Optimal Intervention in an Economy with Trade Unions*

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Abstract

The paper studies the theory of optimal intervention in an economy with trade unions. It is shown that the traditional remedy, a flat employment subsidy in the union sector, cannot produce a first-best welfare optimum. But non-linear wage and employment subsidies can generate a full social optimum, and the paper examines their optimal structure. One appealing form turns out to be a wage subsidy schedule which is an increasing and concave function of union employment. Employment subsidy schedules and statutory wages policy are also discussed.

Optimal Intervention in an Economy with Trade Unions

Andrew J. Oswald

1. Introduction

It seems to be rather widely believed that the existence of trade unions has deleterious effects on an economy. Unions are sometimes blamed, for example, for the recent high levels of unemployment in Western countries.[1] It is also often argued that the activities of labour monopolies distort the allocation of resources and create inequities amongst different types of workers. If this is true, and as yet it seems too early to be certain, the government faces the complicated problem of how to intervene in an economy of this sort.

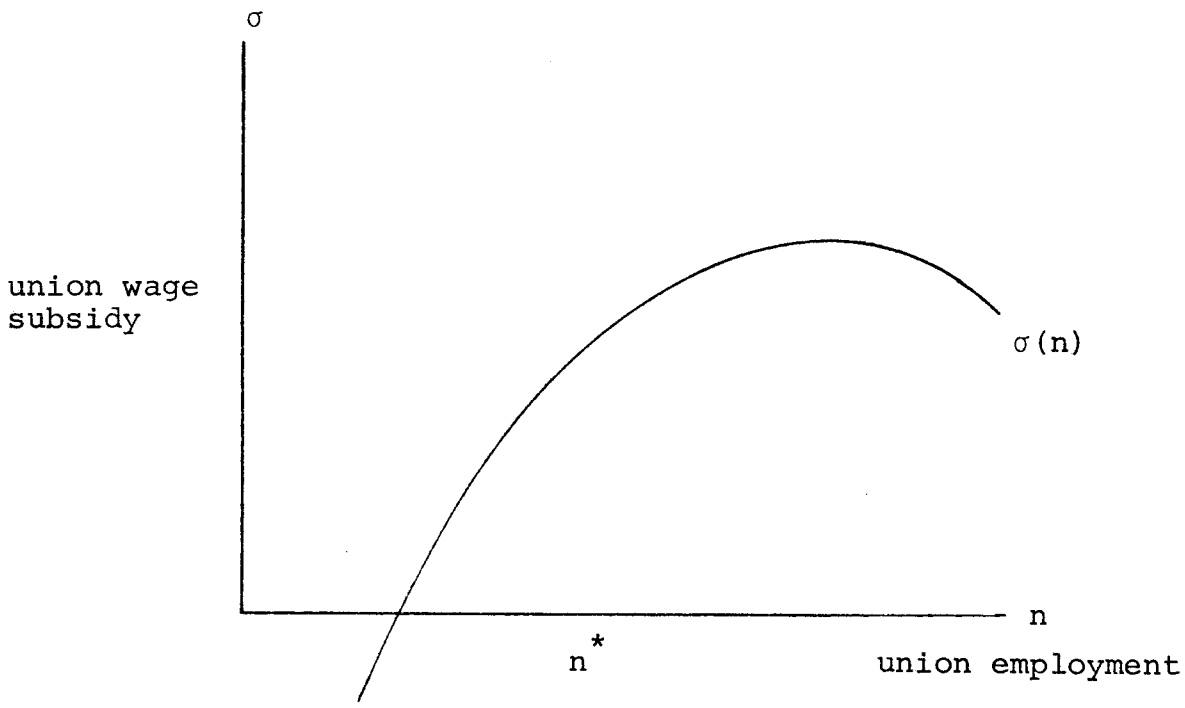
This paper studies the theory of optimal intervention in an economy in which there are trade unions. Its main purpose is to show that in a simple economy non-linear employment and wage subsidies can generate a first-best optimum. A number of different schemes are examined. One rather appealing policy turns out to be for the government to set a wage subsidy - on union work - which is an increasing and (normally) concave function of employment in the union sector. An example is given in Figure 1. In practical terms this means that the government offers the employees in the unionised sector an inducement to raise their numbers, because by doing so the members of that union can draw a higher state subsidy per man, but that that inducement diminishes as union employment increases. There is a way in which this strategy can produce a full social

optimum. Say that, by designing the non-linear subsidy schedule appropriately, it were possible to reward the union in fixed proportion to the value of social welfare. Then rational union behaviour would lead to social optimality: the private and social optima would coincide. This is the intuitive explanation for the success of the non-linear wage subsidy. The details are explained in a later section.

The paper also shows that a first-best optimum, or arbitrarily close to one, can be reached by a non-linear employment subsidy paid to firms. This is more difficult to derive analytically. The reason is that government intervention has to work more indirectly than in the case of subsidies paid to union workers; it must operate by changing the employment choices of firms and thereby influencing the wage decisions of the union which those firms face. It will be shown that this employment subsidy schedule has quite a different form from the non-linear wage subsidy.

Before these results are derived, however, a more traditional remedy is examined. It is that of simple flat employment subsidies. In this case, because the size of the subsidy is independent of any decisions by firms or unions, it makes no difference whether the subsidy is paid directly to workers or employers. The paper proves that this conventional and fairly widely advocated type of intervention will not produce a full first-best welfare optimum. The intuition is reasonably easy to see. A flat union employment subsidy - £x per man per week, for example - has two principal effects on a unionised economy. First, it tends to raise employment and

Figure 1



An Optimal Non-linear Wage

Subsidy Schedule

output in the union sector, because firms there find that the net cost of their labour has fallen. Second, a flat subsidy will normally create higher union wages, as unions discover that the labour demand curve which they face has shifted to the right. The first of these effects is beneficial; the value of national output increases. The second is detrimental to social welfare; it worsens the distribution of income. If the government restricts itself to a flat subsidy on unionised labour, therefore, it is inevitably faced with a trade-off between efficiency and equity. But non-linear subsidies can get around this problem. They allow the government to reconstruct the agents' choice sets in such a way as to make privately rational behaviour consistent with socially optimal actions.

The literature on optimal government policy in a unionised economy is not a large one. It falls into four parts. First, some work has been done by writers on international economics: Hagen (1958), Bhagwati and Ramaswami (1963), Bhagwati and Srinivasan (1974), Corden (1974), (1981) and Anand and Joshi (1979) have all contributed to the literature known as the general theory of distortions and welfare. A survey on factor market distortions can be found in Magee (1973). Second, there have been a number of recent papers - Johnson (1980) and Jackman and Layard (1980), for example - on the case for employment subsidies in economies with (i) inflexible wage rates and (ii) distortions caused by unemployment insurance payments that are not fully 'experience-rated'. Third, there is a paper by Calvo (1978) which incorporates trade unions into the Harris-Todaro model of unemployment and migration.[2] Fourth, there are recent theoretical papers on unions by

Jackman and Layard (1982), Pissarides (1982), Sampson (1982) and Shah (1982) - all of which discuss government schemes to raise real national output.[3]

The literature does not seem to answer all the questions in which one might be interested. It might be criticised in a number of ways. Some writings, for example, assume that trade unions behave in a rather simple fashion: the most common assumption is that the union has an exogenously fixed real wage. This means that the union does not change its wage demands as the government alters the levels of taxes and subsidies on labour. Almost the whole of the literature also ignores the fact that governments care about the distribution of income (the exception is Anand and Joshi (1979)). In general, therefore, efficiency is taken to be the single goal; equity is put to one side. This seems to be unsatisfactory, and certainly contrasts sharply with the rest of modern public economics. Finally, some of the literature fails to say exactly how tax revenue is raised and to whom subsidies are paid.

It seems natural to go about the problem in a slightly different way. The analysis set out in later sections has the following main features. First, the trade union is treated as a rational agent: it is assumed that the union maximises its members' combined utilities (subject to various constraints), which means that the union has utilitarian preferences. Second, an explicit social welfare function is used, and for consistency this is also utilitarian. Third, a government budget constraint is specified. The government's problem is then to maximise social welfare under the conditions that agents behave rationally and the government budget constraint is satisfied. As in other

areas of public economics the difficulty for the government is that it has to optimise subject to others' optimisation decisions.

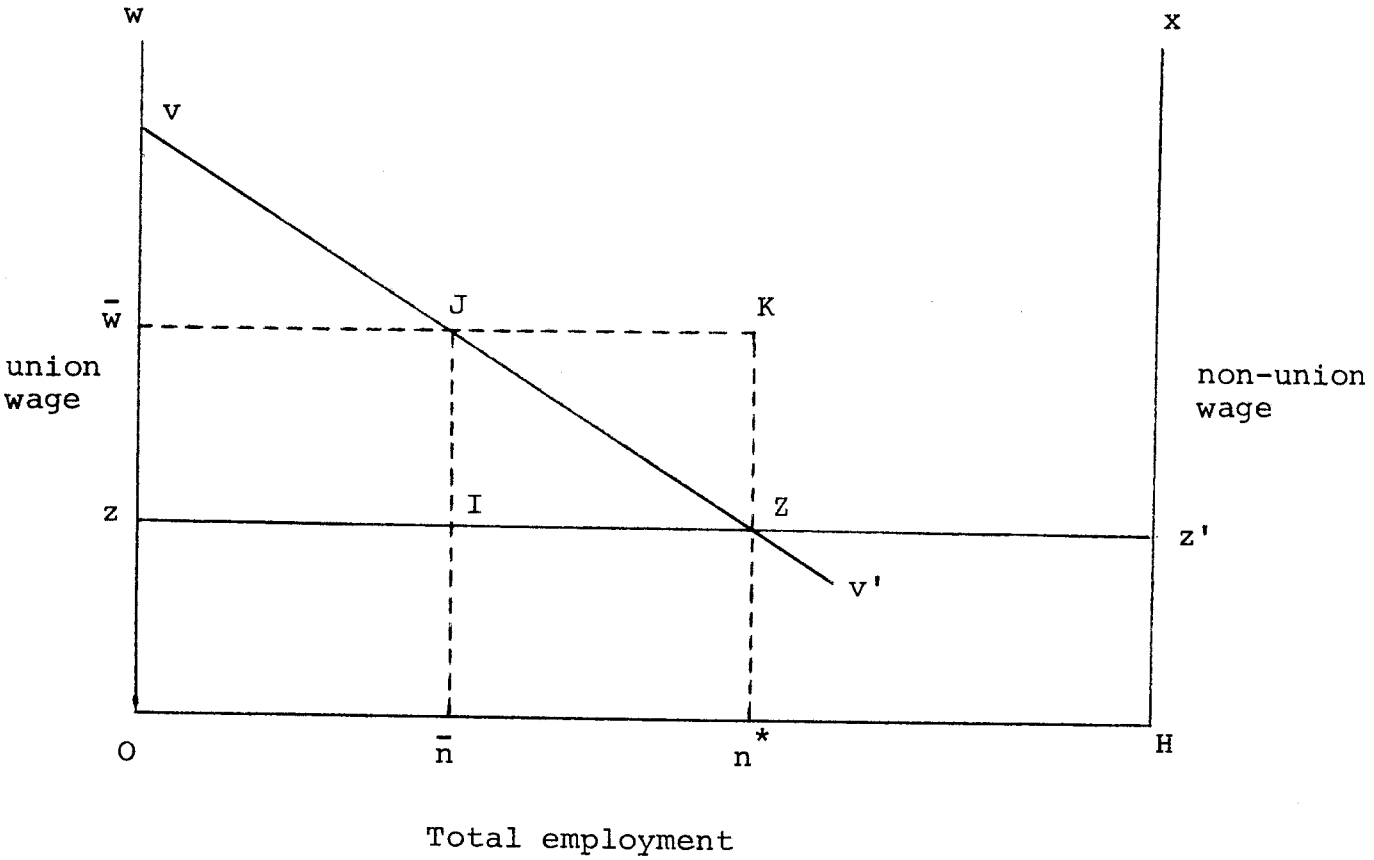
2. The Model of the Economy

The paper uses a simple and conventional general equilibrium model. Imagine an economy in which there are two types of output and two factors of production. Assume that one factor, labour, can move between the two industries, but that the second factor, capital, is immobile. Let total factor supplies and output prices be given exogenously (the latter might be thought of as determined on world markets). Assume that one sector is unionised whilst the other has a competitive labour market. Let the production function in the unionised sector be a strictly concave, increasing and appropriately differentiable function $f(n)$, where n is the number of employed union men. Assume a fixed marginal product in the other (non-unionised) sector. Let the price of output of the union sector be normalised at unity.

If we were now to follow the literature on the theory of distortions - Corden (1974) and Anand and Joshi (1979), for example - we would assume that the trade union specifies a fixed wage rate at which its members will work, $\bar{w}$, and is able to prevent non-union men from entering the sector to drive the wage below this level. If firms maximise profits, equilibrium in this sort of world is then described by Figure 2. The labour force, OH, is measured horizontally. The value of the marginal product is given by line vv' in the unionised sector (drawn as linear for simplicity) and by the constant zz' in the non-unionised sector. The fixed union wage is measured

Figure 2

A Traditional Model



by $O\bar{w}$. Without government intervention, therefore, there is full employment: the level of employment in the unionised sector is $O\bar{n}$ and that in the other sector is $\bar{n}H$. However, technical efficiency is obviously not attained (that is, the value of the economy's output is below its maximum), because that requires that the value of the marginal product be the same in each sector, which occurs only at employment n^*, namely point Z. To reach Z, according to this literature, the government must pay a flat subsidy to firms of KZ per worker. The net gain in the value of national output is then represented by the area JZI (the dead-weight loss from unionisation), and the minimum government revenue which is necessary to achieve this subsidy per worker is given by JZK.

If $\bar{w}$ is fixed in real terms, so that it is not possible for the government to make a union worker's income less than this amount, there are two sources of government revenue - profits, given by $vJ\bar{w}$, and non-unionised workers' incomes, given by $Iz'H\bar{n}$. If the sum of these areas is less than JKZ, technical efficiency cannot be attained by a labour subsidy. It is still second-best, however, to attempt by the same method to get as close as possible to Z.

To go beyond this traditional type of analysis one needs a model of the trade union. This paper assumes that the trade union attempts to maximise the sum of its members' utilities. Hence let the union have the utilitarian objective function[4]

$$U = nu(c) + (m-n)u(x)$$

where U is the union's utility function, n is the level of employment of union men, m is the membership of the union,

$u(c)$ is the utility of a union member who is employed in the unionised sector, and $u(x)$ is the utility available to any worker in the non-union sector. Membership, m, is assumed to be exogenous.[5] Workers cannot freely enter the union sector, because the trade union is a monopoly, but those workers displaced from the unionised part of the economy can always find jobs in the other (competitive) labour market. The worker's utility $u(.)$ is strictly concave and increasing. Consumption levels are c and x in, respectively, the union and non-union sectors. This form of union utility function is now widely used: see, for example, McDonald and Solow (1981), Oswald (1982) and Sampson (1982), inter alia. It is equivalent - as can be checked by division by the constant m - to the assumption of expected utility maximisation.

Assume that the union cannot negotiate individually with each of the many competitive firms who employ its members. It must then fix a wage subject to the labour demand constraint imposed by those firms' actions. If there is no government subsidy, therefore, the union's problem is to

$$\begin{aligned} &\text{Maximise} \quad U = nu(c) + (m-n)u(x) \\ &\text{subject to} \quad c = f'(n) \ , \end{aligned} \tag{1}$$

where the constraint is the solution to the firms' profit maximisation decisions. At an interior optimum this implies that

$$\frac{u(c) - u(x)}{u'(c)} = - nf''(n) \ . \tag{2}$$

The left hand side gives the gain in utility (valued in terms of output) from a transfer of one member from the non-union sector

to the union sector. The right hand side measures the consequent fall in consumption of those already employed in the unionised sector. At an interior maximum the union equates these two quantities. Figure 3 sketches this micro-economic model in a diagram: I_0 and I_1 are convex union indifference curves[6] (level curves of equation (1)), and the labour demand curve is $c = f'(n)$, which is again drawn as linear.

It is also necessary to specify a government budget or real output constraint. In this model it is

$$f(n) - nc - (x-z)(1-n) \geqslant 0 \qquad (3)$$

where z is, as before, the fixed marginal product of labour in the non-union industry, and the total labour force of the economy (H in the earlier diagram) is normalised at unity. This equation states that the net profit of the union sector must be at least as large as the total value of subsidies paid to the non-union sector. It implies that any revenue left after union firms have been paid any subsidies can be taxed in a lump sum way and distributed as poll transfers to the non-unionised segment of the economy. Constraint (6) can normally be treated as a strict equality, and will be throughout this paper.

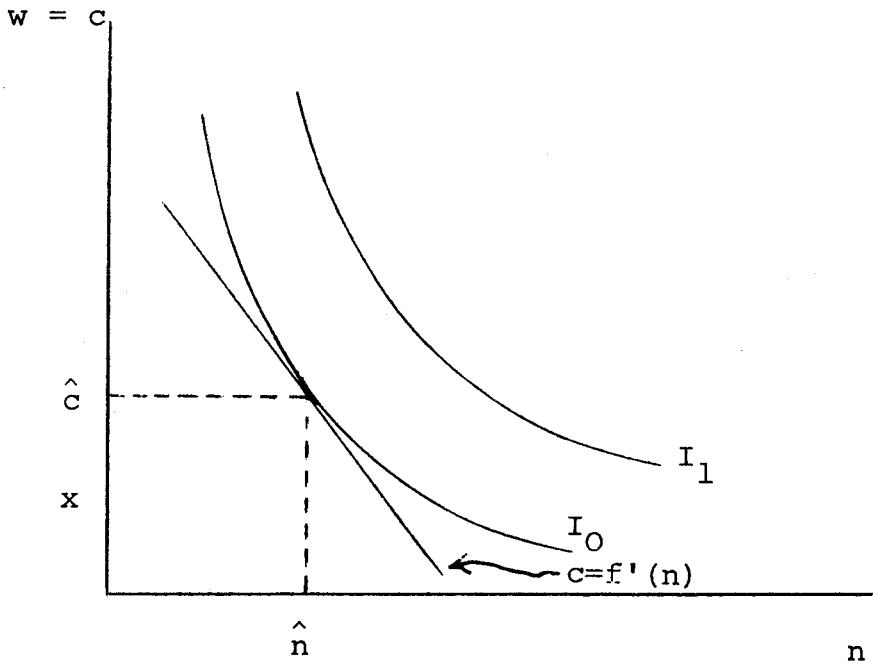
Finally, we need to define a social welfare function. A utilitarian welfare function will be used here, so let welfare be

$$W = nu(c) + (1-n)u(x) \ . \qquad (4)$$

This does not seem, for this economy, to be too restrictive: people are identical, so the maximisation of (4) is equivalent

Figure 3

The Union's Optimal Wage/Employment Combination (under no intervention)



to the maximisation of the typical person's expected level of utility.

The rest of the paper is concerned with how a government might maximise (4) under the constraints (1), (2) and (3). In order to tie the paper's analysis into the literature the remaining sections begin with a discussion of the traditional remedy of flat employment subsidies.

3. Flat Employment Subsidies and Second Best Optima

Let s be a poll subsidy paid by the government to either union workers or unionised firms. It will make no difference which side of industry nominally receives it; in both cases the worker's consumption becomes

$$c = f'(n) + s \ . \tag{5}$$

Three interesting questions can now be asked. First, how should a government choose s in order to maximise social welfare? Second, can a flat subsidy of this type push the economy to a first best optimum? Third, is the optimal s always positive (in other words, might it ever be necessary to have a tax, $s < 0$, on union labour)?

The government has to solve the following problem:

$$\underset{s}{\text{Maximise}} \quad W = nu(c) + (1-n)u(x) \tag{6}$$

$$\text{subject to} \quad f(n) - nc - (x-z)(1-n) \geqslant 0 \tag{7}$$

$$u(c) - u(x) + nf''(n)u'(c) = 0 \tag{8}$$

$$c - f'(n) - s = 0 \ . \tag{9}$$

The second constraint, that of rational union behaviour, is

the condition derived from the maximisation of $U = u(c)n + (m-n)u(x)$ subject to $c = f'(n) + s$ (the labour demand curve). The third constraint is thus redundant, so we can work with a Lagrangean

$$L = nu(c) + (1-n)u(x) + \lambda\{f(n) - nc - (x-z)(1-n)\}$$
$$+ \mu\{u(c) - u(x) + nf''(n)u'(c)\} . \qquad (10)$$

It will do little harm to assume differentiability and to ignore corners. First-order conditions for an interior optimum then include

$$c : n[u'(c) - \lambda] + \mu[u'(c) + nf''(n)u''(c)] = 0 \qquad (11)$$

$$n : u(c) - u(x) + \lambda[f'(n) - c + x - z]$$
$$+ \mu u'(c)[f''(n) + nf'''(n)] = 0 \qquad (12)$$

$$x : (1-n)[u'(x) - \lambda] - \mu u'(x) = 0 , \qquad (13)$$

where λ and μ are multipliers. These implicitly define the optimal flat employment subsidy, s. Note that $\lambda \geqslant 0$ by construction: it is the multiplier on the inequality (7).

The following result can be now proved.

<u>Proposition 1</u> <u>Assume that the government is restricted to a flat employment subsidy, lump-sum profits taxes and poll income transfers to workers. Then</u>

<u>(i) a first-best is not attained,</u>

<u>(ii) union workers must be better off than non-union workers,</u>

<u>(iii) the optimal subsidy, s^*, is not necessarily positive,</u>

<u>(iv) the multiplier μ is non-negative.</u>

<u>Proof</u> By equation (8), because $nf''(n)u'(c)$ is unambiguously

negative, we know that $u(c) > u(x)$. This establishes (i) and (ii): $u(c) = u(x)$ at the first-best. The optimal subsidy is equal to $c - f'(n)$. This cannot be signed unambiguously. To prove (iv), assume the reverse, namely $\mu < 0$, in order to establish a contradiction. Then equation (11) implies that

$$u'(c) - \lambda > 0 , \tag{14}$$

and equation (13) implies that

$$u'(x) - \lambda < 0 , \tag{15}$$

so taken together we have

$$u'(c) - u'(x) > 0 . \tag{16}$$

But $u(.)$ is strictly concave, which means that (16) states that $x > c$. This contradicts part (ii) of the proposition. Hence $\mu \geqslant 0$.

There is also an instructive special case which makes it clear that in well-behaved problems we should expect to find that $s > 0$ at the optimum subsidy level.

<u>Proposition 2</u> <u>If the union sector's production function is $f(n) = \beta n^{\gamma}$, where $\gamma \in [0,1]$ and $\beta > 0$, and if the assumptions of Proposition 1 hold, then the optimal flat employment subsidy, s^*, is strictly positive.</u>

<u>Proof:</u> Assume $f(n)$ takes the constant elasticity form βn^{γ}. Then

$$f''(n) + nf'''(n) = \beta n^{\gamma-2}\gamma(\gamma-1)^2 > 0. \tag{17}$$

Hence, by equation (12) and parts (ii) and (iv) of Proposition 1,

$$f'(n) - c + x - z < 0 , \tag{18}$$

which implies that $x - z < c - f'(n)$. Substitute this into (7) to give

$$f(n) - nc = (x-z)(1-n) < [c - f'(n)](1-n) \tag{19}$$

or, after simplification,

$$f(n) - f'(n)n < c - f'(n)$$

But $f(n) - f'(n)n = \beta n^{\gamma}(1-\gamma) > 0$, so $c - f'(n) > 0$. The subsidy is defined by $c = f'(n) + s^*$. Hence $s^* > 0$.

What is the intuitive explanation for these results? There seem to be two forces at work. First, the government would like to raise real national output (that is, to get closer to technical efficiency, $z = f'(n)$). It can do this by paying a positive employment subsidy, because that lowers the real cost of labour and encourages union firms to take on more men. Second, the government wants to improve the distribution of income: without intervention the unionised workers have higher wages than those in the non-union sector. It can do this by taxing union men, or their employers, and it therefore wishes to pay a negative employment subsidy. These two forces obviously conflict; there is a trade-off between equity and efficiency. This is the explanation for parts (i) and (iii) of the Proposition. Part (ii), the result that union men must necessarily retain an advantage over non-union workers (even at this second-best optimum), is also straightforward: it is not socially optimal for the government to go the whole way to full equality, because the efficiency gains are too high.

The government must find the ideal compromise between technical efficiency and distributional equality.

The conclusion from this analysis, then, is that a simple employment subsidy will not produce a first-best optimum, and that, in extreme cases, it might even lower social welfare. The rest of the paper discusses alternative policies.

4. First-Best Optima

This section focuses upon the different effects of wages policy, non-linear wage subsidies and non-linear employment subsidies.

4.1. Statutory Wages Policy

There are no informational difficulties for the government of this economy. It understands the structure of the economy and can identify union and non-union workers. In addition, the government has been assumed so far to have the power to use both lump sum taxes on profits and also poll taxes and subsidies on individuals. But we shall assume that it cannot simply ban trade unions (perhaps because of some political constraint). There is then an obvious route to a first-best optimum, namely to legislate directly about wage levels and to pay out all profits as poll income transfers to workers.

Proposition 3 A first-best optimum, $c^* = x^*$ and $f'(n) = z$, can be reached by the combination of statutory wages policy, lump sum profits taxes and poll income transfers to workers.

Proof The proof is straightforward. If the government can set wage rates by statute then it need only solve the planning problem:

$$\underset{c,x}{\text{Maximise}} \quad W = nu(c) + (1-n)u(x) \tag{21}$$

$$\text{subject to} \quad f(n) - nc - (x-z)(1-n) = 0 \ , \tag{22}$$

which implies $c^* = x^*$ and $f'(n) = z$, where c^* is the socially optimal consumption level of workers in the unionised industry and x^* is that for individuals in the non-union sector. The first-best thus combines technical efficiency ($f'(n) = z$) and full equality ($c = x$).

What the government does here, in effect, is to override the union's ability to set wage rates. It then calls out those wage levels which would exist at competitive equilibrium, which means that the value of the marginal product of labour is the same in both sectors, and it finally shares out the economy's profit equally amongst the workers.

4.2. Non-Linear Wage Subsidies

This section of the paper trys to show that a first-best optimum can be produced by a special type of tax and subsidy scheme. It is helpful to begin in an informal way.

Imagine that the government could design a form of subsidy which ensured that union workers always received the same income as those in the non-unionised half of the economy. Then $c = x = y$, where y is that common income per head. In that case the union's maximisation decision would be

$$\text{Maximise} \quad U = nu(c) + (m-n)u(x)$$

$$= mu(y) \ , \tag{23}$$

so that the union would merely try to maximise y, if it could. Now say that we could make y be monotonically related to real

national output. The result would be that privately rational union actions would produce a first-best level of employment: the union would maximise its utility by setting employment equal to n^*, the first-best level. Real national income is $f(n) + z(1-n)$. Hence a scheme of the sort just suggested would generate an equilibrium like that in Figure 4. Union indifference curves are horizontal in the diagram, because the union does not mind, as $c = x$, whether its member work in the union or non-union sector. The value of national income is a concave function with a turning point at $n = n^*$.

The next proposition shows exactly how a wage subsidy schedule can do this.

<u>Proposition 4</u> <u>(i) A first-best optimum can be reached by the combination of a non-linear poll subsidy $\sigma(n)$ to unionised workers, lump sum profits taxes, and poll income transfers to non-union workers.</u>

<u>(ii) The optimal non-linear poll subsidy on unionised workers is</u>

$$\sigma(n) = - f'(n) + f(n) + z(1-n) \quad . \tag{24}$$

<u>Proof</u> The union worker's consumption is now

$$\begin{aligned} c &= f'(n) + \sigma(n) \\ &= f(n) + z(1-n) \end{aligned} \tag{25}$$

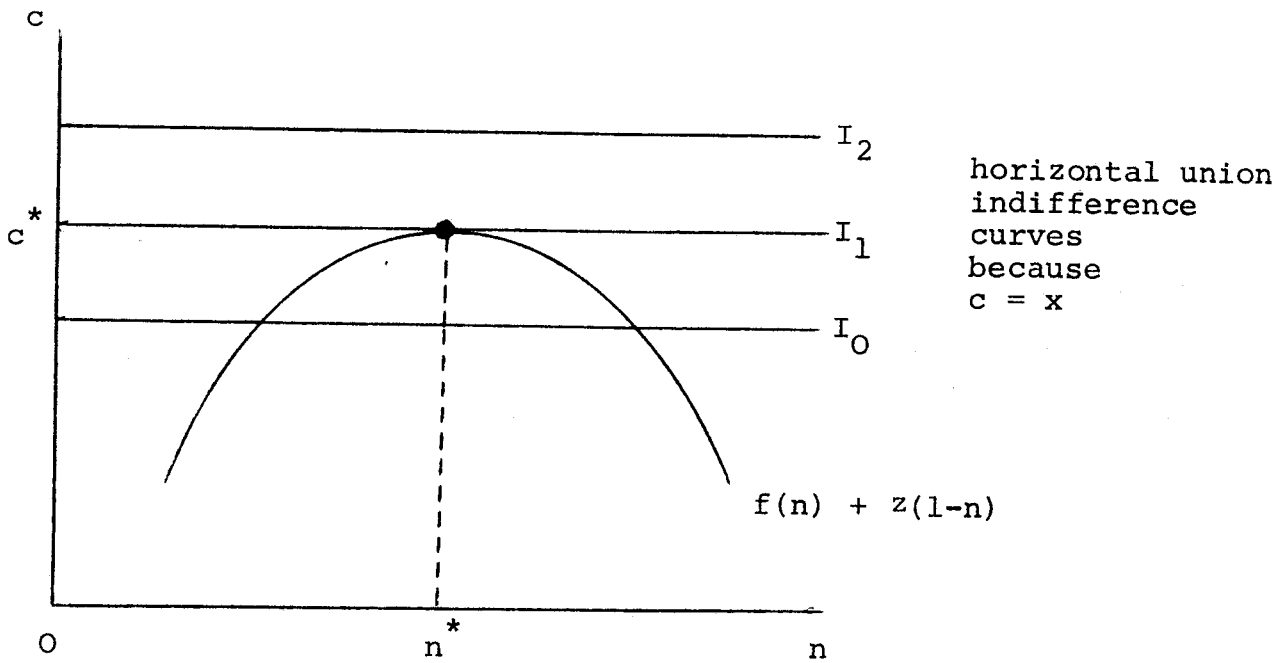
Hence the union solves the problem

$$\underset{u}{\text{Maximise}} \quad U = u(f(n) + z(1-n))n + u(x)(m-n) \quad . \tag{26}$$

Figure 4

A Non-Linear Poll Subsidy to Men in

The Union Sector: Its Effects



Now feasibility requires output equals consumption, so that

$$f(n) + z(1-n) = cn + x(1-n) \quad , \tag{27}$$

which implies $c = x$ combined with (25). This establishes that the equilibrium is equitable. The solution to (26) then guarantees full efficiency. To see why, solve the problem stated in equation (26) to give

$$nu'(c)[f'(n) - z] + u(c) - u(x) = 0 \quad , \tag{28}$$

or more simply, because $c = x$ by equations (25) and (27), $f'(n) - z = 0$. The second-order condition is also satisfied. Hence employment is set at its first-best level.

It is possible to learn a good deal about the optimal non-linear wage subsidy schedule. Its form is

$$\sigma(n) = f(n) - f'(n) + z(1-n) \quad , \tag{29}$$

so that, if $f(0) = 0$, which seems reasonable,

$$\lim_{n\to 0} \sigma(n) = z - \lim_{n\to 0} f'(n) \quad , \tag{30}$$

which will be negative in any interesting model. Next, if we impose the condition that $f'(n) \to 0$ as $n \to \infty$, then

$$\lim_{n\to\infty} \sigma(n) = \lim_{n\to\infty} [f(n) + z(1-n)] \quad , \tag{31}$$

which may be a small number. Moreover, as long as $\sigma(n)$ is differentiable, we know that

$$\sigma'(n) = f'(n) - f''(n) - z \quad , \tag{32}$$

so that around the first-best level of employment

$$\sigma'(n^*) = - f''(n^*) > 0 \quad (\text{at } z = f'(n)) \tag{33}$$

Moreover,

$$\sigma''(n) = f''(n) - f'''(n) \ , \tag{34}$$

which can take either sign in the general case, but is likely, for the usual types of production functions, to be negative. Hence $\sigma(n)$ will probably look like the curve in Figure 1.

Some numerical examples are given in Figures 5, 6 and 7. They rest on the following assumptions.

$f(n)$	=	βn^{γ}	Union output
β	=	2	Productivity parameter
γ	=	$\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$	Elasticity parameters
z	=	1.6	Non-union marginal product

The three different cases show the effects of variations in the elasticity γ. The first-best levels of emoloyment in the three instances are $n^* \cong 0.2$ for $\gamma = \frac{1}{4}$, $n^* \cong 0.4$ for $\gamma = \frac{1}{2}$, and $n^* \cong 0.8$ for $\gamma = \frac{3}{4}$.

In conclusion, this section of the paper has shown that there is a way to push a partially unionised economy to a first-best optimum. The government has only to set the appropriate non-linear union wage subsidy. In other words, it announces that union workers will receive from the state a poll subsidy, σ, which will depend upon the number of men employed in the unionised sector. The government fixes the shape of this schedule according to the optimal rule given in equation (24). Unions are thus induced to allow more employment in their sector, because the extra government subsidy more than compensates for

Figure 5

Optimal Schedule when $\gamma = \frac{1}{4}$

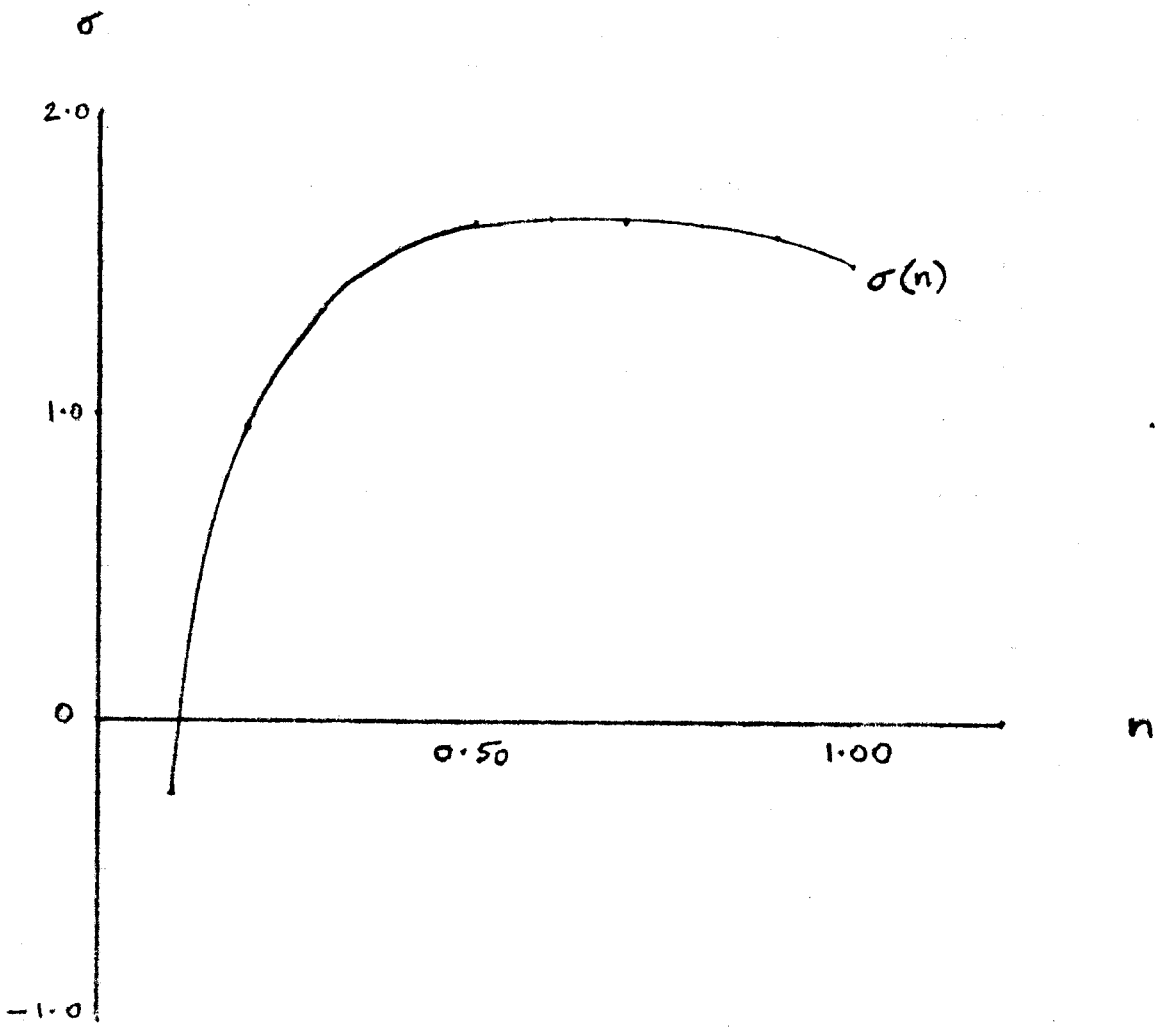


Figure 6

Optimal Schedule when $\gamma = \frac{1}{2}$

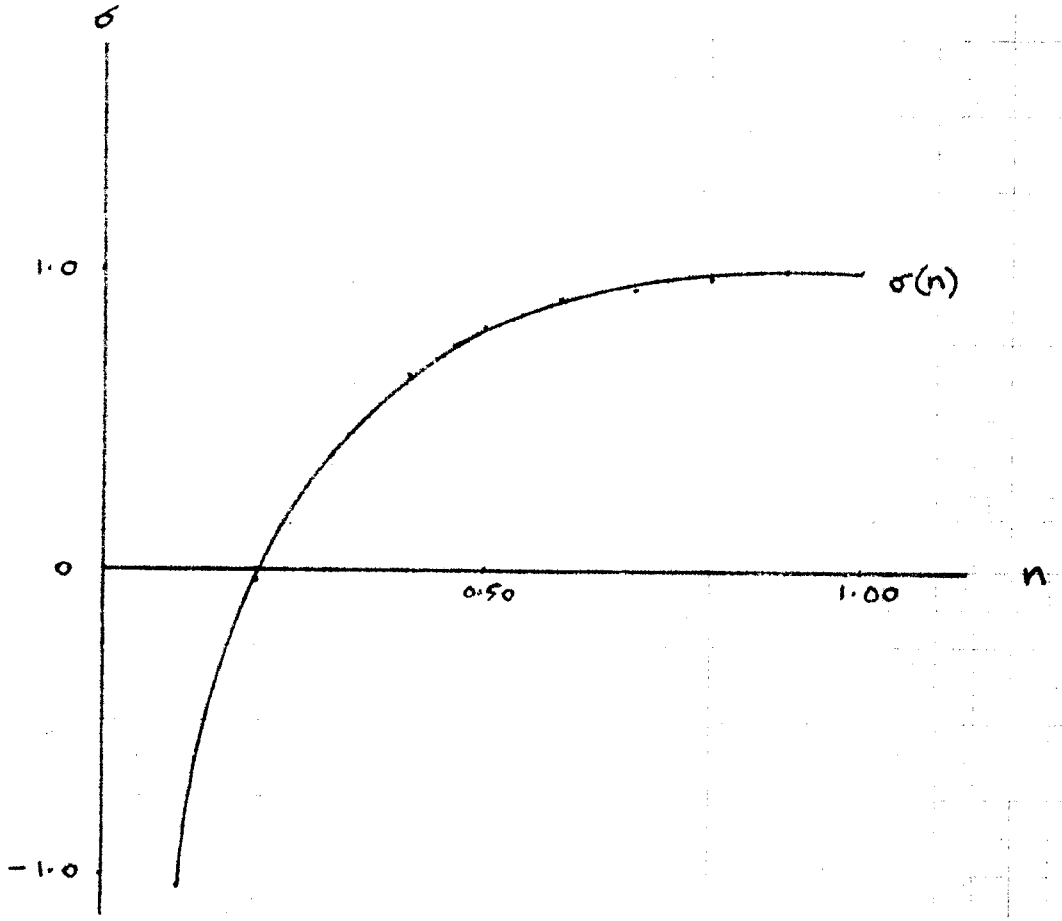
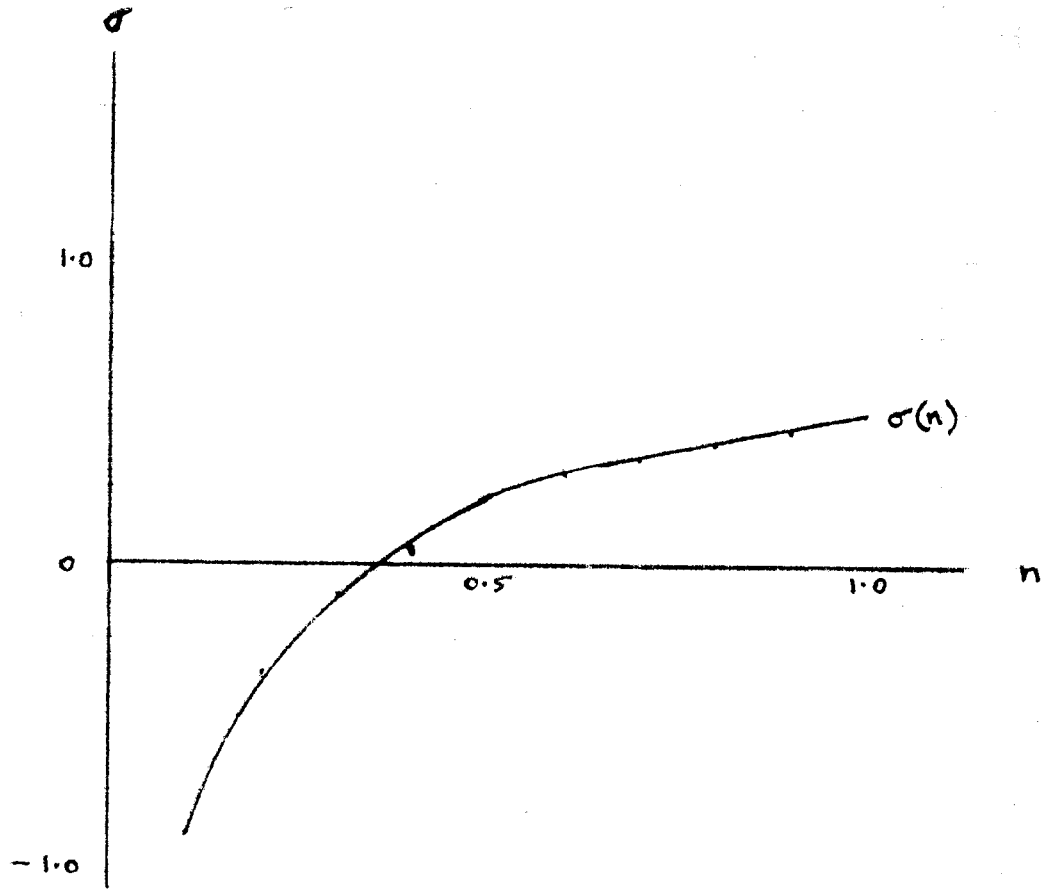


Figure 7

Optimal Schedule when $\gamma = \frac{3}{4}$



the fall in their wage rate (the marginal product of labour is of course reduced as a consequence of the greater employment). By setting $\sigma(n)$ optimally the government can change the shape of the trade union's choice set in such a way as to make the union's rational decisions coincide with those which would achieve a first-best. Hence a non-linear wage subsidy can eliminate the distributional and allocational distortions caused by the existence of trade unions.

Finally, it is useful to emphasise that the rationale for non-linear subsidies here is quite different from that in the literature on optimal non-linear taxation. The latter makes the assumption that people's characteristics are not observable by the government. See Mirrlees (1976), for example.

4.3. Non-Linear Employment Subsidies

It is also possible to design a first-best employment subsidy schedule. Let it be $s(n)$, where s is the poll subsidy paid (per man employed) to unionised firms by the government.

Firms now solve the problem

$$\text{Maximise} \quad \pi = f(n) - wn + s(n)n \ , \tag{35}$$

which produces an interior maximum, if everything is appropriately differentiable, where

$$f'(n) - w + s'(n)n + s(n) = 0 \tag{36}$$

and

$$f''(n) + s''(n)n + 2s'(n) \leqslant 0 \ . \tag{37}$$

Equation (36) is just an unconventional type of labour demand

curve. This non-linear subsidy does not just shift the labour demand curve (the effect of the $s(n)$ term alone); it twists it in a complicated way (the effect of the $s'(n)n$ term). The equation is best written as

$$c = f'(n) + s'(n)n + s(n) \quad , \tag{38}$$

where $c = w$ is a union man's consumption level.

The union still has to maximise against the labour demand curve. It finds a solution to the following:[7]

$$\underset{n}{\text{Maximise}} \qquad U = nu(c) + (m-n)u(x) \tag{39}$$

$$\text{subject to} \qquad c = f'(n) + s'(n)n + s(n) \quad , \tag{40}$$

which has as a first-order condition

$$nu'(c)\left[f''(n) + s''(n)n + 2s'(n)\right] + u(c) - u(x) = 0 \ . \tag{41}$$

But the second-order condition for the firm to be at a maximum is (37), and it seems sensible to insist that it hold as a strict inequality (otherwise there is no guarantee that the firm would ever choose any particular n). Hence, by (41) and (37),

$$u(c) - u(x) > 0 \tag{42}$$

under any differentiable employment subsidy function $s(n)$. This proves the fifth result.

<u>Proposition 5</u> <u>There is no differentiable employment subsidy schedule $s(n)$ that can produce a first-best welfare optimum.</u>

This makes the task seem harder than one might have expected. There are two ways out. One is to study approximate first-

best optima; the other is to try to design a non-differentiable subsidy function. Both approaches are followed here.

For the moment it will pay to retain the criterion that $s(n)$ be differentiable. Consider the differential equation

$$s''(n)n + 2s'(n) + f''(n) + \varepsilon = 0 , \tag{43}$$

where ε is an arbitrarily small positive number. This must satisfy

$$s''(n)n + 2s'(n) + f''(n) < 0 , \tag{44}$$

so that the second-order condition for profit maximisation is fulfilled. Moreover, it looks likely, from (41), that

$$\lim_{\varepsilon \to 0} \{u(c) - u(x)\} = 0 . \tag{45}$$

Thus it appears to be possible to derive an approximate first-best schedule.

<u>Proposition 6</u> <u>(i) An approximate first-best optimum can be reached by the combination of a non-linear employment subsidy $s(n)$ to union firms, lump sum profits taxes and poll income transfer to workers.</u>

<u>(ii) One optimal employment subsidy schedule is</u>

$$s(n) = c^* + \frac{k}{n} - \frac{f(n)}{n} - \tfrac{1}{2}\varepsilon n , \tag{46}$$

<u>where k is an arbitrary constant and ε is an arbitrarily small positive number.</u>

<u>Proof</u> Firms maximise the profit function $\pi = f(n) - wn + s(n)n$,

which in this case, using equation (46), is

$$\pi = (c^* - w)n + k - \tfrac{1}{2}\varepsilon n^2 \ . \tag{47}$$

The function is maximised at

$$\pi_n = c^* - w - \varepsilon n = 0 \ , \tag{48}$$

and it is important to check that

$$\pi_{nn} = - \varepsilon < 0 \ , \tag{49}$$

which ensures that employers are at a maximum.

The union then maximises subject to (48), which means it faces the problem

$$\underset{n}{\text{Maximise}} \quad U = u(c^* - \varepsilon n)n + (m-n)u(x) \ . \tag{50}$$

This has a turning point at

$$u(c^* - \varepsilon n) - u(x) - u'(c^* - \varepsilon n)n\varepsilon = 0 \ . \tag{51}$$

It is a maximum, as required, because

$$- 2u'(c^* - \varepsilon n)\varepsilon + u''(\gamma - \varepsilon n)n\varepsilon^2 < 0 \ . \tag{52}$$

Hence, by (48), as $\varepsilon \to 0$ so $c \to c^*$. But this means that

$$\lim_{\varepsilon \to 0} \{u(c^* - \varepsilon n) - u(x) - u'(c^* - \varepsilon n)n\varepsilon\}$$
$$= u(c^*) - u(x) = 0 \ , \tag{53}$$

which can only occur, by technological feasibility, if x is at x^*. Hence, as ε gets arbitrarily small, the economy converges to an approximate first-best optimum. This completes the proof.

The trick, of course, was first to solve the differential equation given in (43). The solution, which is easier to check than to find,[8] is

$$s(n) = j + \frac{k}{n} - \frac{f(n)}{n} - \tfrac{1}{2}\varepsilon n^2 , \tag{54}$$

where j and k are any constants. Then by setting j to be c^*, the first-best consumption level, the first-best subsidy function follows.

What are the characteristics of the optimal non-linear subsidy schedule? If

$$s(n) = c^* + \frac{k}{n} - \frac{f(n)}{n} - \tfrac{1}{2}\varepsilon n^2 , \tag{55}$$

then

$$\begin{aligned} \lim_{n\to 0} s &= \infty - \lim_{n\to 0} \left[\frac{f(n)}{n}\right] , \\ &= \infty - \lim_{n\to 0} \left[f'(n)\right] , \\ &= \infty \end{aligned} \tag{56}$$

as long as f'(n) is bounded above at n equal to zero. We also know that

$$s'(n) = -\frac{k}{n^2} - \frac{f'(n)}{n} + \frac{f(n)}{n^2} - \varepsilon n . \tag{57}$$

Now, by concavity of f(n),

$$-\frac{f'(n)}{n} + \frac{f(n)}{n^2} \geqslant 0 . \tag{58}$$

Hence s'(n) can take either sign, although it is certainly negative if f(n) is close to linear. Furthermore,

$$\lim_{n\to\infty} s(n) = -\infty , \tag{59}$$

using L'Hopital's rule again, so s(n) appears to have a point of inflection when

$$k + (2n-1)f'(n) - n^2 f''(n) - \varepsilon n^2 = 0 . \tag{60}$$

Apart from this it is hard to say much about the function's curvature. One possible s(n) shape is depicted in Figure 8.

This section has concentrated on a rather complicated differentiable subsidy schedule. However, a simpler non-differentiable employment subsidy can generate a full first-best.

Proposition 7 (i) A first-best optimum can be achieved by a non-differentiable employment tax/subsidy schedule, lump sum profits taxes and poll income transfers to workers.

(ii) One optimal tax/subisidy schedule is $s = 0$ (if $n = n^*$), s = a large negative number (if $n \neq n^*$).

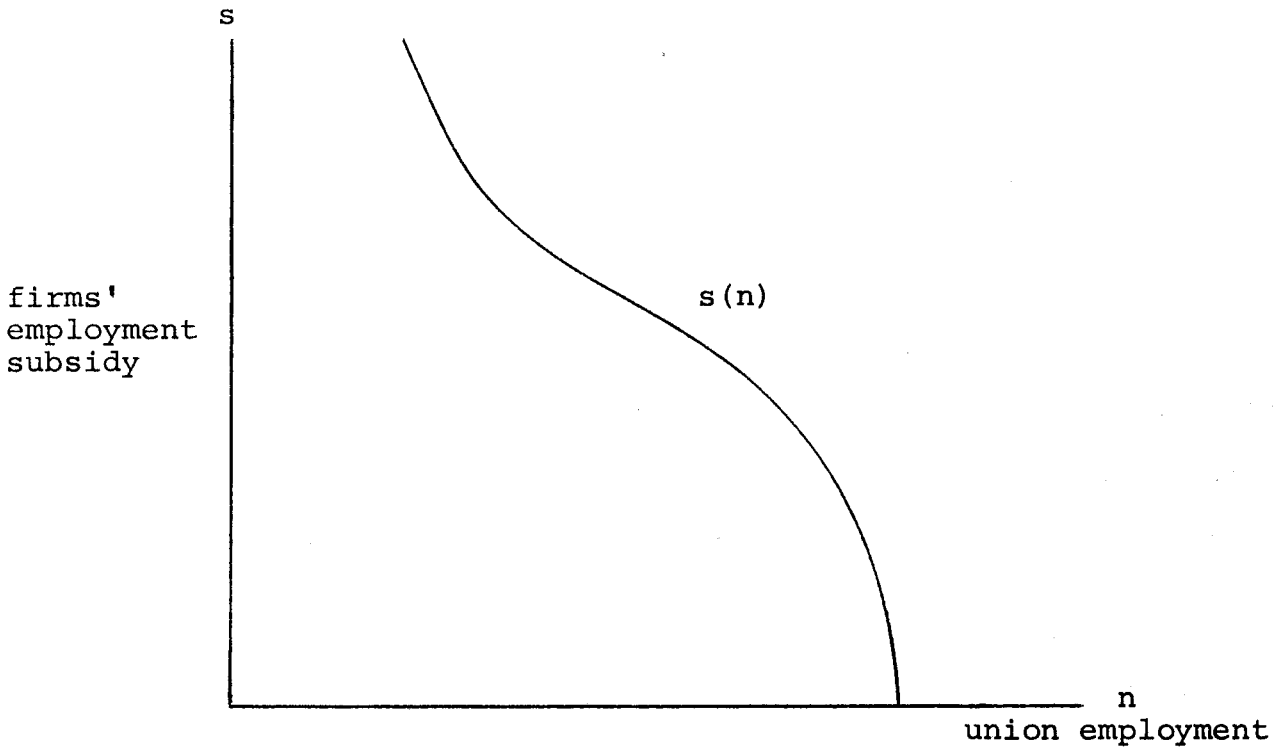
Proof If the level of employment is not n^*, the first-best level, the government should impose a tax large enough to drive the firms out of business. Hence employment must be n^*.[9] The union, therefore, knows that it can drive the wage up to the point at which the firms will go bankrupt. That is its optimal strategy. The government then uses poll income transfers to equate the utility of union and non-union men.

5. Extensions

The model presented in this paper is a very simple one, but the main idea - that a flat employment subsidy will not be

Figure 8

A Possible Optimal Non-Linear Employment Subsidy Schedule



optimal - will continue to hold in a much more complicated framework. There are a number of ways in which the analysis can be extended, and further results will be reported in another paper.

1. It is useful to consider other ways in which the tax revenue for subsidies might be raised. This paper has assumed that the government can use lump sum profits taxes and poll income taxes (or subsidies) on workers.[10] This keeps the issues clear, but is not especially realistic.

2. The production structure can be made more general. In particular, it seems natural to assume that capital is mobile between sectors. A model of this sort is outlined in Carruth and Oswald (1981).

3. A different social welfare function can be assumed. It is straightforward, for example, to extend the results to the case where the government uses a generalised utilitarian welfare function (where the government enters some concave function $\nu(.)$, rather than true preferences $u(.)$, into the social maximand).

4. Uncertainty can be introduced.[11] This does not alter the main argument for non-linear intervention.

5. A different type of model of a trade union can be introduced - the 'cooperative' model of Oswald and Ulph (1982), say. Then the analysis can be extended to unions of the same type as studied in McDonald and Solow (1981), and Nickell (1982), in which unions and firms agree on efficient bargains. See MaCurdy and Pencavel (1981) for empirical work along these lines.

6. Interdependent utility functions can be assumed: it seems to be widely believed, for example, that unions follow other groups' wage demands because they care about wage parity. The model can be changed to allow for the idea that union workers might have a utility function $u(c,x)$ where, perhaps because of 'jealousy', u_x is negative.[12]

7. A more sophisticated union - one that bears in mind general equilibrium repercussions on the non-union wage - can be allowed into the model. This has some interesting and important effects, and complicates the analysis.

6. Conclusions

This paper has studied the theory of optimal intervention in an economy in which there are trade unions. The government has been assumed to maximise a utilitarian social welfare function subject to utility-maximising behaviour by trade unions, profit-maximising behaviour by firms, and a government budget constraint. To avoid other issues the paper has assumed that it is feasible to use lump sum profits taxes on firms and poll taxes and subsidies on workers.

The paper suggests one major conclusion. In a world in which trade unions exist there is a case for non-linear government subisides in the labour market.[13] These can be paid either to union workers or to unionised firms. If they are wage subsidies paid to workers then in a large class of cases the subsidy schedule should be an increasing and concave function of union employment. If they are employment subsidies to firms then rather less can be said at a general level, but under certain circumstances the

optimal subsidy should be a declining function of employment in the unionised sector. A discontinuous schedule may also be desirable.

A number of more precise results have also been derived. The main ones are summarised below.

1. A traditional flat employment subsidy will not produce a first-best optimum. This is because it raises the (highly paid) union workers' consumption levels, which worsens the distribution of income. However, a flat employment subsidy increases employment in the unionised sector, and this makes real output greater. There is a gain in technical efficiency. Hence flat employment subsidies are a second-best policy: they improve efficiency but worsen equity.

2. A differentiable wage subsidy schedule can produce a first-best welfare optimum.

3. A non-differentiable employment subsidy schedule can generate a first-best optimum. An approximate first-best (that is, to within ε) can be reached by a differentiable employment subsidy function.

4. Statutory wages policy - if politically feasible - can also lead the economy to a first-best optimum.

The principal implication for economic policy seems to be that non-linear employment and wage subsidies are more valuable than has apparently been realised.

Footnotes

1 See, for example, the conclusions of Minford (1982). Oswald and Ulph (1982) suggests a theoretical counter-argument, and Ulph and Ulph (1982) examines the consequences of union activities in a model with many types of labour.

2 This paper is not as well known as it deserves to be, although its assumptions about trade union preferences are more restrictive than those used in the analysis to follow.

3 Sampson (1982) and Shah (1982) both discuss the effects of flat employment subsidies. They assume that unions maximise against their labour demand curves. Jackman and Layard (1982) show that in a simple model a tax on wage inflation can raise employment. Pissarides (1982) sets out a model in which unions create a distortion by another route from that studied here. A slightly different approach is taken in Layard and Nickell (1980), which advocates marginal employment subsidies.

4 Farber (1978) is one of the earliest articles to employ something like this form of union utility function. McDonald and Solow (1981) and Oswald (1982a) use an equivalent maximand. Early work on unions includes Atherton (1973), Cartter (1959), and Fellner (1949). Mulvey (1978) is a standard textbook. Grossman (1982) uses a more complicated voting model. Some recent empirical work on models of unions can be found in Dertouzos and Pencavel (1981).

5 This is a normal assumption in the literature, but may be an important restriction. Grossman (1982) suggests a way in which union membership could be allowed to be endogenous.

6 The union utility function is quasi-concave in wages and employment as long as u(.) is concave. Oswald (1982a) gives a proof.

7 It is analytically helpful to set it up with n as a choice variable, although one can equally well think of the union as choosing the wage.

8 There is a standard formula for this: see Adams and White (1968), p. 852, for example.

9 Of course this assumes that the union believes that if $n \neq n^*$ the government will stick to its announced policies.

10 These are common assumptions in this literature, but are not always made explicit.

11 Very little has been written on this. Oswald (1982c) tries to prove one or two results on unions' actions under uncertainty.

12 This sort of assumption has significant effects in the literature on optimal tax theory (Oswald (1982b)).

13 These are not the same as marginal employment subsidies, where the government sets a subsidy function

$$s = s(\dot{n}) \tag{61}$$

in which $\dot{n}$ is the rate of change of employment. A subsidy of the sort described by equation (61) is also much more difficult to handle analytically, because the firm's optimisation problem is no longer a static one.

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