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Kenneth F. Wallis

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SUMMARY

Dynamic models of the relations between economic variables often rest on theories about how unobservable expectations are formed. In this paper the "adaptive expectations" and "rational expectations" hypotheses are compared and contrasted. The main distinctions concern the size of the information set on which expectations are based, and the optimality or otherwise of expectations, given that information set. Expectations that are optimal with respect to the given information may be defined as rational. With this definition, some bivariate dynamic rational expectations models are presented, incorporating the appropriate time series forecasting rules for either a single forecast or an infinite sum of discounted forecasts. Model identification problems are discussed, and it is shown how they may be resolved by joint estimation of the bivariate process.

1. Introduction

Models of the relations between economic variables that are based on time series data typically include one or more lagged values of one or more variables. Such models are variously termed dynamic models, distributed lag models, transfer function models, and so forth, and for our present purposes we take the term dynamic models as generic. A common justification for the presence of lagged variables is that the behavior of economic agents, which the model attempts to capture, is affected by their expectations or anticipations of the future value of a relevant variable; in the absence of data on agents' expectations, the expectation-formation process is modelled by assuming that expectations depend on past values of relevant variables. In this paper we first compare, contrast, and reconcile the "adaptive expectations" and "rational expectations" approaches to modelling, and then consider the specification and validation of some simple dynamic models based on this reconciliation.

2. Adaptive expectations hypothesis

As a simple example, we assume that an economic agent's behavior (decision rule) with respect to the variable y depends on the one-step-ahead forecast of the variable x, and so we write

$$y_t = \alpha + \beta \hat{x}_{t+1} + u_t$$

where u is a random disturbance term. For example, a retailer's inventory level or re-order level depends on anticipated sales. The adaptive expectations hypothesis is in widespread use in applied econometrics; it is that $\hat{x}$ is formed as follows:

$$\hat{x}_{t+1} = \hat{x}_t + (1-\theta)\ (x_t - \hat{x}_t), \quad 0 < \theta < 1.$$

By repeated substitution we see that this is equivalent to the exponentially weighted moving average scheme

$$\hat{x}_{t+1} = (1-\theta) \sum_{j=0}^{\infty} \theta^j x_{t-j} \quad .$$

A transformation of the original equation yields the relation

$$y_t = \alpha(1-\theta) + \beta(1-\theta)x_t + \theta y_{t-1} + v_t \, ,$$

which can be estimated from time-series data on y and x. Since $v_t = u_t - \theta u_{t-1}$, maximum-likelihood methods for moving average estimation are required if $\{u_t\}$ is a white noise process, although ordinary least squares can be applied if the disturbance is generated by the first-order autoregression $u_t = \theta u_{t-1} + \varepsilon_t$, which seems to be implicitly assumed very frequently.

When first introduced, the adaptive expectations hypothesis was simply advanced as a plausible description of expectation formation, in which the previous expectation is modified in response to observed errors. It was criticised for its apparent ad hoc nature, for example, it appeared not to rest on any optimising behavior. Subsequently it was shown (Muth,1960) that the hypothesis does indeed yield optimal forecasts, in the sense of minimum mean squared error, if x follows an ARIMA(0,1,1) process

$$x_t - x_{t-1} = \varepsilon_t - \theta \varepsilon_{t-1} \quad .$$

For other x-processes adaptive expectations may or may not provide a good approximation to the optimal forecast. Irrespective of this, time series analysts have often criticized the resulting regression model for its overly simple dynamics. In Section 5 we consider the application of an optimal forecast assumption to more general x-processes.

3. Rational expectations hypothesis

A further criticism of adaptive expectations is that in basing the forecast purely on the past values of the variable in question, valuable information provided by other variables with which that variable interacts is neglected. In contrast, the rational expectations hypothesis introduced by Muth (1961) assumes that "expectations, since they are informed predictions of future events, are essentially the same as the predictions of the relevant economic theory" and hence depend "specifically on the structure of the relevant system describing the economy." Unfortunately the example used by Muth has a number of special features that result in the rational expectations variable obeying an adaptive expectations scheme, and this may have led to an unjustified over-emphasis on the latter in subsequent empirical work.

Muth's example is a simple model of the market for a commodity which cannot be stored:

demand function	$d_t = -\beta p_t$
supply function	$s_t = \gamma p_t^e + u_t$
market-clearing	$d_t = s_t$.

The market price in period t is denoted p_t, and p_t^e is the expectation of p_t implied by the model, conditional on information Ω_{t-1} available at time t-1, that is, $p_t^e = E(p_t|\Omega_{t-1})$. Variables are defined as deviations from equilibrium values, and there is a <u>single</u> random input, u_t. Equating the right-hand sides of the demand and supply functions gives

$$p_t = -\frac{\gamma}{\beta} p_t^e - \frac{1}{\beta} u_t ,$$

and on taking expectations across this equation we obtain

$$p_t^e = -\frac{1}{\beta+\gamma} \hat{u}_t ,$$

writing $\hat{u}_t$ for $E(u_t|\Omega_{t-1})$. Assuming that u_t follows a random walk this forecast is given as

$$\hat{u}_t = u_{t-1}$$

$$= -\beta p_{t-1} - \gamma p^e_{t-1} ,$$

and so the rational expectation p^e_t follows the scheme

$$p^e_t = (1-\theta)\, p_{t-1} + \theta\, p^e_{t-1} , \quad \theta = \frac{\gamma}{\beta+\gamma} .$$

As Muth remarks, the only difference from previous applications of adaptive expectations is that the "coefficient of adjustment" in the expectations formula depends on the demand and supply parameters. The equivalence between rational expectations and adaptive expectations in this example rests on the presence in the model of but one autocorrelated input variable, which follows a random walk. Although the attention given to this example may have sustained interest in the empirical application of adaptive expectations to a greater extent than was justified, more recently increasing attention has been devoted to the rational expectations hypothesis in its full generality. For discussion of its econometric implications, and references to earlier literature, see Wallis (1980).

A basic framework used in that paper is that of a static simultaneous equations econometric model, in which unobserved expectations of endogenous variables are included among the explanatory variables:

$$\underset{\sim}{B}\, \underset{\sim}{y}_t + \underset{\sim}{A}\, \underset{\sim}{y}^e_t + \underset{\sim}{\Gamma}\, \underset{\sim}{x}_t = \underset{\sim}{u}_t .$$

The vectors $\underset{\sim}{y}_t$ and $\underset{\sim}{x}_t$ of endogenous (output) and exogenous (input) variables are observed, while the expectations $\underset{\sim}{y}^e_t$ and random disturbances $\underset{\sim}{u}_t$ are not. If the expectations of certain endogenous variables do not appear in the model, then the corresponding columns of the square matrix $\underset{\sim}{A}$ are identically zero. Assuming that the random disturbances are non-autocorrelated, the rational expectations hypothesis $\underset{\sim}{y}_t^e = E(\underset{\sim}{y}_t | \Omega_{t-1})$ then gives

$$\underset{\sim}{y}_t^e = -(\underset{\sim}{B}+\underset{\sim}{A})^{-1} \underset{\sim}{\Gamma} \hat{\underset{\sim}{x}}_t ,$$

where $\hat{\underset{\sim}{x}}_t = E(\underset{\sim}{x}_t | \Omega_{t-1})$. It might appear that the rational expectations hypothesis has simply transferred the problem from forecasting the endogenous variables to forecasting the exogenous variables, but the key ingredient is the information about the relevant economic structure contained in the parameter matrices $\underset{\sim}{B}$, $\underset{\sim}{A}$ and $\underset{\sim}{\Gamma}$, that is, about the links between endogenous and exogenous variables. Moreover, the specification of the model indicates what information is relevant in forming $\hat{\underset{\sim}{x}}_t$: since these are exogenous variables, not explained by the model, the relevant information is $\underset{\sim}{x}_{t-1}, \underset{\sim}{x}_{t-2}, \ldots$, assuming that there is not a third group of variables outside the model and independent of $\underset{\sim}{u}$ that nevertheless helps to forecast $\underset{\sim}{x}$. On substituting the appropriate expression for $\hat{\underset{\sim}{x}}_t$ in terms of past x's into the above expression for $\underset{\sim}{y}_t^e$, and substituting this in turn into the original model, we again obtain a dynamic model. There is no apparent dynamic element in the original model, such as lags in adjustment, merely the passage of time between the formation of expectations and the realisation of actual values, but this is sufficient to provide a dynamic model.

It is well known that a dynamic linear model together with an ARMA model for the input variables implies the existence of a univariate ARMA representation for each output variable. For example, the simple model

$$y_t = \beta_o x_t + \beta_1 x_{t-1} + u_t$$

where u_t is white noise, together with the AR(1) process

$$x_t = \phi x_{t-1} + \varepsilon_t$$

implies that the univariate representation for y is of the form

$$y_t - \phi y_{t-1} = \eta_t - \theta \eta_{t-1} ,$$

where $\{\eta_t\}$ is another white noise process and θ is a function of the original parameters. There then exist two possible approaches to forecasting y : either we may insert a forecast of x based on its own past into the dynamic model to obtain a forecast of y, or we may forecast y from its own past, using the implied univariate representation. It can be shown that the latter, purely extrapolative, forecast of y in general has greater mean squared error than the model-based forecast. Likewise, in the rational expectations model, an alternative extrapolative mechanism exists for calculating expectations of endogenous variables, which might in simple cases be of the adaptive expectations form. Again this is less efficient than the rational expectation, if the model contains at least two random inputs (exogenous variables or disturbances) having different stochastic properties. In Muth's example the rational expectation and extrapolative predictor of p coincide because there is only a single input u; if u is generated by a scheme other than a random walk, the optimal extrapolative predictor is no longer of the adaptive expectations form.

4. Reconciliation

At first sight, a simple distinction between the models discussed in the previous two sections might be drawn by assuming that the variable about which expectations are formed in Section 2 is an exogenous variable, whereas expectations of endogenous variables enter the model of Section 3. However we argue that this is a difference of degree rather than a difference of kind.

As seen in the previous section, the rational expectations hypothesis makes use of the economic structure to specify how expectations consistent with that structure are formed, nevertheless the time series forecasting problem is that of forecasting the exogenous variables. In Section 2 the forecasting problem concerns a single exogenous variable. Thus the difference is simply between the size of the information set. The rational expectations hypothesis makes use of the given economic structure and the data generation process of the exogenous variables to form optimal forecasts of the endogenous variables, but in the single-equation example of Section 2 the only relevant information on which to base the unobserved expectations variable is the set of past values of the variable itself. Nevertheless we can define expectations that are optimal relative to the available information set as rational. Then in the simple model of Section 2 the rational expectations hypothesis is that the economic agent knows the data generation process of the x-variable and uses this information optimally in constructing minimum mean squared error forecasts. This usage accords with that of Hansen and Sargent (1980), who discuss examples featuring little economic structural information about the variables whose expectations enter the model.

In early empirical applications of the rational expectations hypothesis, it was often contrasted with the adaptive expectations hypothesis, but our argument is that the appropriate distinction is between optimal and sub-optimal forecasts, relative to a given information set. In the general model of Section 3, adaptive expectations or any other purely extrapolative predictor of the endogenous variables is clearly sub-optimal. In the single-equation model of Section 2, with an ARIMA(0,1,1) x-process, we may describe adaptive expectations as rational. Of course for other x-processes adaptive expectations are no longer rational, and in the next section we consider the necessary generalisations and the dynamic models that result.

5. Models with a single forecast variable

We consider a model of the previous form

$$y_t = \alpha + \beta \hat{x}_{t+1} + u_t \quad ,$$

in which x is generated by the ARMA(p,q) model

$$\phi(L)\, x_t = \theta(L)\, \varepsilon_t \quad ,$$

where $\phi(L)$ and $\theta(L)$ are polynomials in the lag operator L,

$$\phi(L) = 1 - \phi_1 L - \phi_2 L^2 - \ldots - \phi_p L^p \quad ,$$

$$\theta(L) = 1 - \theta_1 L - \theta_2 L^2 - \ldots - \theta_q L^q \quad ,$$

obeying the usual stationarity and invertibility conditions. It is convenient to work with the pure autoregressive representation for x_t,

$$\frac{\phi(L)}{\theta(L)}\, x_t = \left(1 - \frac{\theta(L)-\phi(L)}{\theta(L)}\right) x_t = \varepsilon_t \quad ,$$

since we may then write

$$x_t = \psi(L)\, x_{t-1} + \varepsilon_t \quad , \qquad \psi(L) = \frac{\theta(L) - \phi(L)}{L\, \theta(L)}$$

so that the minimum mean squared error forecast, given information up to time t, is

$$\hat{x}_{t+1} = \psi(L)\, x_t \quad ,$$

which generalises the exponentially weighted moving average scheme. Substitution into the original equation gives

$$y_t = \alpha + \beta\, \psi(L)\, x_t + u_t \quad ,$$

and on multiplying through by $\theta(L)$ we have

$$\theta(L)\, y_t = \alpha' + \beta\, \frac{\theta(L) - \phi(L)}{L}\, x_t + v_t \quad ,$$

giving the dynamic regression equation

$$y_t = \alpha' + \theta_1 y_{t-1} + \ldots + \theta_{\hat{q}} y_{t-q} + \beta(\phi_1 - \theta_1) x_t + \beta(\phi_2 - \theta_2) x_{t-1} + \ldots + v_t .$$

The number of x-values entering this equation is max(p,q). If p=q=1, the model of Section 2 is obtained as $\phi_1 \to 1$. A moving average component in the x-process, that is, $q \neq 0$, causes the presence of the lagged dependent variables and requires moving average estimation methods to be employed, subject to coefficient restrictions, if u is non-autocorrelated. If the x-process is a finite autoregression, then a finite distributed lag model results, which is relatively easy to handle.

The specification of this simple model can be varied in a number of ways. Different assumptions are possible concerning the time at which the forecast is calculated and hence the information on which it is based, and also concerning the period to which the forecast relates. If the net effect of variations in these assumptions is to retain a one-step-ahead forecast in the model (for example by entering $\hat{x}_t$ in the original equation and assuming that it is based on data available up to time t-1), then the dynamic relation between y and x is of the same form as above, except that the x-values are shifted forward or back in time as appropriate (all x-subscripts are decreased by 1 in the example just given). However if a forecast further ahead is required, then the way in which the ϕ- and θ- coefficients appear in the regression equation changes.

To illustrate, we amend the original model to

$$y_t = \alpha + \beta \hat{x}_{t+h} + u_t$$

and assume that the h-step forecast is based on data available up to time t. A convenient expression for $\hat{x}_{t+h}$ is obtained by writing x_{t+h} in terms of $\varepsilon_{t+h}, \varepsilon_{t+h-1}, \ldots, \varepsilon_{t+1}$ and $x_t, x_{t-1}, \ldots$. We do this by writing the infinite moving average representation as

$$x_t = \frac{\theta(L)}{\phi(L)} \varepsilon_t = \xi(L)\,\varepsilon_t \quad ,$$

and defining $\mu(L)$ as the first h terms of the polynomial $\xi(L)$:

$$\mu(L) = \sum_{j=0}^{h-1} \xi_j L^j \quad .$$

Then x_{t+h} is given as

$$\begin{aligned} x_{t+h} &= \mu(L)\,\varepsilon_{t+h} + \{\,\xi(L) - \mu(L)\,\}\,L^{-h}\,\varepsilon_t \\ &= \mu(L)\,\varepsilon_{t+h} + \frac{\xi(L) - \mu(L)}{L^h\,\xi(L)}\,x_t \quad , \end{aligned}$$

so that the optimal h-step forecast is given by the second term on the right-hand side, namely

$$\hat{x}_{t+h} = \frac{\theta(L) - \mu(L)\,\phi(L)}{L^h\,\theta(L)}\,x_t \quad .$$

Substituting into the original equation and multiplying through by $\theta(L)$ gives

$$\theta(L)\,y_t = \alpha' + \beta\,\frac{\theta(L) - \mu(L)\,\phi(L)}{L^h}\,x_t + v_t \quad .$$

In the resulting dynamic regression equation, we see first that the same lagged values of the dependent variable appear, with the same coefficients, irrespective of the value of h. On the right-hand side, the polynomial $\theta(L) - \mu(L)\phi(L)$ has leading term $\xi_h L^h$, from the definition of $\mu(L)$,

so division by L^h gives the leading term in the distributed lag function as $\beta\,\xi_h\,x_t$. The number of x-values entering is max $(p+h-1,q) - h + 1$, so if q is relatively small the values that enter are

$x_t, x_{t-1}, \ldots, x_{t-p+1}$, again irrespective of the value of h. Convenient expressions for the coefficients of the x's in terms of the θ's and ϕ's are not available in the general case, although an algorithm for their recursive calculation can be obtained from the relation

$\xi(L)\phi(L) = \theta(L)$.

Attention is concentrated in this paper on dynamic models that arise from expectations hypotheses, but it is clearly possible for the original decision rule to contain lagged variables, as a result of costs of adjustment, for example. That is, the relation at the beginning of this section might also include terms in x_t and x_{t-1}, say. Little is changed by this, however, since these terms are simply incorporated with those that result from the expectation-formation process. Nevertheless the problem of disentangling the underlying parameters becomes more difficult as a number of different sources of dynamics are confounded.

A further generalisation is to include a number of forecasts in the decision rule, $\hat{x}_{t+1}$, $\hat{x}_{t+2}$, ... , say. In practice this is not a common feature, except for the case in which a discounted sum of expected future values enters the decision rule, so that forecasts over all horizons, calculated as of time t, appear in the model, and an example of this is presented in Section 7.

6. Model identification

In many empirical applications of dynamic models that rest on expectations hypotheses, the investigator is content with a dynamic regression equation that fits the data well according to conventional statistical criteria, and seldom investigates the underlying parameterisation. However if the rational expectations hypothesis, as interpreted in Section 4, is part of the underlying theoretical framework, then parameter restrictions that can be exploited in estimation and testing may arise once a joint process for x and y is considered.

In general the parameters of the decision rule, α and β in our example, cannot be identified unless knowledge of the forecasting rule, based on knowledge of the x-process, is available. For example, in the simple case

$$y_t = \alpha + \beta \hat{x}_{t+1} + u_t$$

with an ARMA(1,1) x-process, the dynamic regression is

$$y_t = \alpha(1-\theta_1) + \theta_1 y_{t-1} + \beta(\phi_1-\theta_1)x_t + v_t ,$$

and the three regression coefficients cannot yield estimates of the four parameters. This problem is solved by joint estimation with the time series model

$$x_t = \phi_1 x_{t-1} + \varepsilon_t - \theta_1 \varepsilon_{t-1} ,$$

and now a parameter restriction occurs, that might form the basis of a specification test. Note that in the simple adaptive expectations model the dynamic regression equation is of the above form, but the assumption that $\phi_1 = 1$ identifies the remaining three parameters.

Since the dynamic regression is essentially a reduced form equation, model identification problems might arise, and in practice data might not be very informative about the details of timing relationships. In the models of the previous section, different forecast horizons produce similar dynamic regressions, and while joint estimation subject to cross-equation restrictions may permit some discrimination, a counter example is provided by an ARIMA (0,1,1) x-process, for then the optimal forecast is the same for any forecast horizon, so the same dynamic regression is obtained whatever the value of h.

Failure to separate the parameters of the decision rule from those of the x-process is a major source of Lucas' (1976) criticism of conventional econometric policy evaluation exercises. In the present context, a "conventional exercise" would comprise the specification and estimation of a dynamic regression equation from a given sample of data, followed by the use of the estimated equation to predict the dependent variable in some new actual or hypothetical circumstances. If the new circumstances are represented by a change in the structure of the x-process, the predictions will clearly be in error, since such a change necessitates a change in the dynamic regression specification under the rational expectations hypothesis. To overcome this difficulty separation of the decision rule and its parameters from the optimal forecasting rule for x is needed, as discussed in the preceding paragraphs. Without this separation the effect of a required change in the optimal forecasting rule on the dynamic regression and hence on the predictions of the dependent variable cannot be perceived. With this separation a prediction exercise proceeds by inserting the forecasting rule appropriate to the new x-process into the estimated decision rule, and using the resulting dynamic model to predict in the usual way.

7. A "present-value" example

As a final example we consider an economic agent's decision rule that relates the variable y to the expected present value of future x, with interest or discount rate ρ :

$$y_t = \alpha + \beta \sum_{j=o}^{\infty} \rho^j \hat{x}_{t+j} + u_t$$

and in which x follows an AR(p) process, $\phi(L)x_t = \varepsilon_t$. The optimal forecasts $\hat{x}_{t+j}$ given data up to time t are functions of x_t, x_{t-1}, ... , x_{t-p+1} , so the dynamic model relates y_t to these p values of x, with coefficients that are functions of β, ρ and ϕ_j. The x-forecasts can be calculated recursively from the relations

$$\hat{x}_{t+1} = \phi_1 x_t + \phi_2 x_{t-1} + \dots + \phi_p x_{t-p+1}$$

$$\vdots$$

$$\hat{x}_{t+j} = \phi_1 \hat{x}_{t+j-1} + \dots + \phi_{j-1} \hat{x}_{t+1} + \phi_j x_t + \dots + \phi_p x_{t-p+j}, \quad 1 < j \le p,$$

$$\vdots$$

$$\hat{x}_{t+j} = \phi_1 \hat{x}_{t+j-1} + \dots + \phi_p \hat{x}_{t+j-p}, \qquad j > p.$$

To express the infinite summation of future discounted expectations in terms of p observed x-values, consider

$$\phi(\rho) \sum_{j=1}^{\infty} \rho^j \hat{x}_{t+j} = (1-\phi_1\rho-\phi_2\rho^2-\dots-\phi_p\rho^p) \sum_{j=1}^{\infty} \rho^j \hat{x}_{t+j}$$

$$= \sum_{j=1}^{\infty} (\rho^j - \phi_1 \rho^{j+1} - \phi_2 \rho^{j+2} - \dots - \phi_p \rho^{j+p}) \hat{x}_{t+j}$$

$$= \sum_{k=1}^{\infty} \rho^k \hat{x}_{t+k} - \phi_1 \sum_{k=2}^{\infty} \rho^k \hat{x}_{t+k-1} - \phi_2 \sum_{k=3}^{\infty} \rho^k \hat{x}_{t+k-2} - \dots - \phi_p \sum_{k=p+1}^{\infty} \rho^k \hat{x}_{t+k-p}$$

$$= \sum_{k=p+1}^{\infty} \rho^k \left(\hat{x}_{t+k} - \phi_1 \hat{x}_{t+k-1} - \dots - \phi_p \hat{x}_{t+k-p} \right)$$

$$+ \sum_{k=1}^{p} \rho^k \hat{x}_{t+k} - \phi_1 \sum_{k=2}^{p} \rho^k \hat{x}_{t+k-1} - \phi_2 \sum_{k=3}^{p} \rho^k \hat{x}_{t+k-2} - \dots - \phi_{p-1} \rho^p \hat{x}_{t+1}.$$

The first term is identically zero, and the remaining terms are equal to

$$\rho \hat{x}_{t+1} + \rho^2 (\hat{x}_{t+2} - \phi_1 \hat{x}_{t+1}) + \rho^3 (\hat{x}_{t+3} - \phi_1 \hat{x}_{t+2} - \phi_2 \hat{x}_{t+1}) + \dots ,$$

which the forecasting equations give, in terms of observed x-values, as

$$\rho \sum_{j=o}^{p-1} \phi_{j+1} x_{t-j} + \rho^2 \sum_{j=o}^{p-2} \phi_{j+2} x_{t-j} + \dots + \rho^p \phi_p x_t$$

$$= \sum_{j=o}^{p-1} \left(\sum_{k=1}^{p-j} \phi_{j+k} \rho^k \right) x_{t-j} .$$

Hence the dynamic model between y and x is given as

$$y_t = \alpha + \beta \left(x_t + \frac{1}{\phi(\rho)} \sum_{j=o}^{p-1} \left(\sum_{k=1}^{p-j} \phi_{j+k} \rho^k \right) x_{t-j} \right) + u_t$$

$$= \alpha + \frac{\beta}{\phi(\rho)} \left(x_t + \sum_{j=1}^{p-1} \left(\sum_{k=1}^{p-j} \phi_{j+k}\, \rho^k \right) x_{t-j} \right) + u_t \quad .$$

Clearly if p=1 this simply gives a scaling of the current x-variable from which none of the underlying parameters can be deduced. If p=2 the joint process is

$$y_t = \alpha + \frac{\beta}{1-\phi_1\rho-\phi_2\rho^2} (x_t + \phi_2\, \rho\, x_{t-1}) + u_t$$

$$x_t = \phi_1\, x_{t-1} + \phi_2\, x_{t-2} + \varepsilon_t \quad ,$$

which allows recovery of $\alpha, \beta, \rho, \phi_1$ and ϕ_2. Higher values of p lead to cross-equation parameter restrictions, since the number of coefficients in the two equations is 2p+1 while the number of basic parameters is p+3. Note finally that if the dynamic model is simply taken to be a relation between y_t and x_t, x_{t-1}, ..., x_{t-p+1}, with no attention paid to its parameterisation, it is indistinguishable from that resulting from a decision rule in which y depends on (at least) one x-forecast, of any forecast horizon.

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