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ABSTRACT

This paper applies simple duality theory to integrate some elements of the theory of public finance into the theory of economic policy in open distorted economies.

Results on the effect of reductions in distortions are derived which generalize the results of Dixit (Journal of Public Economics, 1975). Extensions of the theory of immiserising growth are made, focussing on the role of shadow factor prices, following Bhagwati, Srinivasan and Wan (Economic Journal, 1978).

The results most relevant for piecemeal policy making (Dasgupta and Stiglitz, 1974; Findlay and Wellisz, 1976; Srinivasan and Bhagwati, 1978; all Journal of Political Economy) are shown to hold strictly only in small open economies in which all goods are internationally traded.

Introduction

In this paper the increasingly well-known techniques of simple duality theory are applied to models of competitive production in open economies with the aim of integrating some elements of the theory of public finance into the well-established theory of economic policy in open distorted economies. Income redistribution problems are ignored: the focus is purely on "efficiency" questions.

The structure of the paper is as follows. In the next section the details of the model are set out and the basic relations describing consumption, production and foreign trade are derived. This model is then used to analyse the effects of some particular types of tax changes, to discuss optimal policy in the presence of fixed distortions, to show the relevance of these issues to the theory of cost-benefit analysis, and to clarify the theory of immiserising growth. Throughout it is emphasised that optimal public policy is particularly simple when all goods are traded at fixed world prices.

The structure of the model

There are p inputs which are exogenously supplied and which can therefore be assumed to have no direct effect on individuals' welfare. (For the sake of brevity, I shall refer to these inputs as "resources," though we shall see they can have a wider interpretation.) There are n other goods, which may be inputs and outputs, of which the first m are internationally traded and the remaining n-m are nontraded. Production may be undertaken in two sectors, which I shall call the public and private sectors, though for some interpretations they are better thought of as two private sectors, such as rural and urban production, facing possibly different prices.

The following notation is adopted:

v	$p \times 1$ vector of resources
w	$1 \times p$ private producer resource price vector
w^g	$1 \times p$ public producer resource price vector
$x = (x_t, x_u)$	$n \times 1$ vector of private net outputs
$g = (g_t, g_u)$	$n \times 1$ vector of public net outputs
$c = (c_t, c_u)$	$n \times 1$ aggregate consumption vector
$z = c_t - x_t - g_t$	$m \times 1$ net import vector
π	$1 \times m$ world price vector
$p = (p_t, p_u)$	$1 \times n$ private producer output price vector
$p^g = (p_t^g, p_u^g)$	$1 \times n$ public producer output price vector
$q = (q_t, q_u)$	$1 \times n$ consumer price vector
$t = (t_t, t_u)$	
$= q - p$	$1 \times n$ consumer tax vector
$s = (s_t, s_u)$	
$= p^g - p$	$1 \times n$ public producer output subsidy vector
$\tau = p_t - \pi$	$1 \times m$ trade tax vector.

In each case the partitioning of an n-vector is an (m, n-m) partition where the first part refers to the traded goods and the second to the non-traded goods. Clearly $c_u = x_u + g_u$. Resources are allocated intersectorally so that $v = v^x + v^g$ in the obvious notation, though for most of the analysis it is natural to assume that resources are not intersectorally mobile. With four sets of prices, we have three sets of taxes and subsidies defined as differences between prices. Exactly how these taxes are defined is arbitrary. Above I have chosen the convention of defining the taxes and subsidies as the differences between consumer prices, public producer prices and world prices on the one hand and private producer prices on the other. In fact, as the analysis proceeds, this convention is modified at times.

Private production is assumed to be perfectly competitive and to take place under constant returns to scale. Given resource endowments v^x the production set $X(v^x)$ describes the technologically feasible set of net outputs. We define the revenue function

$$r(p,v^x) = \max_x \{px \mid x \in X(v^x)\} \qquad (1)$$

and assume that it exists and is twice differentiable.

(The implications of this assumption and the technical modifications required when it is not satisfied are discussed in the Appendix.)

The supply functions $x(p,v)$ and the competitive resource prices $w(p,v)$ are given by

$$x(p,v^x) = \frac{\partial r(p,v^x)}{\partial p} \qquad (2)$$

$$w(p,v^x) = \frac{\partial r(p,v^x)}{\partial v} \qquad (3)$$

where (2) is the well-known "envelope" result, and (3) follows from prices being competitive. $r(p,v)$ is homogeneous of degree 1 in p, so

$$r(p,v^x) = px(p,v^x) \qquad (4)$$

while constant returns implies that r is homogeneous of degree 1 in v^x, so

$$r(p,v^x) = w(p,v^x)v^x \qquad (5)$$

From (3) and (4),

$$w(p,v^x) = p \frac{\partial x(p,v^x)}{\partial v} \qquad (6)$$

and from (2) and (5)

$$x(p,v^x) = \frac{\partial w(p,v^x)}{\partial p} v^x, \qquad (7)$$

while (2) and (4) imply

$$0 = p \frac{\partial x(p,v^x)}{\partial p} \qquad (8)$$

so x is homogeneous of degree 0 in p. (3) and (5) imply

$$0 = \frac{\partial w(p,v^x)}{\partial v} v^x \qquad (9)$$

so w is homogeneous of degree 0 in v^x.

$r(p,v^x)$ is convex in p so $\partial^2 r/\partial p^2 = \partial x/\partial p$ is positive semi-definite; while the fact that r is concave in v implies that $\partial^2 r/\partial v^2 = \partial w/\partial v$ is negative semi-definite. The symmetry of cross-partial derivatives implies

$$\frac{\partial x(p,v^x)}{\partial v} = \frac{\partial^2 r(p,v)}{\partial p \partial v} = \frac{\partial w(p,v^x)}{\partial p} \qquad (10)$$

The significant restriction placed above on the technology is the assumption of constant returns, but this need not imply a loss of generality if we interpret, following McKenzie (1959), the set of resources as including property rights to the profits of decreasing returns activities.

In much of the discussion below, I shall treat the "public" sector as a second private sector with commodity prices p^g and resource endowments v^g and assume that relationships like (1)-(10) apply to this sector also. These relationships are referred to as (1g)-(10g) without necessarily being explicitly written.

Transactions with foreigners take place at prices π. The fact that international payments must balance in a timeless world is written as

$$\pi z = 0 \tag{11}$$

In general prices depend on transactions so $\pi = \pi(z)$. In this case, it proves convenient to define a "trade revenue function":

$$t(\pi^*) = \max_z\{\pi^* z \mid \pi(z)z \leq 0\} \tag{12}$$

which is the function that would be maximised by a monopoly state trading organisation maximising its profits at the fixed prices π^* subject to the net supplies offered by foreigners. Analogously to the revenue function $r(p,v)$ it has the properties that

$$z(\pi^*) = \frac{\partial t(\pi^*)}{\partial \pi^*}, \tag{13}$$

that $t(\pi^*)$ is convex and homogeneous of degree 1 and $z(\pi^*)$ is homogeneous of degree 0, so

$$t(\pi^*) = \pi^* z(\pi^*) \tag{14}$$

and

$$0 = \pi^* \frac{\partial z(\pi^*)}{\partial \pi^*} = \pi^* \frac{\partial^2 t(\pi^*)}{\partial \pi^{*2}} \tag{15}$$

(Again, the implications of the assumption that the trade revenue function is twice differentiable are discussed in the appendix.)

The relationship between the trade revenue function and more conventional presentations of optimal tariff theory in a many good model (for example, Dixit and Norman (1980, p. 152) or Smith (1980, pp. 12-13)) can be seen by observing that the first-order conditions for solving the maximisation problem in (12) are that π^* should be proportional to $\pi + \tau^*$ where

$$\tau^*(z) = z' \frac{\partial \pi(z)}{\partial z} \tag{16}$$

gives the vector of optimal tariffs. The function t and its derivatives thus describe the foreign offer set in terms of the price hyperplanes supporting it, while the prices π^* reflect the true opportunity costs of foreign trade.

Individuals are identical (or, equivalently, perfect lump-sum redistribution is undertaken), so we describe the economy as if there is a single individual, facing prices q, with money income I and choosing consumption c to maximise his utility subject to the constraint $qc \leqq I$. c may include negative terms representing supply, say of labour.

It is convenient to describe consumer behaviour by the expenditure function:

$$e(q,u) = \min_c \{qc \mid u(c) \geqslant u\} \tag{17}$$

Then the compensated demand functions are

$$c(q,u) = \frac{\partial e(q,u)}{\partial q} . \tag{18}$$

The expenditure function is concave in q and homogenous of degree 1 in q and the compensated demand functions are homogeneous of degree 0 in q, so

$$e(q,u) = qc(q,u) \tag{19}$$

and

$$0 = q \frac{\partial c(q,u)}{\partial q} = q \frac{\partial^2 e(q,u)}{\partial q} \qquad (20)$$

Utility maximisation implies that

$$I = e(q,u) \; . \qquad (21)$$

Since we shall discuss government policy without explicit reference to the government's budget constraint it is worth confirming that, following Walras' law, this constraint is satisfied if consumers and producers satisfy constraints already set out, and foreign trade is balanced.

Government revenue consists of the rents earned by the public sector $w^g v^g$ plus that part of the rents of privately owned resources wv^x which is not left in the hands of consumers as income I, plus net revenue from taxes and subsidies. But

$$w^g v^g + wv^x - I = p^g g + px - qc = sg - tc - \tau z \qquad (22)$$

Using (4g), (5g), (4), (5), (19), (21) and the relationship between the different prices and quantities, so that the government's budget is balanced.

The effects of tax changes : world prices fixed, all goods traded

Suppose that there are no nontraded goods (so n = m) and that world prices π are fixed. There are n market equilibrium conditions, one for each good:

$$c(q,u) = x(p,v^x) + g(p^g, v^g) + z \; . \qquad (23)$$

Changes in q, p and p^g for given y^x and y^g imply that changes in u and z must satisfy

$$c_u du = - Cdq + Xdp + Gdp^g + dz \tag{24}$$

where $C = \partial c/\partial q$, $X = \partial x/\partial p$ and $G = \partial g/\partial p^g$. Premultiplying by π gives

$$(\pi c_u)du = - \pi Cdq + \pi Xdp + \pi Gdp^g + \pi dz \tag{25}$$

$$= (t + \tau)Cdq - \tau Xdp - (s + \tau)Gdp^g + \pi dz \tag{26}$$

$$= (t + \tau)C(dt + d\tau) - \tau Xd\tau - (s + \tau)G(ds + d\tau) + \pi dz \tag{27}$$

from (19), (8), (8g) and the definitional relationships between prices and taxes.

It is now easy to see, following Kemp (1968) and Hatta (1976) that $\pi c_u \geq 0$ is required for stability of equilibrium. Informally, the point is that $\pi c_u = qc_u - (t + \tau)c_u = (1/\lambda)(1 - (t + \tau)c_I)$ where $\lambda = (1/qc_u)$ is from (21) seen to be the marginal utility of income and c_I to be the effect of income on demand. If the government were to increase by one unit the lump sum income I of consumers keeping all prices constant, then the change in tax revenue received would be $(t + \tau)c_I$ units. If $1 - (t + \tau)c_I < 0$ the government's revenue would actually increase, so leading to further increases in consumers' expenditure. The equilibrium is unstable.

More formally, (26) or (27) with all prices constant show that $(\pi c_u)du = \pi dz = d(\pi z)$, so that at an equilibrium with $\pi c_u \leq 0$, a positive perturbation of real income with prices constant would cause a change in net trades which would not violate the balance of payments constraint, and would thus be feasible. This is incompatible with stability of the equilibrium so in (26) and (27) we can assume that $\pi c_u > 0$ and that (11) is satisfied with equality, so that $\pi dz = 0$ giving

$$(\pi c_u)du = (t + \tau)Cdq - \tau Xdp - (s + \tau)Gdp^g \tag{26a}$$

$$= (t + \tau)C(dt + d\tau) - \tau Xd\tau - (s + \tau)G(ds + d\tau) \tag{27a}$$

Now consider reductions in taxation of the following form. Let $dt + d\tau = (t + \tau)d\psi^c$, $d\tau = \tau d\psi^x$, and $ds + d\tau = (s + \tau)d\psi^g$ where ψ^c, ψ^x and ψ^g are positive scalars. Then (27a) becomes

$$(\pi c_u)du = [(t + \tau)C(t + \tau)]d\psi^c - [\tau X\tau]d\psi^x - [(s + \tau)G(s + \tau)]d\psi^g \tag{28}$$

and since C, -X and -G are negative semi-definite matrices, reductions in ψ^c, ψ^x and ψ^g will each independently raise welfare. Hence if we define "distortions" as meaning divergences between a set of domestic prices and world prices we have:

Proposition 1 If all goods are traded at fixed world prices, a proportionate reduction in (i) all consumption distortions, or (ii) all private production distortions, or (iii) all public production distortions, with, in each case, all other distortions constant, will raise welfare at a stable equilibrium.

From (26a) or (27a) we can deduce the corollary which justifies the use of the term "distortions":

Corollary 1 (i) A necessary condition for optimality is that each of $(t + \tau)C$, τX, $(s + \tau)G$ be zero;

(ii) A necessary condition for optimality if C, X and G are of rank n-1 is that all taxes be zero.

Of course, in Corollary 1(ii) we are implicitly extending our terminology to include as zero taxes those trivial taxes which do not change relative prices.

The most significant feature of Proposition 1 is that we can discuss the effects of changes in some distortions, keeping others fixed. The source of this apparent contradiction of the general negative second-best theorem is that foreign trade at fixed prices essentially separates decisions in the three sectors. It is well known, see Negishi (1972) and Boadway and Harris (1977) for example, that separability assumptions allow considerable progress to be made in second-best theory.

The effects of tax changes : the general cases

The introduction of nontraded goods or variability in world prices destroys the separability which was the source of Proposition 1. It also introduces an ambiguity into the meaning of reductions in distortions. With world prices fixed, a distortion change implies an equal price change. In general, however, this is not so.

We have $n(> m)$ equilibrium conditions

$$c(q,u) = x(p,v^x) + g(p^g, v^g) + \{z,0\} \qquad (29)$$

whence for given v^x, v^g price changes satisfy

$$c_u du = - Cdq + Xdp + Gdp^g + \{dz,0\} \qquad (30)$$

so that if we define $p^* = (\pi^*, p_u)$ we have

$$(p^* c_u) du = p^* Cdq + p^* Xdp + p^* Gdp^g + \pi^* dz \qquad (31)$$

From the definition of $t(\pi^*)$ in (12) it is immediately evident that a change dz, at an equilibrium, satisfying $d(\pi(z)z) \leqq 0$, must satisfy $\pi^* dz \leqq 0$ (or, equivalently, one can observe that $d(\pi(z)z) = (\pi + \tau^*)dz)$ so that, by the same argument as in the previous section, $p^* c_u > 0$ is required for stability and we can assume that $\pi^* dz = 0$.

Defining t_z, s_z and τ_z as the distortions measured with respect to the prices p^*, that is $q = p^* + t_z$, $p^g = p^* + s_z$, $p = p^* + \tau_z$, we can write (31) as

$$(p^* c_u) du = t_z Cdq - \tau_z Xdp - s_z Gdp^g \,. \qquad (32)$$

Recalling that π^* is equal (up to a factor of proportionality which can be eliminated by choice of numeraire) to world prices plus optimal tariffs, we see that τ_z is the vector of differences between actual tariffs and optimal tariffs, while $t_z = t + \tau_z$ and $s_z = s + \tau_z$.

Now consider changes in taxes which change the distorted prices in proportion to the distortions in each sector, that is changes of the form $dq = t_z d\psi^c$, $dp^g = s_z d\psi^g$, $dp = \tau_z d\psi^x$ where ψ^c, ψ^x and ψ^g are scalars. (31) becomes

$$(p^* c_u) du = [t_z C t_z] d\psi^c - [\tau_z X \tau_z] d\psi^x - [\tau_z G \tau_z] d\psi^g \qquad (33)$$

Then the negative semi-definiteness of the matrices C, -X and -G ensures that a change in prices in any one sector proportional and opposite in sign to the distortions in that sector, with prices in other sectors constant, will be welfare improving.

In fact it is fairly easy to see that the same argument goes through if either of the domestic producer price vectors is taken as the "base" against which distortions are measured. We can derive from (30) and (13)

$$(pc_u)du = t\hat{C}dq - sGdp^g + \tau_z Ddp^* \qquad (34)$$

where D is the positive semi-definite matrix with $\partial^2 t/\partial\pi^{*2}$ as its leading mxm submatrix and zeros elsewhere, and letting $dq = td\psi^c$, $dp^g = sd\psi^g$, $d\pi^* = -\tau_z d\psi^x$ in (34) gives the required result since $pc_u > 0$, is, by the same argument as before, required for stability. Symmetrically, the argument goes through with the prices p^g as the base prices.

Hence whether the base prices are taken to be the "world efficiency prices" π^* for traded goods and private producer prices for nontraded goods with distortions defined as the differences between those prices and the respective sets of domestic prices, or the base prices are taken to be one of the sets of domestic producer prices with distortions defined as the differences between them and, respectively, the other sets of domestic prices and the set of world efficiency prices, we have:

<u>Proposition 2</u> A change in all of the prices in any one sector proportional and opposite in sign to the distortions in those prices, with all other distorted prices constant, will raise welfare at a stable equilibrium.

From (32) we have

Corollary 2 (i) A necessary condition for optimality is that each of t_zC, τ_zX and s_zG be zero.

(ii) A necessary condition for optimality if C, X and G are each of rank n-1 is that all domestic taxes be zero and trade taxes be optimum tariffs.

Slight variants of Corollary 2 could be derived from (34) and expressions symmetrical to (34) but with different "base" prices, but they are omitted.

The results summarised in Proposition 2 may, at first sight, seem as strong as the results in Proposition 1, but this appearance is illusory. Proposition 1 describes genuinely piecemeal policy changes, where taxes in only one sector at a time are being changed. The policy change described in Proposition 2 involves a change in only one set of prices at a time, but to hold prices constant elsewhere would generally require active policy intervention in all sectors. For example, a change in consumer taxes will change world prices and thereby producer prices unless producer taxes are changed. Further, as Dixit (1975) has pointed out in commenting on his Theorem 1 of which Proposition 2 is a generalisation, the calculation of tax changes throughout the economy which will change one set of prices in the desired way and keep other prices constant would impose heavy computational and informational demands on the policy-maker.

So it is natural to turn to an alternative generalisation of Proposition 1, where it is changes in distortions rather than prices which are discussed. (30) can be rewritten using (13) as

$$c_u du = - Cdt_z + Xd\tau_z + Gds_z + Hdp^* \qquad (35)$$

where $H = D + X + G - C$, and p^*, t_z, s_z, and τ_z are as before.

It is reasonable to assume that, although each of X, G, C and D is singular, H is nonsingular if the distortions are nontrivial, so that H is positive definite. (The consequences of H being singular are discussed in the appendix.) With H nonsingular, (35) can be rewritten as

$$H^{-1}c_u du = - H^{-1}(Cdt_z - Xd\tau_z - Gds_z) + dp^* \qquad (36)$$

Writing the first row of H^{-1} as h^*, we can write the first row of (36) as

$$(h^* c_u) du = - h^*(Cdt_z - Xd\tau_z - Gds_z) + dp_1^* \qquad (37)$$

We can choose good 1 as the numeraire so that $p_1^* = \pi_1^* = 1$. Recall that this price is the marginal value of good 1 in foreign trade. (37) shows that with $dt_z = ds_z = d\tau_z = 0$ a rise in real income will raise the domestic market clearing value of p_1^* above 1 if $h^* c_u > 0$. But that is just what ought to be the case at a stable equilibrium. Thus $h^* c_u > 0$ is required for stability. Setting $dp_1^* = 0$ in (37) gives

$$(h^* c_u) du = - h^*(Cdt_z - Xd\tau_z - Gds_z) \ . \qquad (37a)$$

(Note that h^* plays a role similar to p^* in the previous argument: it is rather like a price vector, but it is easy to see from simple examples, that h^* need not be nonnegative. There are, however, circumstances in which h^* is nonnegative: in particular, if all the off-diagonal elements of H are negative.)

Now consider a proportional change in all distortions: $dt_z = t_z d\psi$, $ds_z = s_z d\psi$, $d\tau_z = \tau_z d\psi$ where ψ is a scalar. Then, using (20), (8) and (8g),

$$(h^* c_u)du = - h^*(Ct_z - X\tau_z - Gs_z)d\psi$$

$$= h^*(C - X - G)p^* d\psi \qquad (38)$$

From the definition of H, $H^{-1}(C - X - G) = H^{-1}D - I$ so $H^{-1}(C - X - G)p^* = H^{-1}Dp^* - p^* = - p^*$ since the definition of D implies that $p^*D = 0$ from (15). Hence $h^*(C - X - G)p^* = - p_1^* = -1$ and (38) becomes

$$(h^* c_u)du \; = \; - d\psi \qquad (39)$$

Thus:

<u>Proposition 3</u> A proportionate reduction in <u>all</u> the distortions in the economy will raise welfare at a stable equilibrium.

It would be desirable to look at the issue of whether reductions in <u>taxes</u> t, τ and s, rather than <u>distortions</u>, would give a similar result. One would then have to look at how the τ^* vary as taxes changes, which leads to some considerable complications, and this question is not tackled here.

<u>Constraints on prices and distortions</u>

Optimal policy when the policy-maker is constrained to fix nonoptimally the values of certain quantities has been extensively discussed. (For references and further discussion see Smith (1980, pp. 16-17).) But many constraints are more plausibly expressed in terms of prices: the need to hold down the price of a politically sensitive consumer good, the need to raise the price of an imported good in order to raise the incomes of owners of resources used intensively in the production of substitutes, and so on. Then the dual approach used here proves fruitful.

Consider first the case of fixed world prices and no non-traded goods. The relevant equations are (26a) and (27a). It is immaterial whether we think of the constraints on the government's taxing power as being a fixed price, or a fixed tax rate with respect to world prices, as both are the same.

If part of all of dq is constrained to be zero in (26a) the first order conditions with respect to dp and dp^g are $\tau X = 0$ and $(s + \tau)G = 0$, which are satisfied only by setting $s = \tau = 0$, if X and G are of rank n-1. If only part of dq is constrained to be zero the first order conditions for the remaining q are equations of the form $(t + \tau)(\partial c/\partial q_j) = 0$ a set of $r(< n-1)$ equations which will in general be solved by choosing nonzero values for the r unconstrained t_j. A similar argument holds when part or all of the p or p^g vectors are constrained.

In fact this result is simply another way of looking at Proposition 1: it is optimal to remove all distortions in any sector in which this is possible, independently of the presence of distortions in other sectors.

The way that Propositions 2 and 3 relate to Proposition 1 then indicates what is the optimal policy with fixed distortions when there are nontraded goods or world prices are variable.

If it is <u>prices</u> which are constrained to be fixed the argument is essentially unchanged. Equation (32) shows that if, for example, part of q is fixed so that part of dq is zero, the remaining first-order conditions are that distortions in production should be removed, but that in general all consumer prices should be distorted. This could also be directly deduced from Proposition 2.

If, however, it is <u>distortions</u> which are constrained, the conclusion is different. The relevant equation is (37a) which can be written in several different ways:

$$(h^* c_u)du = - h^* Cdt_z + h^* Xd\tau_z + h^* Gds_z \qquad (40)$$

$$= - h^* Cdt + h^* (X - C)d\tau_z + h^* Gds_z \qquad (41)$$

$$= - h^* Cdt_z + h^* (G + X)d\tau_z + h^* Gds \qquad (42)$$

$$= - h^* Cdt + h^* (G + X - C)d\tau_z + h^* Gds \qquad (43)$$

$$= - h^* Cdt - h^* Dd\tau_z + h^* Gds \qquad (44)$$

where (41)-(43) follow from the definitions of the distortions, while (44) follows from the fact that $H^{-1}(G + X - C) = H^{-1}D - I$ and the choice of numeraire, implying that $\tau_{z1} = 0$. It is simplest initially to suppose that all goods are traded.

Suppose now, for example, that part or all of the t_z vector is constrained, but τ_z and s_z are freely chosen. From (40) we derive first order conditions for optimal choice of τ_z and s_z to be $h^* X = h^* G = 0$. Thus taxes should be chosen so that $p = p^g$ with each being proportional to h^*. (Recall that h^* need not be non-negative, so the possibility exists that elements of p and p^g should be negative.) However, h^* will be proportional neither to p^* nor to q. This follows from the fact that

$$h^*(D - C) = h^*(D + X + G - C) = (1,0,\ldots,0)$$

so that, for example, if $h^*D = 0$ (that is, if h^* were proportional to p^* so $p = p^g = p^*$) this would imply that $h^*C = (1,0,\ldots,0)$, so that $h^*Cq = q_1$. But $Cq = 0$, and we have a contradiction. There is a symmetrical argument to rule out the possibility that $q = p^g = p$.

Similarly, if s_z is fixed, consumers and the private sector should face the same prices, given by h^*, these prices being different both from p^* and p^g. If τ_z is fixed, $q = p^g \neq p \neq p^*$. If it is t which is fixed, from (44) we see that it is optimal to have public producers face world prices, so $p^g = p^* \neq q \neq p^g$. If s is fixed, $q = p^* \neq p \neq p^g$. Finally, if t-s, the set of distortions between consumers and public producers, is fixed, from a slight manipulation of (44) one can derive the optimal policy to be $p^* = p \neq p^g \neq q$.

When not all goods are traded there is the additional problem that part of τ_z is constrained to be zero. This leads to some complications which can be dealt with by further manipulation of equations (40)-(44), but the structure of the optimal policy rules is unchanged. Thus we have:

<u>Proposition 4</u>(i) If some prices in a sector of the economy are constrained to be fixed, it is optimal to have no distortions between the remaining sectors of the economy.

(ii) If between prices in two sectors of the economy there are distortions which are constrained to be fixed, it is optimal to have no distortions between the remaining sectors.

Two slightly fanciful examples may clarify the difference between the two sorts of cases discussed above. Suppose the government believes that cigarette smoking is more damaging than smokers realise, that all attempts to persuade smokers of this have failed, and that the political power of smokers prevents the price of cigarettes being raised above a certain level. The optimal policy for the paternalistic government is to impose other consumer taxes (in a way that could presumably be interpreted as subsidising substitutes and taxing complements). It should not, however, impose any production or trade taxes, as the introduction of production inefficiency does nothing to alleviate the problem caused by the fixed prices. If, on the other hand, the constraint took the form of a fixed tax on consumer purchases from private producers, public production of cigarettes should be at world efficiency prices but private production should be subject to distortionary taxation, with the efficiency loss being traded off against the improving effects on consumer prices given the fixed consumer taxes.

Public sector cost-benefit-analysis

The discussion in the previous section is directly relevant to the theory of public sector cost-benefit analysis in distorted economies. It may be more revealing to change the mode of analysis slightly, by leaving public sector changes in quantity terms, and by allowing resource changes, so that (32) is replaced by

$$(p^* c_u) du = t_z C dq - \tau_z X dp + w^* dv^x + p^* dg \qquad (45)$$

$$= t_z C dq - \tau_z X dp + w^* dv + (p^* dg - w^* dv^g) \qquad (46)$$

where $w^* = p^*(\partial x/\partial v)$, the value marginal products of resources in private production evaluated at the prices p^*.

It ought to be clear from the earlier discussion that the case where all goods are traded at fixed world prices will be a significant special case. Proposition 1 shows that sector-by-sector efficiency improvements are welfare improvements, so that it follows that public sector projects should be valued on the basis of world prices. This is the argument of Little and Mirrlees (1974, Chapter 19), more formally treated by Dasgupta and Stiglitz (1974, section 5). It is also essentially the same as Proposition 3 in Bhagwati (1971). The argument can be made explicit by rewriting (46) with $p^* = \pi$ as

$$(\pi c_u)du = (t + \tau)C(dt + d\tau) - \tau X d\tau + w^{\pi}dv + (\pi dg - w^{\pi}dv^g) \qquad (47)$$

where, obviously $w^{\pi} = \pi(\partial x/\partial v)$, the prices that Findlay and Wellisz (1976) and Srinivasan and Bhagwati (1978) call foreign-exchange-equivalent shadow prices. With fixed distortions ($dt = d\tau = 0$) and given resources available to the economy as a whole ($dv = 0$) a public sector project producing net outputs of traded goods dg requiring resource inputs dv^g should be undertaken if and only if it is profitable at world prices for traded goods, foreign-exchange-equivalent prices for resources.

It ought to be equally clear from the earlier discussion that when not all goods are traded at fixed world prices policy prescriptions are much more complicated. <u>If all consumer and producer prices are fixed</u> then (46) shows that appropriate shadow prices are (p^*, w^*): that is net outputs of traded goods are valued at world efficiency prices, net outputs of nontraded goods at private producer prices, and resource inputs at "foreign-exchange-equivalent" shadow prices which value their private marginal products at the prices p^*. This extends the Findlay-Wellisz concept in two directions - world efficiency prices rather than world prices, and private sector prices for nontraded goods. (This second extension makes the terminology somewhat inexact.)

(We could alternatively take p as the "base" prices and modify (34) to become

$$(pc_u)du = tCdq + \tau_z Ddp^* + wdv + (pdg - wdv^g) \qquad (48)$$

but the problems noted below would apply equally to this case.)

The problem with this approach is that it is extremely unlikely that consumer and private producer prices should be unchanged as a result of the public sector project , notably if the project is a net producer or consumer of nontraded goods. But even if all goods are traded, p and q will remain unchanged only if world prices are constant. The only other possibility in either case is that the government deliberately alters taxes so as to keep the prices constant.

The structure of the argument in the previous section should suggest an alternative approach. Suppose that τ_z and t_z are fixed. Then

$$c_u du = - Cdq + Xdp + Ddp^* + (\partial x/\partial v)dv$$
$$+ (dg - (\partial x/\partial v)dv^g) \qquad (49)$$
$$= - Cdt_z + Xd\tau_z + Kdp^* + (\partial x/\partial v)dv$$
$$+ (dg - (\partial x/\partial v)dv^g) \qquad (50)$$

where $K = D + X - C$. Writing k^* as the first row of K^{-1}, and recalling that $dp_1^* = 0$, we have

$$(k^* c_u)du = - k^* Cdt_z + k^* Xd\tau_z + w^k dv$$
$$+ (k^* dg - w^k dv^g) \qquad (51)$$

where $w^k = k^*(\partial x/\partial v)$. Thus for fixed t_z and τ_z and given v, the appropriate shadow prices for goods are given by k^*, and for resources by their private marginal products valued at the prices k^*. The analogy with the "prices" h^* in the earlier argument should be clear. In particular note that k^* is not necessarily a non-negative vector. In general k^* will not equal p^*, p or q. Not even for traded goods is there any reason to suppose that the relative shadow prices are equal to relative world, or world efficiency, prices. This suggests that nontraded goods and variable world prices make it unlikely that there is a simple relationship between world prices and shadow prices.

It seems doubtful that the "shadow prices" k^* can be cast in a way that would suggest how they could be practically implemented in a real-world setting without a full general-equilibrium computation, so they are of dubious practical value.

There may, however, be situations where yet another approach might be fruitful. This could be regarded as a more straightforward generalisation of the Findlay-Wellisz approach than the above. Suppose that we can find prices p_u^*, w^π for nontraded goods and resources such that they would be competitive private producer prices if private producers faced the prices π^* for traded goods. That is to say, suppose that $p^\pi X = 0$ and $w^\pi = p^\pi(\partial x/\partial v)$ where $p^\pi = (\pi^*, p_u^\pi)$. Define $\tau^\pi = p - p^\pi$, the vector of "shadow tariffs" and $t^\pi = q - p^\pi = t + \tau^\pi$. Then

$$du = (qc_u)du = -qCdq + qdc \tag{52}$$

$$= t^{\pi}dc + p^{\pi}(dx + dg + \{dz,o\}) \tag{53}$$

$$= t^{\pi}dc + p^{\pi}Xdp + p^{\pi}(\partial x/\partial v)dv^{X} + p^{\pi}dg \tag{54}$$

$$= t^{\pi}dc + w^{\pi}dv + (p^{\pi}dg - w^{\pi}dv^{g}) \tag{55}$$

and, with v given, we have a relatively simple rule for cost-benefit analysis, requiring the use of the "world-equivalent" shadow prices (p^{π}, w^{π}) but also requiring an estimate to be made of the actual change in consumption, and the tax and "shadow tariff" revenue effect of this change to be counted as an additional benefit. This seems related to the rule proposed by Boadway (1975, p. 426) but it is not exactly the same and has the advantage of being generalised to the case of variable world prices and of being a generalisation of the Findlay-Wellisz rule too. (It is also in agreement with the argument of Warr (1977) that world prices are the appropriate shadow prices for traded goods even in the presence of nontraded goods, but Warr does not discuss the pricing of the nontraded goods themselves.)

There are two problems raised by this approach. First, can we find prices satisfying the equations defining p^{π} and w^{π}? Scott (1976), Srinivasan (1978) and Smith (1980) have discussed technologies in which one can, showing that at least some extension of Findlay-Wellisz is possible, but the treatments of Bhagwati and Wan (1979) and Bertrand (1979) have emphasised the fact that w^{π} may not be uniquely defined in many situations.

More fundamentally, what is the value of a rule like that embodied in (55)? The point of doing cost-benefit analysis in a distorted world is that one separates one set of decisions out from the general problem of planning for the whole economy to move towards optimality. Once we introduce nontraded goods and variable world prices, the separability that permits simple, piecemeal decision-making breaks down (or, to put it another way, the assumptions used in Diamond and Mirrlees (1976) to derive simple relationships between shadow prices and the prices in a constant-returns sector are violated). The calculation of shadow prices requires either solution of a full general equilibrium set of equations to derive price vectors such as k^* above, or else the calculation of actual consumption shifts (as in (55) above) which too requires a general equilibrium computation.

One point worth serious consideration is that there may be circumstances in which the project analyst is able to make rough guesses of consumption changes using judgement rather than knowledge of a large number of elasticities. Putting the information requirements in the form of (55) may then be the most useful way to ask a question that can only, in a world of imperfect knowledge, be answered imperfectly.

This contrasts somewhat with the views expressed in Hammond (1979) where the difficulties of using world prices as shadow prices are recognised but perhaps overemphasised in that inadequate attention is given to the extent to which, with suitable assumptions, distortions in one sector do not affect the desirability of optimality in other sectors, that is the various ways described in this paper in which second-best rules are similar to first-best rules. It should be noted in particular that the paradoxical example presented in Hammond's Figure 2 violates the stability condition discussed on pp. 8-9 above. Hammond's solution, further, of using (in the single consumer case) consumer prices to value actual consumption changes would require the public planner to solve a full general equilibrium computation in each calculation. Whatever this is, it is not project-by-project cost-benefit analysis.

Immiserising growth

Just as the reduction in some distortions may not be welfare improving in the presence of other distortions, so an expansion in production possibilities need not, in the presence of distortions, be welfare improving. Growth, in other words, may be immiserising. The power of the dual approach in analysing distortion changes should suggest that it be applied to the theory of immiserising growth. It turns out not only that the phenomenon of immiserising growth can be compactly and revealingly analysed in terms of prices but also that the dual approach reveals that not all distortions can give rise to immiserising growth.

Consider first the case where all goods are traded at fixed world prices. Allowing exogenous changes in v^x and v^g gives in place of (26a):

$$(\pi c_u)du = (t + \tau)Cdq - \tau Xdp - (s + \tau)Gdp^g + w^\pi dv^x + w^{g\pi}dv^g \qquad (56)$$

where, as before, $w^\pi = \pi(\partial x/\partial v)$ the private sector marginal products valued at world prices of resources, while $w^{g\pi} = \pi(\partial g/\partial v)$ the corresponding public sector value marginal products. Thus the effects of resource growth on welfare are measured by the Findlay-Wellisz-Srinivasan-Bhagwati shadow factor prices.

Although $w = p(\partial x/\partial v)$ and $w^g = p^g(\partial g/\partial v)$ are non-negative, being market prices, there is no assurance that w^π and $w^{g\pi}$ are non-negative if p and p^g are different from π, since

$$\begin{aligned} w^\pi &= w - \tau(\partial x/\partial v) \\ w^{g\pi} &= w^g - (s + \tau)(\partial g/\partial v) \end{aligned} \qquad (57)$$

Thus (56) shows that resource increases are immiserising if production distortions are sufficiently great to make the corresponding shadow factor prices negative. That negative shadow factor prices are the dual expression of immiserising growth was observed by Bhagwati, Srinivasan and Wan (1978).

It should be observed that this argument identifies <u>production</u> distortions as the source of immiserising growth whereas earlier discussion has implied, if not explicitly stated, that growth could be immiserising in the presence of <u>any</u> distortions. The point is that consumption distortions appear in (56) only in the term πc_u on the left, but at a stable equilibrium this term must be positive so that consumption distortions cannot be a source of immiserising growth.

Turning to the more general case, with variable world prices and nontraded goods, we can simply add the appropriate terms to (32) to get

$$(p^* c_u)du = t_z Cdq - \tau_z Xdp - s_z Gdp^g + w^* dv^x + w^{g*} dv^g \qquad (58)$$

where $w^* = p^*(\partial x/\partial v)$ and $w^{g*} = p^*(\partial g/\partial v)$. Then the prices w^* and w^{g*} could be taken as identifying the effects of resource growth on welfare, but only if q, p and p^g were unchanged by the resource growth, an unlikely outcome.

So instead it seems more appropriate to add resource growth to equation (40) (or any one of (40)-(44)) to get

$$(h^*c_u)du = - h^*Cdt_z + h^*Xd\tau_z + h^*Gds_z + w^h dv^x + w^{gh}dv^g \qquad (59)$$

where $w^h = h^*(\partial x/\partial v)$ and $w^{gh} = h^*(\partial g/\partial v)$. Thus if it is distortions which are fixed, it is the shadow factor prices w^h and w^{gh} which determine whether growth in any resource is immiserising. Now we have even less ground to rule this out as an unlikely possibility, for the goods "price" vector h^* may itself have negative elements.

Finally note the possibility of formally analysing in this framework the effects of inflows of foreign-owned factors of production, as discussed in Bhagwati (1979). Only the simplest case is considered here, as an indication of how the analysis could be developed. Suppose all goods are traded at fixed world prices. In addition, the private sector can obtain at market prices resources from foreigners so that $v^\pi = v^1 + v^2$ where v^2 are the foreign-owned resources. The balance of payments constraint is

$$\pi z + wv^2 = 0 \qquad (60)$$

Suppose w is fixed. Then equilibrium-preserving changes must satisfy

$$c_u du = - Cdq + Xdp + (\partial x/\partial v)(dv^1 + dv^2) + Gdp^g + (\partial g/\partial v)dv^g + dz \qquad (61)$$

and

$$(\pi c_u)du = (t + \tau)Cdq - \tau Xdp - (s + \tau)Gdp^g + w^\pi dv^1 + w^{g\pi}dv^g + (w^\pi - w)dv^2 \qquad (62)$$

$$= (t + \tau)Cdq - \tau Xdp - (s + \tau)Gdp^g$$

$$+ w^{\pi}dv^1 + w^{g\pi}dv^g - \tau(\partial x/\partial v)dv^2 \qquad (63)$$

Where (62) follows from (60) and (61) while (63) is derived using (57). Thus in a small economy whose trade is distorted by tariffs, the value of imported factors of production is given by $-\tau(\partial x/\partial v)$. In particular we can at once derive the result of Bhagwati (1979, p. 11) that in a two good model a positive tariff protecting a capital-intensive importable will imply that capital imports are immiserising: we have $\tau_2 > 0$, $\partial x_2/\partial v_k > 0$ (the Rybczynski result for the capital intensive good), so that the coefficient of dv_k^2 is negative.

Conclusion

This paper has not attempted to analyse the full range of second-best issues: there are, for example, problems discussed in Bhagwati (1971) for which the dual approach is not especially insightful and which have therefore not been discussed here; there are a much broader range of possible restrictions on taxation, as discussed in Stiglitz and Dasgupta (1971), than those considered here; questions of income distribution have been left to another paper; and the distortions which have been discussed have been entirely of the form of taxes, or "policy-imposed" distortions, as opposed to the "endogenous" distortions that would arise from market failure of different kinds. Among the specific issues that could be, but have not been, discussed in this framework is the impact of quantitative trade restrictions. It should be clear, however, that the methods of analysis can be extended to a wider range of issues than those considered here.

The principal policy conclusions can be summarised as follows. The results most relevant to piecemeal policy-making in a distorted economy hold strictly only in small open economies in which all goods are internationally traded, for they rely on the separability created by fixed world prices. In other circumstances, second-best policies are genuinely different from, though clearly related to, first-best policies.

APPENDIX

The purpose of this appendix is to consider two technical matters which were left aside in the paper itself: the differentiability of the revenue functions, and the nonsingularity of the matrices H and K.

Nondifferentiable revenue functions

In the paper it was assumed that the revenue functions were twice differentiable everywhere. This is an assumption which is quite likely to fail to be satisfied. If the number of resources is less than the number of production activities the transformation set has a ruled surface so that for any given p there are a set of values of x which maximise r. (See Kemp, Khang and Uekawa, 1978, and Chipman, 1980.) Well known examples are the simple Ricardian model with one factor and two outputs, when the transformation curve is a straight line, and the two-factor three-good variant of the Heckscher-Ohlin-Samuelson model whose Lerner-Pearce diagram shows clearly the indeterminacy of output at any set of prices at which all three goods are produced. In such a situation it is clear that (2) cannot be satisfied for there is not a well-defined supply function. In fact the revenue function does not have a derivative function defined independently of the direction of differentiation.

Similar problems arise with the trade revenue function. It is easy to imagine circumstances in which some though not all relative prices are fixed on the world market, so the foreign offer set has a ruled surface. The case of all world prices being fixed is the extreme case.

Rigorous treatment of these problems would be possible either by considering prices as functions of quantities or by using the subdifferential functions of Rockafellar (1970). The latter approach would be close to the methods adopted in the paper, though considerably more demanding technically. Here I only sketch a treatment of this problem.

In the paper we have dealt with one case of nondifferentiability, when all world prices were fixed, and it turned out to be the simplest case to analyse. Clearly the method adopted there will work if any <u>one</u> of the trade revenue function , the private production revenue function, the public production revenue function and the expenditure function is not differentiable while the rest are. We take the prices of the non-differentiable sector as the "base" prices and derive from (24) equations analogous to (25)-(27). The analysis proceeds much as in the case of fixed world prices.

This then leaves the case where more than one of the functions is not differentiable at the equilibrium under consideration. Consider the simplest possible example: the two good Ricardian model with fixed world prices, no public sector and no consumption distortions. Let the initial distortion be sufficiently great that the country specialises in the "wrong" good. Reductions in the production distortion have no effect (because all supply price-elasticities are zero) until they bring the domestic producer price to equality with the slope of the production possibility curve. At this point output is indeterminate as is the level of welfare: an infinitesimal distortion reduction brings welfare from its initial level up to the free trade optimal level. Further distortion reductions have no effect. The story is slightly changed if the world price ratio is equal to the slope of the transformation curve. Then it is the final infinitesimal distortion reduction which makes welfare jump to its optimal level, and in the final equilibrium production and trade are indeterminate, though their sum, being the level of consumption, and the value of welfare are determined.

Switching to a slightly more complicated example, the three-good two-factor model, still with fixed world prices, no public sector and no consumption distortions,we see the possibility of yet another complication. If the distorted domestic producer prices are such that only two goods are produced then all is determined and distortion reductions lead to positive welfare increases. However if the domestic prices are compatible with production of all three goods, that is if they coincide with any one of the lines of which the transformation surface consists, then the equilibrium values of production and trade are indeterminate, and, except at the optimum when domestic producer prices equal world prices, consumption and welfare are indeterminate. A distortion reduction that shifts the economy from one set of possible equilibria to another cannot definitely be said to raise welfare (unless the final equilibrium is the free trade optimum). However, at any one set of prices there is an output plan (in which there is specialisation in only two goods) which gives a higher utility level than all others (and also one with a different two-good specialisation, which gives the lowest utility level) and we can say that distortion reductions do lead to welfare improvements in the sense that the maximum level of utility available rises as distortions are reduced (and the minimal level rises also).

It should be fairly clear that introducing consumption distortions, a public production sector, some variable world prices, preferences that are not strictly convex, and larger numbers of goods and factors will lead to richer possible combinations of the complications arising in the above example but no fundamental changes. It should also be clear that the discussion above is some distance away from being rigorous. But these examples should also suggest that only the technical details and not the general shape or economic significance of the arguments are altered by these problems.

Singular H and K matrices

Let us consider only the possibility that the matrix $H = D + X + G - C$ is singular, since the arguments involving the matrix $K = D + X - C$ are very similar.

Let Λ be the diagonal matrix of latent roots of H. These roots are non-negative but not all positive since H is positive semidefinite, but singular by hypothesis. Let M be the orthogonal matrix which diagonalises H : $M'HM = \Lambda$. Define Λ^+ to be the diagonal matrix with zeros on the diagonal where Λ has zeros and λ_i^{-1} on the diagonal where Λ has λ_i, so that $\Lambda\Lambda^+\Lambda = \Lambda$ (and Λ^+ is the generalised inverse of Λ). Let $H^+ = M\Lambda^+M'$. Then using the orthogonality of M,

$$HH^+H = MM'HM\Lambda^+M'HMM' = M\Lambda\Lambda^+\Lambda M$$

$$= M\Lambda M' = H$$

(so H^+ is the generalised inverse of H). Thus premultiplying (35) by $(I - HH^+)$ we have

$$(I - HH^{+})c_u du = - (I - HH^{+})Cdt_z + (I - HH^{+})Xd\tau_z$$
$$+ (I - HH^{+})Gds_z$$

but if we consider changes in distortions of the form $dt_z = t_z d\psi$, $d\tau_z = \tau_z d\psi$, $ds_z = s_z d\psi$, we have

$$(I - HH^{+})c_u du = (I - HH^{+})(X\tau_z + Gs_z - Ct_z)d\psi.$$

But $X\tau_z + Gs_z - Ct_z = - (X + G - C)p^* = - Hp^*$ and since $(I - HH^{+})H = 0$ we have

$$(I - HH^{+})c_u du = 0$$

implying that $du = 0$ since $(I - HH^{+})$ is nonsingular.

This result is easy to understand when one realises that H will be singular only in trivial cases where, essentially, taxes lead to changes in the scale of price vectors but to no real changes so that they cause no real distortions and removal of such "distortions" has no effect.

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