ERROR CORRECTION MECHANISMS

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1. INTRODUCTION

The interface between economic theory and applied econometrics is often one of uneasy compromise, with the pragmatic justification for many accepted procedures resting on a tenuous theoretical base. This paper examines the surprisingly strong arguments that exist in terms of economic theory, for the use of error correction mechanisms in the specification of short run dynamic adjustment.

A common heresy exists that while economic theory provides a detailed analysis of comparative static equilibria it can offer no guidance as to the appropriate specification of dynamic adjustment towards an equilibrium. Perhaps in consequence it is not uncommon to find examples where the necessary dynamic specification is achieved by "tacking" onto an existing equilibrium specification some relatively *ad hoc* short run adjustment scheme. The intercession of stochastic arguments in this process is confused and critical implications are frequently ignored in practice, but perhaps more importantly there will typically be no guarantee that the dynamic specification is consistent with the prescribed equilibrium. Consistency in this sense requires that the short run dynamic adjustment be directed by the perceived disequilibrium and that eventual convergence to the equilibrium position be ensured. That two separate theoretical arguments, co-exist within the final specification is the root cause of many difficulties both theoretical and empirical.

Concern for this problem is not new; the literature on adjustment costs in the investment decision (see Lucas [11], Treadway

[16] and particularly Gould [6]) raised the issue in a specific application and more recently an important series of papers, Hendry and Anderson [8], Davidson, Hendry, Srba and Yeo [5], (DHSY), Hendry [7], Hendry and Von Ungern Sternberg [9] (HUS) and Currie [3] have discussed the general problem. The result has been to emphasise the use of a particular econometric specification known as an error correction mechanism that may be constructed so as to achieve both requirements for the consistency in short run behaviour described above. In this paper it is shown that a family of error correction mechanisms may be developed and that the appropriate choice from this set depends on the dynamic properties of the equilibrium. Further in most of the applications cited above the development of the error correction specification was formally derived through the optimization of a cost function in which particular equilibrium conditions served as a target for the short run dynamic adjustment. When the equilibrium specification is itself derived through some dynamic optimization exercise it is a small but theoretically important step to integrate both aspects within the same dynamic optimization problem. A second objective of the paper is to indicate certain general features of the cost function in such an optimization problem that ensure that the resulting reaction function incorporates dynamic adjustment that is consistent with the equilibrium. Moreover by allowing for random errors in optimizing behaviour the resulting specification leads to white noise error terms at the point of estimation. Misspecification detected through residual autocorrelation may then be traced back to an inappropriate specification of the cost function.

2. SHORT RUN ADJUSTMENT AND STEADY STATE ERROR

In this section we consider the question of the consistency of the short run adjustment given a deterministic environment and given an equilibrium target. In the following section we then remove these restrictions and apply the general principles developed here to the specification of the cost function for the "integrated" dynamic optimization problem.

The fundamental issue of concern is whether it is possible to make general statements regarding short run dynamic adjustment mechanisms that ensure some desired long run position. While we abstract, for the present, from determining the path of desired values x_t^*, we assume that the economic agent is able to recognise deviations between this state and his current position. The motivating force in short run adjustment will then be transmitted through some general dynamic reaction function on the observed disequilibrium. Thus we may write:

$$x_t = A(L)(x_t^* - x_t) \tag{1}$$

and

$$e_t = x_t^* - x_t \tag{2}$$

where $A(L)$ is an arbitary rational polynomial in the lag operator L, and e_t is the error between the current and desired or target position. We obtain the conventional partial adjustment hypothesis by setting

$$A(L) = \gamma / \left\{ (1 - \gamma)(1 - L) \right\} .$$

Formally we require that the short run adjustment mechanism achieve a zero steady state error given the chosen target. This is a basic issue in the analysis of dynamic control systems and in this section we present results on discrete time models that are developed from corresponding results for continuous time models in the control/systems analysis literature. To make our meaning clear, by steady state behaviour we mean behaviour after any transient part of the solution has died away. Nothing is implied regarding, for example, stationarity or constancy by the expression "steady state"; it may indeed represent <u>any</u> dynamic equilibrium path.

Equations (1) and (2) imply that

$$e_t = \frac{x_t^*}{1 + A(L)} \tag{3}$$

and we now consider the steady state behaviour of the error for different assumptions on the dynamic behaviour of the x_t^* .

The final value theorem of z transforms (see R.G.D. Allen [1]) enables us to consider the value of a time subscripted variable as t goes to infinity, so that in a linear model, analysis of the steady state may be separated from that of the transient component. Using this theorem we may write the steady state error as

$$e_{ss} = \lim_{z \to 1} (1 - z^{-1}) E(z) \tag{4}$$

where $E(z)$ is the z transform of expression (4) . Thus

$$e_{ss} = \lim_{z \to 1} (1 - z^{-1}) \frac{X^*(z)}{1 + A(z)} , \qquad (5)$$

which clearly depends on both the dynamic behaviour of the equilibrium value x_t^* and the adjustment mechanisms $A(L)$. We first consider the response to different paths for x_t^* . We assume that x_t^* has a constant value, which without loss of generality we take to be zero, up to some time point again taken to be $t = 0$, and thereafter consider three cases.

(a) <u>Static Equilibrium</u>: $x_t^* = k, t > 0$

Then the z transform of the equilibrium path may be written

$$X^*(z) = \frac{k}{(1 - z^{-1})}$$

and the steady state error is given as

$$e_{ss} = \frac{k}{1 + \lim_{z \to 1} A(z)} \qquad (6)$$

(b) <u>Constant Growth</u>: $x_t^* = kt, t > 0$

In this case

$$X^*(z) = \frac{k}{(z - 1)(1 - z^{-1})}$$

$$e_{ss} = \lim_{z \to 1} (1 - z^{-1}) \frac{k}{(z - 1)(1 - z^{-1})(1 + A(z))}$$

$$= \frac{k}{\lim_{z \to 1} (z - 1) A(z)} \qquad (7)$$

(c) <u>'Dynamic' Growth:</u> $x_t^* = \frac{1}{2}kt^2, \ t > 0$

In this case

$$X^*(z) = \frac{\frac{1}{2}kz(z+1)}{(z-1)^3}$$

and so

$$e_{ss} = \lim_{z\to 1}\left[\frac{(1-z^{-1})\ \frac{1}{2}kz(z+1)}{(z-1)^3\ (1+A(z))}\right]$$

$$= \lim_{z\to 1}\left[\frac{\frac{1}{2}k(z+1)}{(z-1)^2(1+A(z))}\right] = \frac{k}{\lim\limits_{z\to 1}(z-1)^2 A(z)} \qquad (8)$$

Expressions (6), (7) and (8) provide the required results on steady state error for three alternative choices for the dynamic characteristics of the equilibrium path given an arbitrary rational transfer function $A(L)$. We now consider specializations of the transfer function, that is different specifications of the short run adjustment process. Of particular relevance is the classification of models according to the number of poles at unity in the denominator polynomial of $A(L)$. Particular steady state error properties are then seen to be determined, for a given equilibrium path, by the particular type of transfer function or short run adjustment specification.

We consider transfer functions that may be written in the form

$$A(z) = \frac{(z - z_1)(z - z_2) \ldots\ldots (z - z_m)}{(z - 1)^s (z - p_1)(z - p_2) \ldots (z - p_n)}$$

$$\equiv \frac{A^*(z)}{(z - 1)^s} \tag{9}$$

where p_i, the non-unity poles are assumed to satisfy $|p_i| < 1$, $i = 1 \ldots n$, so that $A^*(z)$ is a stable transfer function. A model may be said to be of type s if there are s poles at unity in the transfer function, and if s is non zero, to possess integral action through s integral effects. Returning to our general form for the model, expression (1), we can see the following classification of typical specifications according to model type.

<u>TYPE 0</u> $x_t = A(L)\ (x_t^* - x_t)$ (10)

<u>TYPE 1</u> $\Delta x_t = A^*(L)\ (x_{t-1}^* - x_{t-1})$ (11)

<u>TYPE 2</u> $\Delta^2 x_t = A^*(L)\ (x_{t-2}^* - x_{t-2})$ (12)

The importance of integral action and the number of poles at unity can be seen immediately by looking at the consequent cancellations when (9) is substituted into expressions (6), (7) and (8) for the steady state error. This information on the combined effect of adjustment process and target dynamics is collected in the following table.

STEADY STATE ERROR

Model Type s	Static Equilibrium i = 1	Constant Growth i = 2	'Dynamic' Growth i = 3
0	k_1	∞	∞
1	0	k_2	∞
2	0	0	k_3

where k_i are finite, (typically) non zero, constants.

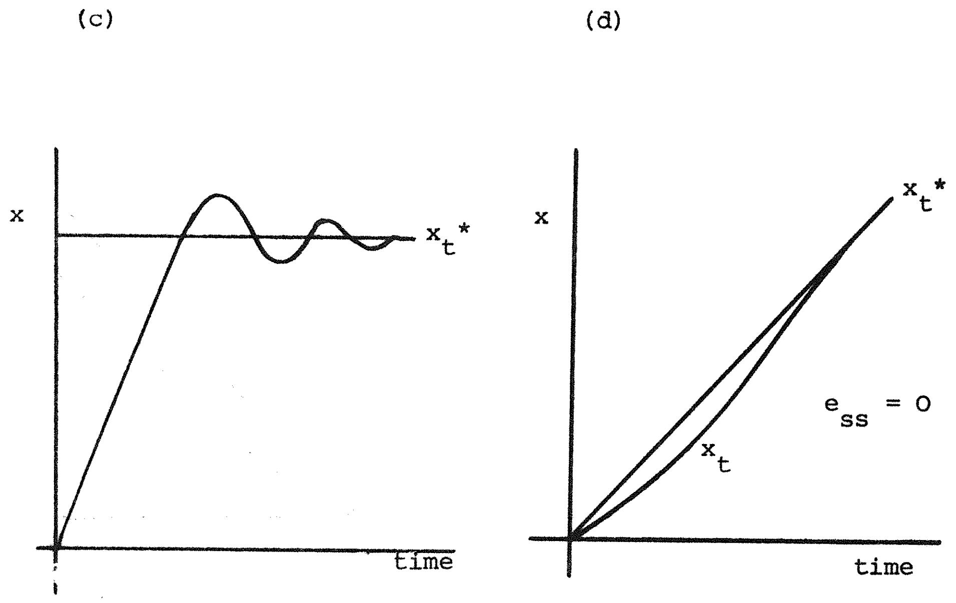


FIG. 1

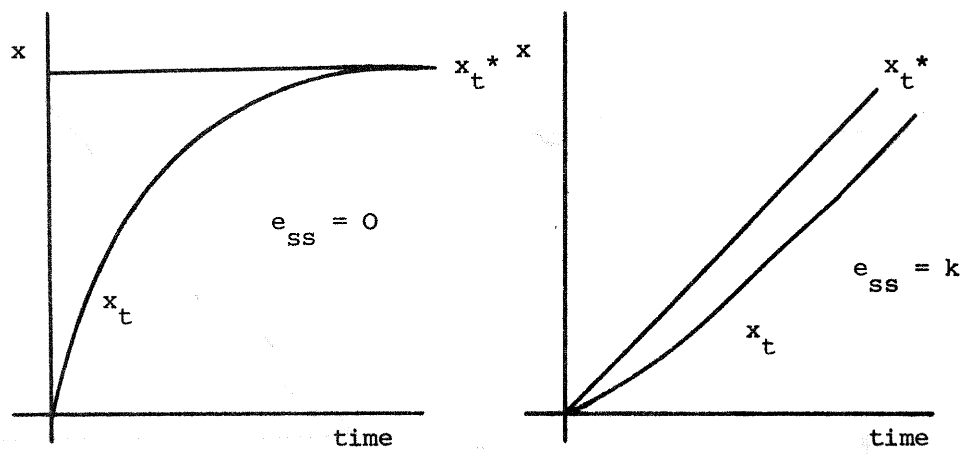
Type 2, Static Equilibrium — Type 2, Constant Growth

One interesting case that is not plotted in figure 1 is when the model is of type one. We separate out this case because it corresponds to the usual error correction mechanism, one example of which is the following implementation of the partial adjustment hypothesis

$$x_t - x_{t-1} = \gamma(x_t{}^* - x_{t-1}) \tag{13}$$

The two graphs in figure 2 show the response from this partial adjustment model to static and constant growth paths

(a) (b)



Type 1, Static Target — Type 1, Constant Growth

FIG. 2

PARTIAL ADJUSTMENT MODEL

Thus, use of the standard partial adjustment mechanism will only ensure the achievement of the target level if that level is constant in steady state. If the target path is constantly growing the partial adjustment model with lead to a fixed offset and will thus never satisfy the target level even in steady state. For higher order time paths the partial adjustment mechanism will diverge from the target path. If the target path switches between two constant growth paths the partial adjustment model will follow the switch but will still not converge to the target in steady state. The actual size of steady state error (when it is finite) can of course be calculated *a priori* and for any particular case is given by expressions (6), (7) and (8). Typically it is directly related to the steady state level or growth rate and inversely related to $\lim_{z \to 1} A(z)$. There is of course no reason why this offset should be small.

As a final comment on the approach taken in this section it is perhaps appropriate to emphasise the crucial importance of the choice of the target. Given a particular theory any distinctive feature of the implied steady state behaviour could serve as a target, although it is likely that a specification with only a single target would only provide a partial characterization of the equilibrium and hence only partially identify the underlying theory. For instance, homothetic preferences lead to a unit long run income elasticity in the consumption function so although this condition could serve as a target further conditions would be required to distinguish the life cycle from the permanent income hypothesis. A further problem can arise when a target is specified in such a way that it does not uniquely define the type of adjustment model. Again the unit elasticity condition provides an example since it simply reflects long run proportionality and this in itself does not uniquely define the adjustment mechanism. Thus in Figure 3 below $(\frac{C}{Y})^*$ is assumed to be the desired equilibrium position such that $C = k^*Y$ and this is clearly consistent with a long run unit elasticity. However so is the steady state $(\frac{C}{Y})^{\dagger}$ defined by $C = k^{\dagger}Y$ towards which an adjustment path converges. So although the target, if defined in terms of the unit elasticity, is achieved, the actual equilibrium k^* is never reached.

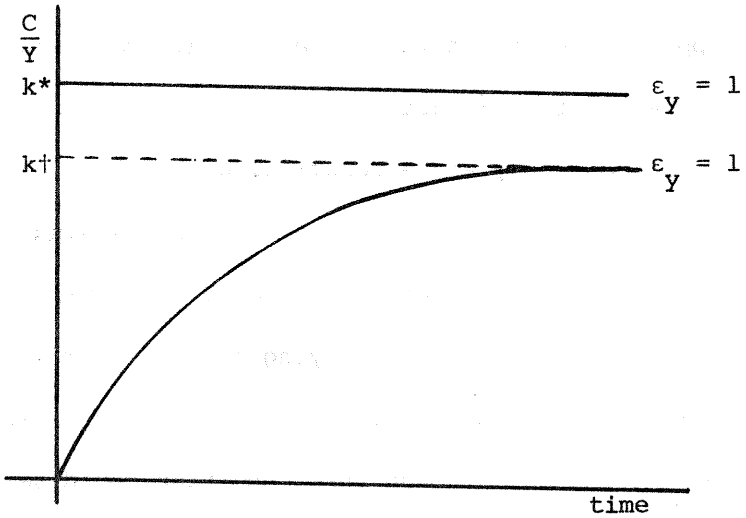


FIG. 3

3. THE INTEGRATION OF SHORT RUN DYNAMIC ADJUSTMENT AND EQUILIBRIUM BEHAVIOUR

Most econometric relations correspond to the control rule or reaction function of a nominal decision maker faced with a dynamic optimization problem, thus consumption expenditure is taken as a control variable in a utility optimization exercise. The weights on the variables in such a control rule would be designed by a control engineer but are estimated as parameters by the econometrician. One of the most basic requirements in the design of control mechanisms in physical systems is the achievement of zero steady state error, and the reason why many econometric specifications fail to satisfy this basic requirement can be traced to a failure in the formulation of the economic problem. The specification of the cost function in the dynamic optimization problem plays a crucial role. In particular it seems that the usual approach to the treatment of behaviour under uncertainty leads to decision rules that have a limited capacity for contingent action given unforeseen events. Fully contingent plans naturally return zero steady state error so that the design of these decision rules necessarily involves integral action as discussed in the previous section and they will then automatically integrate both equilibrium and short run behaviour. To clarify these points we shall consider alternative approaches to the development of control rules in economics.

3.1 Cost Functions and Control Rules

Two approaches to the control of physical systems may be found in the control literature; classical control techniques that rely heavily on proportional, integral and derivative (PID) feedback effects based on the observed output error, and modern control theory that tends to rely on state variable feedback and also substantially on an optimal control formulation of the problem. Typically economic theory has relied on the dynamic optimization or modern control approach but it is certainly not invalid to formulate PID forms of reaction functions and indeed they provide considerable insight and may correspond more directly to behaviour under uncertainty than the usual optimal formulations. We now briefly outline the relative merits of these two distinct approaches to the construction of economic reaction functions, or plans, before showing that the solution to our problem is gained by recognising the opportunity to combine certain features of both.

3.1.1 PID reaction functions

The basic form of a PID control rule in continuous time is as follows,

$$x(t) = k_p e(t) + k_i \int e(t)dt + k_d \frac{de(t)}{dt} \tag{14}$$

where the error $e(t) = x^*(t) - x(t)$, $x^*(t)$ is the target or desired position and k_p, k_i, k_d are the proportional, integral and derivative feedback gains respectively. Phillips discussed the nature of PID control rules in the context of stabilization policy in two classic papers in this journal in 1954 and 1957, [13], [14].

Transforming expression (14) into discrete time we have

$$x_t = k_p e_t + k_i \sum_{s=0}^{t} e_s \tau + k_d \left(\frac{e_t - e_{t-1}}{\tau}\right) \tag{15}$$

where τ is the sampling interval. Lagging one period gives

$$x_{t-1} = k_p e_{t-1} + k_i \sum_{s=0}^{t-1} e_s \tau + k_d \left(\frac{e_{t-1} - e_{t-2}}{\tau}\right) \tag{16}$$

setting $\tau = 1$ and subtracting (16) from (15) gives

$$\Delta x_t = k_p \Delta e_t + k_i e_t + k_d \Delta^2 e_t$$

$$\Delta x_t = k_p \Delta (x_t^* - x_t) + k_i (x^*_t - x_t) + k_d \Delta^2 (x_t^* - x_t) \tag{17}$$

The righthand side of expression (17) may be rearranged as follows;

$$(1 - L)x_t = (k_p(1 - L) + k_i + k_d(1 - L)^2)(x_t^* - x_t)$$

$$= ((k_p + k_i + k_d) - (k_p + 2k_d)L + k_d L^2)(x_t^* - x_t)$$

$$= (\alpha_0 + \alpha_1 L + \alpha_2 L^2)(x_t^* - x_t)$$

So $$(1 - L)x_t = A^*(L)(x_t^* - x_t) \tag{18}$$

indicating that the PID control rule is in fact a model of type 1 and thus ensures a zero steady state error against static equilibria, but not against more general dynamic growth paths. If we wish the PID control rule to be able to account for these higher order growth paths we must, following the discussion in the previous section, increase the model type. In terms of the PID specification this would be equivalent to adding a further term of higher order of integration, i.e. the integral of the integral, to (14). In many cases behaviour of the transient component may be affected by the introduction of further integral terms, indeed there may potentially be a conflict between the attainment of steady state properties and reasonable (i.e., relatively non-oscillatory) short run properties in the resulting model (see Phillips [13] p. 297).

We should emphasise that this discussion relates to an examination of the properties of a PID control rule from the design point of view. Given the preceding arguments an equivalent PID reaction function could be considered to represent reasonable economic behaviour and equation (17) could be put forward for estimation with a stochastic error term appended. However there is clearly a choice as to whether the stochastic term should originally enter equation (15) rather than (17). Given the transformation from (15) to (17), the stochastic implications of the two options are clearly not equivalent. This confusion was one of the objections raised regarding current practice in the introduction. What is crucial for equation (17) to yield a zero steady state error in a static equilibrium are the restrictions on the parameters that lead to a dependent variable appearing in differenced form with an explanatory, disequilibrium, variable in levels. So although the direct use of a PID rule for econometric analysis may in principle deliver the required steady state error properties, there still remains the question of appropriate stochastic specification. We now investigate whether the modern or optimal control approach can return such a restricted specification without the ambiguity in stochastic specification.

3.1.2 'Optimal' reaction functions

Consider the general linear economic model written in state space form as follows

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cu \end{aligned} \tag{19}$$

where at the risk of some confusion with the previous notation we have adopted standard control notation, so that x, u and y represent vectors of state, control and observed output variables respectively, A, B and C are constant matrices. In the traditional economic optimization problem we wish to minimise a general cost function J,

$$J = \int_0^\infty (x'Qx + u'Ru)dt \tag{20}$$

subject to (19)

The optimal control rule that follows may be written (see [2])

$$u^* = R^{-1}B'Px \tag{21}$$

This function would correspond, for example to the implied consumption function in a life cycle model in which a discounted utility stream represented the cost. Note that the cost function in (20) could equally be expressed in terms of deviations of the actual state from a desired state position, in which case the control rule can be written

$$u^* = R^{-1}B'(Px + b) \tag{22}$$

where b is a function of the desired state.

It is important to recognise the dynamic nature of the state variables in this analysis. In order to express a general dynamic model in state space form, the state vector would be defined in such a way that the dynamics of the original model would be reduced to the first order, as expressed in equation (19). This necessarily involves increasing the number of variables in the state vector as higher order derivatives become defined as new state variables. Two points are then important to bear in mind in what follows. In the first place, the state vector will, in our case, only consist of endogenous variables and such derivatives of these variables as are required in the economic model. State variable feedback decision rules, such as (21) and (22), expressed as linear functions of this state vector, will then reflect the effect of derivatives, and implicitly integrals, of the underlying economic variables only to the extent to which the state vector is so defined. No further derivative or integral operations arise in structure of a rule such as (22); R, B and P are constant matrices. Secondly as it stands most optimal formulations of this sort in economics rest totally on the determination of equilibrium behaviour and frequently the associated state vector includes no higher derivative terms. To the extent then that such 'optimal' equilibrium specifications ignore dynamic adjustment we can see that they will typically lead to models such as (21) or (22), which have no integral effects and hence, in terms of our previous discussion are of type zero and so yield non zero steady state error.

A PID control rule for the same problem, but not formulated as an optimization exercise, could also have been developed and would be written as

$$u^{\dagger} = k_p e + k_i \int e \, dt + k_d \dot{e} \tag{23}$$

where e represents the error between the target and actual output variable y_t. In most cases of relevance to econometrics the entire state vector will be observable so that the distinction between state and output variables vanishes. However apart from this question there are important differences between the two formulations. In particular the optimal control rule, as discussed above, will only incorporate proportional and derivative effects whereas the PID control rule will, by its constuction, also incorporate integral action on the observed output error. This crucial difference between the two approaches is unaffected by whether the cost function in the optimal problem is framed in terms of the deviation from the desired path and although in this case the control rule (22) will be a function of the desired state, through b, no integral action is implied. It may be intuitively obvious, at this stage, that the problem with the optimal approach lies in the nature of the state variables or in other words what are considered to be the appropriate economic variables in the cost function.

3.1.3 A Synthesis

Relatively recent developments suggest that integral action may be introduced into an optimal formulation in one of two ways, either by extending the state vector to include a new state variable that is in effect the integral of output error, or alternatively by introducing into the cost function the derivative of control action, see Kwakernaak and Sivan, Sect. 3-7 (1972) or Anderson and Moore, Chapter 10 (1971). Once these adjustments have been made the optimal control rule leads to an equivalent PID control rule. So in terms of our preceding discussion regarding the specification of short run adjustment we can see two general classes of cost function specification that will ensure that PID reaction functions and hence error correction mechanisms represent the <u>optimal</u> response of economic agents.

Typically the partial adjustment model is derived from the minimisation of the following one period cost function.

$$J_1 = \alpha_1(x_t - x_t^*)^2 + \alpha_2(x_t - x_{t-1})^2 \qquad (24)$$

where the first term represents the cost of the error or disequilibrium and the second term, the cost of adjusting towards equilibrium. Nickell (1980) extends this class of cost function to consider minimising, at time t

$$J_2 = \sum_{s=0}^{\infty} \alpha^s \Big[\lambda_1(x_{t+s} - x^*_{t+s})^2 + (x_{t+s} - x_{t+s-1})^2 - 2\lambda_3(x_{t+s} - x_{t+s-1})(x^*_{t+s} - x^*_{t+s-1})\Big] \qquad (25)$$

The third term indicates that the loss is attenuated if the decision variable moves in the same direction as the target variable, x_t^*.

HUS consider a similar loss function to J_2. Since cost functions such as J_2 include terms in the derivative of control action, the resulting optimal reaction functions will incorporate integral action of some degree. The preceding discussion suggests that introducing as a separate variable in the cost function the integral of output error could also lead to a reaction function with integral effects. In many economic contexts this new state variable would be a stock variable, such as assets or wealth, corresponding to an equilibrium specification in the underlying flow variables.

The cost functions (24) and (25) are set up with the sole consideration of determining optimal short run adjustment functions, given an equilibrium target, although the cost function in the general control problem discussed in 3.1.2 was quite unrestricted and would represent for instance the underlying utility function. So by adding terms representing either the integral of output error or the derivative of control action to cost functions that at present only lead to equilibrium relations we may derive through a single theoretical argument, economic reaction functions that incorporate both optimal equilibrium behaviour and optimal short run adjustment consistent with that equilibrium.

Consider the extended cost function

$$J = \int_0^{\infty} (x'Qx + u'Ru + \dot{u}'S\dot{u})\,ds \tag{26}$$

and its optimisation subject to (19). We may then define an extended state vector z by

$$z = \begin{bmatrix} x \\ u \end{bmatrix}$$

with an associated state equation

$$\dot{z} = \begin{bmatrix} \dot{x} \\ \dot{u} \end{bmatrix} = \begin{bmatrix} A & B \\ O & O \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} + \begin{bmatrix} O \\ I \end{bmatrix} \dot{u} \qquad (27)$$

The cost function may then be rewritten as

$$J = \int_0^\infty (z' \begin{bmatrix} Q & O \\ O & R \end{bmatrix} z + \dot{u}' S \dot{u}) ds \qquad (28)$$

Solving this optimal control problem leads to, (see [2] or [10]),

$$\dot{u}^* = F_1 x + F_2 \dot{x}$$

or

$$u^* = \int_0^t F_1 x \, ds + F_2 x \qquad (29)$$

which is an 'optimal' control rule of type one, that incorporates an integral effect in addition to terms proportional to the state vector, unlike (21) and (22). Following our discussion of steady state error in section 2, optimal reaction functions of higher model type could then be derived by introducing further derivatives of the control variable or higher integrals of output error into the cost function. This naturally returns the issue of the specification of the cost function to one of economic theory concerning the selection of the appropriate economic decision variables to be included. In particular this discussion emphasises the crucial importance of including stock variables that reflect integrals of underlying flow relationships in developing a fully consistent dynamic economic theory.

3.2 Uncertainty

The preceding optimal rules have been derived in a deterministic environment and as such have ignored the impact of information flows and uncertainty on the behaviour of the economic agent through the planning period. As a consequence the stochastic specification of decision rules, such as (29), when viewed *ex post* by the econometrician, still remains *ad hoc*. To complete this link it is important that we now consider how the foregoing discussion is influenced by the recognition of uncertainty in the agent's intertemporal planning. A further source of stochastic error arises, of course, at the model/data interface when the theoretical reaction function, being only an approximation to reality, is confronted with the data.

Reaction functions, such as (21) or (22), represent "open loop" plans, in that both the functional form and the values of the determining right-hand side variables over the entire planning horizon are assumed known at the origin of the plan. When the theoretical model becomes stochastic, thereby introducing uncertainty, it is in general impossible to derive analytically a closed form expression for the control rule. However in the L.Q.G. formulation employed above (Linear model, Quadratic cost function and Gaussian disturbances) the certainty equivalence result delivers exactly the same reaction function, (21), as in the deterministic case. Once again the functional form of the reaction function is fully determined at the origin of the plan, but now it is expressed in terms of the expected values of the uncontrollable exogenous inputs. As time proceeds through the planning period the actually realised values of these variables are substituted into the fixed control rule and hence this feedback of current information leads to a "closed loop" plan. Such closed loop behaviour tends to ameliorate the effect of discrepancies that arise between the expected future states and exogenous inputs and their realised values.

If the economy were linear, as assumed in the models used above, then imperfect expectations would lead to an additive error in the reaction function with properties determined by the process used to generate the future expectations. So that if, for instance, rational expectations had been employed the errors would be characterised by their conditional independence of the information set that generated the expectations. From the point of view of the econometrician attempting to identify the reaction function by observing the empirical effects of the plan *ex post*, there would in this case then be none of the ambiguity in stochastic specification that arose when considering the direct PID model in section 3.1.1. Examples of this argument are found in HUS [9] and Hendry and Anderson [8] where "integral inducing" cost functions, such as J_2, of section 3.1.3, are optimized, leading to parametric restrictions in the reaction function that may then be rewritten as an error correction mechanism with an additive error. If the economic environment were non linear then clearly the impact of stochastic errors would be more complicated and most probably non additive, even if we assumed the linear model as an approximation. However employing a Linear-Quadratic-Gaussian framework, with a suitable cost function returns as an optimal reaction function an error correction mechanism with additive stochastic errors determined by the expectational errors in the plan.

While this is the basic result we require to justify empirical analysis of error correction mechanisms, the argument that follows makes an even stronger case for their use in representing behaviour under uncertainty.

In the foregoing characterization of decision making through time it was assumed that the agent would not attempt to rederive his reaction function on the arrival of updated information. Clearly if the agent responded in this "time inconsistent" manner then the econometrician would fail to identify a single structurally invariant relationship for the reaction function and the preceding argument regarding the additive nature of the stochastic errors would become irrelevant. How the agent interprets the source of the uncertainty in this problem will crucially affect the way he responds in formulating his reaction. In the stochastic control problem described above the undertainty is assumed to be of such a form that the disturbances follow a normal distribution with a zero mean. Such stochastic disturbances can be distinguished from "one off" or deterministic disturbances that may occur during the plan. Moreover even if a series of such "one off" disturbances were seen, *ex post*, to obey some stochastic law there is no reason to believe that the agent would not respond to each one in isolation, attempting to account for its particular effect on his plan. Crucially, with regard to our previous discussion, the optimal reaction functions are not the same in either case e.g. (22) and (29) . For instance when driving a car that is hit by a gust of wind it is probably inadvisable to respond given only the knowledge that in general such wind gusts are normally distributed around a point of zero effect, particularly if the wind gust lasted until the end of the plan, which in this case might be earlier than anticipated. The point is that in economic decision problems where the environment may be poorly understood, the nature of uncertainty may in some cases be inadequately represented by such long run notions of uncertainty that are captured by probalistic laws. What is required is some contingent plan that describes how to respond to disturbances as they arise and it is precisely this capacity, through their inclusion of integral action that has led to the predominance of PID control rules for physical systems. Notice that (29) could be treated as a stochastic control rule and hence would represent response to both characterizations of uncertainty. The integral terms have the effect of responding to the disturbance in a manner that ensures zero steady state error. Thus a plan may be derived at the origin of the planning period that is robust to particular types of future disturbances. Indeed it is precisely the analysis of section 2 that determines the response of a dynamic reaction function to the three forms of shock in the desired target position that may, in fact, have been induced by such a disturbance. If the agent employs a contingent plan that is robust to disturbances in this sense we may once again be assured of a structurally constant reaction function with additive errors for empirical analysis. It is therefore argued that there exists a separate justification for error correction mechanisms in representing contingent behaviour in an uncertain economic environment, over and above the question of consistency in short run dynamic adjustment. Notice once again the crucial role played by stock variables in representing the integral effects, but now with regard to planning under uncertainty. 1/

1/ The issues raised in this section are discussed in more detail in Salmon [15] .

4. Conclusion

The main objective of this paper has been to clarify the principles behind error correction mechanisms and to emphasise their role in the theoretical integration of short run and equilibrium behaviour. The following are the main points that have emerged.

(1) The notion of model type may be useful in determining the steady state properties of econometric models, where the model type essentially specifies the degree of integral action in an equation. The dynamic behaviour of a target variable determines the degree of integral action required to ensure zero steady state error. The appropriate specification of an error correction mechanism should then vary according to the behaviour of the target.

(2) When either the integral of error or the derivative of control action are included in a cost function the resulting optimal reaction function includes integral action. It is then straightforward to incorporate within the same dynamic optimisation problem both the determination of equilibrium behaviour and short run dynamic adjustment towards that equilibrium. Thus the common view that dynamic specifications are simply *ad hoc* may be challenged since the resulting error correction mechanisms represent the optimal response of economic agents.

(3) One basic form of error correction mechanism is the PID rule.

This form of reaction function exists independently, without requiring any interpretation as the result of an optimization exercise and may therefore serve when there is a poor understanding or even complete absence of an adequate "optimal" theory.

(4) The presence of integral action in decision rules makes them, to a degree, robust to disturbances and thus they may reflect more accurately economic behaviour under uncertainty than decision rules that do not possess integral action.

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