposition 1</u>

If preferences are such that

$$U^h = U^h (f^h (g^h (\underline{x}^h) , x_o^h), e^h) \quad h = i, 2$$

and for fixed x_o^h, the indirect subutility function $\hat{g}^h(\ )$ satisfies

$$\hat{g}^h(P_o, \underset{\sim}{P}, T^h + \bar{x}^h_{N+1}, x^h_o) = \frac{T^h + \bar{x}^h_{N+1} - P_o x^h_o - a^h(\underset{\sim}{P})}{b(\underset{\sim}{P})} \qquad h = 1, 2$$

then optimal 'indirect' tax rates are all zero, and the optimal 'direct' rate satisfies

$$t_o^* = -\frac{1}{\mu}\left(\frac{r_1 S^2_{oo} + r_2 S^1_{oo}}{S^2_{oo} + S^1_{oo}}\right) .$$

The first remark to be made is that these conditions are identical to those derived by Deaton (1979) for uniformity of commodity taxation, and require that the indifference map in the $\underset{-}{\bar{x}}^h$ space be of a special form. It must be independent of x_o^h and e^h and must be quasi-homothetic.

The second point is that this result is a partial generalisation of Green and Sheshinski (1976) who find for their model ($N + 1 = 3$ and utility linear in income) that preferences must be additively separable in the following way for zero indirect taxes ;

$$U^h = f(x_o^h, e^h) + g(x_1^h) + x_2^h \qquad (15)$$

It should be noted, however, that (15) does not satisfy weak separability in e^h and therefore Green and Sheshinski's result is in this sense more general than Proposition 1 above.

However it is established below that one can do without weak separability in e^h only in the special case of no income effects; when income effects are allowed for, it is shown by counter-example that indirect taxes are non-zero for preference structures satisfying all the other hypotheses of Proposition 1 except for weak separability in e^h . Therefore, it would appear that Green and Sheshinski's condition (15) cannot be generalised to the case of non-zero income effects, although it may be possible to extend it to the case of many commodities.

Finally, we may note that when optimal indirect taxes are zero the formula for the optimal direct tax is identical to the one first derived by Diamond((1973) equation 10) for the case where the government has <u>only</u> direct taxes at its disposal. This establishes that the approach in this paper is consistent with earlier analysis.

<u>Proof of Proposition 1</u>

By hypothesis, the constrained demand functions <u>for a fixed</u> $\hat{x}_o^h$ can be written

$$\hat{x}_i^h = \alpha_i^h \; (\underset{\sim}{P}) + \beta_i \; (\underset{\sim}{P}) \; (T^h - P_o x_o^h + \bar{x}_{N+1}^h) \qquad (16)$$

$i = 1....N + 1$, $\qquad\qquad h = 1, 2.$

Clearly $\dfrac{\partial \hat{x}_i^h}{\partial M^h} = \beta_i \; (\underset{\sim}{P})$ for all i and h.

Now, by Theorem 5 of Goldman and Uzawa (1964) we know that

$$S^h_{oi} = \phi^h \frac{\partial x^h_i}{\partial M^h} \qquad i = 1,\ldots N + 1, \quad h = 1, 2 \tag{17}$$

where $\frac{\partial x^h_i}{\partial M^h}$ is the derivative of the relevant unconstrained (x^h_o variable) demand function.

Using the analysis of Neary and Roberts (1978) and the fact that preferences are separable in $\underset{\sim}{x}^h$ we can derive

$$\frac{\partial \hat{x}^h_i}{\partial M^h} = \frac{\partial x^h_i}{\partial M^h} \left(1 + \hat{P}^h_o \frac{\partial x^h_o}{\partial M^h} \right) \qquad i = 1\ldots.N + 1, \quad h = 1,2 \tag{18}$$

where $\hat{P}^h_o$ is the 'virtual price' or marginal willingness to pay for the ration level x^h_o. As $\frac{\partial \hat{x}^h_i}{\partial M^h}$ is independent of x^h_o, at that level which would have been chosen by the consumer h at the given market prices and income, which gives $\hat{P}^h_o = P_o$ and thus

$$S^h_{oi} = \phi^h \left(1 + P_o \frac{\partial x^h_o}{\partial M^h}\right)^{-1} \beta_i (\underset{\sim}{P}), \quad \text{or} \tag{19}$$

$$S^h_{oi} = \gamma^h \beta_i, \quad i = 1\ldots N + 1, \quad h = 1, 2.$$

Now by the homogeneity of degree zero of the compensated demand functions, we obtain

$$\gamma^h \hat{\beta} \; + \; S^h_{oo} \; = \; 0 \qquad h = 1,2 \qquad (20)$$

where

$$\hat{\beta} \; = \; \sum_{i=1}^{N} \; \frac{P_i \beta_i \; + \; \beta_{N+1}}{P_o} \qquad (21)$$

Using (19), (20) and (21) we can write the N equations in (12) and (13) in matrix form

$$\left[\begin{array}{c} -\hat{\beta} \\ \hline \underset{\sim}{\beta} \end{array}\right] \frac{\gamma_1 r^2 \; + \; \gamma_2 \, r^1}{\gamma_1 \; + \; \gamma_2} \; \frac{1}{\mu} \; = \; \left[\begin{array}{c|c} -\hat{\beta} & \underset{\sim}{\beta}' \\ \hline \underset{\sim}{\beta} & A \end{array}\right] \left[\begin{array}{c} \underset{\sim}{t_o} \\ \hline \underset{\sim}{t} \end{array}\right]$$

where $\underset{\sim}{\beta}$ is the column vector $(\beta_1 \ldots\ldots \beta_N)$, $\underset{\sim}{t}$ is the column vector $(t_1 \ldots\ldots t_N)$, and the ij^{th} element of A , a_{ij} is defined as

$$a_{ij} \; = \; (\, S^1_{ij} \; + \; S^2_{ij} \,) \; / \; (\gamma_1 \; + \; \gamma_2 \,) \; .$$

Then solving for t_i ($i = 0 \ldots . N$)

using Cramer's rule we have, as long as the matrix is non-singular,

that $t_i \; = \; 0$ ($i = 1 \ldots\ldots N$) and that $t^*_o \; = \; - \frac{1}{\mu} \; \frac{(\gamma_2 r_1 \; + \; \gamma_1 r_2 \,)}{\gamma_1 + \gamma_2}$

Substituting from (20) we obtain the desired result.

In exactly the same way, we can show that if $S_{oi} = 0$ for all $i = 1....N$ then $t_i = 0$ and t_o^* satisfies Proposition 1 : this proves the assertion above. It should be noted however that $S_{oi} = 0$ $(i = 1 ....N)$ does not require either separability in goods $1.....N$ or the Gorman aggregation conditions, so that it is not a special case of Proposition 1. We have

Proposition 2

If $S^h_{oi} = 0$ for $i = 1 ....N$ and all h then the optimal 'indirect' taxes are all zero and t_o^* is as in Proposition 1.

We may now extend the analysis by considering the implications of dropping the assumption of separability in e^h. This is best done by means of an example: we will first analyse the 'bench mark' case of separable externalities, and then consider the more complicated non-separable case.

Consider the two pairs of utility functions

$$U_1 = \tfrac{1}{2} \log x_o^1 + \frac{1}{4} \log x_1^1 + \frac{1}{4} \log x_2^1 + \frac{1}{2} \log e^1$$
$$U_2 = \frac{1}{2} \log x_o^2 + \frac{1}{4} \log x_1^2 + \frac{1}{4} \log x_2^2 + \log e^2 \qquad (22)$$

and

$$U_1 = \frac{1}{2} \log (x_o^1 + \frac{1}{2} e^1) + \frac{1}{4} \log x_1^1 + \frac{1}{4} \log x_2^1$$
$$U_2 = \frac{1}{2} \log (x_o^2 + e^2) + \frac{1}{4} \log x_1^2 + \frac{1}{4} \log x_2^2 \qquad (23)$$

It is easily seen that the utility functions in (22) satisfy all the hypotheses of Proposition 1, and those in (23) satisfy all the hypotheses except for weak separability in e^h (h = 1,2).

Let us suppose that the relevant budget constraints are

$$
\begin{aligned}
P_o x^1 + P_1 x_1^1 + x_2^1 &\leqq 1 + T_1 \\
P_o x_o^2 + P_1 x_1^2 + x_2^2 &\leqq 1 + T_2
\end{aligned}
\qquad (24)
$$

Here we may interpret good 2 as leisure, and we require at the optimum that $T_1 > -1$, $T_2 > -1$.

Now define $Z_o = \frac{1}{P_o}$, $Z_1 = \frac{1}{P_1}$.

The social welfare function can be written as the sum of the indirect utility functions, which are themselves derived from maximising (22) subject to (24).

We have

$$W = \frac{1}{2}\log Z_1 + \frac{5}{2}\log Z_o + 2\log(1 + T_1) + \frac{3}{2}\log(1 + T_2). \qquad (25)$$

The revenue constraint is

$$\frac{1}{2}(1 - Z_o C_o) + \frac{1}{4}(1 - Z_1 C_1)(2 + T_1 + T_2) - (T_1 + T_2) \geqq 0 \qquad (26)$$

where C_o , C_1 are the (fixed) marginal costs of production of goods O and 1.

The tax problem is to maximise (25) subject to (26), the control variables being Z_o , Z_1 and T_1 , T_2 .

The first-order conditions are

$$\frac{5}{Z_o} = \mu \, . \, C_o \, (2 + T_1 + T_2) \tag{27}$$

$$\frac{2}{Z_1} = \mu \, . \, C_1 \, (2 + T_1 + T_2) \tag{28}$$

$$\frac{2}{1 + T_1} + \mu \left[\frac{1}{2} (1 - Z_oC_o) + \frac{1}{4} (1 - Z_1C_1) - 1 \right] = 0 \tag{29}$$

$$\frac{3/2}{1 + T_2} + \mu \left[\frac{1}{2} (1 - Z_oC_o) + \frac{1}{4} (1 - Z_1C_1) - 1 \right] = 0 \tag{30}$$

$$\left\{ \frac{1}{2} (1 - Z_oC_o) + \frac{1}{4} (1 - Z_1C_1) \right\} (2 + T_1 + T_2) = T_1 + T_2 \tag{31}$$

We can easily eliminate the multiplier ; from (29), (30) and (31) we obtain

$$\emptyset = \frac{7}{4} \tag{32}$$

Now substituting (32) into (28) we find, remembering that $(\frac{1}{Z_1} - C_1)$ $= t_1$ that $t_1 = 0$ if and only if

$$T_1 + T_2 = - \frac{6}{7} \tag{33}$$

We can now show that condition (33) can be deduced from the first-order conditions alone, implying that it is a feature of the solution of equations (27) - (31). This is easily done by substituting for Z_oC_o and

and Z_1C_1 from (27) and (28) respectively into (31), whereupon we obtain $T_1 + T_2 = -\frac{6}{7}$ 5/.

This is both an illustration of the validity of Proposition 1, and a useful comparison with the non-separable case where, as will be seen below, this property of the solution no longer holds.

Non-separability implies dependence of demands on externality levels. These 'conditional' demands are obtained by maximising (23) subject to (24) and are then solved for the Nash equilibrium to obtain

$$x_o^1 = \frac{8}{14} \frac{1}{P_o} \left\{ 1 + T_1 + \frac{1}{4} (1 + T_2) \right\} \tag{34}$$

$$x_o^2 = \left(\frac{8}{14} \right) \frac{1}{P_o} \left\{ (1 + T_2) + \frac{1}{2} (1 + T_1) \right\} \tag{35}$$

$$x_1^1 = \left(\frac{1}{4} \right) \frac{1}{P_1} \left\{ \frac{8}{7} (1 + T_1) + \frac{2}{7} (1 + T_2) \right\} \tag{36}$$

$$x_1^2 = \left(\frac{1}{4} \right) \frac{1}{P_1} \left\{ \frac{8}{7} (1 + T_2) + \frac{4}{7} (1 + T_1) \right\} \tag{37}$$

The social welfare function is then

$$W = \log Z_o + \frac{1}{2} \log Z_1 \tag{38}$$

$$+ \frac{1}{2} \log \left[\frac{9}{4} (1 + T_1)^2 + \frac{5}{8} (1 + T_2)^2 + \frac{21}{8} (1 + T_1) (1 + T_2) \right] \tag{39}$$

$$+ \frac{1}{4} \log \left[\frac{32}{49} (1 + T_1)^2 + \frac{16}{49} (1 + T_2)^2 + \frac{24}{49} (1 + T_1) (1 + T_2) \right].$$

5/ Also, from (29) and (30) we obtain $3 T_1 - 4 T_2 = 1$. Combined with (33) this implies that at the optimum $T_2 = -\frac{4}{7}$, $T_1 = -\frac{2}{7}$ so that $T_1 , T_2 > -1$ is satisfied.

One easily obtains from the first-order conditions for a constrained maximum of (3) that

$$\frac{t_1}{C_1} = \frac{\mu}{2} \left\{ (1 + T_1) \frac{12}{7} + \frac{10}{7} (1 + T_2) \right\} - 1 \tag{40}$$

(where μ is the multiplier on (39). Also, one can establish that $\mu = \frac{3}{4}$ from the first-order conditions, so that the condition for zero 'indirect' taxation is

$$\frac{12}{7} T_1 + \frac{10}{7} T_2 = -\frac{10}{21} \tag{41}$$

However, using the first-order conditions for variables Z_o and Z_1 and substituting in (39) one can obtain

$$\frac{2}{7} T_1 + \frac{1}{14} T_2 = \frac{5}{14} \tag{42}$$

It is here that the problem arises. We have followed exactly the same procedure as in the separable case, but (41) and (42) are not linearly dependent, which implies that <u>imposing</u> $t_1 = 0$ requires an additional linear restriction independent of the first-order conditions. This implies that given that there are one or more solutions to the system of equations given by the first order conditions, the system given by the first-order conditions plus (42) is overdetermined. Thus it is easy to assume (42) and obtain a contradiction implying that $t_1 \neq 0$ at the optimum.

From (41) and (42), we have

$$T_1 = -\frac{5}{3}, \quad T_2 = \frac{5}{3} \tag{43}$$

Using (43), we find that $t_o = 0$ at the optimum so that from the first-order conditions,

$$\frac{\partial W}{\partial T_1} = \mu = \frac{3}{4} . \tag{44}$$

However, evaluating $\frac{\partial W}{\partial T_1}$ at $T_1 = -\frac{5}{3}$, $T_2 = \frac{5}{3}$ we find that

$$\frac{\partial W}{\partial T_1} = \frac{18}{7} + \frac{1}{16} ,$$ a contradiction.

This example has been rather involved, but it has demonstrated the important points; first, that the main result of this paper depends crucially upon the separability of preferences in e^h, and second, that Green and Sheshinski's result depends in a similar way upon the assumption of no income effects. It is therefore probably fruitless to search for general conditions on preferences, in the absence of separability, which imply that indirect taxes are not necessary.

3. Conclusions

It remains to be asked if some of the restrictive assumptions of the analysis can be relaxed somewhat.

The first is that there is assumed to be only one externality: however it does not seem that Proposition 1 can be extended to the case of many externality-causing goods. The reason for this is that when there are many externalities, an optimally uniform tax on all non-externality-causing goods when the externality-causing goods are all untaxable is no longer equivalent to optimal 'direct' tax on each of the externality-causing goods as it imposes an equal tax on each of them, and generally the 'direct' taxes will be different at the optimum. The second assumption is that the government has the ability to make lump-sum transfers between individuals; making the assumption allows us to isolate those features of an optimal commodity tax system which 'correct' externalities from those features which play a redistributional role. However in any realistic economic environment with externalities the optimal commodity tax system will play both roles. In terms of the model above, if the government has only a poll-tax at its disposal then not surprisingly Propositions 1 and 2 no longer hold - it will be optimal to tax or subsidise other goods for reasons of equity.

We may remark finally that under very special circumstances, Proposition 2 holds for the case of many externalities.

Suppose the first $0 \ldots . k$ goods are externality-causers, and the $k+1 \ldots . N+1$ goods are purely private.
Then if for any good $0 \leq i \leq k$, and $k+1 \leq j \leq N+1$, $S_{ij} = 0$ and for any $0 \leq i \leq k$, $0 \leq j \leq k$, $S_{ij} = 0$, then it can be proved that all 'indirect' taxes will be zero at the optimum. Also, the optimal 'direct' tax on the i^{th} externality-causing good can be found to be

$$t_i^* = -\frac{1}{\mu}\left(\frac{r_1 S_{ii}^2 + r_2 S_{ii}^1}{S_{ii}^1 + S_{ii}^2}\right) .$$

However, this is clearly a very special case.

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