Three Theorems on Inflation Taxes
and Marginal Employment Subsidies

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This paper is circulated for discussion purposes only and its contents should be considered preliminary.
Abstract

The paper studies the micro-economics of inflation taxes and marginal employment subsidies. It proves that under very weak assumptions (i) an inflation tax will reduce the long-run equilibrium wage or price and (ii) that a marginal employment subsidy will raise the long-run equilibrium employment level. The theorems are illustrated with examples. The paper also proves (iii) that in special circumstances a tax on inflation is exactly equivalent to a marginal employment subsidy.
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1. Introduction

Most Western nations are now experiencing high levels of unemployment. Traditional economic policies have apparently either helped to produce this or have not been able to eliminate the problem. Thus some economists have tried to suggest new ways in which governments might intervene in the labour market to raise the equilibrium level of employment. Two of the most widely known schemes are (i) a tax on inflation and (ii) marginal employment subsidies. This paper examines the micro-economic effects of these policies.

The paper will try to make a number of points. One of the most fundamental is that of the formal similarity between an inflation tax and a subsidy on marginal employment. Both work to raise the long run level of employment by changing the cost of adjusting from one short run equilibrium to another. An inflation tax imposes a fine on firms or workers when wage rates rise too fast; its aim is to reduce wages and so raise employment. A marginal employment subsidy is designed to reward firms or workers when employment rises fast; its purpose, too, is to increase the level at which employment will be in equilibrium. Technically, then, these are both examples of the same sort of analytical problem. Each one requires that the relevant decision-maker bear in mind that his life-time utility depends on how quickly he alters his present behaviour.
In other words, each is a problem in the calculus of variations, not one in simple calculus.

There is now quite a large literature on the case for an inflation tax. Although it dates back at least twenty years, the idea has become topical because of the work of Richard Jackman and Richard Layard - most notably in Layard (1982). Other recent papers include Beath (1979), Isard (1973), Kotowitz and Portes (1974), Latham and Peel (1977), Nichols (1979), Oswald and Rosewell (1981), Pemberton (1982), Seidman (1978), (1979) and Slitor (1979). However, it does not seem unreasonable to argue that the literature is largely a collection of special cases. Some authors study a unionised labour market; others focus on monopsony; others examine special bargaining solutions. These are all important as examples, but it would be valuable to know what can be said at a more general level. The first objective of this paper is to prove a theorem on the effects of an inflation tax in which very little structure is put on the problem. The result turns out to be that under rather weak assumptions a tax on price rises (where the price could be that of labour) will reduce the long run equilibrium price. Hence a wage inflation tax, for example, will lower the equilibrium wage level. Then it becomes obvious that in models in which there is a downward-sloping labour demand curve the tax scheme will increase employment, whereas in alternative models, like monopsony, this conclusion may be reversed.

An important assumption will be made throughout the paper. It will be assumed that the inflation tax is designed to fall only on real wage or price increases. This accords
with Jackman and Layard's treatment, but that is not in itself
the reason for the assumption. If an inflation tax is levied
on nominal wage and price changes, as under some proposals,
a rather strange thing happens to the structure of the economy.
There is no longer a simple natural rate of unemployment:
the real and monetary halves of the economy become intertwined. ²
This is because such a tax system would link a real economic
variable, a tax rate, to a nominal economic variable, the
rate of change of prices. Then the classical dichotomy could
not hold.

There has been some confusion in the literature on
the meaning of a tax on the rate of wage (or price) inflation.
Such a scheme could be designed in two different ways. The
obvious and natural one is to let some tax rate, say an employ-
ment tax on firms or a personal tax on workers' incomes, be a
function \( \tau = \tau(\dot{w}) \), where \( \dot{w} \) is the rate of change of wages
(or prices). The alternative, which is not an inflation tax,
is to set \( \tau = \tau(w) \), where \( w \) is the level of wages. The latter
is simply a sort of wage or income tax, of course, and has
little to do with inflation. Confusion probably stems mainly
from attempts to set up the model in discrete time, although
it can be done properly, as in Jackman and Layard's appendix
to Layard (1982). This paper is concerned with a tax on the
rate of inflation. To clarify the difference between a tax
on a wage or price level and that on a rate of inflation the
analysis will use continuous time.

One other slight difficulty with the analysis of an
inflation tax is that the issues tend to become blurred by
what happens to the revenue raised by the tax. In Jackman
and Layard's work, for example, the tax revenue is used to pay for employment subsidies. Then it turns out to be hard to disentangle the effects of the subsidies from those stemming from the inflation tax. The subsidies appear to do part of the work. Although this is perfectly reasonable if one's goal is to suggest a workable economic policy, it does not help us to be clear about the influence of the inflation tax alone. On balance, therefore, it seems useful to ignore the question of how revenue raised by the tax is spent. One can think of it as being used to make lump sum grants, or to finance some separate scheme to raise employment.

Marginal employment subsidies are discussed in a later section. There has been much less work on this topic: the main recent paper is Layard and Nickell (1980), which is rather Keynesian in spirit. It is natural, though, as these authors agree, to see a marginal employment subsidy as an essentially neo-classical type of intervention - one which lowers the cost of increasing the size of the labour force and thereby alters the equilibrium rate of unemployment. Profit-maximising firms who face such a scheme have to solve a complicated optimisation problem. When they raise the numbers of men they employ, that increases their short-run costs (the men have to be paid); but, simultaneously, they get an immediate subsidy and lose a chance in the future to take that subsidy. Their profits tomorrow thus depend in part on what they do today.

The paper's second result is about the effects of a marginal employment subsidy. It is shown here that under weak and rather plausible assumptions a marginal employment subsidy - a subsidy to the firm which is linked to its rate
of change of employment - will raise the long run employment level. The formal proof is identical in structure to that used to establish the previous theorem on an inflation tax. This is another example of the policies' technical similarity.

Finally, the paper examines the question of whether an inflation tax could ever be analytically equivalent to a marginal employment subsidy. The intuitive explanation for this possibility is of the following sort. A tax on inflation penalises wage (or price) rises. In a world with a downward sloping demand curve that is the same as penalising falls in employment (or output). But this is exactly what a marginal employment subsidy does: it discriminates against firms who lower the numbers they employ and in favour of those who have a growing employment level. On the face of it, therefore, the two schemes look surprisingly similar. The obvious difference is that one requires that the government raise the necessary revenue whilst the other assumes that the government can disburse it. As long as there are non-distortionary taxes and subsidies (presumably of a lump sum kind) which are capable of providing or distributing that revenue, however, this difficulty can be ignored. Of course this is unrealistic, so in practice the two types of government policy will turn out to be somewhat different. But what this paper can show - the third theorem - is that there exist conditions under which an inflation tax and a marginal employment subsidy are exactly equivalent.

The paper is divided into five more sections. Section 2 proves a theorem on inflation tax, and Section 3 gives some applications and examples to try to illustrate the result. In Section 4 a mathematically similar theorem about marginal
employment subsidies is established. Section 5 demonstrates that under certain conditions a tax on inflation is equivalent to a subsidy on marginal jobs. The final section summarises the argument.

2. A Theorem on Inflation Tax

Imagine any sort of economic agent who has utility $V(w,\tau)$ in each period in which there is some price $w$ and some tax level $\tau$. A natural interpretation is that of $V(\ldots)$ as a union's indirect utility function, $w$ as the wage rate which it wishes to impose, and $\tau$ as an employment or income tax. But $w$ could be the price of a firm's output, $V(\ldots)$ its profit function, and $\tau$ the rate of corporation tax. Other interpretations are also possible.

Assume that $\tau$ is initially fixed at zero. Then at the optimum the agent will set $V_w \leq 0$ (where this holds as a strict equality when the optimal price, $w^*$, is strictly positive) as long as the function is suitably differentiable, which will be assumed. Now consider the effects of the tax rate being tied to the degree of price changes. Let $\tau = \tau(\dot{w})$, for example. Then the agent's decision-making is more complicated, because its choice of $w$ in period $t$ affects its opportunities in periods $t+1$, $t+2$ ... If the agent's discounted utility is $\int V(w,\tau(\dot{w}))e^{-rt}dt$, where $t$ is time and $r$ is the discount rate, the problem can be solved by conventional variational methods. Assume that $\bar{w}$ is the steady-state solution. The interesting question is whether this is lower than it would be without the tax scheme. To ensure that this is so, in fact, it is only necessary to assume that $V_{\tau} < 0$, which means that the
agent should dislike higher tax rates. The proof is given below.

**Theorem 1** Assume that there is an economic agent who chooses a price path over time \( \{w_t\} \) to maximise a functional

\[ I = V(w, T(w))e^{-rt} dt, \]

where \( T(w) \) is an (increasing) tax function defined on the time rate of change of prices, and \( r \) is a strictly positive discount rate. Assume that \( V(\cdot) \) is decreasing in \( T \), and that \( T(0) = 0 \). Assume that there exists a unique optimum price \( w^* > 0 \) for the case \( T(\bar{w}) = 0 \) (the previous equilibrium, where there is no inflation tax). Then, if everything is appropriately differentiable, any steady-state equilibrium price \( \bar{w} \) produced by the introduction of the tax is lower than \( w^* \).

**Proof** The agent solves the problem

\[
\text{Maximise } \int F(w, \bar{w}, t) dt \quad (1)
\]

where \( F = V(w, T(w))e^{-rt} \). Therefore, by differentiability, the optimum price path satisfies the Euler equation

\[
\frac{d}{dt}(F_w) - F_w = 0 \quad (2)
\]

This can be written in full as

\[
\bar{w}F_{\bar{w}} + \bar{w}F_{\bar{w}} + F_{\bar{t}} - F_w = 0 \quad (3)
\]

For a steady state price, namely \( w = \bar{w} \) for all \( t \),

\[
F_{\bar{w}t} - F_w = 0 \quad (4)
\]
Now there exists, by assumption, an optimum price \( w^* \) for the maximisation problem when \( \tau(\dot{w}) = 0 \), which is the agent's problem before the introduction of the tax scheme. Thus \( F \) must be concave in the price \( w \) (at least around the optimum under \( \tau = 0 \)), because otherwise there would be no original equilibrium. Hence, by concavity and equation (4), if \( F_{\dot{w}t} > 0 \) then \( \ddot{w} < w^* \). This is because \( F_w \) must be positive if \( F_{\dot{w}t} \) is, and \( F_w \) can only be positive at a lower wage than \( w^* \). But

\[
F_{\dot{w}} = V_{\tau \tau}'(\dot{w})e^{-rt},
\]

so that

\[
F_{\dot{w}t} = -rV_{\tau \tau}'(\dot{w})e^{-rt}.
\]

However, in long run equilibrium, \( \ddot{w} = 0 \), so that

\[
F_{\dot{w}t} = -rV_{\tau \tau}'(0)e^{-rt}
\]

at the steady state price \( \ddot{w} \). Now \( \tau'(.) > 0 \) and \( V_{\tau} < 0 \). Thus \( F_{\dot{w}t} > 0 \), and \( F_{\dot{w}t} \) is strictly positive if \( r \) is strictly positive. This completes the proof. Figure 1 sketches the change in optimum prices.

The theorem suggests, for example, that a utility maximising trade union will set a lower equilibrium wage after a government introduces a wage inflation tax. The attractive thing about this result is that it emerges from the basic mathematical structure of the problem: the theorem can be applied under any sort of union utility function and any sort of tax (as long as it reduces workers' utilities, either directly or indirectly). All that it has been necessary to assume, in
Figure 1

The Geometry of the Tax Scheme

\[ F_{\dot{w}t} = F_w > 0 \]

\( E = \) the original equilibrium

\( \hat{E} = \) the equilibrium after the inflation tax.
fact, is that a rise in $\tau$ reduces the agent's utility, that $\tau$ goes up with inflation, and that $F(w,0)$ is concave around the pre-tax optimum. The result also goes through for a firm setting an output price; only the meanings of the symbols need be changed.

What is the explanation for the theorem? The result implies that, by making it costly to raise the price, the government can ensure that the relevant economic agent aims at a lower target price. To get some feel for this, imagine that, before any tax scheme, the price-setter wishes to raise its price to $w^*$ for time periods from 0 onwards. Let the initial price be $w_o$, as in Figure 2. Then there is no penalty for doing this as quickly as is feasible: it is optimal to jump directly to $w^*$. After the inflation tax, however, it does not pay to adjust in this way. If there is a positive discount rate it will be optimal for the agent to raise prices quickly at first and more gradually later on. But the important thing is that the target, originally $w^*$, will now be lower. This is because higher prices have an extra cost; they are 'bought' by paying the additional marginal cost of the government's tax.

Exactly the same is true in the literature on investment theory: the cost of adjusting the capital stock affects its long run equilibrium level. Other examples can be found.

3. Some Illustrations of Theorem 1

This section sketches a few applications of Theorem 1. They are confined to models of the labour market, but it can be checked that product market examples are easy to construct.
Figure 2

Possible Price Paths over Time

\[ W^* \] = the original equilibrium price
\[ \bar{w} \] = the new steady-state price
Case I. A Tax on Employers: The Utilitarian Monopoly Union

Say that the agent is a union choosing a wage schedule over time to maximise the utilitarian functional

\[ \int_0^T F dt = \int_0^T [v(w)N(w+\tau(w)) + u(b)M-N(w+\tau(w))M]e^{-rt} dt \]  

where

\[ v(w) = \text{the individual's utility from work} \]
\[ w = \text{the wage rate} \]
\[ \tau(\dot{w}) = \text{an employment tax on firms which is linked to the rate of wage change} \]
\[ N(w+\tau) = \text{the demand for labour} \]
\[ M = \text{the membership of the union} \]
\[ u(b) = \text{the individual's utility from unemployment} \]
\[ b = \text{a government payment to the unemployed}. \]

Let \( N(.) \) be a decreasing function, and assume \( v(w) > u(b) \).

Then

\[ F_{\dot{w}t} = \frac{2}{\dot{t}} N'(.)\tau'(.)[v(w) - u(b)]e^{-rt} \]

\[ = -rN'(.)\tau'(.)[v(w) - u(b)]e^{-rt} > 0. \]

The larger this is, the greater is the reduction in the equilibrium wage rate. Hence an inflation tax on firms will lower the long run equilibrium wage demanded by a utilitarian monopoly union.

Case II. A Tax on Employers: The Cooperative Union

Imagine that the union and firm negotiate a wage and employment combination which lies on a contract curve. This is equivalent, for a simple firm and a union which has utilitarian preferences, to the solution of a problem like
Maximise
\[ \int_{t}^{t+1} f d\pi = \int \{ v(w)N + u(b)(M-N) \} e^{-rt} dt \]  \hspace{1cm} (11)

subject to
\[ p f(N) - \left[ \bar{w} + \tau(\bar{w}) \right] N \geq c \]  \hspace{1cm} (12)

where
\[ \tau(\bar{w}) = \text{an employment tax linked to wage rises} \]
\[ p = \text{the price of output} \]
\[ f(N) = \text{output} \]
\[ c = \text{the firm's minimum profit level}. \]

The constraint, equation (12), can again be written in the form
\[ N = N(w + \tau(\bar{w})) \]. Therefore
\[ F_{\pi t} = -r \left[ v'(w) - u(b) \right] \left[ pf'(N) - w - \frac{\tau}{N} \right] e^{-rt} \]  \hspace{1cm} (13)

Before the tax, however, the equilibrium is characterised by the static first-order condition
\[ v'(w) + \left[ v'(w) - u(b) \right] pf'(N) - w = 0 \]  \hspace{1cm} (14)

which is the solution to
\[ \text{Maximise} \hspace{1cm} v(w)N + u(b)(M-N) \]
\[ \text{subject to} \hspace{1cm} pf(N) - wN \geq c . \]

Equation (14) makes it clear that \( pf'(N) - w \) is negative, because \( v'(w) \) and \( v(w) - u(b) \) are both positive. Hence for small tax changes around the initial static equilibrium it must still be true that \( pf'(N) - w < 0 \). Then equation (13) can be signed unambiguously: \( F_{\pi t} \) is positive. For large tax changes, however, the results are less clear cut.
These examples illustrate the effects of the most plausible type of wage inflation tax - one levied on firms. In both cases the long run wage level after the introduction of the tax is lower than that before the tax. Moreover, because there is an inverse relationship between wages and employment in each of the two examples, equilibrium employment is raised by the inflation tax scheme.

**Case III. A Poll Tax on Workers: The Utilitarian Monopoly Union**

The inflation tax can be levied on workers. In this case the utilitarian monopoly union solves

\[
\text{Maximise } \int_{\{w_t\}} F dt = \int \left\{ v(w - \tau(w))N(w) + u(b)[M - N(w)] \right\} e^{-rt} dt, \quad (15)
\]

where \( \tau \) is now a poll income tax on men's earnings. It can then be checked that

\[
F_{\dot{w}_t} = rv'(\cdot)\tau'(\cdot)Ne^{-rt} > 0. \quad (16)
\]

Therefore the same result goes through: the inflation tax lowers the long run equilibrium wage rate.

**Case IV. A Poll Tax on Workers: The Cooperative Union**

For the Cooperative union the analysis is similar to that in case III. Around the old equilibrium an inflation tax will reduce the long run wage rate.

**Case V. A Tax on Employers: Monopsony**

The same result holds for a conventional monopsonistic labour market. Let the firm's maximand be
\[
\int F dt = \int \left[ pf(N(w)) - \left[ w + \tau(\dot{w}) \right] N(w) \right] e^{-rt} dt \tag{17}
\]

where

\[
\begin{aligned}
p &= \text{the price of output} \\
f(N) &= \text{output} \\
N(w) &= \text{the workers' labour supply curve} \\
\tau(\dot{w}) &= \text{an employment tax linked to the rate of change of wages}
\end{aligned}
\]

Theorem 1 states that it is only necessary to check the sign of

\[
F_{\dot{w}t} = r\tau'(\cdot) N e^{-rt} > 0. \tag{18}
\]

Thus the new steady state wage, \( \bar{w} \), lies below the old one, \( w^* \). But in this case, of course, employment is lower after the introduction of the tax scheme, as long as the labour supply curve, \( N(w) \), has a positive slope.

4. A Theorem on Marginal Employment Subsidies

The alternative scheme considered here is one which rewards employment growth. Thus if \( s \) is an employment subsidy per man, which the government wishes to pay to firms, then a marginal employment subsidy is of the general type \( s = s(N) \), where \( \dot{N} \) is the time rate of change of employment. A firm's discounted profits might then be

\[
\hat{\pi} = \int \left[ pf(N) - \left[ w - s(N) \right] N \right] e^{-rt} dt, \tag{19}
\]

using the same sort of notation as before. But again it is possible to prove a fairly general result which requires only weak assumptions.
Theorem 2  Assume that there is an economic agent who chooses a time path for employment \( \{N_t\} \) to maximise a functional
\[
\int V(N_t, s(N)) e^{-rt} dt,
\]
where \( s(N) \) is an increasing subsidy defined on the time rate of change of employment, and \( r \) is a strictly positive discount rate. Assume that \( V(\cdot) \) is increasing in \( s \), and that \( s(0) = 0 \). Assume that there exists a unique optimum employment level \( N^* > 0 \) for the case \( s(N) = 0 \) (the previous equilibrium, where there is no marginal employment subsidy). Then, if everything is appropriately differentiable, any steady-state equilibrium employment level \( \bar{N} \) produced by the introduction of the subsidy is higher than \( N^* \).

Proof  The proof is identical to that for Theorem 1 and is omitted.

The formal similarity between an inflation tax and a marginal employment subsidy is clear. The two theorems are technically identical. An illustration of the theorem is given by the problem in equation (19): as long as \( s'(N) > 0 \) the new steady-state employment level lies above the old one.

5. An Equivalence Theorem

For a particular special case it can actually be shown that a marginal employment subsidy is equivalent to a tax on wage inflation. Equivalence, here, will mean that both create the same consumption choice set for individuals. Thus agents would see the two as indistinguishable.

Assume that the government can use lump sum taxes and subsidies. Let a worker's consumption be \( c \) and his wage be \( w \).
Assume that employment, N, is the only input, and that f(N) is output. Firms will be taken to maximise profits. Let there be a wage inflation poll tax on workers of τ(\dot{w}). Then firms set
\[ f'(N) - w = 0, \quad (20) \]
and workers receive the wage rate less tax, which means that their consumption is
\[ c = f'(N) - τ(\dot{w}). \quad (21) \]
Differentiate equation (20) with respect to time, to give
\[ f''(N)N - \dot{w} = 0, \quad (22) \]
and combine this with equation (21) to produce
\[ c = f'(N) - τ(f''(N)N). \quad (23) \]

This suggests the following equivalence result.

**Theorem 3** Under special assumptions an inflation tax can be exactly equivalent to a marginal employment subsidy.

**Proof** It is necessary to show that there exists a marginal employment subsidy which will create the same consumption opportunities as (23). Assume that workers receive a poll subsidy on employment growth of size s(N,N). Then their consumption is
\[ c = f'(N) + s(N,N) \quad (24) \]
Now define
\[ s(N,N) \equiv -τ(f''(N)N) \quad (25) \]
and note that \( s'_N = -\tau'(\cdot)f''(\cdot) > 0 \), so that, as we might require, the subsidy must be an increasing function of job growth. A subsidy function of the sort in (25) is identical to the wage inflation tax. This proves the result.

A trade union setting wages in this sort of world would find that its optimal actions were the same under \( \tau(\dot{w}) \) as under \( s(N,N) \). The same would be true for workers in a competitive labour market.

This is not a general result: some very restrictive assumptions have been made. Nevertheless, it may eventually be possible to prove other equivalence propositions under weaker conditions.

6. Conclusion

This paper has studied the micro-economics of inflation taxes and marginal employment subsidies. Three theorems have been presented. First, under very weak assumptions a tax on wage or price changes will reduce the long run equilibrium wage or price level. Second, by a formally identical result it is also true that under weak assumptions a subsidy on employment increases will raise the long run equilibrium level of employment. Third, in very special circumstances an inflation tax can be exactly equivalent to a marginal employment subsidy.

One of the interesting points to emerge from these conclusions is that an inflation tax will increase employment when there is an inverse relationship between the wage rate and the numbers of men employed. This appears to be a reasonably weak condition, although it is not normally satisfied in a
monopsonistic labour market, so in that particular case an inflation tax would raise unemployment. This point aside, an inflation tax looks likely to increase employment in a wide range of circumstances.

Perhaps the other main message from the analysis is that a tax on inflation has much in common with a marginal employment subsidy. Both mean that the relevant economic agents have to solve quite complicated optimisation problems (of a formally similar kind). Moreover, as the third theorem shows, the two policies can have the same sorts of effects on individuals' choice sets. This is because, if the demand curve for labour is a declining function of the wage rate, both schemes reward employment increases and penalise wage rises.
Footnotes

1 In a related paper Pissarides (1981) studies a wage tax rather than a wage inflation tax.

2 This is discussed in Oswald and Rosewell (1981).

3 This is not sufficient. There are also extra corner conditions, a Legendre second-order condition and transversality conditions; but for our purposes these do not have to be treated in detail.

4 If the discount rate is zero it is clear that \( w^* = \bar{w} \).

5 This type of objective function is used in McDonald and Solow (1981), Oswald (1982a) and elsewhere.

6 Oswald and Ulph (1982) and McDonald and Solow (1981) concentrate on union equilibria of these sort.

7 This is demonstrated in Pemberton (1982).

8 The case for non-linear subsidies is discussed in Oswald (1982b).
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