SYNOPSIS : PRICING AND QUALITY CONTROL UNDER GOODWILL LOSS, I

Clive D Fraser

No.193

WARWICK ECONOMIC RESEARCH PAPERS

DEPARTMENT OF ECONOMICS

UNIVERSITY OF WARWICK
COVENTRY

SYNOPSIS : PRICING AND QUALITY CONTROL UNDER GOODWILL LOSS, I

Clive D Fraser

No.193

July 1981

Synopsis : Pricing and Quality Control under Goodwill Loss, I

Extant theoretical studies of goodwill have, with one exception, explored the relationship between advertising and sales. We construct a simple model of the firm's pricing and quality control when it loses goodwill, hence future sales, should it produce defective commodities. Given a deterministic relationship between defectives and the future demand schedule, the impact of the firm's time preference and record of producing defectives upon its current pricing and quality control is examined, together with the conditions for the firm to be driven from the market. Risk is then introduced into the relationship between defectives produced and future goodwill loss and the consequences for price and quality of increased risk are examined.

Clive D Fraser
University of Warwick
July 1981

Pricing and Quality Control under Goodwill Loss, I

I Introduction

Despite the considerable importance of goodwill in commercial life, it is a topic which has been largely neglected by economic theorists. Even the handful of eminent authors who have addressed the issue have been primarily concerned with the impact of advertising upon goodwill ([2], [3], [6]) interpreted as an increased awareness of the product, rather than with the creation of goodwill interpreted as a ceteris paribus increase in sales arising from an enhanced reputation for producing reliable commodities.

A notable exception to this rule was the recent work of John Hey ([4]). However, by concentrating almost wholly upon the replacement/refund strategies of a retailer, Hey largely neglected two issues of crucial importance in the creation or maintenance of goodwill, issues which, furthermore, seem logically prior to the replacement/refund decision. These issues are the firm's pricing strategy and its investment in quality control or more reliable technology in order to influence the level of defective items produced.

In this paper we will be concerned with the firms strategies to maintain some initially given level of goodwill, taken to be embodied in its initial (pre-production) demand function. We will explore aspects of the following simple situation: Suppose one imagines a newly-established single-product firm confronted with initial demand function D(p). D(p) will be maintained in each period it does not produce any defective items. If it produces defective items in any period, however, there is a loss of goodwill for subsequent periods which depends upon the number of defectives produced. The effect of this goodwill loss is to reduce the demand at any price from the level that would have been given by the initial demand function.

One conjectures that pricing strategy affects goodwill loss in two ways. Firstly, it can be used to counteract goodwill loss insofar as a price reduction increases sales. Secondly, and simultaneously, a price reduction expands the population of potential defectives produced in any period and, for a given level of quality control, the expected number of defectives produced that period and hence incremental goodwill loss for the succeeding period. The incentive for quality control exists, on the other hand, because quality control affects the *probability distribution* of defectives. Of particular relevance for the risk-neutral firm is that, for a given level of output, increased expenditure upon quality control can be anticipated to reduce the expected number of defective items sold in any period, and hence the expected levels of goodwill lost for subsequent periods.

We will utilize what seems to be the simplest possible *ad hoc* specification of the process of goodwill loss confronting the firm. The resulting model is used to study, firstly, the impact of a firm's preference for current rather than future profits upon its current pricing and quality control; secondly, the likely impact of accumulated goodwill loss upon the same. With respect to the former, we find that decreased time preference induces the firm to engage in more quality control and to increase its price. With respect to the latter, we find that accumulated goodwill loss induces both a decrease in price *and* a decrease in quality control. These results are not surprising. Perhaps more surprising is the finding that, in the absence of per period fixed costs, or some other source of increasing returns, the firm experiencing goodwill loss will not be driven from the market, except asymptotically. We then introduce risk about the future goodwill loss associated with any given history of defectives into the model. It is demonstrated that an increase in this risk induces the firm to reduce both its current price and quality control.

The paper is structured as follows. Section II considers some general issues in the modelling of goodwill loss, as well as some candidate specifications. Section III presents a basic model by introducing the *ad hoc* explicit functional forms for the firms demand and probability distribution of defectives which we adopt. Section IV considers the existence and uniqueness of a solution to the firm's optimization problem which results from these specifications, while V presents some of the comparative statics of such a solution. Section VI introduces uncertainty about the future goodwill loss associated with any given level of defectives produced. We explore the impact upon current pricing and quality control of increased uncertainty about future goodwill loss. Section VII concludes.

II Some General Issues : The Sure Goodwill Loss Case

Although we will ultimately be concerned only with the solution of a two periods model, it seems worthwhile to consider two issues at least as relevant in a more dynamic formulation of the firm's problem.

The first issue relates to the modelling of the process of goodwill loss. It seems plausible to assert that the amount of goodwill lost by the firm from the initial period up to any point in time will be some increasing function of the number of defectives produced at that time.

Suppose ζ_t is the number of defective items sold by the firm up to time t. Then one plausible hypothesis on the amount of goodwill lost, and the one which we shall employ, is that the firm's tth periods demand function is given by $\overline{D(p)} \equiv D(p) - \zeta_t$ (*Specification A*). A possible justification for such an asumption is the two-armed bandit models of individual market behaviour which have been analysed in different contexts by, eg, Rotschild([7]) and Viscusi([8]). The prediction from such models is that consumers or workers will withdraw for good their custom or services from a producer or employer after a sufficiently unfavourable experience with an unreliable product or job hazard. Specification A assumes, therefore, that in a market of size D(o) for our specimen firm's product, once a customer is sold a defective item he is lost for good.[2]

A notable feature of Specification A is that, for finite D(o) ,

$\bar{D}(o)$ might be expected to reach zero in finite time and hence that the firm s.t. specification A would be driven from the market in finite time. However, it is relatively easy to demonstrate that, given perfect divisibility of the commodity, and in the absence of per period fixed costs, no firms which experiences goodwill loss according to Specification A would be driven from the market, except asymptotically(R1). This conclusion arises as follows :

In descrete time, the equation of motion of the system of goodwill loss confronting the firm is

1) $\zeta_t = \zeta_{t-1} + r_{t-1}$,

r_{t-1} being the number of defectives produced in t-1.

In the absence of fixed costs we assume constant returns to scale, thus the per unit output cost of the commodity is a constant, c. Suppose the firm is producing (profitably) in t-1. The condition for production to be everywhere unprofitable in t is

2) $c \geq p_t = D^{-1}(\bar{q}_t)$, $\forall \bar{q}_t \geq 0 (\bar{q}_t \leq D(0) - \zeta_t)$,

where $\bar{D}^{-1}(\bar{q})$ is the goodwill loss-adjusted inverse demand function. Now, as $\bar{D}^{-1}$ is decreasing in q under normal circumstances, if (2) is satisfied at $\bar{q}_t = 0$, it is satisfied everywhere else. If it is <u>not</u> satisfied at $\bar{q}_t = 0$, there exists some positive price and output at which the firm could more than cover its costs. As

A1) $\bar{D}(p) = D(p) - \zeta$,

thus,

$$p_t = D^{-1}\left\{\bar{D}(p) + \zeta\right\} = D^{-1}\left\{\zeta\right\}$$

when $\bar{D}(p_t) = 0$, the condition for nowhere profitable production is

$$c \geqq p_t = D^{-1}\left\{\zeta\right\}$$

Inverting, this becomes,

3) $$D(c) \leqq D(p_t) = \zeta_{t-1} + r_{t-1} \Longleftrightarrow r_{t-1} \geqq D(c) - \zeta_{t-1} ,$$

using (1) and the fact that D(.) is monotonically decreasing. But

$$r_{t-1} \leqq D(p_{t-1}) - \zeta_{t-1} = \bar{D}\ (p_{t-1})$$

4) $$< D\ (c) - \zeta_{t-1}$$

if profitable production occurred in t-1 . However, 4) violates 3). Therefore, if profitable production occurred in t-1 , it will also occur with certainty in period t . Induction extends this argument from the initial period onwards. Figure 1 illustrates the situation for the case of a linear D(p) (3) .

In Figure 1, p(q) is the initial inverse demand function. Production of ζ defectives induces a parallel shift in this to $p(\bar{q})$, by assumption. Now suppose, instead, that the firm is confronted with the inverse schedule ABC at the start of profitable production in any t . For it to produce sufficient defectives to be driven from the market in t + 1 it must produce at least at the level $r = c$B in the Figure in t . But with constant average and marginal cost at c , profitable production in t entails the firm producing somewhere along the interior of the segment A B. In other words, less than c B.

In the figure c(q) represents unit costs as a function of output in the case where there are decreasing returns to scale; $c + q^{-1}$ represents the normalised unit cost when there is a per period fixed cost. It

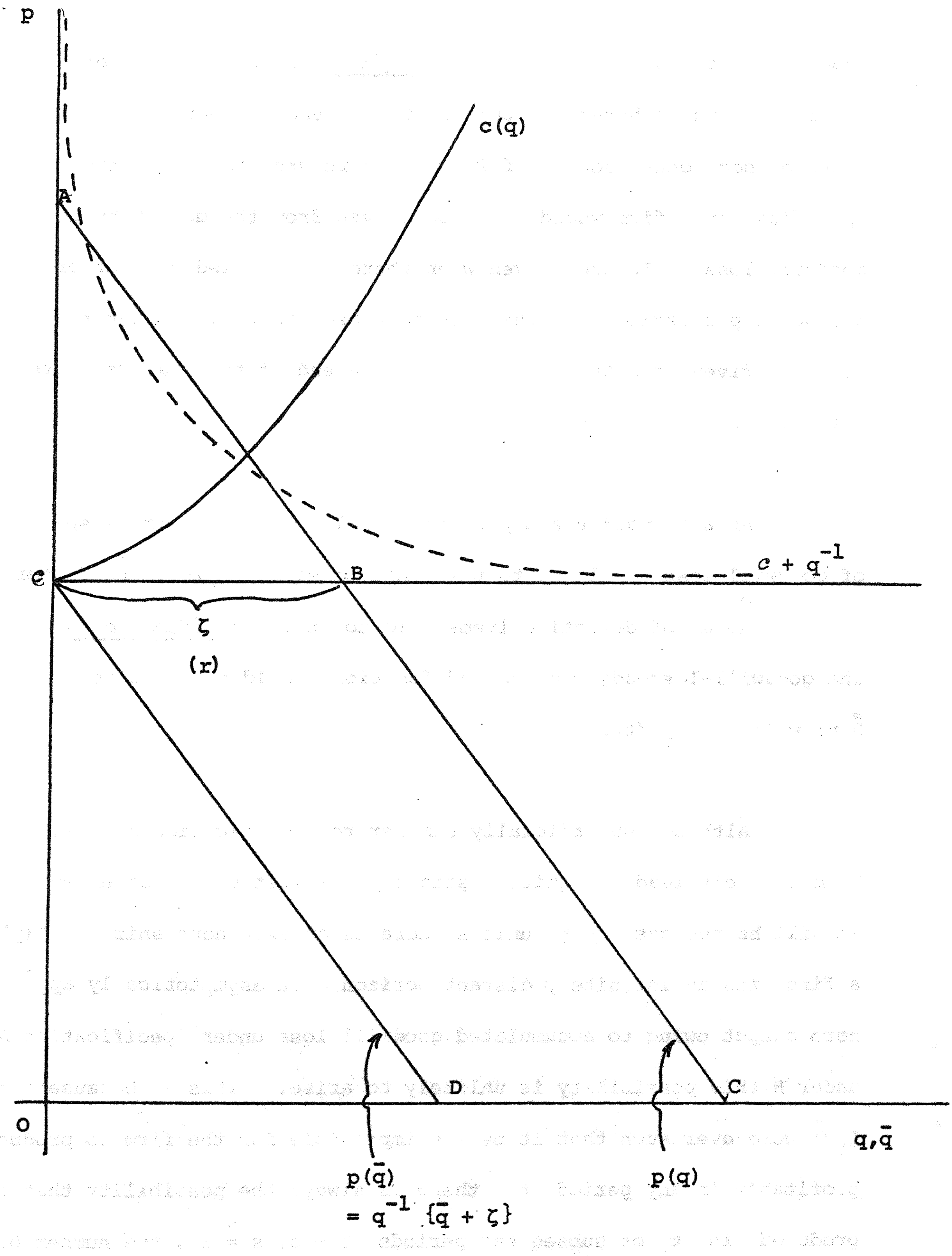


Figure 1 : No Exit from the Market

should be apparent that (R1) holds, *a fortiori*, for the former case. We can conclude, therefore, that in the absence of per period fixed costs or some other source of increasing returns to scale, the Specification A firm would never be driven from the market by accumulated goodwill loss. In fact, even when there is a fixed cost of production, but not a per period one, there is no reason to believe that the firm will be driven from the market before the end of the "natural life" of the fixed plant or machine.

An alternative and perhaps equally plausible simple specification of goodwill loes would be to take this to equal ζ_t/t, the historical average number of defective items sold to t. (*Specification B*). The goodwill-loss-adjusted demand function would now be given by $\bar{\bar{D}}(p) = D(p) - \zeta_t/t$.

Although superficially similar to Specification A, Specification B immediately leads to quite distinct possibilities. For example, while it will be the case that, unless there is an exogenous shift in D(p), a firm with an infinitely distant horizon must asymptotically approach zero output owing to accumulated goodwill loss under Specification A, under B this possibility is unlikely to arise. This is because even if ζ_t/t were ever such that it became impossible for the firm to produce profitably in any period t, there is always the possibility that if it produced in t or subsequent periods $t + s$, $s \geq 1$, the number of defectives produced would be sufficiently low as to reduce $(\zeta_{t+T})/(t + T)$, $T > 0$, to a level which enabled profitable production again. It is not hard to imagine that such a process could provide part of the explanation for firms continuing in operation ("to recapture their market") despite a perceived short-run inability to trade profitably. These intricacies of specification B will be analysed elsewhere.

The second issue of interest relates to the interpretation and scope of "quality control". In principle this term could embrace considerations as diverse as expenditure upon testing or "proving" the commodity immediately after production via an inspectorate, maintenance in service, or the use of more sophisticated machinery to produce more reliable commodities. Whatever interpretation is given to the term, we will make two further strong assumptions to facilitate the analysis :

(A2) Investment in quality control depreciates completely within the period in which it is made;

(A3) Expenditure upon quality control is "lump sum" and its impact upon f(.), the probability density of defectives, is independent of the level of output, although such expenditure is still subject to diminishing returns. This will be made precise in Section III.

(A2) implies that quality control expenditure in t does not affect the probability distribution of defectives produced in subsequent periods. This is not implausible if quality control is interpreted in the sense in which that term is normally understood. One can imagine the firm varying its employment of quality controllers period by period. The assumption is much less valid in the case of investment in machines, unless we regard the period length as being defined by the life of the machine .

III A Basic Model

III (i) Assumptions

We will additionally assume the following :

(A4) The firm has a two periods planning span, T-1 and T . Alternatively, what follows can be regarded as the analysis of the last two periods of a T periods planning span. ζ_{T-1} is then the inherited goodwill loss.

(A5) The firm has a linear initial demand function $D(p) = A - bp, b>0$. At t the goodwill loss adjusted demand function is therefore $\overline{D(p)} = A - bp - \zeta_t \equiv \overline{A}_t - bp$.

(A6) The firm maintains the following naive probability density for the number of defectives which it produces in t, $t \neq T$:

$$f(r_t \mid e_t, p_t) = \begin{cases} e_t^{-1} \, \overline{D}(p)^{-1}, & r \in (0, \overline{D}(p)] \\ 1 - e_t^{-1}, \quad e_t \geq 1, & r = 0 , \end{cases}$$

where e_t is the expenditure upon quality control in t . Of course, f(.) is merely a modification of the rectangular distribution. (The restriction $e_t \geq 1$ is present only to ensure that the density is well defined and it possesses no economic significance. In particular, it is *not* to be interpreted as a fixed cost.) Note that expenditure upon quality control is subject to diminishing returns insofar as the impact of increased e is in increasing the probability of producing zero defectives and reducing the (equal) probabilities of producing any positive number. The former probability is an increasing, but concave, function of e .

(A7) Despite having only a two periods horizon, the firm recognizes no termination penalties for the accumulated goodwill loss at the end of T . Therefore, the firm's criterion will be taken to be simply the maximization of the discounted sum of its two periods' expected profits. The absence of a termination penalty also means that the firm will be unconcerned with the number of defective items produced in T . It will therefore incur no expenditure upon quality control in T.(4)

III (ii) The Firm's Maximization

The firm's objective is

$$\text{5)} \qquad \max_{\{p_{T-1},\, e_{T-1}\}} J_{T-1}(\zeta_{T-1}) = (\bar{A}_{T-1} - bp_{T-1})(p_{T-1} - c) - e_{T-1} + \beta E_r J^*_T(\zeta_T)$$

s.t. (1) and (A.6)

$\beta \in (0,1)$ is the discount factor which increases as the firm's preference for current over future profits decreases. E_r denotes expectation taken over r_{T-1} . $J^*_T(\zeta_T)$ gives the firm's maximised T th period's profits as a function of ζ_T :

$$\text{6)} \qquad J^*_T(\zeta_T) = \max_{p_T} \left\{ (\bar{A}_T - bp_T)(p_T - c) = \Pi_T(\zeta_T) \right\}$$

As $\Pi_T(\zeta_T)$ is concave in p_T , the first order condition $\bar{A}_T - bp_T - b(p_T - c) = 0$ gives the maximising p_T ,

$$\text{7)} \qquad \hat{p}_T = (\bar{A}_T + bc)/2b .$$

Thus, by substitution,

8) $$J^{*}_{T}(\zeta_T) = b^{-1}\left[(\bar{A}_T - b\ )/2\right]^2 .$$

(7) and (8) indicate that both final periods profits and prices are increasing functions of $\bar{A}_T = A - \zeta_T$, and hence decreasing functions of ζ_T , as one would expect.

Substituting (8) into (5) and using R1 and (A.6) we obtain for the firm's objective

9) $$\text{Max}\quad J_{T-1}(\zeta_{T-1}) = \left\{(\bar{A}_{T-1} - bp_{T-1})\ (p_{T-1} - c) - e_{T-1}\right\} +$$

$$+\ \beta b^{-1}\ 4^{-1} e^{-1}_{T-1}\left[(\bar{A}_{T-1} - bc)^2 - (\bar{A}_{T-1} - bc)\ (\bar{A}_{T-1} - bp_{T-1}) +\right.$$

$$\left. +\ (\bar{A}_{T-1} - bp_{T-1})^2\ 3^{-1}\right] + \quad (1 - e^{-1}_{T-1})\ \beta b^{-1}\ 4^{-1}\ (\bar{A}_{T-1} - bc)^2 .$$

We can now suppress time subscripts, having solved out for p_T . The first-order necessary conditions for the firms maximization are

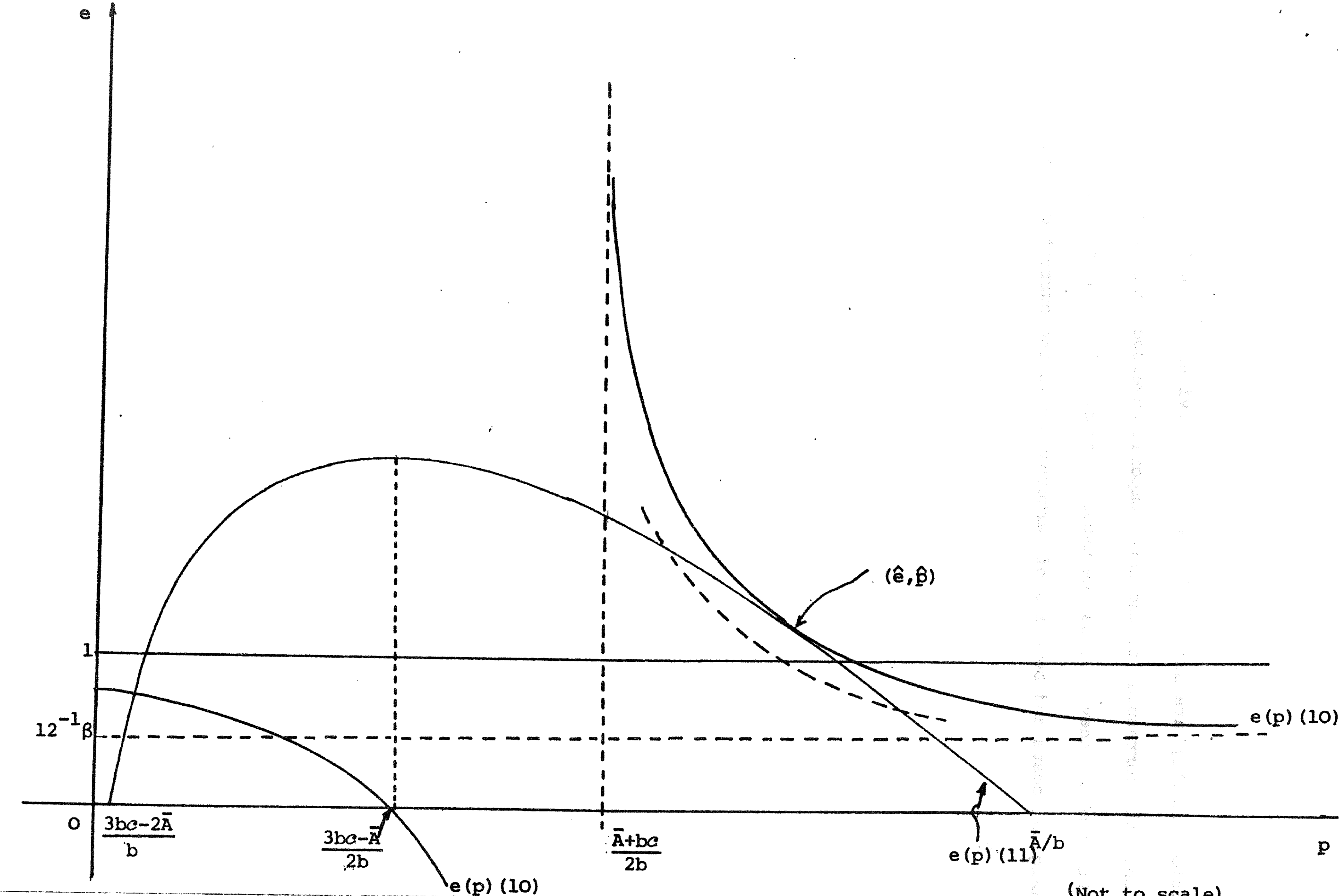
10) $$J_p = \bar{A} - 2bp + bc + e^{-1}\beta\ 12^{-1}\left[3\quad(\bar{A} - bc) - 2\ (\bar{A}-bp)\right] = 0$$

11) $$J_e = -1 - e^{-2}\ b^{-1}\ 12^{-1}\beta\left[(\bar{A} - bp)^2 - 3(\bar{A} - bc)\ (\bar{A} - bp)\right] = 0 .$$

The second order sufficient conditions are

12) $$J_{pp} = -2b + e^{-1}b\ 12^{-1}\ \beta 2 < 0 \quad\text{and}\quad J_{ee} = 2e^{-3}b^{-1}12^{-1}(\bar{A}-bp)\ x$$

$x\ (3bc - 2\bar{A} - bp) < 0$, together with that the determinant of the hessian matrix, $|H| = \begin{vmatrix} J_{pp} & J_{ep} \\ J_{ep} & J_{ee} \end{vmatrix} > 0$. It is easy to verify that the conditions in (12) are satisfied, the first provided that $e > \beta/12$. (10) and (11) correspond to the intertemporal envelope conditions for the firm and they possess the usual interpretations in terms of the marginal costs and benefits of perturbation in the current decision variables.



(Not to scale)

Figure 2 : The Solution of the Basic Model

IV Analysis

An explicit solution to the system (10) and (11) involves finding the roots of a polynomical equation of degree four. While, in principle, such a solution in terms of radicals exists, it would be of sufficient complexity as to be uninterpretable. The relevant qualitative information will be obtained by treating the system geometrically, as in Figure 2.

First, (10) and (11) are solved for e in terms of p :

13) $$e(p)(10) = \beta\, 12^{-1} (\bar{A} + 2bp - 3\, bc)\ (2bp - \bar{A} - bc)^{-1}$$

14) $$e(p)(11) = \sqrt{12^{-1}\, \beta\, [3(\bar{A} - bp)(\bar{A} - bc) - (\bar{A} - bp)^2\,]}\ .$$

These efficiency loci are mapped in Figure 2.

e(p)(10): By inspection, $e(p)(10)$ has an asymptote at $p = (\bar{A} + bc)/2b$. Using L'Hospital's rule, it can be shown that $e(p)(10) \to 12^{-1}\beta$ as $p \to +\infty$. By differentiation, $e'(p)(10) = 12^{-1}\beta(2bp - \bar{A} - bc)^{-2}$ x x $2b\ (2bc - 2\bar{A}) < 0$, as $\bar{A} > bc$ and $(2bp - \bar{A} - bc)^{-2} \geqq 0$. By inspection again, $e(p)(10) \geqq 0$ requires either $p \geqq (\bar{A} + bc)/2b$, $p \geqq (3bc - \bar{A})/b$ simultaneously, or $p \leqq (\bar{A} + bc)/2b$, $p \leqq (3bc - \bar{A})/b$ simultaneously. The figure has been drawn on the assumption $3bc - \bar{A} > 0$, but nothing essential is affected if otherwise. $e''(p)(10)$ can be shown to be $\left\{ \begin{matrix} \text{positive} \\ \text{negative} \end{matrix} \right\}$ as $p \left\{ \begin{matrix} > \\ < \end{matrix} \right\} (\bar{A} + bc)/2b$.

e(p)(11) : Note from (11) that $e(p)(11)$ is well-defined at an interior solution as $(\bar{A} - bp)\ [\,3(\bar{A} - bc) - (\bar{A} - bp)\,] > 0$. $e(p)(11) = 0$

at $p = \bar{A}/b$ and $p = (3bc - 2\bar{A})/b$. $e(p)$ (11) is undefined for $p < (3bc - 2\bar{A})/b$ and $p > \bar{A}/b$. By differentiation, $e'(p)(11) \gtreqless 0$ as $p \lesseqgtr (3bc - \bar{A})/2b$ ($(3bc - 2\bar{A})/b < (3bc - \bar{A})/2b$ as $bc - \bar{A} < 0$). Further differentiation reveals $e''(p)(11) < 0$ everywhere.

By inspection, the two efficiency loci possess at most three real positive intersections: one in the p interval $((3bc - 2\bar{A})/b, (3bc - \bar{A})/2b)$, and at most two in $((\bar{A}+bc)/2b, \bar{A}/b)$. The expression for J_{pp} in (12) suggests that, for e sufficiently small, $J_{pp} > 0$, quite apart from the probabilistic requirement that $e \geqq 1$. The rightmost intersection of the loci in Figure 2 would therefore correspond to a local minimizing choice for p. However, the existence of such a minimum requires $e \leqq B/12$ and we know $e(p)(10) > B/12$ within this interval. Thus if this second intersection existed, it would also correspond to a local maximum. But two adjacent local maxima cannot exist in the absence of a discontinuity in J_{T-1}. The leftmost intersection can likewise be eliminated, and also by the requirement that $p \geqq 0$ (if $3bc - \bar{A} < 0$). These arguments enable us to conclude that a unique J_{T-1}-maximising choice of (e,p) will exist for $p \in ((\bar{A} + bc)/2b, \bar{A}/b)$ at a tangency between the two efficiency loci. Within this interval, $\bar{A} - 2bp + bc < 0$. I.e., in obvious notation, $\partial\Pi_{T-1}/\partial p < 0$. Now, as Π_{T-1} is concave in p_{T-1}, $\partial\Pi_{T-1}/\partial p < 0$ implies p_{T-1} is greater than the Π_{T-1}-maximising choice. The firm is therefore selling less in this period than immediate profit maximising considerations would dictate.

V : Some Comparative Statics (C.S.)

$$(R2) \; : \; \partial\hat{e}/\partial\beta > 0, \; \partial\hat{p}/\partial\beta > 0, \; \partial\hat{p}/\partial\bar{A} > 0 \; , \; \partial\hat{e}/\partial\bar{A} > 0.$$

These can be obtained by the routine procedures. However, the above considerations mean that the CS are particularly straightforward in terms of shifts in the efficiency loci. Consider those for β (see Figure 3). Clearly, a β increase shifts both loci proportionately rightwards/ upwards.

By differentiation, $\partial e(p)(10)/\partial\beta = 12^{-1} (\bar{A} - 3bc + 2bp) \times$ $\times (2bp - \bar{A} - bc)^{-1} = e(p)(10)\beta^{-1}$; $\partial e(p)(11)/\partial\beta = 2^{-1} B^{-\frac{1}{2}} \times$ $\times \sqrt{(\bar{A} - bp)(2\bar{A} - 3bc + bp)b^{-1}12^{-1}} = 2^{-1} e(p)(11)\beta^{-1} < e(p)(10)\beta^{-1}$, evaluating at the original equilibrium where $\hat{e}(p)(11) = \hat{e}(p)(10)$. Thus $e(p)(10)$ is displaced by the greater vertical distance from the original equilibrium (1 in the figure), so the new equilibrium must occur at a point like 2 in the figure, to the right and north of 1.

None of the CS effects are particularly surprising. For example, the less the discounting of future profits, the more will the firm engage in quality control in order to protect expected future profits by reducing expected goodwill loss. Firms with high rates of time discounting (hence low β) can be identified with the "fly by night operators" who would not be expected to be particularly concerned with the reliability of their products. Nonetheless, to the extent that one can generalise from such a simple two-period model, these CS results carry several interesting suggestive implications for, in particular, the firm's intertemporal price profile, and in relation to

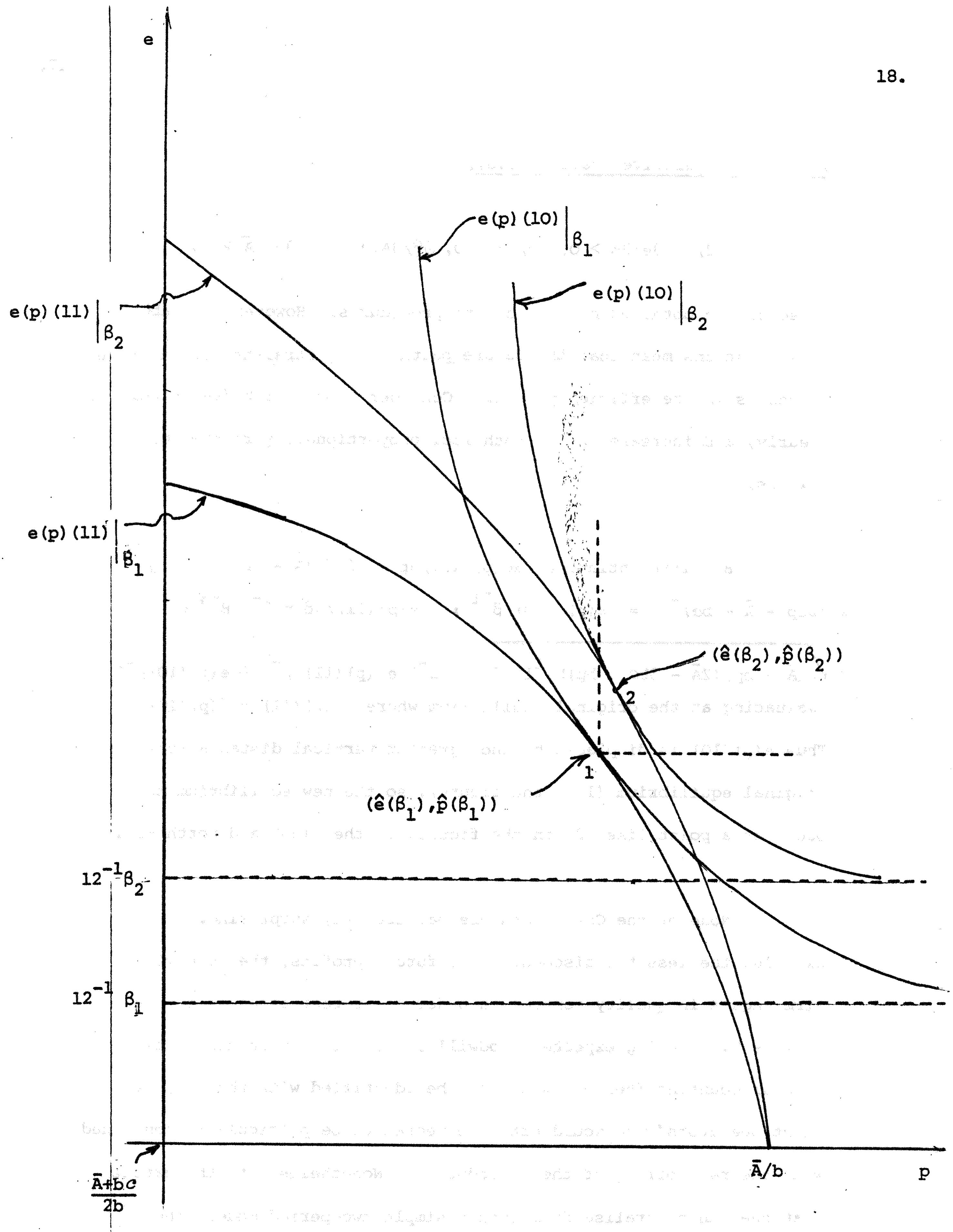


Figure 3 : The Comparative Statics of β Changes

various firms' observed behaviour. For example, the fact that current price is decreasing in accumulated goodwill loss which, in turn, is expected to increase with time, suggests that one might expect to observe a decrease in the (real) price charged over time by the firm suffering goodwill loss as it attempts to counteract the loss of its market. Empirically, this accords with the recent behaviour of, say, British Leyland. Although many aspects of BL's behaviour could not be captured by as crude a model as Specification A, it does not seem too fanciful to believe that its various 'Superdeals' were partly attempts to couteract goodwill loss due to its historical record of producing unreliable cars, rather than the response to other sources of loss of competitiveness such as adverse exchange rate movements. Correct identification of the source of its misfortunes is obviously important for the prescribed remedies.

Again, as the specification A firm does not exit the market except asymptotically, it follows that its price will also asymptotically converge to c, the competitive price, although output moves the other way.[5] (Note that we know that $\hat{P}_{T-1}$ exceeds the Π_{T-1} -maximising price, while $\hat{P}_T$ equals the Π_T -maximising price, is decreasing in ζ_T, and $\zeta_T \geq \zeta_{T-1}$, thus $\hat{P}_{T-1} > \hat{P}_T$ unambiguously.)

The one perhaps mildly surprising CS implication is that $\partial\hat{e}/\partial\zeta (= - \partial\hat{e}/\partial\bar{A}) < 0$: current quality control is decreasing in accumulated goodwill loss. Clearly, this is due to the dominance of an income effect upon the firm's current actions.[6] However, were this result extended to a fully dynamic model, it would suggest that the product quality/reliability should be expected to decline over time.

VI : Riskiness in Expected Goodwill Loss

So far we have assumed that the only source of risk in goodwill loss relates to the production of defectives in a given period. Thus, for example, the firm would know in $t-1$ what its goodwill loss adjusted demand function for t would be if it produced r_{t-1} defectives in $t-1$. However, it is possible that this would not be the case and that the firm would not know *until the start of t* what its effective goodwill loss would be if it produced r_{t-1} defectives in $t-1$.(7) While the risk-neutral, expected profit maximising, firm can be shown to be indifferent to mean-preserving changes in the riskiness of goodwill loss in this second sense, its current pricing and quality control will be influenced by the perceived risk. This feature would obviously be important for any detailed welfare analysis of both sides of a merket in which there is goodwill loss.

VI(i) A Basic Model

A simple modification of our basic model will enable us to present one characterisation, the simplest, of this second form of risk. We will make the following additional assumption:

(A8) If $D(p)$ is the initial demand function, ζ_t the number of defective items sold up to the start of t, then $\bar{D}(p) \equiv D(p) - \varepsilon_t \zeta_t$, $\varepsilon_t \in (0,1]$, $\varepsilon_t \sim g(\varepsilon)$, is the goodwill loss-adjusted demand function perceived for t by the firm. In $t-1$, ε_t is perceived as random with non-degenerate density $g(\varepsilon)$, *but it is known with certainty before production begins at the start of t.*

$$E_{\varepsilon} E_r J_T^* (\varepsilon_T r_{T-1}) = E_{\varepsilon} \Big[b^{-1} 4^{-1} e_{T-1}^{-1} \Big\{ (A-bc)^2 - (A-bc) \times$$

$$\times \varepsilon_T (A-bp_{T-1}) + 3^{-1} \varepsilon_T^2 (A-bp_{T-1})^2 \Big\} + (1-e_{T-1}^{-1}) b^{-1} 4^{-1} (A - bc)^2 \Big]$$

$$16) = b^{-1} 4^{-1} e_{T-1}^{-1} \Big\{ (A-bc)^2 - (A-bc) E(\varepsilon_T)(A-bp_{T-1}) + 3^{-1} E(\varepsilon_T^2) \times$$

$$\times (A-bp_{T-1})^2 \Big\} + b^{-1} 4^{-1} (1-e_{T-1}^{-1}) (A-bc)^2 .$$

Although the precise form of the density $g(\varepsilon)$ is unimportant, it is clearly important that the expectations $E(\varepsilon_T)$ and $E(\varepsilon_T^2) = \sigma_\varepsilon^2 + \{E(\varepsilon_T)\}^2$ exist (σ_ε^2 being the variance of the r.v. ε_T). This we assume.

Substituting (16) into (15) and suppressing time subscripts yields the new maximand:

$$17) \quad J = (A-bp)(p-c) - e + \beta b^{-1} 4^{-1} e^{-1} \Big\{ (A-bc)^2 - (A-bc) E(\varepsilon)(A-bp) + 3^{-1} E(\varepsilon^2) (A-bp)^2 \Big\} + \beta b^{-1} 4^{-1} (1-e^{-1}) (A-bc)^2 .$$

The first order conditions (assuming interiority) are

$$18) \quad J_p = A-2bp+bc + \beta b^{-1} 4^{-1} e^{-1} \Big\{ (A-bc)^2 - (A-bc) b E(\varepsilon) - 3^{-1} E(\varepsilon) 2 (A-bp) b \Big\} = 0$$

$$19) \quad J_p = -1 - e^{-2} \beta b^{-1} 4^{-1} \Big\{ -(A-bc) E(\varepsilon)(A-bp) + 3^{-1} E(\varepsilon^2)(A-bp)^2 \Big\} = 0 \quad .$$

The second order sufficient conditions are precisely analogous to those previously analyzed and they will be taken to be satisfied.

Solving (18) and (19) for the efficiency loci $e(p)$, we obtain

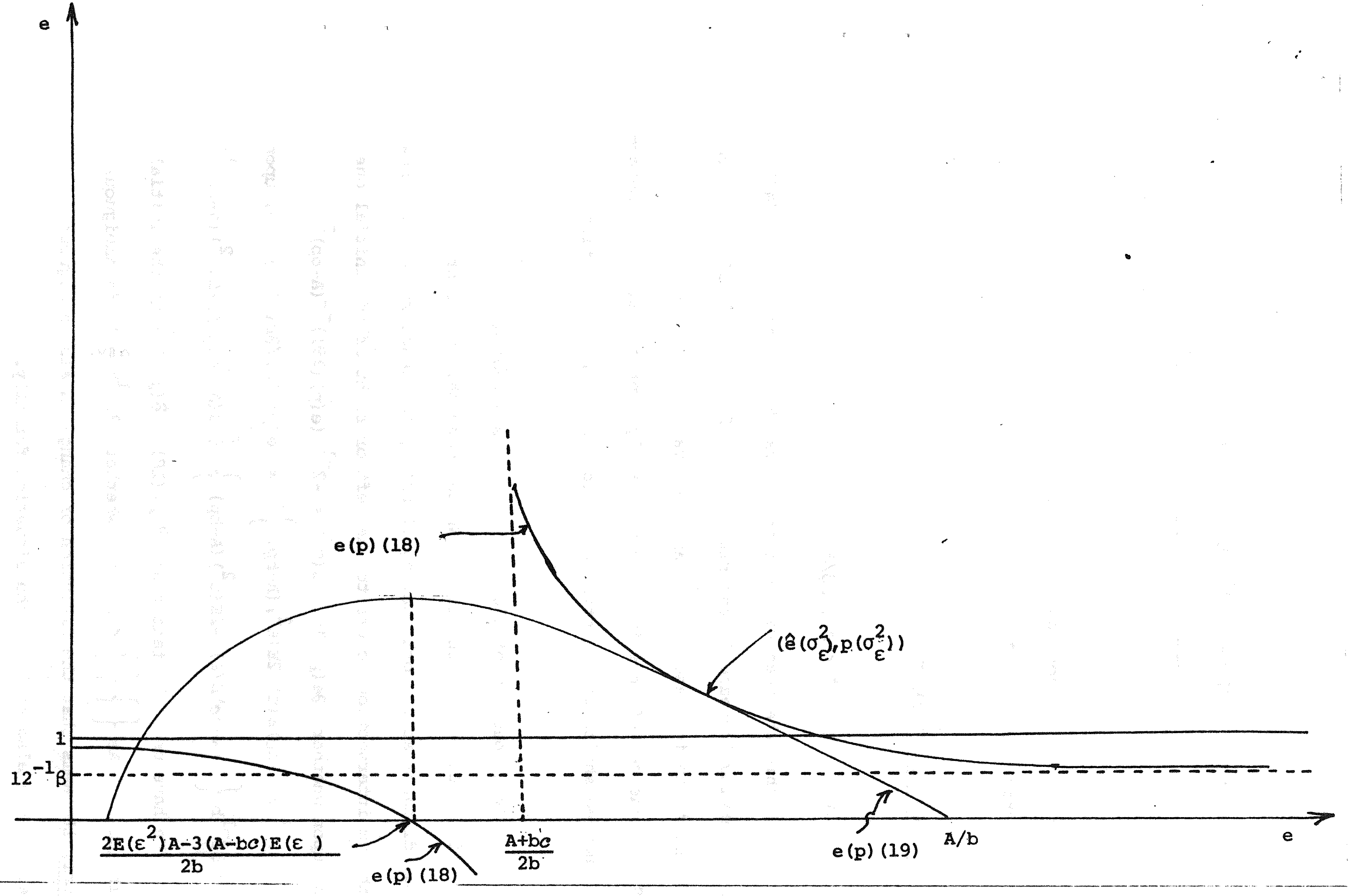


Figure 4 : The Risky Goodwill Loss Case

20) $$e(p)(18) = (2bp-A-bc)^{-1}\ \beta\ 12^{-1} \left\{ 3(A-bc)E(\varepsilon)-2\ E(\varepsilon^2)(A-bp) \right\} ;$$

21) $$e(p)(19) = \sqrt{\beta b^{-1}\ 12^{-1} \left\{ 3(A-bc)\ E(\varepsilon)-E(\varepsilon^2)(A-bp) \right\} (A-bp)} \ .$$

Identical arguments to those used previously enable us to represent the solution of this system by Figure 4.

VI (iii) The CS of a Risk Change

$$(R3)\ :\ d\hat{e}/d\sigma_\varepsilon^2 < 0\ ,\ d\hat{\beta}/d\sigma_\varepsilon^2 < 0\ .$$

Within the mean-variance framework which emerges from (A8), a mean-preserving increase in the riskiness of future goodwill loss simply corresponds to an increase in σ_ε^2 and, equivalently, in $E(\varepsilon^2)$. The CS consequences of this increase are easily represented as the outcome of the induced shifts in the efficiency loci to a new intersection.

By inspection of (20) and (21) we see that an increase in $E(\varepsilon^2)$ shifts both loci southwards. Which locus shifts the greater vertical distance at the original equilibrium, and hence whether the new equilibrium intersection occurs to the left or right of the initial one, depends upon whether $\partial e(p)(19)/\partial E(\varepsilon^2) = -2^{-1}\ (e(p)(19))^{-1}(A-bp)^2$ $\gtreqless -2(A-bp) \left\{ 3(A-bc)E(\varepsilon)-2E(\varepsilon^2)(A-bp) \right\}^{-1} = \partial e(p)(18)/\partial E(\varepsilon^2)$; ie, upon whether $3\beta^{-1}b \left\{ 3(A-bc)E(\varepsilon)\ -2E(\varepsilon^2)(A-bp) \right\} \left\{ 3(A-bc)E(\varepsilon)-E(\varepsilon^2)(A-bp) \right\}^{-1}$ $\lesseqgtr 1$. (We have used the fact that $\hat{e}(p)(18) = \hat{e}(p)(19)$ at the initial equilibrium.) As $\{\ \}\{\ \} < 1$, but whether $3\beta^{-1}b \gtreqless 1$ is ambiguous *a priori*, it seems that the direction of change is also ambiguous. However, we can also analyse the situation formally:

Formally,

$$\text{22)}\qquad \begin{pmatrix} \partial\hat{p}/\partial E\ (\varepsilon^2) \\ \partial\hat{e}/\partial E\ (\varepsilon^2) \end{pmatrix} = - \left|H\right|^{-1} \begin{pmatrix} J_{ee} & J_{pe} \\ J_{pe} & J_{pp} \end{pmatrix} \begin{pmatrix} J_{pE(\varepsilon^2)} \\ J_{eE(\varepsilon^2)} \end{pmatrix} .$$

Here, $J_{pe} = e^{-2}\ \beta\ 12^{-1} \left\{ 2\ E\ (\varepsilon^2)(A\text{-}bp)\ -3(A\text{-}bc)\ E(\varepsilon) \right\}$,

$J_{pE(\varepsilon^2)} = -\beta e^{-1}\ 12^{-1}\ 2(A\text{-}bp) < 0$, and $J_{eE(\varepsilon^2)} = -e^{-2}\beta b^{-1} 12^{-1} (A\text{-}bp)^2 < 0$. As $\varepsilon \in (0,1]$, $E(\varepsilon) \geq E(\varepsilon^2)$. $A\text{-}bp < A\text{-}bc$ as production is profitable in T-1 . Therefore $J_{pe} < 0$. Substituting into $\partial p/\partial E(\varepsilon^2) = -\left|H\right|^{-1} \times \left\{ J_{ee} J_{p\ E(\varepsilon^2)} + J_{pe} J_{e\ E(\varepsilon^2)} \right\}$ and noting that $-\left|H\right|^{-1} < 0$ and $J_{ee} < 0$ enables us to conclude $\partial\hat{p}/\partial E(\varepsilon^2) < 0$. It can likewise be shown that $\partial\hat{e}/\partial E(\varepsilon^2) < 0$. We therefore conclude that an increase in the riskiness of future goodwill loss encourages the firm to produce and sell more in the current period and to incur less expenditure upon quality control. Both tendencies act to increase the expected goodwill loss, for any realization of ε_T , for the subsequent period.

A direct relationship between risk and impatience is clear in our model in that price and quality responses to an increase in either are qualitatively the same.

The result $\partial\hat{p}/\partial E(\varepsilon^2) < 0$ can also be related to the findings of the traditional single period analysis of the firm facing constant marginal cost, random demand, and possessing linear risk preference (summarised in Horowitz [5], Chapter 13) . In this it is typically assumed that the firm sets price and quantity prior to the resolution of risk, and not knowing whether all demand will be satisfied, or more than satisfied, at that price and pre-determined output. It is found that a price lower than in the riskless demand case will be charged. No prediction can normally be made about the direction of change in the predetermined output. Despite the similarity of the price response to risk, this situation differs slightly from the one which we have analysed not only because of the presence of goodwill loss, but also because of the absence of *within* period demand risk in our analysis . One conjectures that, when we extend the analysis of price *and* quantity setting with random demand to a multiperiod setting where future goodwill loss corresponds to current unsatisfied demand at the announced price, the firm will be induced to both charge *higher* prices *and* to produce more (larger inventories) than in the riskless demand case. This extension will be pursued elsewhere.

One word about welfare. The determination of goodwill loss and its riskiness should, of course, properly be treated as endogenous in an equilibrium context and be based upon explicit consumer maximization. However, taking out model as it stands, we can say that provided consumers suffer disutility from having purchased a defective article, not only will expected goodwill loss be greater as the firm's perceived riskiness of goodwill loss increases, but also that consumer welfare will be lower. As firms are indifferent to the degree of risk, this suggests that risk-sharing arrangements which reduced the riskiness of goodwill loss would be social welfare-improving.

VII Conclusion

We have constructed a very rudimentary, partial equilibrium model of the firm's pricing and quality control when it loses goodwill (future sales) if it produces defective items. Our specifications, although plausible, were largely *ad hoc* and chosen in order to obtain determinate conclusions. Clearly, therefore, there is a need for generalisation. However, our results, such as they are, possess the sensible implications detailed in the text. They are not surprising once we make the obvious analogy between the firm's problem and an individual's investment-consumption problem starting from an initial endowment. The difference is that, of course, unlike the consumer, the firm can continue to both eat its cake and yet have it, provided it does not produce defectives.

Perhaps the most interesting results are the "no exit" ones, (R.1) and (R.1'). These were derived under perhaps the least favourable assumptions about the goodwill loss confronting the firm : that it is one-for-one with the number of defective items produced. These results have to be related to Akerlof's demonstration ([1]) that is the firms which produce or supply the more reliable commodities which would be driven from a market for goods of uncertain quality.

In view of (R.1) and (R.1'), the government's imposition of minimum quality standards should perhaps also be seen in a new light. Within the context of our model, such a quality standard would take the form of a minimum per period expenditure upon quality control and, hence, a minimum probability of producing zero defectives in any period. But this mandatory expenditure would also take the form of a fixed cost. Therefore, not only would it serve to improve the reliability of the typical item being produced, but also it could be interpreted as a deliberate device to enforce the exit from the market of firms with sufficiently bad records of producing defectives and thus accumulated goodwill losses.

Clive D Fraser
University of Warwick
July 1981

Bibliography

1. Akerlof, G., The Market for Lemons, Quarterly Journal of Economics, Vol.84 (1970), pp.488-500.

2. Arrow, K.J. and Nerlove, M., Optimal Advertising Policy under Dynamic Conditions, Economica, Vol.29 (1962), pp.129-142.

3. Dorfman, R., and Steiner, P.O., Optimal Advertising and Optimal Quality, American Economic Review, Vol.44 (1954), pp.826-836.

4. Hey, J.D., Goodwill-Investment in the Intangible, in Microeconomic Analysis, edited by D.Currie, D.Peel and W.Peters, Croom Helm, London 1981, pp.196-229.

5. Horowitz, I., Decision Making and the Theory of the Firm, Holt, Rinehart and Winston, Inc., New York, 1970.

6. Jacquemin, A.P., and Thisse, J., Strategy of the Firm and Market Structure : An Application of Optimal Control Theory, in Market Structure and Corporate Behaviour, edited by K.Cowling, Gray-Mills Publishing Co.Ltd., London, 1972, pp.61-84.

7. Rotschild, M., A Two-armed Bandit Theory of Market Pricing, Journal of Economic Theory, Vol.9 (1974), pp.185-202.

8. Viscusi, W.K., Employment Hazards: An Investigation of Market Performance, Harvard University Press, London, 1979.

Footnotes:

1 I am indebted to participants at an Economic Theory Workshop at Warwick and to Paul Weller, for helpful suggestions. The usual exonerations from my errors apply.

2 It also assumes, much more strongly and implausibly, that price in any period is such that those individuals previously sold defectives would otherwise have purchased in the current period. This will indeed be the case if price decreases temporally, but not necessarily so otherwise. The assumption is made in order to avoid having to identify the consumers who are sold defectives. The random goodwill loss case treated in Section VI below could be interpreted as a way of partly circumventing the objection to this implicit assumption of Specification A. In that case, ε_t there would correspond to the proportions of those sold defectives in period earlier than t who would have purchased in t.

3 Note that this conclusion and the ones immediately related below are actually independent of the nature of $D(p)$, except that $D'(p) < 0$, and are also independent of the probability distribution governing the incidence of defectives. The technical significance of (R1) (and (R1') below) for our model is that when we come to calculate future expected profits as a function of current price, by taking expectations over r, we need not restrict the range of integration to a subinterval of $(0, \bar{D}(p)]$.

4 This assumption is not essential. What follows could easily be modified to accomodate some statutory minimum per unit or lump sum expenditure upon quality controllor testing. The discussion of R1 would then become very pertinent.

However, if the firm is to be sold as a going concern at the end of T, the absence of a termination penalty is implausible because the final value of the firm's goodwill, in a strict accounting sense, will be dependent upon the goodwill lost in T. However, the assumption is defensible if the firm is simply going to be closed and the potential consumers are unaware of the firm's planning horizon. If they were aware that the firm plans to close at the end of T, this would be expected to radically alter T's demand schedule. This is because the rational consumer would predict poorer quality/less reliable commodities being produced during T. In any case, inclusion of a termination penalty merely complicates the algebra inordinately, without promising new insights, and the need for one will be obviated in a subsequent infinite horizon analysis.

5 This need not be the case in a model where sufficient loss of initial goodwill by one firm enables others to enter the market to produce related items.

6 In fact, e could hardly be expected to be increasing in ζ indefinitely, otherwise total revenues and quality control expenditure would converge.

7 The firm might not even know this much at the start of t. It might have to choose both its price and output at the start of t without knowing its demand function, except probabilistically. This slightly different case of pricing with a random current demand function has, however, been extensively treated in the literature (see, e.g. [5]). and we will not explicitly discuss it here. See also footnote 2.

ADDENDUM (to footnote 4)

Note that the somewhat bothersome requirement $e \geqq 1$ in the density (A.6) can easily be dispensed with by taking $(1 + e)^{-1}$ and $1-(1 + e)^{-1}$ $e \geqq 0$ as the factors in e. While the resulting algebra is necessarily more complicated, the procedure for characterising the solution remains identical.