Public Enterprise Pricing, Taxation and Market Structure

by

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Number 180

WARWICK ECONOMIC RESEARCH PAPERS

DEPARTMENT OF ECONOMICS

UNIVERSITY OF WARWICK
COVENTRY

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Number 180

October 1980

Paper prepared for the Public Choice Society Annual Meeting, March 14-16, 1980.

Thanks for helpful comments are due to Avinash Dixit, Vidar Christiansen, John Shoven and participants in the Cornell Economics Research Seminar.

This paper is circulated for discussion purposes only and its contents should be considered preliminary.

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Introduction

It is possible to identify two distinct approaches to the problem of how to set pricing rules for public enterprise. The first, typified by the paper of Baumol and Bradford (1970), regards the problem as identical to that of setting optimal taxes. The second, first expounded in the paper of Boiteux (1956), recognises that in practice, and for a variety of reasons, governments impose budget constraints upon public enterprises, and that pricing of public sector outputs is quite likely to be determined independently of the tax structure.

Our aim is to focus on the latter approach, paying particular attention to an aspect of the problem which has been somewhat neglected. We wish to take account of the fact that when a public enterprise adjusts prices, there are general equilibrium repercussions which take place in the private sector. These effects can be neglected by resorting to the expedient of assuming constant producer prices. This assumption would be inconsistent with the existence of pure profit in a competitive private sector, which is one of the cases we analyse. We consider also the implications of having a monopolistically competitive private sector, and as a polar case the situation where a single many-output monopolist operates in the private sector.

We apply our analysis to the situation where public enterprise prices are being set in the presence of an arbitrary tax structure, in order to make a comparison with the familiar results when taxes are simultaneously optimised. The analysis thus far is concerned only with questions of efficiency. We consider briefly the influence of distributional objectives and conclude by drawing a parallel between the pricing problem we have analysed and the question of devising optimal taxes when some goods are untaxable. This last question has been examined by Stiglitz and Dasgupta (1971), but in the context of a model whose assumptions on technology turn out to imply constant producer prices. Our analysis will proceed in the context of a model in some respects more simple (cf. Munk (1978), Atkinson and Stiglitz (1980)), but avoiding the assumption of constant producer prices.

1. The Model

There are $n + 1$ privately produced or supplied commodities x_i, $i = 0, 1, \ldots, n$. Commodity x_0 is labour. In addition, there are m commodities, z_j, produced by a public enterprise. The $(n + 1)$ - vectors q and p are consumer and producer price vectors respectively of the privately produced commodities. The m-vector r is a consumer price vector for the outputs of the public enterprise.

We assume that there is a single representative consumer whose preferences are represented by the indirect utility function $V(q, r, I)$. I represents the amount of any lump-sum income the consumer may receive. The only source of such income we shall consider will be the profits of firms in the private sector. Then $I = (1 - \tau)\pi$, where τ is some given rate of profit taxation, and π is aggregate private sector profit.

On the production side we assume that labour is the only input to production. The labour required to produce $x = (x_1, x_2, \ldots, x_n)$ is $f(x)$, and similarly $g(z)$ is the labour required to produce $z = (z_1, z_2, \ldots z_m)$. We choose labour as the numeraire and so set $p_0 = 1$.

2. Pricing in the Absence of Taxation

2.1. Perfect competition

We examine first the case where the private sector is competitive and faces a constant returns to scale technology. By the latter assumption we mean specifically that the function f is homogeneous of degree one, and not that each output of the private sector is produced separately under constant returns to scale. It is easy to see that under our assumption that there is only a single input to production, if the latter interpretation of constant returns to scale were taken, the famous Samuelson non-substitution theorem would guarantee fixed producer prices under competitive conditions. We do not wish to make this assumption, since it must necessarily be abandoned when we consider the case of positive profits in the private sector. We are, in effect, allowing for the possibility of joint production, since it is still true that we require each firm to produce under constant returns to scale, in order to invoke the zero profit condition.

The government is assumed to be restricted in the extent to which it can levy lump-sum taxes, T. It may be that $T = 0$, but certainly the revenue thus gained is not sufficient to cover the deficit incurred if the public enterprise prices its outputs at marginal cost. In the tradition of Boiteux (1956), we face the public enterprise with a budget constraint. The problem for the public enterprise is to select a price structure which maximises utility subject to the budget constraint imposed by the government. It wishes to

$$\max_{r} \; V(q,r,T) \quad \text{subject to} \quad \sum_{j=1}^{m} r_j z_j - g(z) - T = 0 \tag{1}$$

The first point to notice about (1) is that when public enterprise prices are adjusted, there will be repercussions on private sector prices. Individual demands for goods depend on q, r, I, so $x = x(q,r,I)$. If this is inverted (with labor as numeraire then $q = q(x,r,I)$. However, both x and I are endogenous variables and so will depend upon r. We shall write the total derivative of q_i with respect to r_k as $\frac{dq_i}{dr_k}$, so that

$$\frac{dq_i}{dr_k} = \sum_{i=1}^{n} \frac{\partial q_i}{\partial x_j} \cdot \frac{\partial x_j}{\partial r_k} + \frac{\partial q_i}{\partial r_k} + \frac{\partial q_i}{\partial I} \cdot \frac{\partial I}{\partial r_k} \tag{2}$$

We may then write the first order conditions

$$\sum_{i=1}^{n} \frac{\partial V}{\partial q_i} \frac{dq_i}{dr_k} + \frac{\partial V}{\partial r_k} + \lambda\left(\sum_{j=1}^{m} (r_j - g_j(z)) \frac{dz_j}{dr_k} + z_j\right) = 0 \qquad k=1,\ldots,m. \tag{3}$$

It is the first term in (3) that may appear unfamiliar, since it would not appear here given either constant producer prices, or optimal taxes in the private sector. It is worth remarking that the problem of ensuring that all markets are in equilibrium is solved first of all by the device of treating q as a reduced form vector of equilibrium private sector prices, and secondly by implicitly equating supply and demand for public enterprise outputs in the constraint.

We know, from the assumption of constant returns to scale, that $\sum_{i=1}^{n} q_i x_i = f(x)$ (zero profit condition), and from the assumption of perfect competition, that $q_i = f_i$, $i = 1,\ldots,n$, (price equals marginal cost). Differentiating the zero profit condition with respect to r_k, we obtain

$$\sum_{i=1}^{n} (q_i - f_i) \frac{\partial x_i}{\partial r_k} + \sum_{i=1}^{n} x_i \frac{dq_i}{dr_k} = 0 , \tag{4}$$

Thus we obtain

$$\sum_{i=1}^{n} x_i \frac{dq_i}{dr_k} = 0 \tag{5}$$

using the fact that $q_i = f_i$, $i = 1,\ldots,n$.

Using Roy's identity, and writing the marginal utility of income as α , we see that

$$\sum_{i=1}^{n} \frac{\partial V}{\partial q_i} \frac{dq_i}{dr_k} = -\alpha \sum_{i=1}^{n} x_i \frac{dq_i}{dr_k} = 0 \tag{6}$$

Equation (3) can now be rewritten as

$$\sum_{j=1}^{m} (r_j - g_j(z)) \left(-\frac{dz_j}{dr_k}\right) = (1 - \alpha/\lambda) \quad z_k \tag{7}$$

which is of course the familiar formula obtained under the assumption of constant producer prices.

It might appear at first sight that assuming zero cross elasticities between public enterprise outputs will generate the rule

$$\frac{r_j - g_j(z)}{r_j} \left(-\frac{r_j}{z_j} \frac{\partial z_j}{\partial r_j}\right) = 1 - \alpha/\lambda \tag{8}$$

which states that the mark-up of price over marginal cost should be inversely proportional to the elasticity of demand. However we see that this is incorrect when we observe that $z = z(q(r), r)$. In other words, demand for the outputs of the public enterprise is affected both directly by r , and also indirectly through the influence of r on private sector prices.

For (8) to hold, we must have

$$\sum_{i} \frac{\partial z_j}{\partial q_i} \cdot \frac{dq_i}{dr_k} + \frac{\partial z_j}{\partial r_k} = 0 \quad \begin{array}{l}\text{for all } k \neq j;\\ \text{for all } j.\end{array} \tag{9}$$

A sufficient condition for (9) to hold is that cross elasticities between all public enterprise outputs should be zero, and that $\frac{\partial z_j}{\partial q_i} = 0$ for all i. The first part of the condition requires that in the absence of unlikely coincidences, income effects are zero in the public sector. The second part of the condition, using the Slutsky equation, implies that

$$\frac{\partial z_j}{\partial q_i} = S_{ji} - x_i \frac{\partial z_j}{\partial I} = 0 \tag{10}$$

and so

$$\frac{\partial x_i}{\partial r_j} = S_{ij} - z_j \frac{\partial x_i}{\partial I} = z_j \frac{\partial x_i}{\partial I} \tag{11}$$

In other words, the feedback effects into the private sector are restricted to being income effects. We can also say something a little more specific about the terms $\frac{dq_i}{dr_k}$. Since we have a single input to production, whose price does not change since we have chosen it as the numeraire, supply decisions in the private sector will depend only upon q. We shall write $x^s = x^s(q)$. Demand will typically depend upon all prices, and so $x^d = x^d(q,r)$. The relationship between q and r is determined implicitly in the equilibrium condition

$$x^s(q) = x^d(q,r) \tag{12}$$

If we denote by a_k the vector $\left(\frac{\partial q_1}{\partial r_k}, \ldots, \frac{\partial q_n}{\partial r_k}\right)$, by b_k the vector

$\left(\frac{\partial x_1^d}{\partial r_k}, \ldots, \frac{\partial x_n^d}{\partial r_k}\right)$, and by J^S and J^D the Jacobians of supply and demand functions (with respect to q alone in the latter case), we find that

$$a_k = \left(J^S - J^D\right)^{-1} b_k \tag{13}$$

which illustrates clearly precisely how the impact on private sector prices of a change in r_k is mediated by the vector of cross-price derivatives of the demand functions.

Now we consider the case where positive profits are earned in the private sector. So we abandon the assumption of constant returns to scale technology, and assume that profits are taxed at some fixed rate τ .

The problem for the public enterprise is now

$$\max V(q, r, (1 - \tau)\pi) \quad \text{subject to} \quad \sum_{i=1}^{m} r_j z_j - g(z) + \tau\pi = 0 \tag{14}$$

The first order conditions are

$$\sum_{i=1}^{n} \frac{\partial V}{\partial q_i} \frac{dq_i}{dr_k} + \frac{\partial V}{\partial r_k} + \frac{\partial V}{\partial I}(1 - \tau) \frac{\partial \pi}{\partial r_k} +$$

$$\lambda \left(\sum_{j=1}^{m} \left(r_j - g_j(z)\right) \frac{dz_j}{dr_k} + z_k + \tau \frac{\partial \pi}{\partial r_k} \right) = 0 \tag{15}$$

Since

$$\pi = \sum_{i=1}^{n} q_i x_i - f(x) \tag{16}$$

we see that

$$\frac{\partial \pi}{\partial r_k} = \sum_{i=1}^{n} x_i \frac{dq_i}{dr_k} \tag{17}$$

Substituting this into (15) and rearranging we find that

$$\sum_{j=1}^{m} \left(r_j - g_j(z)\right) \left(-\frac{dz_j}{dr_k}\right) = \left(1 - \alpha/\lambda\right) \left(z_k + \tau\frac{\partial \pi}{\partial r_k}\right) \tag{18}$$

We must again interpret this formula with care. The demand functions for the outputs of the public enterprise are $z = z(q(r), r, I(r))$, and the notation $\frac{dz_j}{dr_k}$ has been chosen to remind us of the fact that these are not ordinary demand derivatives, but incorporate two indirect effects, one through private sector prices, the other through income from private sector profits.

In the zero cross elasticity case, making the same assumptions as above we see that

$$\left(r_j - g_j\right)\left(-\frac{dz_j}{dr_j}\right) = \left(1 - \alpha/\lambda\right)\left(z_j + \tau \frac{\partial \pi}{\partial r_j}\right) . \tag{19}$$

Notice first that the term $\frac{dz_j}{dr_j}$ can now be interpreted straightforwardly as a demand derivative, since the indirect effects are by assumption zero. The term $\left(z_j + \tau \frac{\partial \pi}{\partial r_j}\right)$ can be interpreted as the total effect on revenue resulting from an adjustment in the price of z_j. We can get a sharper result if we make two further assumptions: first, that $\lambda > \alpha$, or the social cost of raising an extra unit of revenue is greater than the private marginal utility of income. This is likely to be so in most cases of interest: second, that $\frac{dz_j}{dr_j} < 0$. Notice that this is now equivalent to saying that the demand curve for z_j slopes downwards. We find that

$$r_j \gtreqless g_j(z) \quad \text{as} \quad \frac{r_k z_k}{\pi} \gtreqless - \tau \frac{r_k}{\pi} \frac{\partial \pi}{\partial r_k} \tag{20}$$

Pricing below marginal cost may be required where expenditure on the output is small relative to the level of private profit, and if the elasticity of private profit with respect to output price is negative. Notice also that in the absence of a profits tax the pricing rule reduces to the constant returns case.

2.2. Imperfect Markets

We will consider two cases of market imperfection in the private sector. In the first, the private sector is assumed to be monopolistically competitive. In the second, output in the private sector is governed by the decisions of a single monopolist.

2.2.1. Monopolistic Competition

We assume that a Nash equilibrium is established in the private sector. Each individual producer maximises profit taking the output decisions of all other producers and the public enterprise prices as given.

We will assume for simplicity that each firm produces a single output, so that profit for firm i, π_i, may be written

$$\pi_i = q_i\,(\,x(r), r\,)\;\; x_i(r) \;-\; f^i(\,x_i(r)\,) \tag{21}$$

where $f^i(x_i)$ is the amount of labour needed to produce x_i. In equilibrium each firm will set output so that

$$q_i + x_i \frac{\partial q_i}{\partial x_i} - \frac{df^i}{dx_i} = 0 \tag{22}$$

Summing over all firms, we see that

$$\sum_{i=1}^{n} \left(q_i - \frac{df^i}{dx_i} \right) + \sum_{i=1}^{n} x_i \frac{\partial q_i}{\partial x_i} = 0 . \tag{23}$$

Now we consider the effect on total profit $\pi = \sum_{i=1}^{n} \pi_i$ of a change in a public enterprise price.

$$\frac{\partial \pi}{\partial r_k} = \sum_{i=1}^{n} \left(q_i - \frac{df^i}{dx_i} \right) \frac{\partial x_i}{\partial r_k} + \sum_{i=1}^{n} x_i \frac{\partial q_i}{\partial r_k} + \sum_{i=1}^{n} \sum_{j=1}^{n} x_i \frac{\partial q_i}{\partial x_j} \frac{\partial x_j}{\partial r_k} \tag{24}$$

Using (23) we can rewrite (24) as

$$\frac{\partial \pi}{\partial r_k} = \sum_{i=1}^{n} x_i \frac{\partial q_i}{\partial r_k} + \sum_{i=1}^{n} \sum_{j \neq i} x_i \frac{\partial q_i}{\partial x_j} \frac{\partial x_j}{\partial r_k} \tag{25}$$

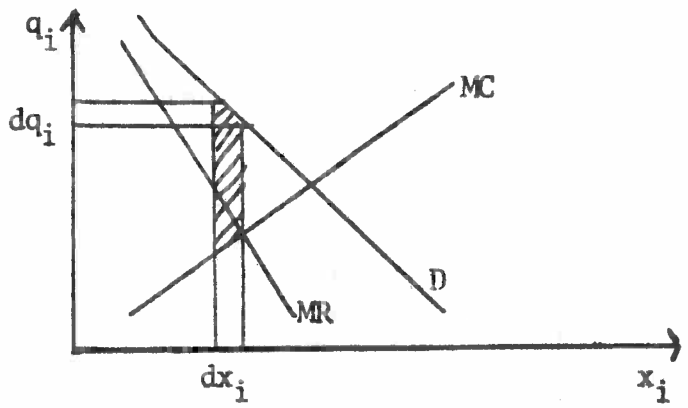
$$= \sum_{i=1}^{n} x_i \frac{dq_i}{dr_k} - \sum_{i=1}^{n} x_i \frac{\partial q_i}{\partial x_i} \frac{\partial x_i}{\partial r_k} \tag{26}$$

The expression differs from the perfectly competitive case by the addition of the term $-\sum_{i=1}^{n} x_i \frac{\partial q_i}{\partial x_i} \frac{\partial x_i}{\partial r_k}$ which from (23) is equal to $\sum_{i=1}^{n} \left(q_i - \frac{df^i}{dx_i} \right) \frac{\partial x_i}{\partial r_k}$.

The first order conditions in (15) remain valid for the new problem, but we now substitute from (26) to obtain

$$\sum_{j=1}^{m}\left(r_j - g_j(z)\right)\left(-\frac{dz_j}{dr_k}\right) = (1-\alpha/\lambda)\left(z_k + \tau\frac{\partial\pi}{\partial r_k}\right) + \alpha/\lambda\sum_{i=1}^{n}\left(q_i - \frac{df^i}{dx_i}\right)\frac{\partial x_i}{\partial r_k}. \quad (27)$$

The right hand side of this expression can be seen as a weighted sum of two terms. The first represents the revenue effect of a price change, both directly through the increase in sales revenue (z_k) and indirectly through the level of profits. The second is the distortion-compensating effect of the price change, and can be thought of as the total change in consumers' and producers' surplus resulting from the output changes. This is illustrated in Figure 1.



The change in consumers' surplus can be written $-x_i(q_i)dq_i$.
The change in monopoly profit is $x_i(q_i)dq_i - q_i dx_i + \frac{df^i}{dx_i}dx_i$.

The weights on these two terms have a natural interpretation. If α/λ is close to one, the social cost of raising revenue by distortionary means is close to the social cost if lump sum taxation were possible. Thus the more important determinant of the structure of public enterprise prices is the distortion-compensating effect. If λ is much larger than α, the revenue effect dominates, and the revenue-raising implications of a particular pricing policy should be of greater concern.

If we consider again the case of zero cross-elasticities, we see that a degree of 'complementarity' between a public enterprise output and outputs of the private sector $\left(\frac{\partial x_i}{\partial r_k} < 0\right)$, may justify pricing at less than marginal cost. This effect becomes more significant the greater the mark-up of price over marginal cost, in other words in markets where there exists a significant degree of market power and demand is relatively inelastic.

2.2.2. Complete Monopoly

Here we assume that the private sector consists of a single monopolist, or alternatively a group who collude to maximise joint profit. There is something of a conceptual problem with the assumption of profit maximisation here, since we must assume that profits are maximised in terms of the numeraire. Certainly, if the monopoly were owned by a group with similar tastes, maximising their utilities would not necessarily correspond to maximising profit in terms of the numeraire. It is probably most convincing to assume that we are dealing with a foreign monopolist who remits profits abroad. We then lose the effect of profit accruing as lump sum income to the consumer, and the pricing rule becomes

$$\sum_{j=1}^{m}\left(r_j - g_j(z)\right)\left(-\frac{dz_j}{dr_k}\right) = \left(1 - \alpha/\lambda\right)\left(z_k + \tau\frac{\partial\pi}{\partial r_k}\right) +$$

$$\alpha/\lambda\left[\sum_{i=1}^{n}\left(q_i - \frac{df^i}{dx_i}\right)\frac{\partial x_i}{\partial r_k} - (1-\tau)\frac{\partial\pi}{\partial r_k}\right] \tag{28}$$

The last bracketed term on the right hand side of the equation tells us that we must offset the loss of after-tax profit remitted abroad against the distortion compensation effect. We have assumed in the analysis above that the foreign monopolist acts as a 'follower' in the sense that it takes public sector prices as given when determining its output decisions. If the reader feels that putting the public enterprise in the position of 'leader' strains credibility somewhat, we observe in concluding this section that a Nash equilibrium between monopolist and public enterprise involves a pricing rule as in (28) except that the term $\sum_{i=1}^{n} \left(q_i - \frac{df^i}{dx_i} \right) \frac{\partial x_i}{\partial r_k}$ disappears.

3. Pricing in the Presence of Taxation

3.1. Arbitrary taxes

The literature has tended to concentrate on pricing rules in the presence of optimal taxes. We shall argue that the rules which emerge for pricing in the presence of arbitrary taxes have a separate application. It is clear that society does best by solving the tax and pricing problems simultaneously. But conditions affecting the optimal tax structure are changing continuously, and yet it is costly for the government to adjust the tax structure frequently. It is reasonable to suppose that it is less costly to adjust a public enterprise price than to change a tax rate. It is certainly true that in practice we observe such price changes much more frequently than we observe tax changes. So we suggest that what the government may wish to do is to use public enterprise prices to bring about a partial adjustment to changed conditions in the economy. Another consideration, relevant particularly in the case of the U.K., is that the tax structure may be determined largely by extraneous considerations. Commitment under the Treaty of Rome to levy Value-Added Tax means that any adjustment to the tax system is constrained to be a major upheaval. Thus responsibility for any fine tuning will devolve upon public enterprises.

We will use the same model as before, but assume now that the government has a fixed revenue requirement R in terms of the numeraire for the provision of public goods. The problem is now

$$\max_{x} V(q, r, I) \quad \text{subject to} \quad \sum_{i=1}^{m} r_j z_j - g(z) + \tau\pi + \sum_{j=0}^{n} t_j x_j = R \tag{29}$$

If we write $T = \sum_{j=0}^{n} t_j x_j$, then it is not difficult to see that the pricing formulae derived in section 2 are in all cases altered only by the addition of a term $\frac{dT}{dr_k}$ on the right hand side. However, it is important to note that this simple result is specific to the case of unit taxes. If we consider ad valorem taxes the picture becomes more complicated, since indirect tax revenue is now $\sum_{j=0}^{n} t_j p_j x_j$ and the t_j are tax rates. Revenue is affected both by quantity changes and by changes in producer prices. What we find now is that there is a close parallel with the formula for monopolistic competition analysed in section 2. We obtain, for the case of perfect competition and constant returns to scale

$$\sum_{j=1}^{m} (r_j - g_j(z)) \left(- \frac{dz_j}{dr_k} \right) = (1 - \alpha/\lambda) \; z_k + \alpha/\lambda \left(-\sum_{i=0}^{n} t_i x_i \frac{dp_i}{dr_k} \right) + \frac{dT}{dr_k} \tag{30}$$

using the fact that $\sum_{i=0}^{n} x_i \frac{dp_i}{dr_k} = 0$.

The term $-\sum_{i=0}^{n} t_i x_i \frac{dp_i}{dr_k}$ appears in exactly the same way as did the term

$\sum_{i=1}^{n} (q_i - f_i) \frac{\partial x_i}{\partial r_k}$. It can be interpreted in the same way as a distortion-compensation term. A reduction in the effective tax on a good can be achieved by reducing the producer price of that good.

The conditions for optimal pricing in the other cases examined in section 1 will all be modified in the same way by the addition of the term

$\alpha/\lambda \left(-\sum_{i=0}^{n} t_i x_i \frac{dp_i}{dr_k} \right)$. So in the monopolistically competitive case we have

a distortion compensation term $\alpha/\lambda \left(\sum_{i=1}^{n} (p_i - f_i) \frac{\partial x_i}{\partial r_k} - \sum t_i x_i \frac{dp_i}{dr_k} \right)$.

3.2 Optimal Taxes

The situation where optimal taxes and public enterprise prices are set simultaneously has been examined extensively in the literature. The problem is given by (29), except that now q , the private sector consumer price vector, is an additional control variable. Therefore we no longer need to take account of the adjustments in these prices as a result of changing the prices of the public enterprise.

If we assume that the private sector faces constant returns to scale, we obtain the familiar Ramsey formula in which public and private sector prices appear symmetrically.

$$\sum_{i=1}^{n} t_i \left(- \frac{\partial x_i}{\partial q_k} \right) + \sum_{j=1}^{m} (r_j - g_j) \left(- \frac{\partial z_j}{\partial q_k} \right) = (1 - \alpha/\lambda)\, x_k \qquad k = 1,\ldots,n.$$

$$\sum_{i=1}^{n} t_i \left(- \frac{\partial x_i}{\partial r_k} \right) + \sum_{j=1}^{m} (r_j - g_j) \left(- \frac{\partial z_j}{\partial r_k} \right) = (1 - \alpha/\lambda)\, z_k \qquad k = 1,\ldots, m. \qquad (31)$$

4. Distribution

So far all our results have been derived from a model in which consumers are identical. We will now consider the implications of introducing distributional considerations. We suppose that the government maximises some welfare function defined on individual utilities. The private sector is competitive and produces under constant returns to scale. The problem of the public enterprise can be expressed as

$$\max W(V^1(q, r), \ldots, V^H(q, r)) \quad \text{subject to} \quad \sum_{h=1}^{H} \sum_{j=1}^{M} r_j z_j^h - g(z) + T = 0 \tag{32}$$

where $z_j = \sum_{h=1}^{H} z_j^h$.

The first order conditions are

$$\sum_{h=1}^{H} \sum_{i=1}^{n} W_h \frac{\partial V^h}{\partial q_i} \frac{dq_i}{dr_k} + \sum_{h=1}^{H} W_h \frac{\partial V^h}{\partial r_k} + \lambda \left[\sum_{h=1}^{H} \sum_{j=1}^{n} (r_j - g_j(z)) \frac{dz_j^h}{dr_k} + \sum_{h=1}^{H} z_k^h \right] = 0 \tag{33}$$

Arguing as before, we see that

$$\sum_{h=1}^{H} \sum_{i=1}^{n} x_i^h \frac{dq_i}{dr_k} = 0 \tag{34}$$

If we introduce the notation $R_k^h = \sum_{i=1}^{n} x_i^h \frac{dq_i}{dr_k}$ we obtain a pricing rule

$$\sum_{h=1}^{H} \sum_{j=1}^{m} \left(r_j - g_j(z)\right) \left(- \frac{dz_j^h}{dr_k}\right) = z_k - \frac{1}{\lambda} \sum_{h=1}^{H} \beta_h \left(R_k^h + z_k^h\right) \tag{35}$$

where $\beta_h = W_h \alpha_h$ is the social marginal utility of income.

Now we know from (34) that $\sum_{h=1}^{H} R_k^h = 0$, but we cannot infer that $\sum_{h=1}^{H} \beta_h R_k^h = 0$. This will only be the case when the government is not concerned about distribution, and $\beta_h = \beta$ all h. The formula then reduces, as we would expect, to what was obtained in the pure efficiency case.

In the zero cross elasticity case we see that the rule

$$\frac{r_k - g_k(z)}{r_k} \left(- \frac{r_k}{z_k} \frac{\partial z_k}{\partial r_k} \right) = 1 - \frac{1}{\lambda} \left[\sum_{h=1}^{H} \beta_h \left(\frac{R_k^h}{z_k} \right) + \sum_{h=1}^{H} \beta_h \left(\frac{z_k^h}{z_k} \right) \right] \tag{36}$$

The last term in the bracket, $\sum_{h=1}^{H} \beta_h \left(\frac{z_k^h}{z_k} \right)$ is what Feldstein (1972) has termed the 'distributional characteristic' of good k. We now have an extra term capturing the effect of the induced price changes in the rest of the economy. We propose calling this term the 'expenditure sensitivity characteristic' of a good. The more sensitive upwards are the expenditures of households with high values of β_h to increase in r_k, the lower the price should be.

It may be instructive at this stage to consider a particular example. Let us assume that individuals do not differ in tastes, but only in terms of productivity. The utility function of household h is

$$u^h = \sum_{i=1}^{n} \gamma_i \log x_i^h + \sum_{j=1}^{m} \gamma_j \log z_j^h + \gamma_0 \log \left(1 + \frac{1}{k^h} x_0^h \right) ; \tag{37}$$

It is easy to show that $x_i^h = (k^h)\bar{x}_i$, $i = 0, 1, \ldots, n$, $h = 1, 2, \ldots, H$. We observe that we may write

$$R_k^h = k^h \sum_{i=1}^{n} x_i \frac{dq_i}{dr_k} = k^h R_k \tag{38}$$

But we know from (34) that $\sum_{h=1}^{H} R_k^h = R_k \sum_{h=1}^{H} k^h = 0$. This must mean that $R_k = 0$. Thus the 'expenditure sensitivity dharacteristic' of the good is equal to zero in this particular case. Notice however that the distributional characteristic of the good is $\sum_{h=1}^{H} \beta_h k^h$, and is not zero.

The example serves to highlight the general equilibrium nature of the effect we are considering. The particular preferences we have considered are consistent with partial equilibrium analysis in all markets.

5. An Alternative Interpretation

Although the preceding analysis has been developed to throw some new light on the public sector pricing problem, it can also be thought of as providing optimal tax formulae when some goods are untaxable. We simply reinterpret the analysis of section 2, with the added assumption that firms in the taxable sector are competitive and produce under constant returns to scale. The expression $\sum_{j=1}^{m} r_j z_j - g(z)$ can then be interpreted as tax revenue, since $g_j(z)$ will be producer prices, and $\sum_{j=1}^{n} g_j(z) z_j = g(z)$.

The above assumptions imply that the analysis can only be applied in situations where imperfectly competitive behaviour appears only in the untaxed sector. This is of course quite unrealistic and further work will need to be done towards relaxing these assumptions.

6. Conclusion

We have argued that it may often be necessary to set public enterprise prices within the framework of a suboptimal tax structure. It then becomes important to take account of the general equilibrium repercussions on prices in the private sector. We have shown that it is possible to derive simple rules which in some respects are significantly different from those normally presented. When the private sector is monopolistically competitive, the pricing rule can be presented in terms of a trade-off between revenue and distortion-compensating effects. We observe that even in the context of a competitive private sector with constant returns to scale technology, if we wish to take account of distributional considerations the pricing rule will differ from the rule which emerges under the assumption of constant producer prices. Finally, we noted that the analysis presented here would be equally applicable to the problem of optimal taxation when the set of taxable commodities is restricted.