A LABOUR-MANAGED FIRM'S REACTION FUNCTION RECONSIDERED

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1. INTRODUCTION

So far it has been thought that a labour-managed (LM) firm's reaction function slopes upward in Cournot's short-run duopolistic situation where the variables acted on are quantities of output (see Vanek, 1970, pp.114-115; Ireland and Law, 1982). In this paper we will make it clear that this statement holds in some limited sense. In inquiring into the shape of the LM firm's reaction function in the short-run situation, it seems that the firm's labour cost function as defined by Meade (1974) is not only assumed to be subject to the increasing marginal labour cost of output everywhere, but is also assumed to have the property that the elasticity of the short-run labour cost curve, which is defined as the proportionate rate of change of labour with respect to output, is greater than unity. However, if we assume that the marginal labour cost is positive everywhere, and is at first declining and then increasing for fixed positive levels of the other factors,[1] or that, even if it is always increasing in the output, the short-run labour cost function has the property that its elasticity is at first less than, equal to, and eventually greater than unity according to an increase in output because of the existence of overhead labour, then what will happen to the LM firm's reaction function?

The former technological assumption is usually adopted in examining the behaviour of a LM firm, competitive or monopolistic, and also includes the property concerning the elasticity of the short-run labour cost curve stated in the latter. Now turning our attention to the case of a modern industrial enterprise, we can see that there exists a large amount of overhead labour such as foremen and office staff who are employed in a more stable manner than the production workers. This fact causes the average labour cost—the amount of total labour required, both production and overhead, per unit of output—to decline. Specifically, as the output level expands from a point of low plant utilisation, the overhead labour staff will be used more efficiently, thereby lowering the average labour cost even though the marginal production labour cost is increasing. This is regarded as one of the important factors that characterise a large-scale industrial enterprise such as General Motors. Also, we will confine ourselves to the case of a duopoly where each firm constitutes a fairly substantial share of a market, which provides further justification for making use of the latter technological assumption.

The purpose of this paper is to work out the shape of a LM firm's reaction function in the short-run duopolistic situation on each of the above-mentioned assumptions concerning the labour cost function. We will begin with Cournot's theory of duopoly whose underlying idea is that of the 'reaction' of one partner assuming the behaviour of the other partner constant. Then, in order to look at what effect a change in the slope of the LM firm's reaction curve has upon a Cournot equilibrium, we will deal with a Cournot 'mixed' duopoly model composed of one LM (net income per worker-maximising) firm called Illyrian and one profit-maximising (PM) firm. Furthermore, following Dixit (1980), we will extend the analysis in order to examine the properties of the LM firm's reaction function in comparison with those of the PM firm's familiar reaction function.

2. COMPARATIVE STATICS OF MIXED DUOPOLY

This section will discuss a Cournot-type mixed duopoly model in which there are one LM firm and one PM firm in an industry.[2] To begin with, we will be concerned with discussing the shape of the LM firm's reaction function in the short run. Then, we will think of the effect on a Cournot equilibrium of a change in the shape of its reaction function.

Let us assume that two firms produce homogeneous products and face a downward-sloping market demand function,

$$P=P(Q_1+Q_2,\alpha)=P(Q,\alpha), \tag{1}$$

where total output, Q, is the sum of the firms' outputs, Q_1 and Q_2, P is the market price of homogeneous products as a function of the total output with $\partial P/\partial Q<0$ and α is a slope-preserving demand shift parameter with $\partial P/\partial\alpha>0$,[3] that is to say, $\partial^2 P/\partial Q\partial\alpha=0$. Let

$$L_i=L_i(Q_i),\ i=1,2, \tag{2}$$

represent the number of members or the number of workers employed which depends solely upon the level of output.[4] At the moment no assumptions are placed on this function. The total revenue of each firm depends upon his own output level and that of his rival,

$$R_i=P(Q_1+Q_2,\alpha)Q_i=R_i(Q_1,Q_2,\alpha),\ i=1,2. \tag{3}$$

Each firm has a level of fixed debt charge, F_i, $i=1,2$. Moreover, each worker in a PM firm is paid the given market wage rate, w. We make subscript 1 refer to a LM firm and subscript 2 to a PM firm throughout the paper.

Following Ward (1958), the appropriate maximand of a LM firm is assumed to be the dividend or net income per worker,

$$y_1=\frac{R_1(Q_1,Q_2,\alpha)-F_1}{L_1(Q_1)}, \tag{4}$$

which is called the Illyrian objective. Let us suppose that the LM firm is free in the short run to choose the number of workers and hence adjusts Q_1, given α, so as to maximise (4).[5] If we make the Cournot assumption that the conjectural variations are zero, then the first- and second-order conditions for maximisation are given by

$$\frac{\partial y_1}{\partial Q_1} = \frac{\frac{\partial R_1}{\partial Q_1} - y_1 \frac{dL_1}{dQ_1}}{L_1(Q_1)} = 0 \tag{5}$$

so that

$$\frac{\partial R_1}{\partial Q_1} = y_1(Q_1, Q_2, F_1)\frac{dL_1}{dQ_1}\ , \tag{5'}$$

and

$$\frac{\partial^2 y_1}{\partial Q_1^2} = \frac{\frac{\partial^2 R_1}{\partial Q_1^2} - y_1 \frac{d^2 L_1}{dQ_1^2}}{L_1(Q_1)} < 0 \tag{6}$$

or using (5)

$$\frac{\partial^2 y_1}{\partial Q_1^2} = \frac{\frac{\partial R_1}{\partial Q_1}}{L_1(Q_1)}\left[\frac{\frac{\partial^2 R_1}{\partial Q_1^2}}{\frac{\partial R_1}{\partial Q_1}} - \frac{\frac{d^2 L_1}{dQ_1^2}}{\frac{dL_1}{dQ_1}} \right] < 0. \tag{6'}$$

Differentiating (5') with respect to Q_1 and Q_2, and then collecting terms yield

$$\frac{dQ_1}{dQ_2} = - \frac{\frac{\partial^2 R_1}{\partial Q_1 \partial Q_2} - \frac{\partial y_1}{\partial Q_2}\frac{dL_1}{dQ_1}}{\frac{\partial^2 R_1}{\partial Q_1^2} - y_1 \frac{d^2 L_1}{dQ_1^2}} \tag{7}$$

which shows the slope of LM firm 1's reaction curve. Since the denominator of (7) is negative from (6), the sign of the slope of the reaction function only depends upon that of the numerator of (7). Since the labour cost elasticity of output, namely the elasticity of the short-run labour cost curve, is defined as the proportionate rate of change of labour with respect to output,

$$\omega_i = \frac{d\log L_i}{d\log Q_i} = \frac{Q_i}{L_i}\frac{dL_i}{dQ_i} = \frac{\text{marginal labour cost}}{\text{average labour cost}}, \quad i=1,2,$$

this changes according to a change in Q_i, so that ω_i can be regarded as a function of Q_i, that is, $\omega_i = \omega_i(Q_i)$.

Letting $a_{11} = \frac{\partial^2 R_1}{\partial Q_1^2} - y_1\frac{d^2 L_1}{dQ_1^2}$ and $a_{12} = \frac{\partial^2 R_1}{\partial Q_1 \partial Q_2} - \frac{\partial y_1}{\partial Q_2}\frac{dL_1}{dQ_1}$ in (7), a_{12} can be rewritten as

$$a_{12} = \frac{\partial^2 P}{\partial Q^2}Q_1 + \frac{\partial P}{\partial Q}(1-\omega_1), \tag{8}$$

from which it follows that the sign of a_{12} depends upon assumptions placed on the shapes of both the demand and labour cost functions. Every possible combination of the assumptions on both functions is given in Table 1, indicating that if the demand function is strictly convex to the origin and the labour cost function satisfies $\omega_1>1$, then the sign of a_{12} is positive, etc. Note, in particular, that a change in the elasticity of the short-run labour cost function, not in its curvature, plays a crucial role in determining the slope of LM firm 1's reaction function.

Consider next what effect a change in the sign of a_{12} will put on comparative statics results in a Cournot equilibrium. Let us assume that the PM firm chooses Q_2 to maximise the profits,

$$\Pi_2 = R_2(Q_1,Q_2,\alpha) - wL_2(Q_2) - F_2. \tag{9}$$

Under the above Cournot assumption the first- and second-order conditions for profit maximisation are given by

$$\frac{\partial \Pi_2}{\partial Q_2} = \frac{\partial R_2}{\partial Q_2} - w\frac{dL_2}{dQ_2} = 0 \tag{10}$$

and

$$\frac{\partial^2 \Pi_2}{\partial Q_2^2} = \frac{\partial^2 R_2}{\partial Q_2^2} - w\frac{d^2 L_2}{dQ_2^2} < 0. \qquad (11)$$

The reaction functions of both firms are determined by solving equations (5) and (10) for Q_1 and Q_2, respectively, and their intersection yields a simultaneous solution of both equations, that is to say, the Cournot equilibrium output levels. The existence of a Cournot equilibrium is assumed. Total differentiation of (5) and (10), then, yields

$$a_{11}dQ_1 + a_{12}dQ_2 = -\frac{\partial P}{\partial \alpha}(1-\omega_1)d\alpha - \frac{1}{L_1}\frac{dL_1}{dQ_1}dF_1, \qquad (12)$$

$$a_{21}dQ_1 + a_{22}dQ_2 = -\frac{\partial P}{\partial \alpha}d\alpha + \frac{dL_2}{dQ_2}dw, \qquad (13)$$

where $a_{21} = \frac{\partial^2 R_2}{\partial Q_1 \partial Q_2}$ and $a_{22} = \frac{\partial^2 R_2}{\partial Q_2^2} - w\frac{d^2 L_2}{dQ_2^2} < 0$ from (11). Given the usual assumption that the commodities are substitutes in the sense that an increased quantity of one partner lowers the marginal revenue for the other partner, that is, $\partial^2 R_i/\partial Q_1 \partial Q_2 < 0$, $i=1,2$, we see that a_{21} is negative and hence that PM firm 2's reaction function slopes downward.

Table 2 tells us how the firms will respond to changes in the parameters, w, F_1, and α (see the Appendix for the derivation of the comparative statics results). Looking at Table 2 carefully, we see that the responses of the firms in Cournot equilibrium to changes in the parameters will be influenced, through a change in the slope of LM firm 1's reaction function, by the assumptions made on the demand and labour cost functions. Specifically, it is well known in the case of a monopoly with one variable input that a slope-preserving increase in demand will stimulate a LM firm to reduce output if the marginal labour cost of output is larger than the average labour cost, but to expand output if the average labour cost is still decreasing, while that in the case of a PM firm it will lead to an increase in output irrespective of the technology assumed (see Ward, 1958; Ireland and Law, 1982), whereas in the case of a mixed duopoly we see not only that the effect on LM firm 1 of a change in the parameter, α, will depend upon the assumptions placed on both demand function and firm 1's labour cost function completely, but also that the response of PM firm 2 to it will be affected by the assumption made on the other partner's, namely LM firm 1's, technology.[6]

3. MORE GENERAL REACTION FUNCTION OF A LABOUR-MANAGED FIRM

So far we have not placed any assumptions on the shape of a labour cost function. The technological assumption usually placed in dealing with the behaviour of a LM firm is that the short-run labour cost function has the positive marginal labour cost of output which is at first declining and then increasing. Note that the labour cost function of this form has the property that the labour cost elasticity of output defined above is at first smaller than, equal to, and eventually greatér than unity as the output increases from the origin. Consider next a labour cost function of the form resulting in that, even if the marginal labour cost curve is rising, the average labour cost curve is U-shaped due to the existence of overhead labour which is one of the chief characteristics of a modern industrial enterprise. It should be emphasised that the labour cost function of this form also has the above property concerning the labour cost elasticity of output. It will be made clear in this section that using the labour cost function of either of the forms leads us to the same conclusion. These two forms of the labour cost function, which will be referred to as forms one and two, respectively, are shown in Fig. 1.[7]

When Vanek (1970, pp.114-116) deals with the short-run duopolistic situation, by arguing that "in the short run the reaction functions should generally be positively sloped", he seems to neglect outcomes which are likely to be derived from the existence of the negatively sloped reaction function. It will be clarified, however, that the case of the negatively sloped reaction function can take place with the not less strong reason than that of the positively sloped one. When we evaluate the slope of the LM firm's short-run reaction function, it is the elasticity of the labour cost curve, not its curvature, that great emphasis should be put on. So we make the assumption that the technology used by the firms is described by a labour cost function of the first or second form.

Meanwhlie, the results on the shape of a LM firm's reaction function derived in the preceding section appear to depend upon the technological assumption and the shape of the market demand function. It will be made clear that the slope of the LM firm's reaction function depends upon an amount of fixed debt charge as well.

Let us first consider the problem above. Differentiating (5') with respect to Q_1 and F_1' yields

$$\frac{dQ_1}{dF_1} = \frac{-1}{L_1 \frac{\frac{\partial R_1}{\partial Q_1}}{\frac{dL_1}{dQ_1}} \left(\frac{\frac{\partial^2 R_1}{\partial Q_1^2}}{\frac{\partial R_1}{\partial Q_1}} - \frac{\frac{d^2 L_1}{dQ_1^2}}{\frac{dL_1}{dQ_1}} \right)} > 0 \tag{14}$$

which means the well-known result that LM firm 1's output level is an increasing function of the fixed debt charge, F_1.

Let us next prove that there exists some level of output, Q_1, which uniquely corresponds to given F_1 and Q_2. Some manipulation of the first-order condition of (5') gives

$$- \frac{\frac{\partial R_1}{\partial Q_1} L_1 - P(Q) Q_1 \frac{dL_1}{dQ_1}}{\frac{dL_1}{dQ_1}} = F_1. \tag{15}$$

Letting $H(Q_1)$ denote the left-hand side of (15) and differentiating it with respect to Q_1, we have

$$\frac{dH(Q_1)}{dQ_1} = -L_1 \frac{\frac{\partial R_1}{\partial Q_1}}{\frac{dL_1}{dQ_1}} \left(\frac{\frac{\partial^2 R_1}{\partial Q_1^2}}{\frac{\partial R_1}{\partial Q_1}} - \frac{\frac{d^2 L_1}{dQ_1^2}}{\frac{dL_1}{dQ_1}} \right) \tag{16}$$

which is positive as long as the second-order condition is satisfied and the marginal revenue is positive. Then, we come to the conclusion that $H(Q_1)$ is a monotonic, increasing function of Q_1 in the properly defined region. Let Q_1'' denote a sufficiently high level of output to make the marginal revenue equal to zero. Note, however, that $dH(Q_1'')/dQ_1>0$ still holds at Q_1'' if $\partial^2 R_1/\partial Q_1^2<0$ is assumed.

Meanwhile, confining our attention to the sign of $H(Q_1)$, its numerator can be rewritten as

$$\frac{\partial R_1}{\partial Q_1}L_1 - P(Q)Q_1\frac{dL_1}{dQ_1} = L_1\{P(Q)[1-\omega_1] + \frac{\partial P}{\partial Q}Q_1\}. \qquad (17)$$

Since $\partial P/\partial Q$ is negative, if LM firm 1 is operating at the interval where $\omega_1 \geqq 1$, (17) is negative, so that $H(Q_1)>0$, while it is possible that the firm is producing a sufficiently small output to lead to the high value of $P(Q)$, and hence to make (17) positive, at the interval where $\omega_1<1$, which implies that there could exist a positive, small value of Q_1, denoted Q_1', at which (17) is equal to zero and hence, $H(Q_1')=0$ holds. So, $H(Q_1)$ can be negative at the interval, $(0, Q_1')$, but it is positive at the interval, (Q_1', ∞), as long as the market price is positive. We can reach the conclusion, therefore, that given the other partner's output level, there exists some positive level of output, Q_1, uniquely corresponding to a certain, positive level of fixed debt charge, F_1, at the interval, (Q_1', Q_1'').[8]

Let F_1^* stand for an amount of fixed debt charge uniquely corresponding to the output level, Q_1^*, at which $\omega_1=1$. Substituting F_1^* and Q_1^* into the first-order condition, rearranging terms and making use of $\omega_1(Q_1^*)=1$ result in

$$-\frac{\partial P^*}{\partial Q}Q_1^{*2}=F_1^*,$$

where $\partial P^*/\partial Q$ represents the slope of the demand function evaluated at Q_1^*, given Q_2. Then, letting $P(Q^*)$ be the value of the demand function at Q_1^* given Q_2, the following can be established.[9]

Proposition 1:

$$\omega_1 \gtreqless 1 \quad \text{if and only if} \quad -\frac{\partial P^*}{\partial Q}Q_1^{*2} \lesseqgtr F_1.$$

Proof: Prove first that $\omega_1>1$ if and only if $-\frac{\partial P^*}{\partial Q}Q_1^{*2}<F_1$.

If $-\frac{\partial P^*}{\partial Q}Q_1^{*2}<F_1$, then, by supposing that $Q_1\leq Q_1^*$, it follows from the first-order condition that

$$0 = [P(Q)+\frac{\partial P}{\partial Q}Q_1]L_1(Q_1) - [P(Q)Q_1-F_1]\frac{dL_1(Q_1)}{dQ_1}$$

$$\geq [P(Q^*)+\frac{\partial P^*}{\partial Q}Q_1^*]L_1(Q_1^*) - [P(Q^*)Q_1^* - F_1]\frac{dL_1(Q_1^*)}{dQ_1} \quad \text{(note that } Q_1\leq Q_1^*)$$

$$> [P(Q^*)+\frac{\partial P^*}{\partial Q}Q_1^*]L_1(Q_1^*) - [P(Q^*)Q_1^* + \frac{\partial P^*}{\partial Q}Q_1^{*2}]\frac{dL_1(Q_1^*)}{dQ_1}$$

$$\text{(by using that } -\frac{\partial P^*}{\partial Q}Q_1^{*2}<F_1)$$

$$> L_1(Q_1^*)[P(Q^*)+\frac{\partial P^*}{\partial Q}Q_1^*][1-\omega_1(Q_1^*)]$$

$$> 0 \qquad \text{(because } \omega_1(Q^*)=1).$$

This is a contradiction, so that $Q_1>Q_1^*$ must hold, which implies that $\omega_1>1$.

Conversely, if $\omega_1>1$, then making use of the first-order condition gives

$$[P(Q)+\frac{\partial P}{\partial Q}Q_1]L_1 > [P(Q)Q_1-F_1]\frac{L_1}{Q_1},$$

so that

$$-\frac{\partial P}{\partial Q}Q_1^2 < F_1.$$

If $\partial^2 R_1/\partial Q_1 \partial Q_2<0$ is assumed,

$$\frac{\partial(-\frac{\partial P}{\partial Q}Q_1^2)}{\partial Q_1} = -\frac{\partial^2 P}{\partial Q^2}Q_1^2 - 2\frac{\partial P}{\partial Q}Q_1$$

$$= -(\frac{\partial^2 R_1}{\partial Q_1 \partial Q_2} + \frac{\partial P}{\partial Q})Q_1 > 0,$$

which implies with $-\frac{\partial P}{\partial Q}Q_1^2>0$ that when $Q_1>Q_1^*$, $-\frac{\partial P}{\partial Q}Q_1^2 > -\frac{\partial P^*}{\partial Q}Q_1^{*2}$.

Therefore, we can obtain

$$-\frac{\partial P^*}{\partial Q}Q_1^{*2} < F_1.$$

The other two cases can be easily proved in a similar way. Q.E.D.

The proposition tells us on which portion of the labour cost function LM firm 1 will be operating in the short run, given the other partner's output level, Q_2. In addition, it can be seen that LM firm 1's choice of the optimal output level depends upon an amount of fixed debt charge as well.

As is easily seen from Table 1, since the situation is very complicated, we will focus on a case throughout the rest of the paper where the market demand function is linear, that is, $\partial^2 P(Q)/\partial Q^2=0$, following Vanek (1970, p.115). (8) is, then, reduced to

$$a_{12}=\frac{\partial P}{\partial Q}(1-\omega_1), \qquad (8')$$

and $\partial^2 R_1/\partial Q_1 \partial Q_2<0$ is also satisfied. In this case Proposition 1 shows that the slope of a LM firm's reaction function will be upward, horizontal, or downward according to a case where the fixed debt charge, F_1, is greater than, equal to, or smaller than $-\frac{\partial P^*}{\partial Q}Q_1^{*2}$, so we come to the conclusion that the slope of the LM firm's short-run reaction function will depend upon not only the assumptions placed on the labour cost and market demand functions but also an amount of fixed debt charge.

Moreover, the LM firm's reaction function derived in this section can be considered to be more general than the one obtained by Vanek (1970, pp.114-115) in that our assumptions on the labour cost function are more general than that which he seems to place in order to deal with an Illyrian duopoly.

It should be emphasised that the elasticity of the short-run labour cost curve, not its curvature, also plays a crucial role in determining the slope of the LM firm's reaction function, and that either of the two forms of the labour cost function leads us to the same proposition. As is well known, a large-scale industrial enterprise is characterised by the existence of a large amount of overhead labour, which in turn causes the technology used by the enterprise to be described by a labour cost function of the second form as defined above. In the case of a LM firm in the short run, this makes it possible for us to argue that the grounds on which the negatively sloped reaction function is built are as solid as those upon which the positively sloped one is based.

In closing this section, let us give examples of the labour cost function of each of the forms as discussed above. It can be approximated by $L_1(Q_1)=(Q_1-1)^3+1$ or $L_1(Q_1)=Q_1^2+1$. It is the latter, however, that particularly concerns us here. That function satisfies $\omega_1<1$ at the interval, $[0,1)$, $\omega_1=1$ at the point, $Q_1=1$, and then $\omega_1>1$ at the interval, $(1,\infty)$. Letting $P(Q)=A-BQ$ where $A>0$, $B>0$ and $Q=Q_1+Q_2$, we can obtain the following equation as LM firm 1's reaction function.

$$Q_1=\frac{(F_1-B)+\sqrt{(F_1-B)^2+(A-BQ_2)^2}}{(A-BQ_2)} \gtreqless 1 \text{ as } F_1 \gtreqless B.$$

This situation is portrayed in Fig. 2 where the arrows mean an increase in F_1.

4. EXTENSION OF THE MODEL

In this section we will modify the above model slightly in order to examine the further properties of a LM firm's reaction function, following Dixit (1980). The assumption of a linear demand function is maintained. Suppose first that firm i already has the capacity of K_i, i=1,2, which is measured in output units and is literally an upper limit on output. This may subsequently be increased, but cannot be reduced. It is assumed that $\partial^2 L_i/\partial Q_i \partial K_i = 0$, i=1,2, that is to say, that capacity has no effect on the marginal labour cost of output. Denote the constant cost per unit of capacity by r. If LM firm 1 is operating within the capacity limit, i.e. if $Q_1 \leqq K_1$, then its net income per worker is expressed by

$$y_1 = \frac{R_1(Q_1, Q_2) - rK_1}{L_1(Q_1)} \quad . \tag{18}$$

On the other hand, if $Q_1 > K_1$, that is, if LM firm 1 expands the capacity level, its objective function can be written as

$$y_1 = \frac{R_1(Q_1, Q_2) - rQ_1}{L_1(Q_1)} \quad . \tag{19}$$

Letting $F_1 = rK_1$ for a given K_1, (18) is reduced to (4). In the case of $Q_1 \leqq K_1$, the results on LM firm 1's reaction function derived in the preceding section apply well.

Differentiating (19) with respect to Q_1 yields

$$\frac{\partial y_1}{\partial Q_1} = \frac{[\frac{\partial R_1}{\partial Q_1} - r]L_1 - [R_1(Q_1, Q_2) - rQ_1]\frac{dL_1}{dQ_1}}{(L_1)^2} . \tag{20}$$

Rearranging terms in the numerator of the right-hand side of (20), it can be rewritten as

$$\frac{L_1}{Q_1}\{Q_1[\frac{\partial R_1}{\partial Q_1} - \frac{R_1(Q_1, Q_2)}{Q_1}] + [R_1(Q_1, Q_2) - rQ_1][1-\omega_1]\}.$$

The revenue function is increasing and concave in each firm's output on the assumption of a linear demand function as long as the marginal revenue is positive. Then, since $\frac{R_1}{Q_1} > \frac{\partial R_1}{\partial Q_1}$, a necessary and sufficient condition to obtain the first-order condition for maximisation of (19) is that (20) is equal to zero at that Q_1 at which $\omega_1<1$, which implies that in this case LM firm 1 is operating on a declining average labour cost portion of the labour cost function.

In addition, setting (20) equal to zero and then differentiating it with respect to Q_1 and Q_2, we have the following equation for the slope of LM firm 1's reaction function,

$$\frac{dQ_1}{dQ_2} = \frac{-\frac{\partial P}{\partial Q}(1-\omega_1)}{\frac{\partial^2 R_1}{\partial Q_1^2} - y_1\frac{d^2 L_1}{dQ_1^2}} . \tag{21}$$

If the second-order condition for maximisation is assumed to be satisfied,[10] (21) is negative, because the denominator of (21) is negative from the second-order condition, and $\omega_1<1$ holds on the basis of the above argument. We then have

Proposition 2: If LM firm 1 wishes to produce a greater output above its previously chosen capacity level, then its reaction function will slope downward.

Consider next how LM firm 1's reaction function would shift with a change in the unit capacity expansion cost, r. Differentiating the first-order condition for maximisation of (18) with respect to Q_1 and r, and rearranging terms, we have

$$\frac{dQ_1}{dr} = \frac{-\frac{K_1}{L_1}\frac{dL_1}{dQ_1}}{\frac{\partial^2 R_1}{\partial Q_1^2} - y_1\frac{d^2 L_1}{dQ_1^2}} > 0.$$

On the other hand, in the case of (19) we obtain

$$\frac{dQ_1}{dr} = \frac{1 - \omega_1}{\frac{\partial^2 R_1}{\partial Q_1^2} - y_1 \frac{d^2 L_1}{dQ_1^2}} < 0,$$

because in this case LM firm 1 must produce output at the interval where $\omega_1 < 1$. It should be noted that there exists a sharp contrast between the above cases concerning the effect on LM firm 1's reaction function of a change in the constant cost per unit of capacity.

Meanwhile, if PM firm 2 is operating within the capacity limit, then its objective function is represented by

$$\Pi_2 = R_2(Q_1, Q_2) - wL_2(Q_2) - rK_2. \quad (22)$$

We wish now to find the relationship between dQ_2 and dr when the equilibrium condition, $\partial\Pi_2/\partial Q_2 = 0$, is maintained. Differentiating the first-order condition, we have

$$\frac{dQ_2}{dr} = 0.$$

On the other hand, if PM firm 2 wishes to produce a larger output above the capacity limit, its profits are expressed by

$$\Pi_2 = R_2(Q_1, Q_2) - wL_2(Q_2) - rQ_2. \quad (23)$$

From the differentiation of the first-order condition we derive

$$\frac{dQ_2}{dr} = \frac{1}{\frac{\partial^2 R_2}{\partial Q_2^2} - w \frac{d^2 L_2}{dQ_2^2}} < 0.$$

Note that PM firm 2's reaction function always slopes downward on the assumption of a linear demand function. We then have

Proposition 3: If each firm is producing output within its capacity limit, an increase in the constant cost per unit of capacity will cause LM firm 1's reaction function to shift upward, but will leave that of PM firm 2 unchanged. On the other hand, if each firm wishes to produce a greater output above the capacity limit, an increase in the constant cost per unit of capacity will result in a downward shift in both LM firm 1's and PM firm 2's reaction functions.

$L_1(Q_1)=Q_1^2+1$ given as an example of the labour cost function in the preceding section suggests the idea that, if the labour cost function takes the form, $L_i(Q_i)=l_i(Q_i)+C_i$, $i=1,2$, where $l_i(Q_i)$ stands for production labour with $dl_i/dQ_i>0$, $d^2l_i/dQ_i^2>0$ and $l_i(0)=0$, and C_i represents constant overhead labour such as foremen and office staff, then making use of this type of labour cost function will also lead to Proposition 1.

Substituting $L_1(Q_1)=l_1(Q_1)+C_1$ into (18) and differentiating the first-order condition for maximisation with respect to Q_1 and C_1 result in

$$\frac{dQ_1}{dC_1} = \frac{-\dfrac{y_1}{L_1}\dfrac{dl_1}{dQ_1}}{\dfrac{\partial^2 R_1}{\partial Q_1^2} - y_1\dfrac{d^2 l_1}{dQ_1^2}} > 0.$$

In the case of (19), the same result as mentioned above is obtained. So, it can be seen that, in the case where the labour cost function takes the above form, overhead labour, C_1, exerts exactly the same influence on LM firm 1's reaction functions as the fixed debt charge, F_1, in the case of (4).

In the case of PM firm 2, we obtain $dQ_2/dC_2=0$ in a similar way. Therefore, we have

Proposition 4: An increase in the overhead labour staff will cause an upward shift in LM firm 1's reaction functions, but those of PM firm 2 will not react at all to a change in the overhead labour staff.

5. CONCLUDING REMARKS

The present analysis has been conducted on the basis of the short-run labour cost function of each of forms one and two, both of which have in common the property that the labour cost elasticity of output is at first less than, equal to and then larger than unity as the output increases. The first form of the labour cost function is the assumption usually made in inquiring into the behaviour of a LM firm, competitive or monopolistic, while the second form describes the technology used by a modern industrial enterprise with a large amount of overhead labour. By making use of the labour cost function with such a property, we have first made it clear that the assumptions made on the labour cost and demand functions affect the shape of a LM duopolist's reaction function, which in turn affects the responses of the duopolists in Cournot equilibrium to cost and demand changes in a mixed duopoly model, and also that it is the elasticity of the short-run labour cost curve, not its curvature, that has a decisive influence in evaluating the slope of the LM firm's reaction function.

Secondly, we have proved under the assumption of a linear demand function that a LM firm's reaction function is positively sloped, horizontal or negatively sloped according as $-\frac{\partial P^*}{\partial Q}Q_1^{*2} \lesseqgtr F_1$, and hence, it should be noted that an amount of fixed debt charge borne by the LM firm also acquires a crucial role in determining the shape of its short-run reaction function. Both forms of the labour cost function have the same property concerning its elasticity, which implies that we will be able to reach Proposition 1 as long as the labour cost function is of the form satisfying the above property. We argue emphatically that in the case of a LM firm there can also occur a case where the short-run reaction function slopes downward, to which little attention has been paid so far.

The LM firm's reaction function derived in this way can be considered to be more general than the one earlier obtained by Vanek (1970) and others.

Finally, it has been clarified with the aid of the model presented by Dixit (1980) that, if a LM firm expands the capacity level above its previously fixed one, then its reaction function will always be negatively sloped. This result is in sharp contrast to the one that, if the LM firm is producing a product below the capacity limit, the slope of its reaction function will change from negative to positive according to the values of the constant cost per unit of capacity and the level of capacity earlier chosen by itself. It is worthwhile mentioning further that there is another sharp contrast, namely, that an increase in the unit capacity expansion cost will lead to a downward shift in the reaction function in the case of a LM duopolist expanding its capacity level, but will cause an upward shift in the reaction function in the case of a LM duopolist operating below the capacity limit. This may be given the interpretation that, although in the former case the LM firm tends to reduce new membership in response to an increase in the unit capacity expansion cost, in the latter case this will cause the LM firm to expand membership in order to spread the cost burden.

MATHEMATICAL APPENDIX

The problem is to determine the responses of both firms in Cournot equilibrium to changes in the parameters, w, F_1, and α, on the assumption that a Cournot equilibrium exists, based on the system of simultaneous equations,

$$a_{11}dQ_1 + a_{12}dQ_2 = -\frac{\partial P}{\partial \alpha}(1-\omega_1)d\alpha - \frac{1}{L_1}\frac{dL_1}{dQ_1}dF_1, \qquad (A1)$$

$$a_{21}dQ_1 + a_{22}dQ_2 = -\frac{\partial P}{\partial \alpha}d\alpha + \frac{dL_2}{dQ_2}dw, \qquad (A2)$$

where $\omega_i = \frac{d\log L_i}{d\log Q_i}$, $i=1,2$,

$$a_{11} = \frac{\partial^2 R_1}{\partial Q_1^2} - y_1\frac{d^2 L_1}{dQ_1^2} < 0,$$

$$a_{12} = \frac{\partial^2 R_1}{\partial Q_1 \partial Q_2} - \frac{\partial y_1}{\partial Q_2}\frac{dL_1}{dQ_1} \gtreqless 0,$$

$$a_{21} = \frac{\partial^2 R_1}{\partial Q_1 \partial Q_2} = \frac{\partial^2 P}{\partial Q^2}Q_2 + \frac{\partial P}{\partial Q} < 0 \quad \text{by assumption},$$

$$a_{22} = \frac{\partial^2 R_2}{\partial Q_2^2} - w\frac{d^2 L_2}{dQ_2^2} < 0.$$

It can easily be seen that $\Delta \equiv a_{11}a_{22} - a_{12}a_{21} > 0$ holds on the basis of the stability analysis of Cournot equilibrium. By Cramer's rule the following results are obtained from (A1) and (A2),

$$\frac{dQ_1}{dw} = -\frac{a_{12}}{\Delta}\frac{dL_2}{dQ_2} \gtreqless 0 \quad \text{as } a_{12} \lesseqgtr 0,$$

$$\frac{dQ_1}{dF_1} = -\frac{a_{22}}{\Delta}\frac{1}{L_1}\frac{dL_1}{dQ_1} > 0,$$

$$\frac{dQ_1}{d\alpha} = \frac{1}{\Delta}[-a_{22}(1-\omega_1) + a_{12}]\frac{\partial P}{\partial \alpha}.$$

This sign cannot be uniquely established. The expression in square brackets is rewritten as

$$-a_{22}(1-\omega_1) + a_{12} = (1-\omega_1)\left(\frac{\partial P}{\partial Q} - a_{22}\right) + \frac{\partial^2 P}{\partial Q^2}Q_1. \quad (A3)$$

In order to establish the sign of (A3) we have to find out the sign of the second parenthetical expression which can be rewritten as

$$\frac{\partial P}{\partial Q} - a_{22} = -\frac{\partial^2 P}{\partial Q^2}Q_2 - \frac{\partial P}{\partial Q} + w\frac{d^2 L_2}{dQ_2^2}. \quad (A4)$$

Since $\frac{\partial^2 R_2}{\partial Q_1 \partial Q_2} = \frac{\partial^2 P}{\partial Q^2}Q_2 + \frac{\partial P}{\partial Q} < 0$ by assumption, if $\frac{d^2 L_2}{dQ_2^2} \geqq 0$, that is, if PM firm 2's marginal labour cost is non-decreasing, then (A4) is positive. This sufficient condition is satisfied by the labour cost function of form two. Suppose now that PM firm 2's $L_2(Q_2)$ has the positive marginal labour cost which is at first declining, but which is eventually increasing. Then, $\frac{d^2 L_2}{dQ_2^2} > 0$ holds at the interval where $\omega_2 \geqq 1$. On the other hand, at the interval where $\omega_2 < 1$, there exists some value of Q_2, denoted $\bar{Q}_2$, to the right of which $\frac{d^2 L_2}{dQ_2^2} > 0$ holds, but to the left of which $\frac{d^2 L_2}{dQ_2^2} < 0$ holds, so that $\frac{d^2 L_2}{dQ_2^2} > 0$ at the interval, $(\bar{Q}_2, \infty)$. Furthermore, even if $\frac{d^2 L_2}{dQ_2^2} < 0$ holds, (A4) is positive as long as $L_2(Q_2)$ satisfies the condition,

$$\frac{\partial^2 P}{\partial Q^2}Q_2 + 2\frac{\partial P}{\partial Q} < \frac{\partial^2 P}{\partial Q^2}Q_2 + \frac{\partial P}{\partial Q} < w\frac{d^2 L_2}{dQ_2^2}.$$

Only if $\frac{\partial^2 P}{\partial Q^2}Q_2 + 2\frac{\partial P}{\partial Q} < w\frac{d^2 L_2}{dQ_2^2} \leqq \frac{\partial^2 P}{\partial Q^2}Q_2 + \frac{\partial P}{\partial Q}$, (A4) is zero or negative.

Therefore, we reach the conclusion that (A4) is most likely to be positive in the vicinity of a Cournot equilibrium.

Since $\frac{\partial P}{\partial \alpha} > 0$ and $\Delta > 0$, the following results are obtained.

In the case where $\omega_1 > 1$, $\frac{dQ_1}{d\alpha} \gtreqless 0$ if $\frac{\partial^2 P}{\partial Q^2} > 0$, but $\frac{dQ_1}{d\alpha} < 0$ if $\frac{\partial^2 P}{\partial Q^2} \leqq 0$.

If $\omega_1 = 1$, then $\frac{dQ_1}{d\alpha} \gtreqless 0$ according as $\frac{\partial^2 P}{\partial Q^2} \gtreqless 0$. In the case where $\omega_1 < 1$,

$\frac{dQ_1}{d\alpha} > 0$ if $\frac{\partial^2 P}{\partial Q^2} \geqq 0$, but $\frac{dQ_1}{d\alpha} \gtreqless 0$ if $\frac{\partial^2 P}{\partial Q^2} < 0$.

Turning to the case of PM firm 2, we have

$$\frac{dQ_2}{dw} = \frac{a_{11}}{\Delta} \frac{dL_2}{dQ_2} < 0,$$

$$\frac{dQ_2}{dF_1} = \frac{a_{21}}{\Delta} \frac{1}{L_1} \frac{dL_1}{dQ_1} < 0,$$

$$\frac{dQ_2}{d\alpha} = \frac{-1}{\Delta}[a_{11} - a_{21}(1-\omega_1)]\frac{\partial P}{\partial \alpha}.$$

This sign cannot be uniquely determined. Since $a_{11} < 0$ and $a_{21} < 0$, if $\omega_1 \geqq 1$ holds, then $\frac{dQ_2}{d\alpha} > 0$, but otherwise $\frac{dQ_2}{d\alpha} \gtreqless 0$.

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Footnotes

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1 It is well known, in terms of a production function rather than a labour cost function, that a LM monopoly will be operating on an increasing returns portion of the production function in the long run. On the other hand, it is still unclear on which portion of the production function the LM monopoly will be producing a product in the short run where in the case of two inputs—labour and capital, for example, capital is fixed. This paper will deal with the problem as well.

2 We will deal with only this type of duopoly model, because so far relatively little attention has been paid to the examination of the behaviour of a LM firm in a capitalist market economy. It is of great interest to consider the interplay of capitalist profit-maximising and labour-managed firms.

3 The slope-preserving demand shift is assumed in Vanek (1970, p.115) which deals with an Illyrian duopoly model composed of two LM firms.

4 This is termed a short-run labour cost function by Meade (1974). This function is used throughout the paper rather than the production function. This is so because in considering the shape of a LM firm's reaction function the former has more advantages than the latter.

5 Note that the symbol, α, will be dropped below when we do not refer to a shift in the parameter.

6 The author has derived the comparative statics results in

the case of an Illyrian duopoly model composed of two LM firms as well, but will omit them in this paper. A copy of the results will be sent on request.

7 When we have a diagram such as Fig. 1-a, we sometimes meet the situation that the marginal labour cost is zero at an inflection point if it exists. However, we rule out this situation and so assume throughout the paper that the labour cost function has a positive marginal labour cost everywhere.

8 It is presumed in this paper that the first- and second-order conditions for maximisation are satisfied and hence that a reaction function exists. This implies that we neglect a case where F_1 is large enough to make a LM duopolist choose that level of output at which the marginal revenue is negative.

9 I am indebted to W. Fukuda for improving the proof of the second half of Proposition 1.

10 In this case the second-order condition is given by

$$\frac{\partial^2 y_1}{\partial Q_1^2} = \frac{\dfrac{\partial^2 R_1}{\partial Q_1^2} - y_1(Q_1, Q_2, r)\dfrac{d^2 L_1}{dQ_1^2}}{L_1} < 0.$$

TABLE 1: Sign of a_{12}

Sign of a_{12}		Market Demand Function		
		strictly convex	linear	strictly concave
Labour Cost Elasticity of Output	$\omega_1 > 1$	+	+	?
	$\omega_1 = 1$	+	0	-
	$\omega_1 < 1$	?	-	-

TABLE 2: Comparative Statics of Cournot Mixed Duopoly

Effect on \ Increase in	w	F_1	α
Q_1 (LM)	(1)	+	(2)
Q_2 (PM)	-	-	(3)

(1) $\frac{dQ_1}{dw} \gtreqless 0$ as $a_{12} \lesseqgtr 0$.

(2)

Sign of $\frac{dQ_1}{d\alpha}$		Market Demand Function		
		strictly convex	linear	strictly concave
Labour Cost Elasticity of Output	$\omega_1 > 1$	?	-	-
	$\omega_1 = 1$	+	0	-
	$\omega_1 < 1$	+	+	?

(3) $\frac{dQ_2}{d\alpha} > 0$ if LM firm 1's labour cost function satisfies the condition that $\omega_1 \geqq 1$.

$\frac{dQ_2}{d\alpha} \gtreqless 0$ otherwise.

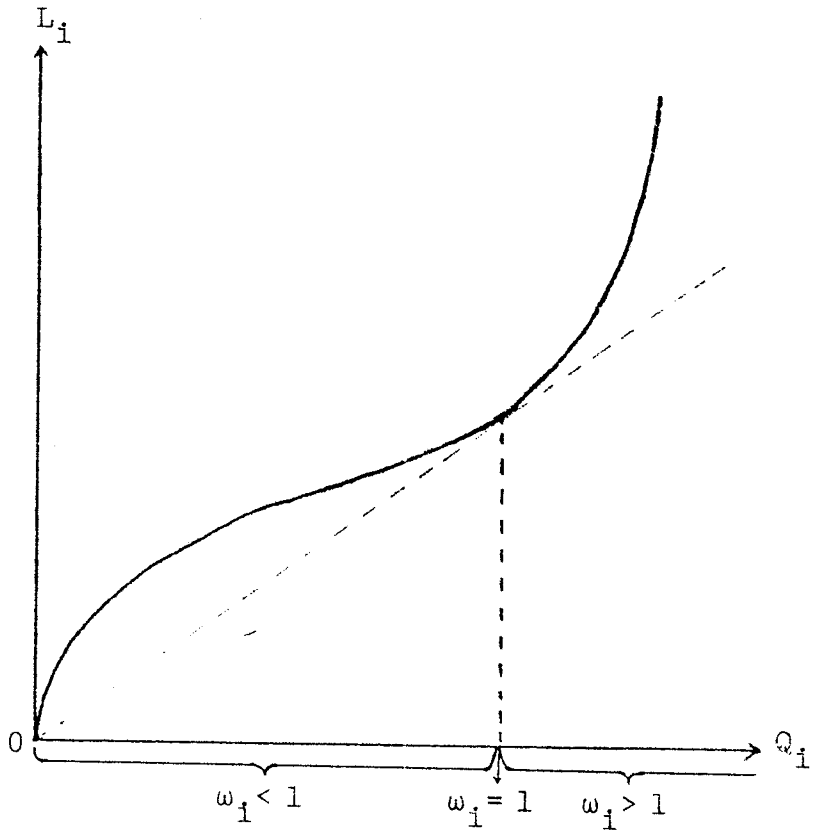

FIG. 1-a: Labour Cost Function of Form One

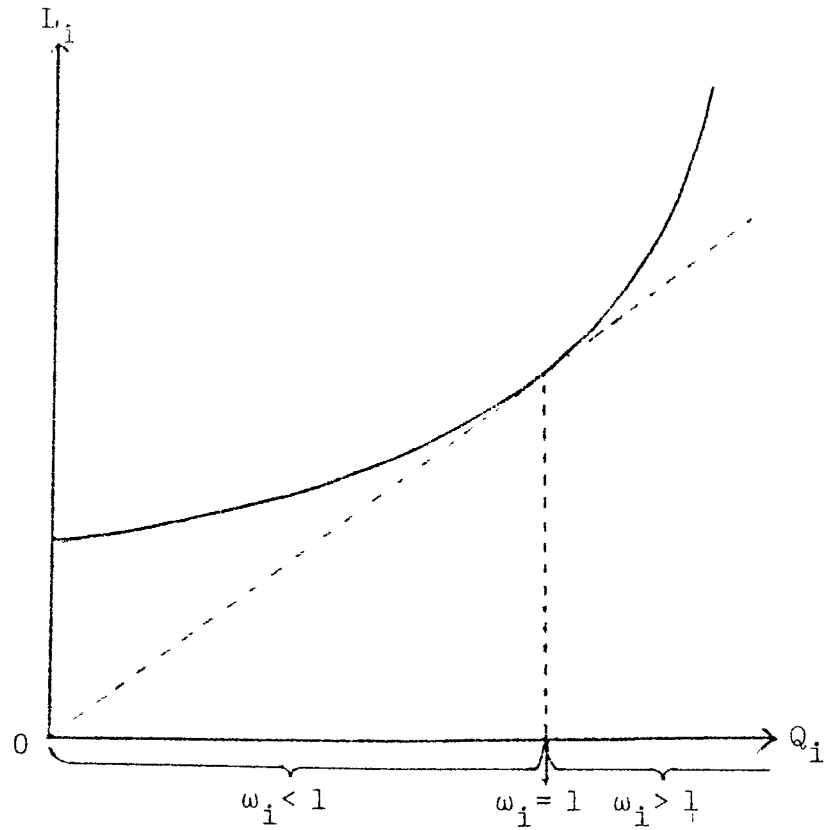

FIG. 1-b: Labour Cost Function of Form Two

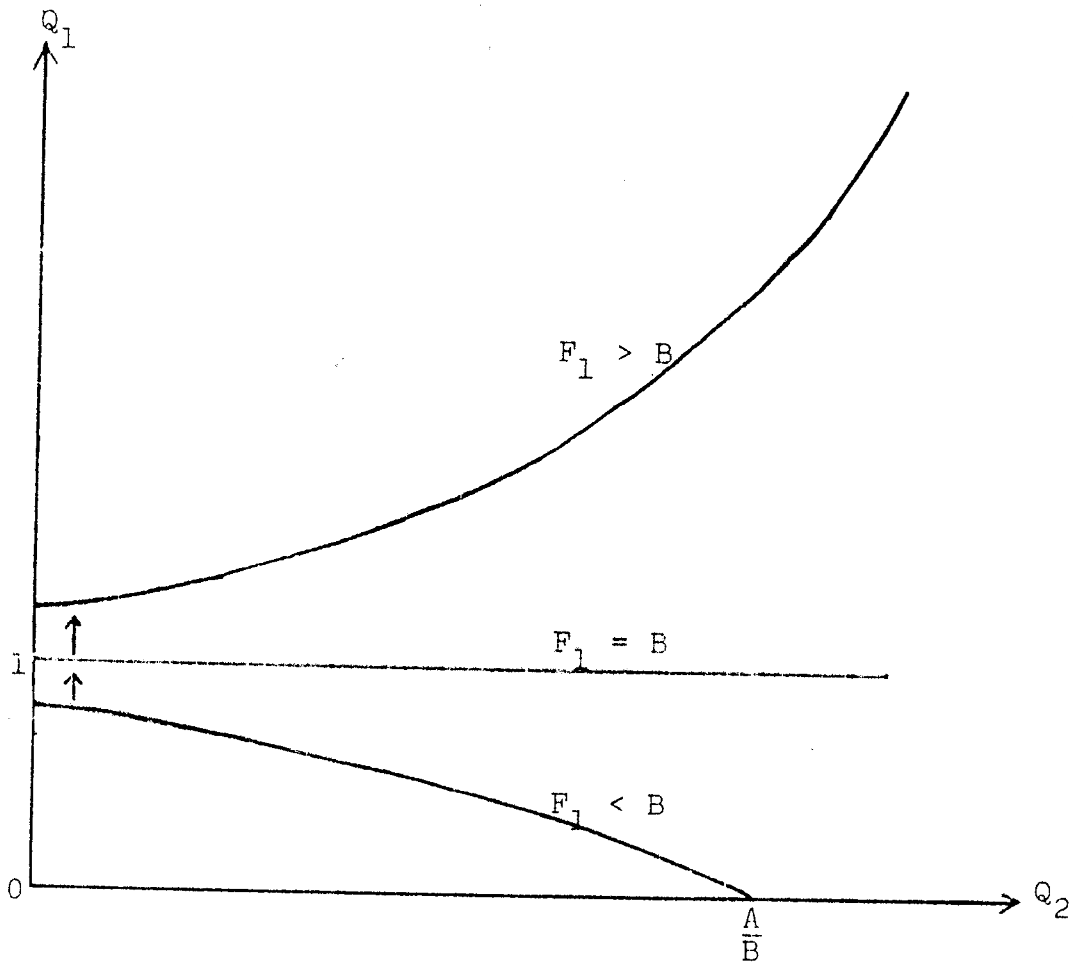

FIG. 2: Example of LM Firm 1's Reaction Function