A Model of Bilateral Bargaining in the Labour Market

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ABSTRACT

In this paper we combine several strands of basic economic theory to provide a structure for analysing bilateral bargaining in the labour market. Using a quadratic revenue function and a Cobb-Douglas utility function largely for expositional ease, we derive isoprofit and isoutility cuves for one firm and union respectively. These curves are used to derive pareto efficient bargaining loci after the manner of Cartter (1959). We then introduce the concept of a bargaining core and demonstrate how different cores deriving from different preferences may be categorized. We then consider four particular solutions to the bargaining process, the monopolist, monopsonist, Nash and "market power" outcomes. The comparative static properties of these solutions are then examined. Finally in an appendix we consider the general case of the problem and demonstrate which aspects of our earlier analysis are not dependent upon the particular functional forms.

1. Introduction

In most analyses of union employer bargaining in the labour market it is either implicitly or explicitly assumed that the bargaining process takes place along one of two special pareto efficient bargaining loci. These loci have clear explanations. The first, the "wage/profit" maximisation locus corresponds to the case where bargaining takes place only about the nominal wage, and once the wage is set the employer adjusts employment to maximise his profits. The second, the "revenue/wage bill" maximisation locus assumes that the union and employer first cooperate to maximise the total revenue earned and then bargain to distribute the revenue between themselves. The purpose of this paper is to analyse a more general approach to the bargaining problem. As a byproduct of this analysis we shall demonstrate that the two special efficiency loci if adopted injudiciously as the basis for an analysis will provide clear cut but very special results and properties which may in certain contexts be incorrect.

In this analysis we shall initially follow the paths taken by Cartter (1959) and Fellner (1960), we shall treat the union employer problem as we would a 2 person 2 good exchange problem, by defining the locus of pareto efficient exchanges and then attempting to determine at what point upon the locus lies the solution. The fundamental difference from the exchange analysis is that one of the "goods" is actually the price of labour, the wage. Consequently we cannot use prices to define our solution but have to utilise actual explanations of the bargaining process, such as that provided by Nash (1950).

2. The Model

The agents in this model are one firm and one union, the union operates a closed shop and labour is the only variable factor of production. It is assumed that the firms demand curve and production function may be jointly summarised by a quadratic revenue function.[1/]

$$R(L) = bL + cL^2 \qquad b > 0, \quad c < 0 \qquad (2.1)$$

$$R'(L) = b + 2cL$$

hence $R'(L) \gtrless 0$ as $L \lessgtr -\frac{b}{2c}$

$$R''(L) = 2c < 0$$

Hence the revenue function has the shape shown in Figure 1.

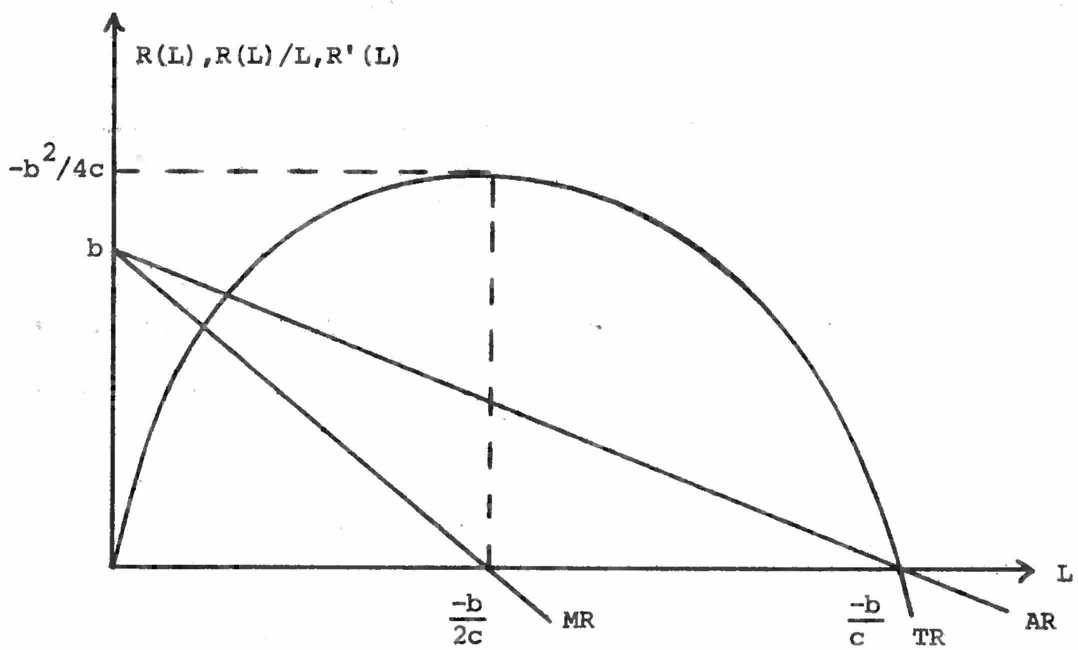

Figure 1

1/ The quadratic revenue function is chosen for ease of exposition. We require only that the revenue function generate convex isoprofit curves. The most general case is examined in the mathematical Appendix A.

(i) The Firms Problem

It is assumed that the firm is a strict profit maximiser and given that labour is the only variable factor in the short-run, its maximand may be written

$$\Pi = R - wL \qquad (2.2)$$

where
Π is total profit
w the homogeneous wage rate
L total labour hours employment
R shorthand for $R(L)$

We define an isoprofit curve as

$$R - wL = \bar{\Pi} \qquad (2.3)$$

substituting in from (2.1) we get (2.4)

$$bL + cL^2 - wL = \bar{\Pi} \qquad (2.4)$$

Taking the total differential and rearranging we obtain

$$\frac{dw}{dL} = \frac{b-w}{L} + 2c \qquad (2.5)$$

hence $\frac{dw}{dL} \gtreqless 0$ as $\left|\frac{b+w}{2c}\right| \lesseqgtr L$

Taking the second total differential

$$\frac{d^2w}{dL^2} = \frac{-\frac{dw}{dL}L - b + w}{L^2} \qquad (2.6)$$

$$= -\frac{2(b-w)-2cL}{L^2} = -\frac{2\bar{\Pi}}{L^3}$$

hence $\frac{d^2w}{dL^2} \gtrless 0$ as $\bar{\Pi} \lessgtr 0$

Hence the family of isoprofit curves have the slope and curvative properties as shown in Figure 2.

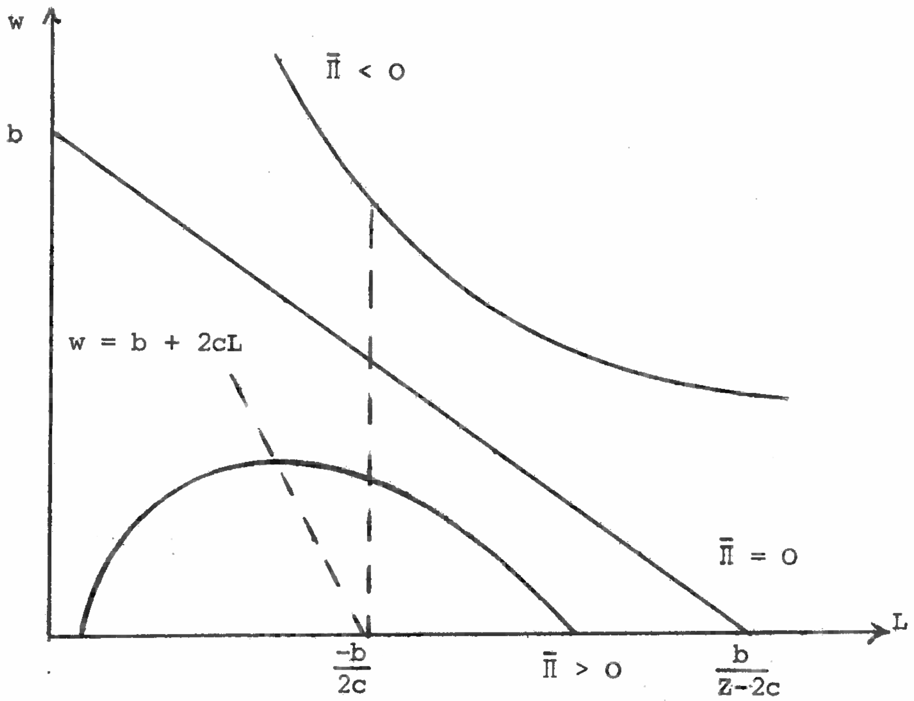

Figure 2

We define Z on the slope of the isoprofit curve when the second derivative (2.6) is zero. $Z < 0$.

We shall only consider cases where the firm obtains non-negative profits hence the isoprofit curves will be quasi-convex. 2/ We now proceed to examining the unions behaviour.

(ii) <u>The Union's Problem</u>

We shall assume that the union's maximand is expressed only over the hourly wage and number of labour hours employment sold. We assume the utility function takes the following form

$$u^u = w^{\alpha} L^{\beta} \qquad \alpha + \beta = 1 \qquad \alpha, \beta \geq 0 \qquad (2.7)$$

the slope of an indifference curve generated by (2.7) is

$$\frac{dw}{dL} = - \frac{\beta}{\alpha} \frac{w}{L} < 0 \qquad (2.8)$$

and curvature

$$\frac{d^2 w}{dL^2} = \frac{\beta}{\alpha} \frac{w}{L^2} > 0$$

The indifference curves are 3/ illustrated in Figure 3.

2/ The linearity of the zero valued isoprofit curve will be useful, but only quasi convexity is required.

3/ Again the only actual requirement is concavity and that the utility function is increasing. In the appendix we demonstrate a more general case.

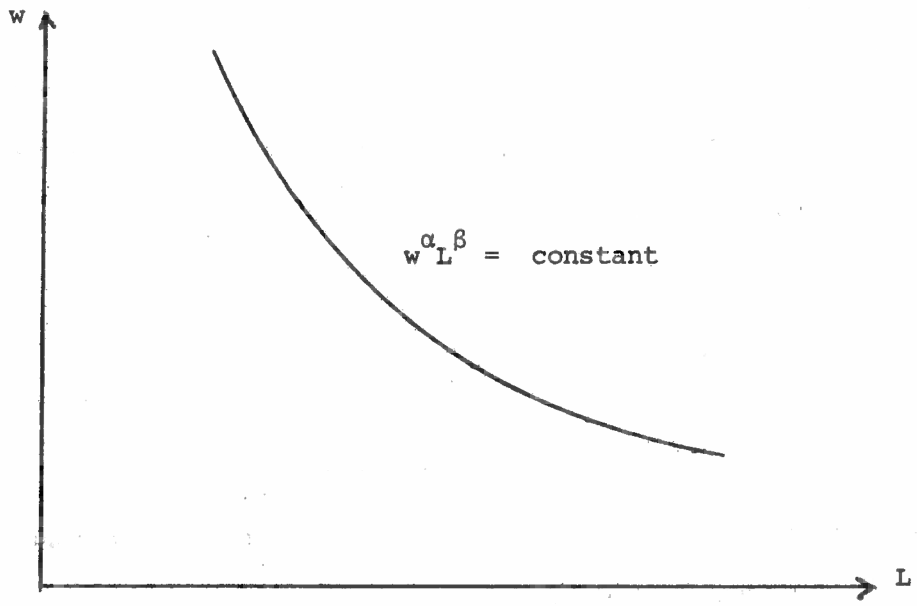

Figure 3

3. Pareto Efficiency and a Bargaining Core

In the previous sections we established isoprofit and indifference curves for the firm and union respectively. Combining these allows us to derive a bargaining lens inside which lies a section of a Pareto efficient bargaining locus, which is of course the bargaining core.

Equating (2.5) and (2.8) and rearranging we get

$$w = \frac{b+2cL}{1-\beta/\alpha} = \frac{R'}{1-\beta/\alpha} \qquad (3.1)$$

This is the locus of Pareto efficient bargaining points and has slope

$$\frac{dw}{dL} = \frac{R''}{1-\beta/\alpha} \gtreqless 0 \quad \text{as} \quad \beta \gtreqless \alpha \qquad (3.2)$$

and curvature

$$\frac{d^2w}{dL^2} = \frac{R'''}{1-\beta/\alpha} = 0 \qquad \forall\ \beta,\alpha \qquad (3.3)$$

Thus using (3.1) and (3.3) we realise that the efficiency loci are rays originating from the point $(w,L) = (0, -b/2c)$.

Defining $U^u = B$ and $\Pi = A$ as the union and employers opportunity costs respectively, we may now represent the bargaining loci, lens and core diagrammatically as in Figure 4.

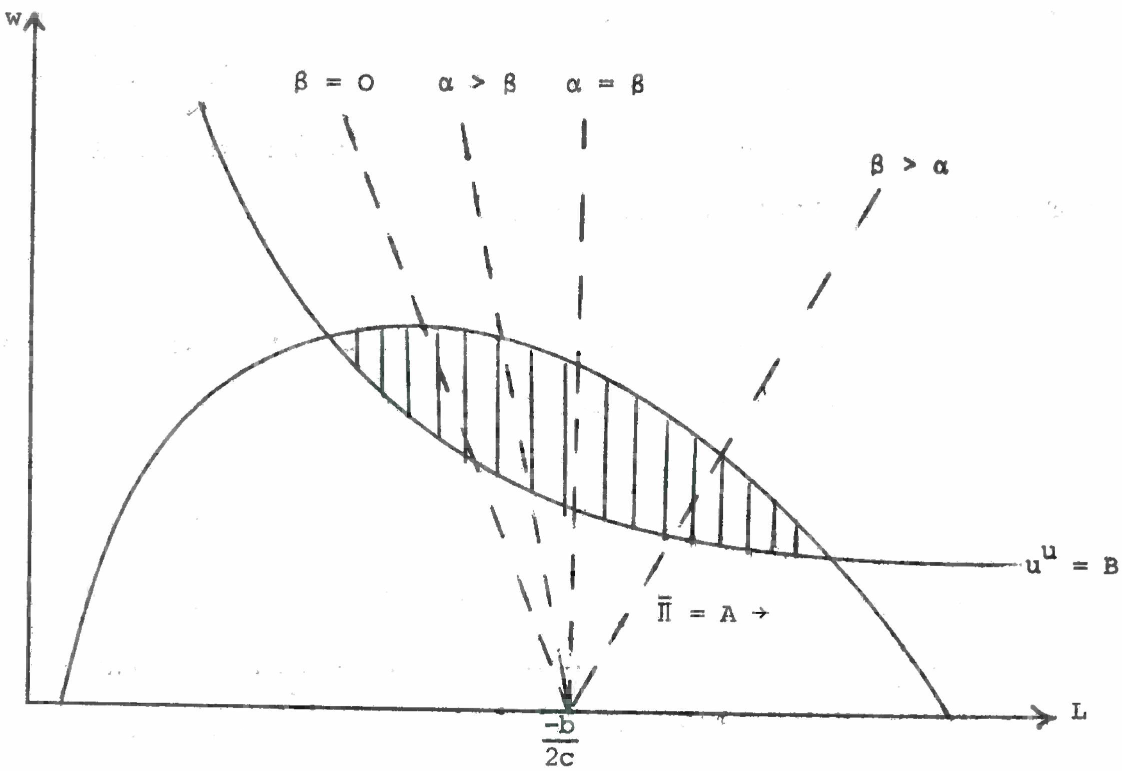

Figure 4

Using (3.2) we see that we may categorise three different sets of efficiency loci according to the relative values of α and β. The two loci $\beta = 0$ and $\alpha = \beta$.

are of special interest[4/] as these are the "profit/wage" and "revenue/wage bill" cases referred to earlier and are defined here as

(i)	$\beta = 0$	$w = R'$	profit/wage locus
(ii)	$\alpha = \beta$	$L = -b/2c$	revenue/wage bill locus

We use these two loci as reference solutions and may thus locate other loci by comparing them to these.

From Figure 4 the interpretation of the bargaining loci is clear, recalling that α and β are elasticities we may state. If the union's utility is more responsive to the level of employment then the locus of Pareto efficient points has positive slope and is located to the right of the "revenue/wage bill" reference locus.

If the union's utility is more responsive to the wage rate then the locus of Pareto efficient points has negative slope and lies betweeen our two reference loci.

We adopted the quadratic and Cobb-Douglas forms for the revenue and utility functions for ease and clarity of exposition. The major advantage these give us is the linearity of the efficient loci we shall now utilise this property to examine actual bargaining solution.

4/ In a more general treatment, as may be found in the appendix, it will be found that only these two reference loci have clear slope and curvature properties. Even so their main use should be to define general solution properties.

The Union as a Monopolist

The union maximises its utility subject to the firms achieving its opportunity cost profit level.

$$\text{Max} \quad U^u = w^\alpha L^\beta$$

$$\text{S.T.} \quad R - wL = A \qquad (3.5)$$

by substitution

$$U^u = \left[\frac{R-A}{L}\right]^\alpha L^\beta \qquad (3.6)$$

using $\alpha + \beta = 1$ and rearranging

$$U^u = (R-A)^{1-\beta} L^{2\beta-1} \qquad (3.7)$$

Maximisation now involves choice of L.

$$\frac{dU^u}{dL} = (1-\beta)(R-A)^{-\beta} R' L^{2\beta-1} + (2\beta-1) L^{2\beta-2} (R-A)^{1-\beta} = 0 \qquad (3.8)$$

$$= (1-\beta) R' L + (2\beta-1)(R-A) = 0$$

Substituting into (3.8) from (2.1) and applying the quadratic formula we obtain

$$L_1, L_2 = \frac{-\beta b \pm \sqrt{(\beta b)^2 + 4c(2\beta A + A)}}{2c} \qquad (3.9)$$

We note that $c < 0$ implies (3.9) must have at least one positive root.

We may also solve for the wage rate by using the positive root of (3.9) into (3.5)

To examine the comparative static properties of the solution we take the total differential of (3.8) and rearrange to give

$$\frac{dL}{dA} = (2\beta-1)/(1-\beta)R' + (1-\beta)R''L + (2\beta-1)R' \qquad (3.10)$$

This comparative static result may be signed as follows. (Proof of which may be found in appendix B).

$$\frac{dL}{dA} \lesseqgtr 0 \quad \text{as} \quad \beta \gtreqless \alpha \qquad (3.11)$$

This agrees with Figure 4 and states that an increase (decrease) in the firm's opportunity cost (perhaps a rise (fall) in interest rates) raises (lowers) employment if the union's employment elasticity is less than its wage elasticity. And does the opposite if the employment elasticity is the greater.

Using (3.1) and (3.11) we write

$$\frac{dw}{dA} = \frac{dw}{dL}\,\frac{dL}{dA} < 0 \qquad \forall \; \alpha,\beta \qquad (3.12)$$

This states that a rise(fall) in the opportunity cost leads to a fall (rise) in the wage rate.[5/]

5/ The clarity of these comparative static results is due to the linearity of the efficiency loci. The result $dw/dA < 0$ will hold up under quite general assumptions.

We summarise these results as

	$\alpha > \beta$	$\beta > \alpha$	$\alpha = \beta$
dL/dA	+	-	O
dw/dA	-	-	-

The Firm as a Monopsonist

The firm maximises profit subject to the union achieving its opportunity cost level of utility. The problem becomes.

$$\text{Max} \quad \Pi = R - wL \tag{3.13}$$

$$\text{S.T.} \quad w^{\alpha}L^{\beta} = B \tag{3.14}$$

By substitution

$$\text{Max} \quad \Pi = R - B^{1/\alpha}L^{1-\beta/\alpha}$$

maximisation now involves choice of L

$$\frac{d\Pi}{dL} = R' - B^{1/\alpha}(1-\beta/\alpha)L^{-\beta/\alpha} = 0 \tag{3.15}$$

which may be solved for L given α,β and would allow us to solve (3.14) for w.

The comparative static properties of this solution are found by taking the total differentiation of (3.15) and rearranging to obtain

$$\frac{dL}{dB} = \frac{1/\alpha B^{1/\alpha-1}(1-\beta/\alpha)L^{-\beta/\alpha}}{R''+(\beta/\alpha)B^{1/\alpha}(1-\beta/\alpha)L^{-(\beta/\alpha+1)}} \qquad (3.16)$$

which is signed as follows (proof in Appendix C).

$$\frac{dL}{dB} \lesseqgtr 0 \quad \text{as} \quad \beta \lesseqgtr \alpha. \qquad (3.17)$$

Hence a rise in the union's opportunity cost increases employment if the union's employment elasticity is greater than its wage eleasticity, and vice versa.

Using (3.17) and (3.1) we obtain

$$\frac{dw}{dB} = \frac{dw}{dL}\frac{dL}{dB} > 0 \qquad \forall\ \alpha,\beta. \qquad (3.18)$$

A rise (fall) in the union's opportunity cost always [6/] leads to a rise (fall) in the wage rate. These results are summarised below.

	$\alpha > \beta$	$\beta > \alpha$	$\alpha = \beta$
dL/dB	−	+	0
dw/dB	+	+	+

The Nash Solution

The Nash solution requires that the two bargainers cooperatively maximise the product of their utility surpluses

6/ Again the clarity of these comparative static results is due to the linearity of the efficiency loci. The result dw/dB > 0 will hold up under quite general assumptions.

$$\text{Max} \quad \text{Prod} = \left[w^{\alpha}L^{\beta}-B\right]\left[R - wL - A\right] \tag{3.19}$$

$$\therefore \quad \text{Prod} = w^{\alpha}L^{\beta}R - w^{\alpha+1}L^{\beta+1} - w^{\alpha}L^{\beta}A - BR + BwL + AB$$

differentiating

$$\frac{\partial\,[\text{Prod}]}{\partial w} = \alpha w^{\alpha-1}L^{\beta}R - (\alpha+1)w^{\alpha}L^{\beta+1} - \alpha w^{\alpha-1}L^{\beta}A + BL = 0 \tag{3.20}$$

$$\frac{\partial\,[\text{Prod}]}{\partial L} = \beta w^{\alpha}L^{\beta-1}R + w^{\alpha}L^{\beta}R' - (\beta+1)w^{\alpha+1}L^{\beta} - \beta w^{\alpha}L^{\beta-1}A \tag{3.21}$$

$$- BR' + Bw = 0$$

To clarify we can solve for L at $A = B = 0$ yielding (3.22). Using (3.20), (3.21) and (2.1) we obtain

$$L = \frac{b}{c}\,\frac{\left[1-\beta + (1-\beta^2)/(2-\beta))\right]}{\left[\beta-2 - (1-\beta^2)/(2-\beta))\right]} \tag{3.22}$$

and from (3.20) and (3.21) we obtain

$$w = bL + cL^2 \tag{5.23}$$

Hence we may solve (3.23) for w given L from (3.22).

The comparative static properties of this solution are found by allowing $A > 0$ and considering changes in A. We may then reduce the F.O.C. (3.20) and (3.21) to

$$\beta R + LR' - (\beta+1)(\alpha/(\alpha+1))(R-A) - \beta A = 0 \tag{5.24}$$

Taking the total differential, using $\alpha + \beta = 1$ and rearranging we obtain

$$\frac{dL}{dA} = \frac{2\beta-1}{\beta R' + LR'' + R' - \frac{1-\beta^2}{2-\beta} R'} \qquad (3.25)$$

which may be signed as follows (see Appendix D)

$$\frac{dL}{dA} \lessgtr 0 \quad \text{as} \quad \beta \gtrless \alpha . \qquad (3.26)$$

Using (3.1) and (3.26) we may immediately write

$$\frac{dw}{dA} = \frac{dw}{dL}\frac{dL}{dA} < 0 \qquad \forall\ \alpha,\beta \qquad (3.27)$$

both (3.26) and (3.27) accord with Figure 4.

To consider the effects of changes in the unions opportunity cost, let $\beta > 0$ and allow it to vary. Using (3.1) and $\alpha + \beta = 1$ we rewrite (3.20) as

$$B = (1-\beta)R'^{-\beta}(1-\beta/\alpha)^{\beta}L^{\beta-1}R - (2-\beta)R'^{1-\beta}(1-\beta/\alpha)^{\beta-1}L^{\beta} \qquad (3.28)$$

To define the comparative statics of changes in B we would totally differentiate (3.28) and sign the resulting expression. This would be very messy. Rather we note that $dA = f(dB)$ where $f(\cdot)$ is some sign changing function defined along any efficiency loci given by (3.1). Clearly then

$$\frac{dL}{dB} \lessgtr 0 \quad \text{as} \quad \alpha \gtrless \beta \qquad (3.29)$$

and using (3.29) and (3.11) we have 7/

7/ Again in a more general treatment only the comparative static effects of the wage rate are clear.

$$\frac{dw}{dB} = \frac{dw}{dL}\frac{dL}{dB} > 0 \qquad \forall, \alpha, \beta \qquad (3.30)$$

We summarise the results as

	$\alpha > \beta$	$\beta > \alpha$	$\alpha = \beta$
dL/dA	+	-	0
dL/dB	-	+	0
dW/dA	-	-	-
dW/dB	+	+	+

Market Power Solution

The Market Power solution is a non-cooperative alternative to the Nash Solution. It is found by taking a weighted average of the monopolist and monopsonist solutions.

Let ϕ represent then unions and $(1-\phi)$ the firms bargaining power.[8/] Hence from (3.8) and (3.15) we obtain

$$\phi\left[R' - B^{1/\alpha}(1-\beta/\alpha)L^{-\beta/\alpha}\right] + (1+\phi)\left[(1-\beta)R'L + (2\beta-1)(R-A)\right] = 0 \quad (3.31)$$

(3.31) is soluble for L and w may then be obtained from (3.1)

To examine the comparative statics of this solution we totally differentiate (3.31) and rearrange the resultant expression and obtain

8/ This solution is crucially dependent upon the linearity of the efficiency loci but may be regarded as a good approximation if the ~~case~~ is small.

(setting $dB = 0$)

$$\frac{dL}{dA} = \frac{(1-\phi)(2\beta-1)}{\phi\left[\bar{R}''+B^{1/\alpha}(1-\beta/\alpha)\beta/\alpha L^{-(\beta/\alpha+1)}\right] + (1-\phi)\left[(1-\beta)R''L + \beta R'\right]} \tag{3.32}$$

We sign (3.32) as follows. Proof may be found in the Appendix E.

$$\frac{dL}{dA} \gtreqless 0 \quad \text{as} \quad \alpha \gtreqless \beta \tag{3.33}$$

Thus a rise in the firm's opportunity cost raises employment if the union's utility is more responsive to changes in the wage rate, and lowers employment if the reverse is true.

Using (3.1) and (3.33) we have

$$\frac{dw}{dA} = \frac{dw}{dL}\frac{dL}{dA} < 0 \qquad \forall\ \alpha,\beta. \tag{3.34}$$

Hence a rise in the firm's opportunity cost always leads to a fall in the wage rate and vice versa.

The comparative statics of changes in the union's opportunity cost are found by equating dA to zero and again rearranging the total differential of (3.31)

$$\frac{dL}{dB} = \frac{\phi^{1/\alpha}B^{1/\alpha-1}(1-\beta/\alpha)L^{-\beta/\alpha}}{\phi R''+\phi B^{1/\alpha}(1-\beta/\alpha)\beta/\alpha L^{-(\beta/\alpha+1)} + (1-\phi)(1-\beta)R''L} \tag{3.35}$$

$$+ \overline{(1-\phi)(1-\beta)R' + (1-\phi)(2\beta-1)R'}$$

the sign of which may be shown to be (see Appendix F)

$$\frac{dL}{dB} \lesseqgtr 0 \quad \text{as} \quad \alpha \gtreqless \beta \tag{3.36}$$

using (3.36) and (3.1) we have

$$\frac{dw}{dB} = \frac{dw}{dL}\,\frac{dL}{dB} > 0 \qquad \forall\ \alpha,\beta \tag{3.37}$$

(3.36) and (3.37) tell us that a rise in the union's opportunity cost always raises the wage rate, whilst its effect upon employment is positive if the union's employment elasticity of utility is greater than the wage elasticity.

Finally we consider the comparative static effect of a shift in market power.[9/] Taking the total differential of (3.31) we obtain ($A = 0$)

$$\frac{dL}{d\phi} = \frac{\beta^{1/\alpha}(1-\beta/\alpha)L^{-\beta/\alpha} - R'}{\phi R'' + \left(\frac{\beta}{\alpha}\right)\beta^{1/\alpha}(1-\beta/\alpha)L^{-(\beta/\alpha+1)} + (1-\phi)(1-\beta)R'L + (1-\phi)BR'} \tag{3.38}$$

the sign of which may be shown to be (see Appendix G)

$$\frac{dL}{d\phi} \lesseqgtr 0 \quad \text{if} \quad \alpha \gtreqless \beta \tag{3.39}$$

and using (3.39) and (3.1) we have

$$\frac{dw}{d\phi} = \frac{dw}{dL}\,\frac{dL}{d\phi} > 0 \qquad \forall\ \alpha,\beta \tag{3.40}$$

9/ A shift in market power may be due to changes in legislation, a greater percentage of the labour force unionised etc.

This tells us that an increase in the union's market power always increases the wage rate and affects employment in a positive manner if $\alpha < \beta$, i.e. the union's utility is more responsive to changes in the level of employment.

We summarise these results as

	$\alpha > \beta$	$\alpha < \beta$	$\alpha = \beta$
dL/dA	+	–	0
dL/d	–	+	0
dL/dϕ	–	+	0
dw/dA	–	–	–
dw/dB	+	+	+
dw/dϕ	+	+	+

6. Conclusions

In this paper we developed a structure for analysing bilateral bargaining in the labour market. A bargaining loci was defined by deriving isoutility and isoprofit curves for the union and firm respectively, and we defined a bargaining core as the section of the locus of pareto efficient bargaining points which lies in the lens. Reference core were defined so that we could categorise possible solutions in relation to this. The reference core were the profit and joint revenue maximisation locireferred to earlier, it was noted that these loci had certain special properties which although useful could not be regarded as general. Finally we derived four different bargaining solutions and examined their comparative static properties, finding that in this model we can always evaluate the effect of changes, in either the union or firms opportunity cost or relative market power, upon wages and employment if we know the ratio of the union wage and employment elasticities.

APPENDIX A

To obtain more general results we do not specify the functional forms of the union utility and firms revenue functions. We assume the unions utility function to be quasi-concave and increasing in all its arguments.

(A1) $$U^u = U^u(w,L)$$

The slope of an indifference curve generated by which is

(A2) $$\frac{dw}{dL} = -\frac{U_L}{U_w}$$

We assume the firms revenue function has the following properties

(A3) $$R = R(L)$$

where (i) R' goes from positive to negative value as L goes to some finite value L^* through positive values of L

(ii) R'' goes from positive to negative values as $R' \to 0$, we then define the point L^{**} as that value of L where $R'' = 0$

(iii) $R''' < 0$ constant $\forall L$.

(iv) Let $R' = R/L$ define the value of L denoted L^{***}.

Thus we have a revenue function as in Figure A1.

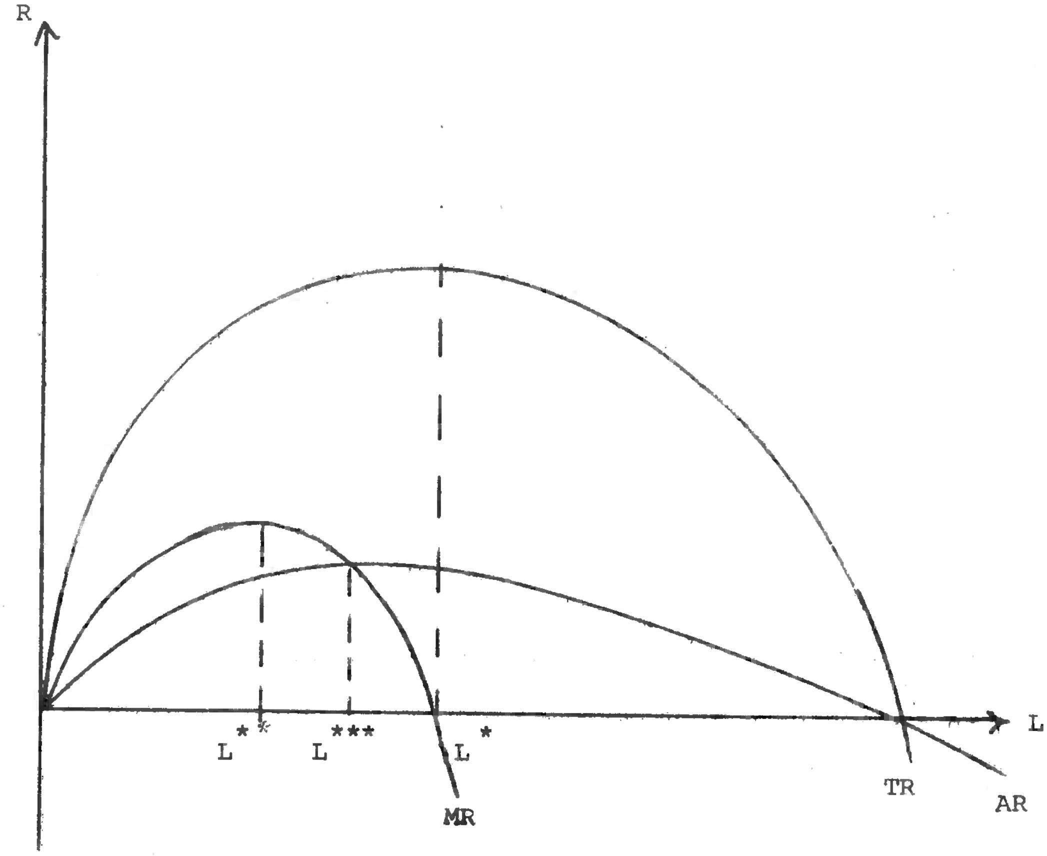

Figure A1

The profit function may be written

(A4) $R(L) - wL = \Pi$

which by differentiating and then substituting in for the value of w from (A4) allows us to write

(A5) $\text{sign} \frac{dw}{dL} = \text{sign} \left| MR-AR + C/L \right|$

where C is a fixed level of Π and (A5) gives the sign of the slope of the isoprofit curve.

Taking the second total derivative of (A4) substituting in for w and rearranging we obtain

$$\text{(A6)} \qquad \text{sign } \frac{d^2w}{dL^2} = \text{sign } 2\,|AR-MR| - 2C/L + R''L$$

Using (A5) and (A6) we may draw a family of non-negatively valued isprofit curves as in Figure A2.

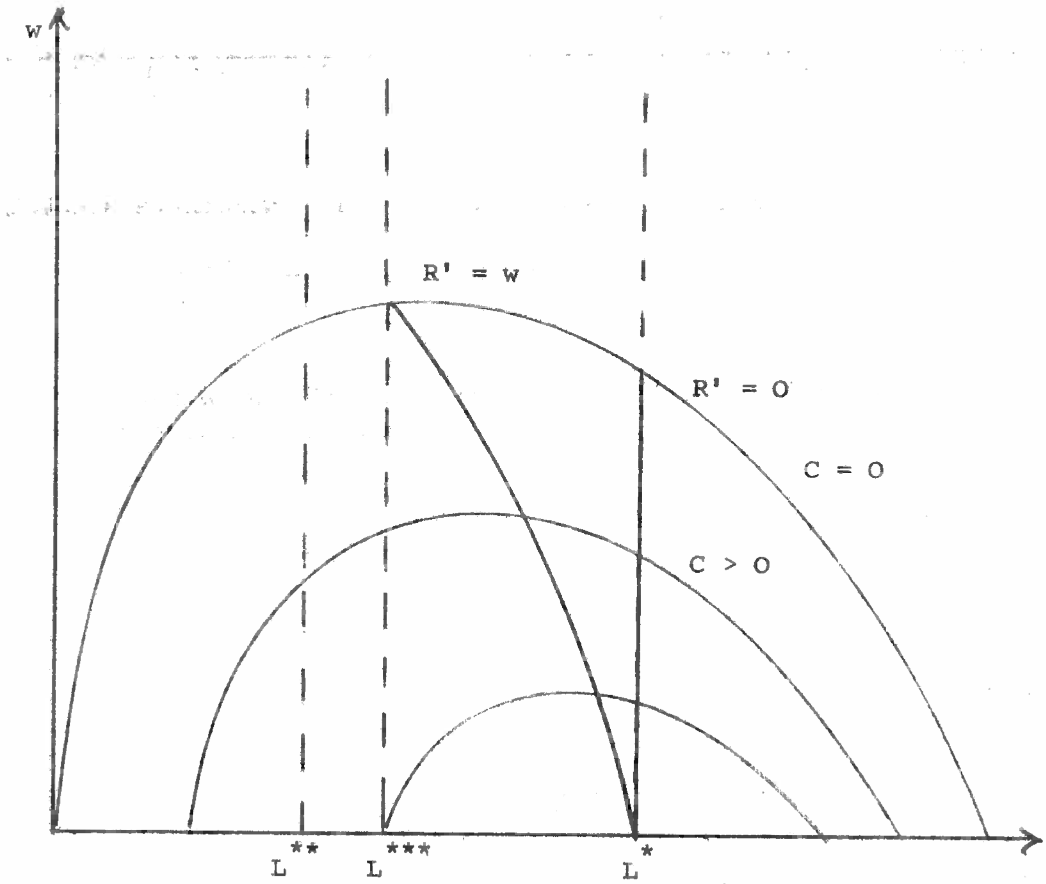

Figure A2

We note that the isoprofit curves may have concave regions. This problem is unavoidable unless we make stronger assumptions about the slope of the marginal revenue curve.

The locus of Pareto efficient bargaining points is given by

(A7) $$L = \frac{Uw}{u_L}(w - R')$$

which has slope

(A8) $$\frac{dw}{dL} = \frac{U_{ww}R' + U_{Lw}U_w W/U_L - U_{ww}W - U_w}{U_L + U_{wL}W - U_{LL}U_w W/U_L + R''U_w + R'U_{wL} - U_{LL}U_w R'/U_L}$$

The sign of (A8) is unclear in the general case, however there are two efficiency loci whose slope and curvature properties are obvious.

(i) If the unionis only concerned with the wage rate (and utility derived from the wage is not state dependent)

Then $U^u = U^u(w)$

and $\frac{dw}{dL} = 0$

hence the locus of Pareto efficiency points becomes

(A9) $R' = w$

and has slope $\frac{dw}{dL} = R'' < 0 \quad \forall L > L^{**}$

This is the "profit/wage" maximisation locus referred to in the main text, and illustrated in Figure A2.

(ii) If the union is concerned only with the maximisation of the total wage bill

Then $U^u = U^u(wL)$

and $\frac{dw}{dL} = -\frac{L}{w}$

hence the locus of Pareto efficient points becomes

(A10) $R' = 0$ hence $L = L^*$

has slope $\frac{dw}{dL} = \infty$

and curvature $\frac{d^2w}{dL^2} = 0$

This is the "revenue/wage bill" maximisation locus seen earlier and is also demonstrated in Figure A2.

These two special cases provide us with a means of dividing up the bargaining space, however it is clear from (A8) that using our reference loci to generally locate other solutions is the limit to our progress in the general case. The slope and curvature properties of any particular efficiency locus are indeterminate in general.

We do however make one further observation. In general the quasi convexity of the unions indifference curves and the convexity of the isoprofit curves will not allow an efficiency locus to be backward bending in the sense that movements out along the loci reduce both employment and the wage. Consequently one of the comparative static effects in each of our bargaining solutions will always be clear.

APPENDIX B

Proof that (3.10) is signed as stated

We require $\frac{dL}{dA} = \frac{(2\beta-1)}{\beta R' + (1-\beta)R''L} \lesseqgtr 0$ as $\beta \gtreqless \alpha$

Using (2.1) we write

(A1) $$\frac{dL}{dA} = (2\beta-1)/\beta b + 2cL$$

by assumption we have $\alpha + \beta b = 1$

hence $\alpha \gtreqless \Rightarrow (2\beta-1) \lesseqgtr 0$ since $2\beta \lesseqgtr 1$.

From (2.1) we know

$$b + 2cL < 0 \quad \text{hence} \quad \beta b + 2cL < 0 \quad \forall\ \beta \lesseqgtr 1$$

hence $\frac{dL}{dA} \lesseqgtr 0$ as $\beta \gtreqless \alpha$.

APPENDIX C

Proof that (3.16) is signed as stated

we require $\frac{dL}{dB} = \frac{1/\alpha B^{1/\alpha-1}(1-\beta/\alpha)L^{-\beta/\alpha}}{R''+(\beta/\alpha)B^{1/\alpha}(1-\beta/\alpha)L^{-(\beta/\alpha+1)}} \lesseqgtr 0$ as $\beta \lesseqgtr \alpha$

Consider the limit cases

(i) $\alpha \to 0$ as $\beta \to 1$

it is clear $\frac{dL}{dB} \to \infty$

(ii) $\alpha \to 1$ as $\beta \to 0$

it is clear $\frac{dL}{dB} \to \frac{1}{R''} < 0$

(iii) $\alpha = \beta = \frac{1}{2}$

then $\frac{dL}{dB} = 0$ since $(1-\beta/\alpha) = 0.$

Recalling the linearity of our efficiency loci it is thus clear that

$$\frac{dL}{dB} \lesseqgtr 0 \quad \text{as} \quad \beta \lesseqgtr \alpha.$$

APPENDIX D

Proof that (3.25) is signed as stated

we require $\frac{dL}{dA} = \dfrac{(2\beta-1)}{\beta R' + LR'' + R' - \left(\frac{1-\beta}{2-\beta}\right)^2 R'} \lesseqgtr 0$ as $\beta \lesseqgtr \alpha$

Consider the limit cases

(i) $\alpha \to 0$ as $\beta \to 1$

$\frac{dL}{dA} \to \frac{1}{2R' + R'L}$ which is clearly negative

(ii) $\alpha = \beta = \frac{1}{2}$

$\frac{dL}{dB} = 0$

(iii) $\alpha \to 1$ as $\beta \to 0$

$$\frac{dL}{dA} \to -\frac{1}{\frac{1}{2}b + 3cL}$$

we know $b + 2cL < 0$

hence $\frac{dL}{dA} > 0$

Again recalling that the efficiency rays are linear

$$\frac{dL}{dA} \gtreqless 0 \quad \text{as} \quad \beta \lesseqgtr \alpha.$$

APPENDIX E

Proof that (3.32) is signed as stated

We require $$\frac{dL}{dA} = \frac{(1-\phi)(2\beta-1)}{\phi\left[R'' + B^{1/\alpha}(1-\beta/\alpha)\beta/\alpha L^{-(\beta/\alpha+1)}\right] + (1-\phi)\left[(1-\beta)R''L + \beta R'\right]} \gtreqless 0$$

as $\alpha \gtreqless \beta$

The limit cases are

(i) $\alpha \to 0$ as $\beta \to 1$

$$\frac{dL}{dA} \to \frac{(1-\phi)}{\phi R'' - \infty + (1-\phi)R'} < 0$$

(ii) $\alpha = \beta = \frac{1}{2}$

$$\frac{dL}{dA} = 0 \quad \text{since } (2\beta-1) = 0$$

(iii) $\alpha \to 1$ as $\beta \to 0$

$$\frac{dL}{dA} \to -\frac{(1-\phi)}{\phi R'' + (1-\phi)R''L} > 0$$

hence $\frac{dL}{dA} \gtrless 0$ as $\alpha \lessgtr \beta$.

APPENDIX F

Proof that (3.35) is signed as stated

we require
$$\frac{dL}{dB} = \frac{\phi 1/\alpha B^{1/\alpha-1}(1-\beta/\alpha)L^{-\beta/\alpha}}{\phi R''+\phi B^{1/\alpha}(1-\beta/\alpha)\beta/\alpha L^{-(\beta/\alpha+1)}+(1-\phi)(1-\beta)R''L +(1-\phi)(1-\beta)R' + (1-\phi)(2\beta-1)R'} \lessgtr 0$$

as $\alpha \gtrless \beta$

Consider the limit cases

(i) $\alpha \to 0$ as $\beta \to 1$

$$\frac{dL}{dA} \to \frac{-\infty}{\phi R'' - \infty + (1-\phi)\beta R'} > 0$$

(ii) $\alpha = \beta = \frac{1}{2}$

$$\frac{dL}{dA} = 0 \quad \text{since} \quad (1-\beta/\alpha) = 0$$

(iii) $\alpha \to 1$ as $\beta \to 0$

$$\frac{dL}{dA} \to \frac{\phi}{\phi R'' + (1-\phi)R''L + (1-\phi)\beta R'} < 0$$

hence $\dfrac{dL}{dA} \gtreqless 0$ as $\alpha \lesseqgtr \beta$.

APPENDIX G

Proof that (3.38) is signed as follows

we require
$$\frac{dL}{d\phi} = \frac{\beta^{1/\alpha}(1-\beta/\alpha)^{-\beta/\alpha} - R'}{\phi R'' + \left(\frac{\beta}{\alpha}\right)B^{1/\alpha}(1-\beta/\alpha)L^{-(\beta/\alpha+1)} + (1-\phi)(1-b)R''L + (1-\phi)\beta R'} \lesseqgtr 0$$

as $\alpha \gtreqless \beta$

Consider again the limit cases

(i) $\alpha \to 0$ as $\beta \to 1$

$$\frac{dL}{d\phi} \quad \frac{-\infty - R'}{\phi R'' - \infty + (1-\phi)R'} > 0$$

(ii) $\alpha = \beta = \frac{1}{2}$

$$\frac{dL}{d\phi} = 0 \quad \text{since } (1-\beta/\alpha) = R' = 0 \quad \text{at this point.}$$

(iii) $\alpha \to 1$ as $\beta \to 0$

$$\frac{dL}{d\phi} \to \frac{B-R'}{\phi R'' + (1-\phi)R''} < 0$$

hence it follows

$$\frac{dL}{d\phi} \gtrless 0 \quad \text{as} \quad \alpha \lessgtr \beta$$

which is what we require.

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