

Dynamic Price Discrimination,  
Competitive Markets and the Matching Process\*

Christopher Doyle  
University of Warwick

No. 229

**WARWICK ECONOMIC RESEARCH PAPERS**

DEPARTMENT OF ECONOMICS

UNIVERSITY OF WARWICK  
COVENTRY

Dynamic Price Discrimination,  
Competitive Markets and the Matching Process\*

Christopher Doyle  
University of Warwick

No. 229

March 1983

\*Many thanks to Kevin Roberts and Nick Stern for some excellent suggestions, and to Jesus Seade, Martin Chalkley and the participants of the Warwick Economic Theory Workshop for providing helpful comments. Support from SSRC gratefully acknowledged. Any remaining errors are my sole responsibility.

This paper is circulated for discussion purposes only and its contents should be considered preliminary.

### ABSTRACT

In a competitive market for an *ex ante* homogenous good where stores and consumers enter in a sequential manner, consumers experience either a good match or a bad match. Upon entry the individual consumer selects a store from which to sample and remains with that store if he experiences a good match. The outcome of a match is determined by an exogenous stochastic process. Consumer uncertainty enables stores to price discriminate against loyal consumers. In the steady state the market will feature two prices, with only one store at any one time charging the low price within a particular location.

## 1. INTRODUCTION

Recent analysis of market equilibria has led to dissatisfaction with the notion of a single market clearing price. Salop and Stiglitz (1977), Braverman (1980), Sadanand and Wilde (1982), and, Chan and Leland (1982) have all shown that competitive markets may display more than one price in equilibria. In such models price dispersion may occur because consumers hold imperfect information about prices and/or quality.

One of the weaknesses with the above models has been highlighted by Varian (1980). He argues that because they model 'spatial' price dispersion it allows some stores to persistently sell their product at a lower price than available elsewhere. This, he argues, ignores the possibility of consumers learning from experience. As an alternative he proposed a model of 'temporal' price dispersion. In such a world stores randomly select their price over time so that they can discriminate between the informed and uninformed sectors of the market. At any one moment in time there can exist a price distribution. He argues that the model could be viewed as an explanation of 'sales'. However, casual empiricism suggests that sales are predictable, regular and well advertised, rather than random, and most stores seem to hold sales simultaneously.

Varian does suggest other factors may cause temporal price dispersion; for example, business cycle effects, advertising, loss-leader behaviour, etc. In this paper we propose a new model of temporal price dispersion which embodies the notion of 'matching'. By matching we follow the definition of Mortensen (1982):

"The term matching refers to any process by which persons and/or objects are combined to form distinguishable entities with some common purpose that none can accomplish alone."

Although matching models have been applied to the labour market; for example, Diamond (1982), Pissarides (1983), there has been no application of the process in modelling competitive markets. This is what we propose to do within a steady state partial equilibrium framework.

The theme of the paper concerns consumers who enter a market and are faced with an *ex ante* homogenous commodity available at any store, but there exists some exogenous stochastic process which determines whether they experience a good match or a bad match. Such a phenomenon brings about an inequality between the consumers *ex ante* and *ex post* valuation of the good. As an example, consider a town with many restaurants each with similar menus. A new consumer entering the market can sample any one of them, but has no information to construct prior judgements. At the restaurant he chooses the consumer may strike a friendship with the owner, or be offended by the waiter, appreciate the atmosphere, dislike the location of his table, etc. If the outcome is an exogenous stochastic process the consumer may experience a good or bad match.

In the model we construct consumers are loyal to the store where they experience a good match. Our main result shows that it is optimal for stores to operate a scheme of intertemporal price discrimination, the outcome of which is a two price steady state market equilibrium. Initially stores price below marginal costs, but at some point in time increase price

to gain positive profits, where this high price lies below the *ex post* good match valuation of consumers. For the case of constant marginal costs stores earn zero discounted profits in the steady state, but have infinite life times. It will also be the case that the number of stores in the market will be steadily growing. We also examine the special case of duopoly and show that a single price steady state equilibrium is inconsistent with profit maximisation. Our interest in duopoly is to counter an assertion by Rosenthal (1982) where he suggested that there would exist a single market price in equilibrium if consumers exhibit store loyalty.

The paper is organised as follows. In Section Two we outline the assumptions and notation used throughout the analysis. Section three considers the behaviour of the individual store, and derives necessary conditions for profit maximisation. In Section Four we discuss the nature of the market equilibrium and present proofs. Section Five examines the special case of duopoly and shows that it is impossible for a single price steady state equilibrium to exist. Welfare considerations are analysed in Section Six, and concluding comments are presented in Section Seven. Finally, some derivation is outlined in the Appendix.

## 2. THE MODEL

We consider a partial equilibrium model of a market for an homogenous good  $x$  in the steady state. There are  $n$  stores, all of which have identical technology. Consumers are identical and flow into the market according to an exogenous Poisson process with mean  $\lambda$ . Once consumers are in the market they die at a rate  $\mu$ . In the steady state the number of new consumers arriving in the market is equal to the number departing. Prices are known across all agents.

For any consumer entering the market all stores offer, *ex ante*, an identical good. Upon entry consumers either experience a good match (GM) or a bad match (BM), which is determined by a stochastic process with an exogenous parameter. Let  $\alpha$  be the probability an entrant experiences a GM. We assume consumers hold a pre-entry reservation price  $\tilde{p} > 0$ , based on conjecture, and sample only at the lowest price available in the market  $p \leq \tilde{p}$ . Each consumer is risk neutral and is assumed to have a von Neumann-Morgenstern expected utility function  $U(H-p)$ , where  $H$  is the monetary valuation of a match, and  $p$  is the price the consumer pays for one unit of  $x$ . Consumers seek to maximise the following expected utility function,

$$\max U = (1-\mu) \int_0^{\infty} e^{-rt} \{U(H-p)\} dt \quad (1)$$

where  $r$  is the discount rate and  $t$  is time. We make the following assumptions about consumers:

Assumption 1.  $U(H-p)$  is a twice differentiable function where  $U' > 0$  and  $U'' = 0$ .

Assumption 2. A consumer experiencing a GM sets  $H = R > \underline{p} > \underline{p}$ .

At a BM,  $H = \underline{p}$ .

Assumption 3. If a new consumer experiences a BM he exits the market. If, however, he obtains a GM at some store  $j$  he continues to purchase one unit of  $x$  at every point in time  $t$ , and only reassesses his loyalty if the store raises its price above  $\underline{p}$ .

Assumptions 1, 2 and 3 imply that there is zero utility for a BM, whereas a GM yields a positive utility. Assumption 3 is essentially imposing a rule-of-thumb for new entrants, but less restrictive assumptions seem to make the analysis too intractable. Those consumers who remain in the market after entry go on to evaluate a subjective probability of gaining a GM at another store  $i \neq j$ , we assume this to be  $\alpha$ . Hence, matches are linear, and the probability any new consumer will experience a GM is independent of  $\lambda$ . The average instantaneous rate at which GM's form is proportional to  $\lambda$ , such that

$$b(\lambda) = \alpha\lambda \tag{2}$$

where  $\alpha\delta t$  is the probability a GM will occur.

Assumption 4. A consumer will not sample from any store that has at some time in the past priced above  $\underline{p}$ .

This assumption is included to capture the effect of 'reputation'. New consumers believe that stores, who at some time priced above  $\underline{p}$ , were unfair to their customers, and as a consequence these stores have a bad



reputation. In contrast, stores who have always charged the low price  $\underline{p}$  are thought to be benevolent and have a good reputation.

Each store has constant marginal costs  $d > 0$  and must pay a once and-for-all licence fee  $F > 0$  to participate in the market. There are no barriers to entry (other than  $F$ ), but any new store entering the market will initially have zero consumers. We assume that stores are risk neutral and maximise expected profits  $\Pi$ , in a Bertrand-Nash manner, discounted at a rate  $r$ . Therefore, firms seek to maximise profits given by,

$$\max \Pi = \int_0^{\infty} e^{-rt} \Pi dt \quad (3)$$

We also assume,

Assumption 5. (a) Every store knows  $F, r, d, \bar{p}, \alpha, \lambda$  and  $\mu$ ; (b) each store also observes that  $F > R > d > \bar{p} > 0$ .

Assumption 5 indicates that stores are fully aware of the parameters consumers construct.

### 3. THE INDIVIDUAL STORE'S PROBLEM

In this section we analyse the behaviour of the individual store. Each store is assumed to maximise expected discounted profits  $\Pi$  which are dependent upon price  $p$ , and the parameters of the model,  $F, R, d, \bar{p}, \alpha, \lambda, \mu$  and  $r$ , denoted by the vector  $X$ . Our aim is to show that the individual store's profits  $\Pi$  are maximised by discriminating against the loyal consumers  $C$ .

We consider a store that is a new entrant to the market, and hence begins with  $C = 0$ . To attract consumers the store will have to charge the lowest price in the market  $\underline{p}$ , which, by Assumption 5 will be on the interval  $(0, \bar{p}]$ . As  $\bar{p} \leq d$ , by Assumption 5, the store will experience non-positive profits. Given the Bertrand-Nash assumption the store will only attract the consumers flowing into the market,  $\lambda$ . The rate of change of  $C$  will be determined by Assumption 3 and the exogenous death rate  $\mu$ :

$$\dot{C} = \alpha\lambda - \mu C \quad (4)$$

Solving (4) gives,

$$C = \frac{\alpha\lambda}{\mu} (1 - e^{-\mu t}) \quad (5)$$

The sum of the discounted profits flowing to the store whilst charging  $\underline{p}$  is,

$$\Pi(\underline{p}; X) = \int_0^T e^{-rt} \{(\underline{p}-d)C\} dt - F \quad (6)$$

where  $\Pi(\underline{p}; X)$  is monotonically increasing in  $\underline{p}$ .

Given the matching technology in equation (2), Assumption 2, and the profit maximising axiom, the store is able to price discriminate against  $C$ , by raising the price above  $\underline{p}$  to the monopoly price  $\bar{p}$ . The monopoly price  $\bar{p}$  is the price which makes loyal consumers  $C$  indifferent about visiting any other store at the new lowest price  $\underline{p}' \geq \underline{p}$ , for all future time. Price discrimination is possible as the uncertainty of getting a GM, held by

consumers, means that search is a costly activity. We assume  $\bar{p} > p'$ , and therefore the store gains no more new consumers. We also assume that the store increases price at some time  $t=T_1$ . From  $T_1$  onwards  $\dot{C}$  is determined solely by  $\mu$ ,

$$\dot{C} = -\mu C(T_1) \quad (7)$$

Solving (7) gives,

$$C = e^{-\mu(t-T_1)} \left\{ \frac{\alpha\lambda}{\mu} (1 - e^{-\mu T_1}) \right\} \quad (8)$$

The sum of discounted profits flowing to the store whilst charging  $\bar{p}$  will be

$$\Pi(\bar{p}; X) = \int_{T_1}^{\infty} e^{-rt} \{(\bar{p}-d)C\} dt \quad (9)$$

where  $\Pi(\bar{p}; X)$  is monotonically increasing in  $\bar{p}$ .

If  $p$  is given, and the store can determine  $\bar{p}$ , it will seek to maximise its discounted profit stream  $\Pi$  by choosing  $T_1$ .  $T_1$  is the time when the store switches price from  $p$  to  $\bar{p}$ . Hence, the store's maximand is the functional,

$$\max_{\{T_1\}} \Pi(p, \bar{p}; X) = \int_0^{T_1} e^{-rt} \{(p-d)C\} dt + \int_{T_1}^{\infty} e^{-rt} \{(\bar{p}-d)C\} dt - F \quad (10)$$

which has the following first order condition,

$$\begin{aligned} \frac{\partial \Pi}{\partial T_1} &= e^{-rT_1} \{(p-d)C(T_1)\} - e^{-rT_1} \{(\bar{p}-d)C(T_1)\} + \\ &\int_{T_1}^{\infty} e^{-rt} \{(\bar{p}-d) (e^{-\mu(t-T_1)} \alpha\lambda)\} dt = 0 \end{aligned} \quad (11)$$

Let  $\tau$  denote the upper bound of integration, such that  $\lim_{\tau \rightarrow \infty}$ . Given that  $\tau$  is unbounded the first order condition in equation (11) becomes,

$$C(T_1)(\underline{p}-\bar{p}) = \frac{\alpha\lambda}{r+\mu} (d-\bar{p}) \quad (12)$$

Equation (12) states that the marginal cost of attracting new consumers at  $T_1$  equals the marginal benefit, the discounted profit stream derived from these additional consumers. Hence, if the store switches price at any other time other than  $T_1$ , say  $T_1 + \delta t$ , it would not be maximising profits. As both functions contained in (6) and (9) are well behaved we assume that the maximum implied by the first order condition is unique and satisfies the second order condition.

Throughout the analysis we have assumed that the store can determine the monopoly price  $\bar{p}$ . Given Assumption 5 each store can calculate  $\bar{p}$ , such that it makes its stock of loyal consumers at  $T_1$  indifferent about visiting any other store at the new lowest price  $p' \geq \underline{p}$ , for all future time.

We derive  $\bar{p}$  applying Bellman's technique of dynamic programming. Each consumer has to make a decision at  $T_1$ , whether to continue his purchases at the present store, or leave and sample a store charging  $p' \geq \underline{p}$ . Assumption 1 states that the consumer is risk neutral, and given that he seeks to maximise expected utility over time, this is equivalent to maximising his net valuation of  $x$ , discounted at a rate  $r$ .

There are three states of the world, any one of which a consumer may be in;

- $V_1$  , a GM at the low price  $\underline{p}$  ;  
 $V_2$  , a GM at the monopoly price  $\bar{p}$  ;  
 $V_3$  , a BM.

A store's objective is to make the consumer indifferent about leaving  $V_2$ . Each consumer is aware of  $\alpha$ ,  $\mu$  and  $1/T_1$ , the latter being the probability per unit time a store raises its price above  $\underline{p}$ . Hence,

$$\left. \begin{aligned}
 V_1 &= (R-\underline{p})\delta t + (1-r\delta t)(1-\mu\delta t)\{(\delta t/T_1)V_2 + [1-(\delta t/T_1)]V_1\} \\
 V_2 &= (R-\bar{p})\delta t + (1-r\delta t)(1-\mu\delta t)V_2 \\
 V_3 &= 0\delta t + (1-r\delta t)(1-\mu\delta t)\{\alpha\delta tV_1 + (1-\alpha\delta t)V_3\}
 \end{aligned} \right\} \quad (13)$$

$V_i$ ,  $i = 1, 2, 3$ , is the net worth to the consumer of occupying one of the three states. If the consumer chooses to depart the store whilst in state  $V_2$  and sample another store, his outcome is uncertain and equal to,

$$L = \alpha\delta tV_1 + (1-\alpha\delta t)V_3 \quad (14)$$

The store will select  $\bar{p}$  such that  $V_2 = L$ . Solving equation system (13) gives,

$$V_1 = \frac{(R-\bar{p})/(r+\mu) + (R-\underline{p})T_1}{(r+\mu)T_1 + 1} \quad (15)$$

$$V_2 = (R-\bar{p})/(r+\mu) \quad (16)$$

$$V_3 = \alpha \left( \frac{(R-\bar{p})/(r+\mu) + (R-\underline{p})T_1}{[(r+\mu)T_1 + 1]^2} \right) \quad (17)$$

As we have already noted, the store will choose  $\bar{p}$  so that equation (13b) equals equation (14). Taking  $\lim_{\delta t \rightarrow 0}$  across this equality the store will choose  $\bar{p}$

such that  $V_2 = V_3$ ,

$$(R-\bar{p})/D = \alpha \left( \frac{R-\bar{p}/D + (R-p)T_1}{A} \right)$$

solving for  $\bar{p}$  gives

$$\bar{p} = R + \frac{\alpha DT_1}{\alpha - A} (R-p) \quad (18)$$

where  $A = [(r+\mu)T_1 + 1]^2$  and  $D = r+\mu$

Given that  $0 < \alpha < 1$ , and  $A \geq 1$ , then  $\bar{p} < R$ . To ensure that  $\bar{p} > p$ , we require,

$$0 < \frac{\alpha DT_1}{A - \alpha} < 1$$

which is always the case. The monopoly price  $\bar{p}$  will therefore lie between the low price  $p$ , and the GM valuation  $R$ .

#### 4. CHARACTERISTICS OF MARKET EQUILIBRIA

We now proceed to examine the features of a market with a linear matching technology.

PROPOSITION 1. Given Assumptions 1-5, no barriers to entry (i.e. zero profit condition), the market will display; (a) two prices in the steady state equilibrium (TPSS),  $p^*$  and  $\bar{p}^*$ , (b) which satisfy the following strict inequalities  $0 < p^* < d < \bar{p}^* < R$ .

Proof. (a) By contradiction. Suppose there are three prices in the steady state;  $p^l < p^m < p^h$ . It must be true that both  $p^m$  and  $p^h$  satisfy equation (18), therefore

$$p^m < p^h = R + \frac{\alpha DT_1}{\alpha - A} (R - p^l) = p^m$$

a contradiction.

(b) By contradiction. Suppose  $\underline{p} > d$ , then by Assumption 5,  $\underline{p} > \bar{p}$ , which implies

$$\dot{c} = 0$$

which means there will be no market, contradicting the profit maximisation assumption. Suppose  $\bar{p} < d$ , then equation (10) is negative, again contradicting profit maximisation and Assumption 5. Finally, suppose  $\bar{p} = R > \underline{p}$ , then from equation (18),  $\alpha = A$ , a contradiction.

Hence,  $\underline{p}^*$  and  $\bar{p}^*$  are steady state equilibrium prices where stores have no tendency to change price, and so these prices are optimal. Substituting  $\underline{p}^*$  and  $\bar{p}^*$  into equation (10) will give  $T_1^*$  the optimal switching point. Therefore,  $\underline{p}^*$ ,  $\bar{p}^*$  and  $T_1^*$  satisfy the zero profit condition and individual optimising behaviour.

PROPOSITION 2. Given Bertrand-Nash behaviour, the number of stores charging  $\underline{p}^*$  will be at most one..

Proof. By contradiction. Suppose there are  $k > 1$  stores charging  $\underline{p}^*$ . Each store will receive  $\lambda/k$  of the new consumers arriving in the market. Given profit maximisation each store calculates  $T_1^*$  from equation (10). Consider some store  $i$ , it reduces price by an infinitesimal amount  $\epsilon > 0$  and captures the whole of the new arrivals,  $\lambda$ , by assumption. Hence, store  $i$  will capture a greater number of loyal consumers  $C$  at a faster rate, which implies that charging  $\underline{p}^* - \epsilon$  yields profits  $\Pi > 0$ , a contradiction.

Proposition 2 suggests that after every time interval  $T_1^*$  a new store enters the market, as  $T$  is unbounded. Therefore,  $\lim_{t \rightarrow \infty} n = \infty$ , implying that the average number of sales by each store in the limit approaches zero.

Each store in the steady state will initially charge a price below marginal costs and hence make losses, whilst steadily building up a stock of loyal consumers. At some point in time this stock will be large enough to discriminate against, and thus the store will recuperate losses by pricing above marginal costs. At this price, which is below the post-sample reservation price, the store will steadily lose its loyal consumers as they die off.

We now consider the effects upon the market equilibrium if Assumption 4 is dropped. In this situation it would be profitable for a store currently charging  $\bar{p}$  to lower price to  $\underline{p}$  at some time  $T$ . This arises because the established store has a cost advantage as it does not have to pay the licence fee  $F$  which a new store must if it wishes to enter the market. However, the qualitative features of the model remain unaffected



and there will still exist a TPSS market equilibrium. For this case there will not be an infinite number of stores in the market, but a constant number over time, with each store adopting a cyclical pricing policy.

5. THE IMPOSSIBILITY OF A SINGLE PRICE MARKET EQUILIBRIUM: THE CASE OF DUOPOLY.

In a recent paper by Rosenthal (1982) on dynamic duopoly with consumer loyalties he suggests that in a steady state model with a birth/death process there would exist a single equilibrium market price. However, he did not consider a matching process, and store loyalty was imposed by assuming that information about prices (or quality) was costly. As his model features consumer loyalty we examine whether our model is consistent with a single price in the steady state (SPSS), for the case of duopoly. To examine this hypothesis we require some modification to the previous analysis.

We assume that there are total variable costs  $T(x)$  and fixed costs  $F$ , with  $T''(x) > 0$ , which implies that there will exist a U-shaped average cost curve. Let  $A(x) = [T(x) + F]/x$  be the average cost curve. Let  $x^* = \inf\{x | A'(x) = 0\}$ , and  $p^* = A(x^*)$ , where  $p^* \leq \bar{p}$ . There are only two stores in the market,  $n = 2$ , and there are no barriers to entry. We assume that both stores price at  $p^*$ , earn zero profits and that we are in a steady state. We now ask: is this consistent with profit maximisation?

PROPOSITION 3. Given Assumptions 1-5 and equation (2), if both stores price at  $p^*$  such that the market exhibits a SPSS equilibrium, then this is inconsistent with the notion of profit maximisation.

Proof. We know that  $\alpha\lambda$  of new consumers will obtain a GM, and by Assumption 2 will set  $H = R$ . Given that  $R > \bar{p} > p^*$  they will derive strictly positive consumer surplus. Consider store  $i$ , it contemplates increasing price by an infinitesimal amount  $\epsilon > 0$  to  $p^* + \epsilon$ . By such action it will gain no new consumers. However, if

$$p^* + \epsilon \leq R(1-\alpha) + \alpha p^* \quad (19)$$

consumers will prefer, or be indifferent, about remaining at the same store. If the weak inequality (19) is satisfied, then by equation (9),  $\Pi(p^* + \epsilon; X) > 0$ , which means that the store is not maximising profits at  $p^*$ .

## 6. WELFARE CONSIDERATIONS

Stores are seen to enforce an intertemporal price discrimination scheme as a consequence of consumers uncertainty about the quality of a match at another store. This leads to the existence of a two price equilibrium in the steady state. This equilibrium is efficient as no individual could be made any better off without some other individual becoming worse off. However, new consumers who experience a GM derive greater instantaneous welfare than those consumers who have occupied the market for at least  $T_1$  time units. This outcome arises because of the price discrimination exercised by stores. Such a market outcome could be viewed as an implicit contract. As searching is a costly activity stores initially sell at a low price which partially insures consumers against uncertainty. Hence, there is some kind of risk sharing agreement, whereby stores forego current profits and consumers foresake a proportion of future consumer surplus.

## 7. CONCLUSION

We have shown that in a market where consumers either experience a good match or a bad match there will exist two prices in the steady state equilibrium.. This occurs because stores price discriminate in an inter-temporal manner. In equilibrium only one store ever charges the low price at any one time, but the number of stores in the market grows steadily. We also examined the case of duopoly and showed that a single price steady state equilibrium would not exist.

In future research we hope to examine a similar market structure using a game theoretic approach. Adopting this approach will enable us to formalise the notion of reputation in a similar manner to that of Kreps and Wilson (1982), and, Milgrom and Roberts (1982), which we hope will strengthen the above analysis.

APPENDIX

Derivation of equation (12):

Given the functional;

$$\max_{\{T_1\}} \Pi(p, \bar{p}; X) = \int_0^{T_1} e^{-rt} \{(p-d)C\} dt + \int_{T_1}^{\infty} e^{-rt} \{(\bar{p}-d)C\} dt - F \quad (10)$$

from equation (8)

$$C = e^{-\mu(t-T_1)} \left\{ \frac{\alpha\lambda}{\mu} (1 - e^{-\mu T_1}) \right\}$$

from  $T_1$  onwards. Applying Leibnitz's rule to equation (10);

$$\begin{aligned} \frac{\partial \Pi}{\partial T_1} &= e^{-rT_1} \left[ \{(p-d)C(T_1)\} - \{(\bar{p}-d)C(T_1)\} \right] \\ &+ \int_{T_1}^{\infty} e^{-rt} \{(\bar{p}-d) (e^{-\mu(t-T_1)} \alpha\lambda)\} dt = 0 \end{aligned} \quad (11)$$

Integrating the improper integral,

$$\lim_{T \rightarrow \infty} \int_{T_1}^T e^{-rt} \{(\bar{p}-d) (e^{-\mu(t-T_1)} \alpha\lambda)\} dt$$

and substituting into equation (11) gives,

$$\frac{\partial \Pi}{\partial T_1} = e^{-rT_1} C(T_1) \{(p-\bar{p})\} + \frac{\alpha\lambda}{r+\mu} (\bar{p}-d) \{e^{-rT_1} - e^{-T(\mu+r)} + \mu T_1\} = 0$$

Therefore,

$$\lim_{T \rightarrow \infty} \frac{\partial \Pi}{\partial T_1} = C(T_1) (p-\bar{p}) - \frac{\alpha\lambda}{r+\mu} (d-\bar{p}) = 0$$

which is equation (12).

Derivation of equations (15), (16) and (17):

This is achieved by taking  $\lim_{\delta t \rightarrow 0}$  across the equation system (13), giving

$$V_1 = \frac{V_2 + (R-\underline{p})T_1}{(r+\mu)T_1 + 1}$$

$$V_2 = (R-\bar{p}) / (r+\mu)$$

$$V_3 = \frac{\alpha V_1}{r+\mu+\alpha}$$

Substitution will obtain equations (15), (16) and (17).

Derivation of equation (18):

It is required that  $V_2 = V_3$ , therefore

$$R-\bar{p}/D = \alpha \left( \frac{R-\bar{p}/D + (R-\underline{p})T_1}{A} \right)$$

hence,  $R-\bar{p}/D - \alpha(R-\bar{p})/DA = \alpha T_1 (R-\underline{p})/A$

$$\Rightarrow \bar{p} \left( \frac{\alpha}{DA} - \frac{1}{D} \right) + \frac{R}{D} - \frac{\alpha R}{DA} = \alpha T_1 (R-\underline{p})/A$$

$$\Rightarrow \bar{p} \left( \frac{\alpha}{DA} - \frac{1}{D} \right) = \alpha T_1 (R-\underline{p})/A - R/D + \alpha R/DA$$

$$\Rightarrow \bar{p} = \frac{\alpha T_1 (R-\underline{p})/A - R/D + \alpha R/DA}{\alpha/DA - 1/D}$$

$$\Rightarrow \bar{p} = \frac{D\alpha T_1 (R-\underline{p}) - AR + \alpha R}{\alpha - A}$$

$$\Rightarrow \bar{p} = \frac{[R(T_1 D\alpha - A + \alpha) - D\alpha T_1 \underline{p}]}{\alpha - A}$$

$$\Rightarrow \bar{p} = R + \frac{\alpha DT_1}{\alpha - A} (R - \underline{p})$$

which is equation (18)

Derivation of equation (19):

Consumers do not envisage price changes, and, therefore, transition probabilities do not arise. The store contemplating the price increase must ensure that its loyal consumers do not leave. This requires the discounted sum of utilities at the high price to equal the sum of expected utilities from leaving. Thus, there are three states of the world:

$$\begin{aligned} J_1 &= \text{GM at } R - \bar{p} \\ J_2 &= \text{BM at } 0, \text{ probability } (1-\alpha) \\ J_3 &= \text{GM at } R - p^*, \text{ probability } \alpha \end{aligned}$$

Defining the improper integrals,

$$U(J_1) = \int_0^{\infty} e^{-rt} (1-\mu) [R - (p^* + \epsilon)] dt = \lim_{N \rightarrow \infty} \int_0^N e^{-rt} (1-\mu) [R - (p^* + \epsilon)] dt$$

$$EU(J_2, J_3) = \int_0^{\infty} e^{-rt} \alpha (1-\mu) (R - p^*) dt = \lim_{N \rightarrow \infty} \int_0^N e^{-rt} \alpha (1-\mu) (R - p^*) dt$$

Hence, the store sets  $p^* + \epsilon$  so that  $U(J_1) = EU(J_2, J_3)$ .

Therefore,

$$\begin{aligned} U(J_1) &= -\frac{e^{-rt}}{r} (1-\mu) [R - (p^* + \epsilon)] \Big|_0^N = -\frac{e^{-rN}}{r} (1-\mu) [R - (p^* + \epsilon)] + \\ &\quad \frac{(1-\mu) [R - (p^* + \epsilon)]}{r} \rightarrow \frac{(1-\mu) [R - (p^* + \epsilon)]}{r} \text{ if } N \rightarrow \infty \end{aligned}$$

Similarly,

$$EU(J_2, J_3) \rightarrow \frac{\alpha(1-\mu)(R-p^*)}{r} \text{ if } N \rightarrow \infty$$

therefore,

$$R - (p^* + \epsilon) - \mu R + \mu(p^* + \epsilon) = \alpha R - \alpha p^* - \alpha \mu R + \alpha \mu p^*$$

$$\Rightarrow (p^* + \epsilon)(\mu - 1) = R(\alpha - \alpha\mu - 1 + \mu) + \alpha p^*(\mu - 1)$$

$$= R(\mu - 1) + R\alpha(1 - \mu) + \alpha p^*(\mu - 1)$$

$$\Rightarrow p^* + \epsilon = R + \alpha p^* + \alpha R \frac{(1 - \mu)}{(\mu - 1)}$$

$$= R(1 - \alpha) + \alpha p^*$$

which is equation (19).

REFERENCES

- BRAVERMAN, A. (1980) "Consumer Search and Alternative Market Equilibria", Review of Economic Studies, 47, 487-502.
- CHAN, Y-S and LELAND, H. (1982), "Prices and Qualities in Markets with Costly Information", Review of Economic Studies, 49, 499-516.
- DIAMOND, P.A. (1982), "Wage Determination and Efficiency in Search Equilibrium", Review of Economic Studies, 49, 217-227
- KREPS, D.M. and WILSON, R. (1982), "Reputation and Imperfect Information", Journal of Economic Theory, 27, 253-279
- MILGROM, P. and ROBERTS, J. (1982) "Predation, Reputation, and Entry Deterrence", Journal of Economic Theory, 27, 280-312
- MORTENSEN, D.T. (1982), "The Matching Process as a Non-Cooperative Bargaining Game", in McCall, J.J. (ed.) The Economics of Information and Uncertainty (University of Chicago Press).
- PISSARIDES, C.A. (1983), "Efficiency Aspects of the Financing of Unemployment Insurance and other Government Expenditure", Review of Economic Studies, 50, 57-69.
- ROSENTHAL, R.W. (1982), "A Dynamic Model of Duopoly with Customer Loyalties" Journal of Economic Theory, 27, 69-76.
- SADANAND, A. and WILDE, L. (1982) "A Generalized Model of Pricing for Homogenous Goods under Imperfect Information", Review of Economic Studies 49, 229-240.
- SALOP, S. and STIGLITZ, L. (1977), "Bargains and Ripoffs: A Model of Monopolistically Competitive Price Dispersion", Review of Economic Studies, 44, 493-510.
- VARIAN, H.R. (1980), "A Model of Sales", American Economic Review, 70, 651-659.