Job Search and the Firm's Wage Offer Decisions : A Model of Null Offers.

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This paper is circulated for discussion purposes only and its contents should be considered preliminary.

ABSTRACT

In this paper a model of a profit maximising firm's responses to job search is developed. This model explains the determinants of a firm's wage offer and the probability that a firm will be found in a state where it is optimal to make no offer (i.e. a 'null' offer). Comparative statics results for the case of constant returns technology are calculated and the implications of the model for a market characterised by search are discussed.

I. INTRODUCTION

Ever since the early work on the implications of search and imperfect information for individual labour market behaviour (see Stigler (1962), McCall (1970)) there has been interest in formulating labour market models consistent with individual job search. This is both a challenging and technically difficult area, but has remained of interest because of the severe limitations of any analysis that concentrates solely on one side of the market. These limitations are most clearly exposited in Rothschild (1973), as are the difficulties involved in modelling simultaneously both sides of a market characterised by search. The theory of individual job search decisions is by now well developed and the economic theorist has a range of models incorporating a large variety of labour market behaviour from which to choose (see Chalkley (1982a) for a survey). The analysis of a firm's responses to a job searching labour force is by comparison very poorly developed.

There are at least two aspects of the job search process that may be emphasised when analysing a firm's profit maximising choices in the presence of a searching labour force.

Firstly imperfect information on the part of workers gives a firm monopsony power. A small decrease in the offered wage will no longer result in an immediate and permanent reduction in labour supply to zero. Whether imperfect information will continue in the market or whether firms will all choose a single wage for homogenous labour and hence eliminate the information imperfection has been the concern of one area of the literature.[1/] (See for example Braverman

(1980)).

Secondly the fact that workers search (and are subject to uncertainty regarding the actual location of different wage offers) means that firms face a stochastic labour supply.

This paper is concerned with the second of these two consequences. In particular the object of this paper is to explain a) why firms choose to offer some job searchers a zero of 'null' offer and b) what determines the wage offer if made. Given answers to the two questions above it is possible to draw some conclusions regarding the operation of a labour market characterised by search and null offers. For example consider the effect of an increase in search costs. The initial reaction of individual job searchers is well known reservation wages fall and <u>cet par</u> individual durations of unemployment shorten. However a shift in reservation wages will alter the wage/employment decisions of firms. A tendency to increase wages and employment will have an offsetting effect. Reservation wages will adjust upwards as the distribution of wage offers becomes more favourable and the probability of receiving a null offer declines. The converse is also true. Clearly the market outcome depends on the decisions of agents on both sides of the market.

The effect on individual search decisions of 'null' offers has already been examined (see Chalkley (1982b)). A specification of the causes of 'null' offers is therefore the next step in studying the operation of a market in which vacancies, unemployment, different wage offers and job searchers co-exist.

As already noted the existing literature in this area is somewhat limited. Eaton and Watts (1977) discuss a complex model of firms behaviour set in discrete time where in each period (and contingent on the current state of labour supply) the firm decides on the number of offers to make and the wage. The Eaton-Watts formulation does not lend itself to analytic solution and numerical analysis whilst possible is by necessity of only limited value and computationally very expensive. Pissarides (1979), (1982) has analysed a labour market where firms are wage takers and must decide on the creation of at most one [2/] vacancy. Once again the framework is one of discrete time.

In the next section a somewhat different framework of analysis is outlined. Whilst job searchers are assumed to search once per period they are assumed to be distributed over time. Converting the firm's decision problem to a continuous time framework enables the simple derivation of the steady state distribution of employment levels. Considering labour input to be similarly continuous further simplifies the model and renders it analytically soluble.

In Section 3 a constant returns technology is assumed and some comparative statics results obtained for the simplified (linear) model. In particular the effect of parameter changes on the probability of receiving a 'null' offer from a firm are examined. Where comparative statics results are ambiguous some numerical results are presented. The section concludes with a brief discussion of the implications of the results for the specification of a model of the labour market where a distribution of wages, null offers and job search co-exist.

2. A MODEL OF THE FIRM'S WAGE/EMPLOYMENT DECISIONS

2.1 The Firms Employment Level

Consider a firm that at time t has a labour force ℓ (integer). In a time interval δt suppose there is a probability $\mu\delta t$ that 1 member of the labour force leaves[3/] the firm. For suitably small δt it is possible to assume that the probability of more than one individual leaving is of order δt^2 and approximately zero. In order to simplify analysis it is assumed that $\mu\delta t$ is independent of ℓ. More realistically it might be expected that μ is an increasing function of ℓ.

Unemployed individuals are assumed to search for new employment opportunities in an optimal fashion. Again for simplicity it is assumed that a) Job searchers search once only in some period h b) that job searchers are distributed uniformly over time, and c) that firms to be searched are selected at random. These assumptions together imply that each firm has a probability $\gamma\delta t$ of being contacted by an individual in time interval δt. Not all searchers who contact the firm and who are offered a wage (per period h) of w will accept. A job searcher will only accept a wage if it is greater than some critical value r - the reservation wage. If $g(r)$ and $G(r)$ are respectively used to denote the probability density (pd) and cumulative distribution (cd) functions of reservation wages, the probability that in δt a firm with wage w will be contacted and its offer (if made) accepted is $\gamma\delta t G(w)$ which will be denoted simply as $\lambda(w)\delta t$ or $\lambda\delta t$.

From a given initial state ℓ at time zero the probability of being in state $\hat{\ell}$ at time $2\delta t$ can be shown by use of the simple probability tree diagram below.

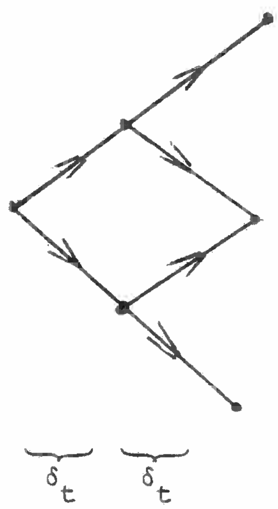
$\ell + 2$

$\ell + 1$

ℓ

$\ell - 1$

ℓ 1 2



$P(\ell+2, 2\delta t) = (\lambda \delta t)^2$

$P(\ell+1, 2\delta t) = 2\lambda\delta t(1-\lambda\delta t-\mu\delta t)$

$P(\ell, 2\delta t) = 2\lambda\mu\delta t^2+(1-\lambda\delta t-\mu\delta t)^2$

$P(\ell-1, 2\delta t) = 2\mu\delta t(1-\lambda\delta t-\mu\delta t)$

$P(\ell-2, 2\delta t) = (\mu\delta t)^2$

FIGURE 1

Taking the limit as $\delta t \to 0$ enables the firm's decisions in continuous time to be studied.

2.2 Production and Profit

A firm is assumed to produce an output Q which is sold at a market price $\bar{p}$. The firm's output is assumed to be a function of labour ℓ and capital k, i.e. $Q = f(\ell, k)$. Whilst output varies continuously with labour over some range capital acts as a constraint. When a given capital stock is fully employed increasing ℓ does not produce further output.

Specifically,

$$(2.1) \qquad Q = f(\ell,k) = F(\ell) \quad \forall\, \ell \leq L$$

$$= F(L) \quad \forall\, \ell > L$$

L can be interpreted at the maximum labour force that can be productively employed with capital stock k.

Specific restrictions on the form of $F(\ell)$ will be introduced later. For now it is simply required that $F_\ell(\ell) \geq 0$ (see Figure 2.

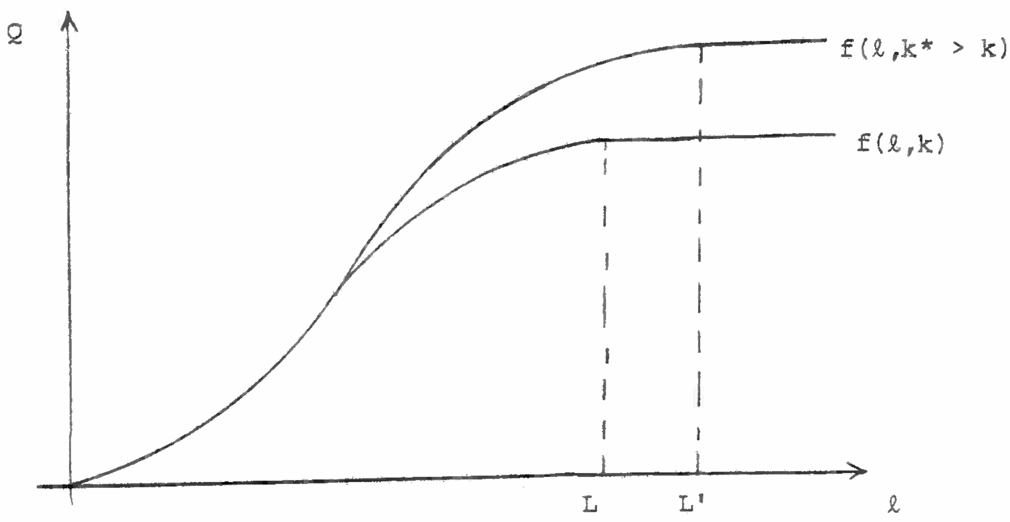


FIGURE 2

The per period profit of the firm can now be defined as

$$\text{(2.2)} \qquad \Pi(\ell) = \bar{p}F(\ell) - w\ell - ck$$

where $k = k(L)$ and c is the cost of capital.

It is clear that a firm will not choose to offer a wage to any job searcher if it currently employs the maximum labour force (L) compatible with the capital stock.

2.3 The Choice Variables and Maximand

The choice variables of the firm are the wage per period (w) and capital stock (k) Since the latter is assumed to be a direct function of L it is more convenient to consider the firm's choices in w,L space.

As with any theory of the firm where time is explicitly considered there remains the problem of specifying a time horizon and discount rate. In general the probability distribution $P(\ell,t)$ of employment states will be a function of time. Here only the steady state will be considered, in which the time dependency of $P(\ell,t)$ is absent. The steady state distribution is denoted $p(\ell)$ and is derived in Appendix A. The time horizon of the firm is assumed infinite and discounting ignored.[4/] These factors taken together imply that the appropriate objective function to study is the expected per period profit.

For discrete ℓ we have

$$\text{(2.3a)} \qquad E[\Pi] = \Phi = \sum_{\ell=0}^{L} \Pi(\ell)p(\ell)$$

whilst for continuous ℓ

$$(2.3b) \qquad E[\Pi] = \Phi = \int_0^L \Pi(\ell)\,p(\ell)\,d\ell$$

2.4 The Steady State Probability Distribution of Employment States

The stochastic process described in Section (a) above is entirely analogous to a simple birth/death process as described in Cox and Miller (1965) and Feller (1968), except that $\ell = 0$ and $\ell = L$ act as 'reflecting barriers'. In Appendix A the steady state probability distributions of employment states are derived as

$$(2.4a) \qquad p(\ell) = p(0)\,(\lambda/\mu)^{\ell}$$

$$p(0) = 1/(1+(\lambda/\mu) + (\lambda/\mu)^2 \ldots (\lambda/\mu)^L)$$

for discrete ℓ

$$(2.4b) \qquad p(\ell) = p(0)e^{b\ell} \qquad b = (\lambda/\mu) - 1$$

$$p(0) = b/(e^{bL} - 1)$$

for continuous ℓ.

Henceforth only the continuous approximation to the discrete model will be considered since this allows for greater analysis. The complete model for the firm is therefore

$$\max_{w,L} E[\Pi] = \int_0^L (F(\ell)\bar{p} - w\ell - c(L))be^{b\ell}/(e^{bL}-1)\,d\ell \tag{2.5}$$

where $b = (\lambda/\mu) - 1$

$\lambda = \lambda(w) = \gamma G(w)$

In the next section the consequences of considering a constant returns technology (where $\bar{p}F(\ell) = \beta\ell$) are considered

3. CONSTANT RETURNS TECHNOLOGY

3.1 Conditions for a maximum

The case of constant returns technology provides a simple illustration, within the context of the model described in Section 2, of the determinants of the wage, capital stock and null offer probability for a given firm. In the following it is assumed that

$$(3.1) \qquad \bar{p}F(\ell) = \beta\ell$$

With constant returns there is a problem that the optimal choice of L may be unbounded. This follows from the fact that the expected employment level $(\bar{\ell})$ is an increasing function of L with increasing slope,[5/] (i.e. $\bar{\ell}_L > 0$, $\bar{\ell}_{LL} >$). It is therefore assumed that the cost of capital is a convex function[6/] of L. A quadratic form will suffice and in the following the simplest case is taken, so that

$$(3.2) \qquad c(L) = aL^2$$

Substituting (3.1) and (3.2) into (2.5) and integrating yields

$$(3.3) \qquad E[\Pi] = \phi = (\beta-w)\bar{\ell} - aL^2$$

where

$$(3.4) \qquad \bar{\ell} = \int_0^L \ell p(\ell)\,d\ell = Le^{bL}/(e^{bL}-1) - 1/b$$

The first order conditions (FOC's) for a maximum with respect to w and L are

$$(3.5a) \qquad \phi_w = (\beta-w)\bar{\ell}_w - \bar{\ell} = 0 \Rightarrow \bar{\ell}_w = \bar{\ell}/(\beta-w)$$

$$(3.5b) \qquad \phi_L = (\beta-w)\bar{\ell}_L - 2aL = 0 \Rightarrow \bar{\ell}_L = 2aL/(\beta-w)$$

For a maximum we also require that the Hessian be negative definite, i.e.

$$\phi_{ww}, \phi_{LL} < 0 \qquad \begin{vmatrix} \phi_{ww} & \phi_{wL} \\ \phi_{Lw} & \phi_{LL} \end{vmatrix} > 0$$

The second order partials of ϕ are

$$(3.6a) \qquad \phi_{ww} = (\beta-w)\bar{\ell}_{ww} - 2\bar{\ell}_w$$

$$(3.6b) \qquad \phi_{LL} = (\beta-w)\bar{\ell}_{LL} - 2a$$

$$(3.6c) \qquad \phi_{Lw} = \phi_{wL} = (\beta-w)\bar{\ell}_{wL} - \ell_L$$

The various partial derivatives of $\bar{\ell}$ are defined in appendix B.

Since (3.5a), (3.5b) form a pair of nonlinear simultaneous equations in w and L explicit solutions of w and L are not possible.

3.2 Comparative Statics Results, w and L

If z is some parameter of ϕ a change in z will simultaneously affect the optimal choice of w and L. The signs of these changes are given by

$$\text{(3.7a)} \qquad \text{sign}\ \frac{dw}{dz} = \text{sign} \begin{vmatrix} -\phi_{wz} & \phi_{wL} \\ -\phi_{Lz} & \phi_{LL} \end{vmatrix}$$

$$\text{(3.7b)} \qquad \text{sign}\ \frac{dL}{dz} = \text{sign} \begin{vmatrix} -\phi_{Lz} & \phi_{Lw} \\ -\phi_{wz} & \phi_{ww} \end{vmatrix}$$

The exogenous parameters of the model as described so far are:

β – The marginal revenue product of labour

a – The cost of capital

γ – The instantaneous probability of arrival of an unemployed individual at the firm

μ – The instantaneous probability of a departure from the firm

To which must be added parameter(s) of the cd function of reservation wages. In order that $G(w)$ has an analytic form it is assumed here that r is distributed exponentially with parameter α so that

$$\text{(3.8)} \qquad g(r) = \alpha \ell^{-\alpha r}, \quad G(w) = (1-e^{-\alpha w}).$$

All the comparative statics results depend upon the sign of ϕ_{wL} as evaluated at the optimal w and L. Since the sign of

ϕ_{wL} is ambiguous results for both possibilities (ϕ_{wL} negative or positive) are reported below

TABLE 1 $\phi_{wL} < 0$

z	β	a	γ	μ	α
dw/dz	+?−	+	−	+	−
dL/dz	?+	−	+	−	+

TABLE 2 $\phi_{wL} > 0$

z	β	a	γ	μ	α
dw/dz	+	−	+?	−?	+?
dL/dz	+	−	+?	−?	+?

A ? indicates changes that cannot be signed theoretically, in these cases the signs given in the tables indicate the results of numerical analysis conducted using the model, some examples of which are included below. Where only one sign is given this indicates that for all reasonable parameter values used the sign was unambiguous.

The results on the responses of the maximum labour force decisions are in any case consistent throughout, favourable changes (increased β, α, γ) increase L unfavourable changes (increased μ, a) decrease L. The wage decision depends far more upon the actual parameter values of the model with a range of responses being possible depending on these values.

3.3 The Probability of a Null Offer

The probability of receiving a 'null' offer from a firm is simply the probability of finding that firm in its full employment state. If q is the probability of a null offer

$$(3.9) \qquad q = p(L) = e^{bL} b/(e^{bL}-1).$$

q is a function of both w and L with $\partial q/\partial L < 0$ and $\partial q/\partial w > 0$. It would therefore appear that very little can be said about the effects of changes in the exogenous variables on q, the only unambiguous result being that $dq/da > 0$ (for $\phi_{wL} < 0$). However q depends directly on α, γ and μ and it is the direct effect of changes in these variables (rather than the secondary effect through w and L) that in practice dominates. The following results were found to hold in all but extreme cases.

$z =$	β	a	γ	μ	α
dq/dz	+?	+	+?	−?	+?

An increase in the rate at which job searchers contact the firm or a favourable (to the firm) shift in the distribution of reservation wages increases the (steady state) probability of a null offer. Conversely a tendency towards more frequent separations between workers and the firm results in a decrease in the probability of a null offer. In general an increase in β raises the null offer probability.

3.3 The Determinants of Market Wages and Null Offer Probabilities

A number of firms as described above will comprise a labour market. Since identical firms as modelled will choose identical wage offers the existence of a wage distribution necessitates some heterogeneity of firms. The most obvious source of differences between firms is in the productivity of labour β. Whilst unambiguous results are not possible it will generally be the case that high β firms will pay higher wages and have higher null offer probabilities. This prediction of the model is in line with Nickell's (1977) *a priori* specification of a model of job search that incorporates null offers.

A distribution of β's over firms will imply a distribution of wages and a distribution of null offer probabilities. The probability of receiving a null offer for any individual who searches firms randomly is the expectation of the latter distribution. The interdependence of wage offers and null offer probabilities should be taken into account when specifying an individual's job search strategy and this task remains to be accomplished, space precludes further discussion here.

TABLE 3 Sample Numerical Solutions of Model

for $\gamma = 0.5$ $\mu = 0.3$ $\alpha = 0.03$

		w^*	L^*	q	$\bar{\ell}$	ϕ	ϕ_{wL}
$\beta = 60$	$a = 1$	24.	7	.10	2.83	56.8	+
$\beta = 80$	$a = 1$	38.83	18	.16	12.18	193.5	+
$\beta = 100$	$a = 1$	40.81	29	.18	23.36	550.9	-
$\beta = 80$	$a = 2$	30.92	6	.16	3.14	76.5	+
$\beta = 80$	$a = 3$	22.51	4	.19	1.65	53.4	+

4. SUMMARY AND DISCUSSION

This paper has considered one possible method of modelling the occurrence of 'null' or zero offers in labour markets characterised by job search. Firms are seen as deciding on a wage and maximum labour force level where both these variables affect the steady state probability distribution of employment states that the firm faces. The relationship between the steady state distribution of employment states and the firms decisions was formally derived.

The simplest case to analyse for this model is where returns to labour input are constant. In this case the objective function can simply be expressed in terms of the expected employment state and first order conditions easily derived. The two decision variables however are determined by a pair of non linear simultaneous equations for which no simple solution can be expressed. Given this difficulty some basic comparative statics exercises were performed and some results obtained. Where the effect on the solution of certain exogenous variables was uncertain numerical analysis was employed to suggest results. Similar methods were used to determine the effect on 'null' offer probabilities. The response of wage offers to changes in exogenous parameters was often ambiguous. This clearly implies a difficulty in modelling a labour market incorporating search. In order for unambiguous results to pertain it is necessary to either restrict attention to particular parameters values or otherwise limit the model (for example by considering only the short run with capital - k,L - fixed).

The model as presented predicts some correlation between the probability of receiving a 'null' offer from a firm and the wage that the firm will offer if it is not fully employed. This correlation whilst suggested in the empirical literature in this area (see Nickell (1977)) has not yet been incorporated into the theoretical literature on individual job search decisions.

Numerous extensions are possible to the work reported here. Obviously it is possible to relax the constant returns assumption. Given the difficulty in signing many changes in the linear case it is to be expected that results will be very difficult in more complex specifications. In fact numerical work has been carried out for the case of a Cobb Douglas (with decreasing returns) technology and whilst not reported here the results are very similar to those studied in some detail above. It is also possible to consider a model in which wages are made state (i.e. employment) contingent. This assumes that firms are able to adjust wage offers instantaneously as their employment state changes. This would yield a model rather similar to that of Eaton and Watts (1977) except that it would only be the offered wage and not the wage of current employees that would be considered state dependent. Such a model would also be considerably more difficult to solve but would have the advantage that identical firms would still produce a wage distribution.

It is also clear that much work remains to be done in completing a market model of search, null offers, unemployment and wages. Recently Pissarides (1979, 1982) has pointed to a number of interesting areas of investigation and attempted some analysis using simplified models. Whether worthwhile extensions to that work are possible within the framework of this paper remains of course to be seen.

FOOTNOTES:

1/ Whilst this literature in fact deals with price search in product markets the central ideas are entirely analogous to the labour market (job search) literature.

2/ The assumption is of one employee firms and the central concern is with the implications of institutions such as Job Agencies for the advertising of a firm's vacancy.

3/ The exact reasons for 'separation' of workers from firms are not specified. Clearly it is possible to argue that the main reasons for separation are mismatching of the individual to certain characteristics of a job that can only be observed by actually working. Such characteristics have been labelled 'experience' characteristics by Nelson (1970) and Wilde (1979). The separation may be initiated by either worker or employer.

4/ Without these assumptions the objective function could only be defined implicitly. In the case of a finite horizon

$$G_t^*(\ell) = \max_{w,L} (\Pi(\ell) + \alpha E_{w,L} (G_{t+1}^*(\ell)))$$

which can be solved numerically commencing at the horizon and recursing backwards. In the infinite horizon case

$$G^*(\ell) = \max_{w',L}(\Pi(\ell) + \alpha E(G^*(\ )))$$

Efficient recursive techniques for solving numerically this sort of functional equation are outlined in Howard (1960) and are employed by Eaton and Watts (1977).

5/ This is the case if $b > 0$ (i.e. if $\lambda > \mu$). Since $\lambda < \mu$ implies a firm that shrinks on average (and hence a firm that will seldom be at its maximum employment level) it is assumed throughout that at the optimal w, $\lambda > \mu$. This is ensured if $\gamma >> \mu$.

6/ This assumption is equivalent to assuming decreasing returns to capital as opposed to decreasing returns to utilised resources (as when $F(\ell)$ is Cobb-Douglas for example).

APPENDIX A

Steady State Employment Distributions

Denoting by $P(\ell,t)$ the probability of being in state ℓ at time t, the discussion of section 2.1 suggests the following relationships

$$\text{(A1)} \quad P(\ell,t+\delta t) = \lambda\delta t P(\ell-1,t) + \mu\delta t P(\ell+1,t) + (1-\lambda\delta t-\mu\delta t)P(\ell,t)$$

for $0 < \ell < L$

$$\text{(A2)} \quad P(0,t+\delta t) = \mu\delta t P(1,t) + (1-\lambda\delta t)P(0,t)$$

for $\ell = 0$

$$\text{(A3)} \quad P(L,t+\delta t) = (1-\mu\delta t)P(L,t) + \lambda\delta t P(L-1,t)$$

for $\ell = L$

Transposing terms of $P(\ell,t)$, $P(0,t)$ and $P(L,t)$ respectively, dividing by δt and taking the limit as $\delta t \to 0$ yields 3 differential equations

$$\text{(A4)} \quad P_t(\ell,t) = -(\lambda+\mu)P(\ell,t) + \lambda P(\ell-1,t) + \mu P(\ell+1,t)$$

$$\text{(A5)} \quad P_t(0,t) = -\lambda P(0,t) + \mu P(1,t)$$

$$\text{(A6)} \quad P_t(L,t) = -\mu P(L,t) + \lambda P(L-1,t)$$

In steady state by definition $P_t(\ell,t) = 0 \ \forall \ \ell$. (A4), (A5) therefore imply by recursion that

$$p(1) \quad = \quad p(0).\lambda/\mu$$

$$p(2) \quad = \quad p(1)\lambda/\mu \quad \lambda/\mu \quad = \quad p(0)\lambda/\mu^2$$

(A7) $$p(\ell) \quad = \quad p(0)\lambda/\mu^{\ell}$$

(where $p(\ell)$ is the steady state probability of employment state ℓ)

Finally we require that

$$p(0)\left[1 + \lambda/\mu + (\lambda/\mu)^2 \ \ldots \ (\lambda/\mu)^L\right] = 1$$

$$p(0) \quad = \quad (1 + (\lambda/\mu) + (\lambda/\mu)^2 \ \ldots \ (\lambda/\mu)^L)^{-1}$$

(A7) and (A8) together specify the steady state distribution for the case of discrete ℓ.

In the case of continuous ℓ we require a continuous analogue to (A7) and (A8).

By analogy to the discrete case

(A9) $$\mu p(\ell+\delta\ell) \quad = \quad \lambda p(\ell)$$

re-arranging terms

$$\text{(A10)} \qquad \mu(p(\ell+\delta\ell) - p(\ell)) = (\lambda-\mu)p(\ell)$$

$$\Rightarrow \delta\ell\mu(p(\ell+\delta\ell) - p(\ell))/\delta\ell = (\lambda-\mu)p(\ell)$$

$$\Rightarrow dp(\ell)/d\ell = \frac{\lambda-\mu}{\mu} p(\ell)$$

Solving for $p(\ell)$

$$\Rightarrow \text{(A11)} \quad p(\ell) = Ce^{b\ell}, \quad b = (\lambda/\mu) - 1$$

with $\quad C = p(0)e^{0} = p(0)$

we also require that $\displaystyle\int_0^L p(0)e^{b\ell}d\ell = 1$

$$\Rightarrow \text{(A12)} \quad p(0) = b/(e^{bL} - 1)$$

(A11) and (A12) specify the steady state distribution of employment states for continuous ℓ

APPENDIX B

The first and second order conditions for a maximum of the model of Section 3 are all defined in terms of partial derivatives of the expected labour supply $\bar{\ell}$. Here these derivatives are defined.

From (3.4) $\bar{\ell}$ is defined as

(B1) $$\bar{\ell} = Le^{bL}/(e^{bL} - 1) - (1/b)$$

Differentiation with respect to w yields

(B2) $$\partial\bar{\ell}/\partial w = \bar{\ell}_w = b_w(b^{-2} - L^2e^{bL}/(e^{bL} - 1)^2) > 0$$

and with respect to L

(B3) $$\partial\bar{\ell}/\partial L = \bar{\ell}_L = (e^{bL} - Lb - 1)e^{bL}/(e^{bL} - 1)^2 > 0$$

If r is assumed to be distributed exponentially with parameter α then

(B4) $$b_w = \partial b/\partial w = \gamma\alpha e^{-\alpha w}/\mu > 0$$

(B3) and (B4) can be differentiated again to yield the following

(B5) $$\partial\bar{\ell}_w/\partial w = \bar{\ell}_{ww} = b_{ww}\bar{\ell}_w/b_w + b_w^2(L^3e^{bL}(e^{bL}+1)/(e^{bL}-1)^3 - 2b^{-3})$$

(B6) $\partial\bar{\ell}_L/\partial L = \bar{\ell}_{LL} = be^{bL}(e^{bL}(bL-2) + bL+2)/(e^{bL}-1)^3$

(B7) $\partial\bar{\ell}_L/\partial w = \bar{\ell}_{wL} = \bar{\ell}_{Lw} = b_w(Le^{bL}(e^{bL}(Lb-2) + Lb+2)/(e^{bL}-1)^3)$

Once again if r is exponentially distributed

(B8) $b_{ww} = \partial b_w/\partial w = -\alpha^2\gamma e^{-\alpha w}/\mu < 0$

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