

A STATIC FRAMEWORK FOR THE ANALYSIS OF
POLICY OPTIMISATION WITH
INTERDEPENDENT ECONOMIES

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Number 235

WARWICK ECONOMIC RESEARCH PAPERS

DEPARTMENT OF ECONOMICS

UNIVERSITY OF WARWICK
COVENTRY

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University of Warwick
August, 1983

Thanks are due to Marcus Miller and Mark Salmon for comments made during the writing of this. As usual all remaining errors are mine.

This paper is circulated for discussion purposes only and its contents should be considered preliminary.

Introduction

The idea of optimal policy design using an explicit loss function was a natural consequence of the development of econometric models of the macroeconomy. Since the economic theory underlying these was, at first the comparative static Keynesian model the techniques used tended to be static also. This approach can be seen in Tinbergen (1956). In addition since most of the early theory of open-economy macroeconomics was within the small, open economy framework single controller models were felt adequate to capture the policy optimisation problem facing a country. As models have come increasingly to concentrate on the dynamics of macroeconomic variables and the interdependence of economies has become more obvious it has become necessary to modify and extend our optimisation methods. The problem of dynamic models is easily dealt with using the standard techniques of optimal control theory i.e. dynamic programming or Pontryagin methods. The problem of interdependence can be handled by use of game theoretic concepts. We propose to set out the standard solution concepts of game theory in a static context because we feel this gives an intuition for the issues which it is difficult to obtain from the more technically demanding dynamic game literature. As an application we will consider throughout the problem of policy optimisation with interdependent economies.

The application of game theoretic ideas to international economics dates back to Hamada (1974) and (1976). Our analysis uses his as a foundation but extends it to consider some new issues such as cheating in Stackelberg games as well as providing a general framework in which a whole range of problems can be analysed. The derivation of the general framework via Lagrange multipliers is interesting in that it provides an almost exact

analogue to the dynamic game case as shown in Miller and Salmon (1983). However we focus on the issue of conflict due to incompatible targets which can be discussed most clearly within a static context.

In our choice of examples we have restricted ourselves to two simple fixed price, fixed exchange rate models. Flexible price models are clearly better dealt with within a dynamic game framework and we feel that this is also true for flexible exchange rate models. This is particularly the case when we are considering the rational expectations response to anticipated policy. We nevertheless feel that there is sufficient interest in these simple models to make them worthy of study, particularly when we wish to examine policy optimisation problems in the very short-run. We also employ the simplifying device of making the economies symmetric since differences in structure are not central to our argument and therefore complicate the analysis unnecessarily. Allowing the economies to be dissimilar is a relatively simple extension and some existing results for this can be seen in Hamada (1976).

Single Controller Problems

Before going on to game theoretic solutions we first consider the single controller or 'game against nature' problem. Here the agent has loss function

$$\frac{1}{2} [(x-x^a)^T A(x-x^a) + u'Ru] \quad (1)$$

where x is a vector of states, x^a is a vector of targets the controller puts on the states and u is a vector of control variables. A quadratic

loss function is assumed because it has convenient properties, in particular it punishes deviations from the targets increasingly as they get larger. It also has the convenient property of certainty equivalence i.e. it does not affect the optimal solution whether or not we include a stochastic element in the state equation or not. For ease of exposition we shall therefore consider the case of a deterministic state equation as given by (2).

$$\mathbf{x} = \mathbf{B}u \quad (2)$$

The Lagrangean is therefore given by (3).

$$L = \frac{1}{2} [(\mathbf{x}-\mathbf{x}^a)^T \mathbf{Q}(\mathbf{x}-\mathbf{x}^a) + u^T \mathbf{R}u] + \mathbf{p}^T [\mathbf{B}u - \mathbf{x}] \quad (3)$$

and the first order conditions with respect to u yield.

$$u = -\mathbf{R}^{-1} \mathbf{B}^T \mathbf{p} \quad (4)$$

to relate the controls to the states rather than the shadow prices we take the derivative with respect to the states (5).

$$\frac{\partial L}{\partial \mathbf{x}} = \mathbf{Q}(\mathbf{x}-\mathbf{x}^a) - \mathbf{p} = 0 \quad (5)$$

and substitute into the control rule (4) to yield the alternative version (6).

$$u = -\mathbf{R}^{-1} \mathbf{B}^T \mathbf{Q}(\mathbf{x}-\mathbf{x}^a) \text{ or } u = [\mathbf{I} + \mathbf{R}^{-1} \mathbf{B}^T \mathbf{Q} \mathbf{B}]^{-1} [\mathbf{R}^{-1} \mathbf{B}^T \mathbf{Q} \mathbf{x}^a] \quad (6)$$

In the following sections we go on to consider cases where there are several independent controllers whose actions go to determine a common state equation.

Static Nash Games

In the case of the Nash game there are several players or controllers each of whom has a loss function of the form:

$$\frac{1}{2} [(x-x_i^a)^T Q_i (x-x_i^a) + u_i^T R_i u_i] \quad i = 1, N \quad (7)$$

The values of the state vector are determined by the actions of all the players.

$$x = \sum B_i u_i \quad i = 1, N \quad (8)$$

In determining the optimal decisions for the individual players we make the assumption that each individual ignores the impact of his choice of control on the choices of the others i.e. the i 'th player assumes $\frac{\partial u_j}{\partial u_i} = 0$ for all j not equal to i . Therefore we can write down decision rules for the players which are similar to those in the single controller problem.

$$u_i = -R_i^{-1} B_i^T p_i \quad (9)$$

Taking the derivative with respect to the states we get:

$$\frac{\partial L_i}{\partial x} = Q_i (x-x_i^a) - p_i = 0 \quad (10)$$

Substituting (10) into (9) we get the Nash reaction functions for the players i.e. their optimal decision rules given fixed controls for the other players and the players own targets.

$$u_i = [I + R_i^{-1} B_i^T Q_i B_i]^{-1} [-R_i^{-1} B_i^T Q_i (\sum_{j \neq i} B_j u_j) + R_i^{-1} B_i^T Q_i x_i^a] \quad (11)$$

Alternatively we could write each players decision rule as a function of the states as in (12) and substitute back into the state equation (8) to find the values of the state given by the system under control.

$$u_i = -R_i^{-1} B_i^T Q_i (x - x_i^a) \quad (12)$$

(13) shows the values of the states in the system under decentralised control

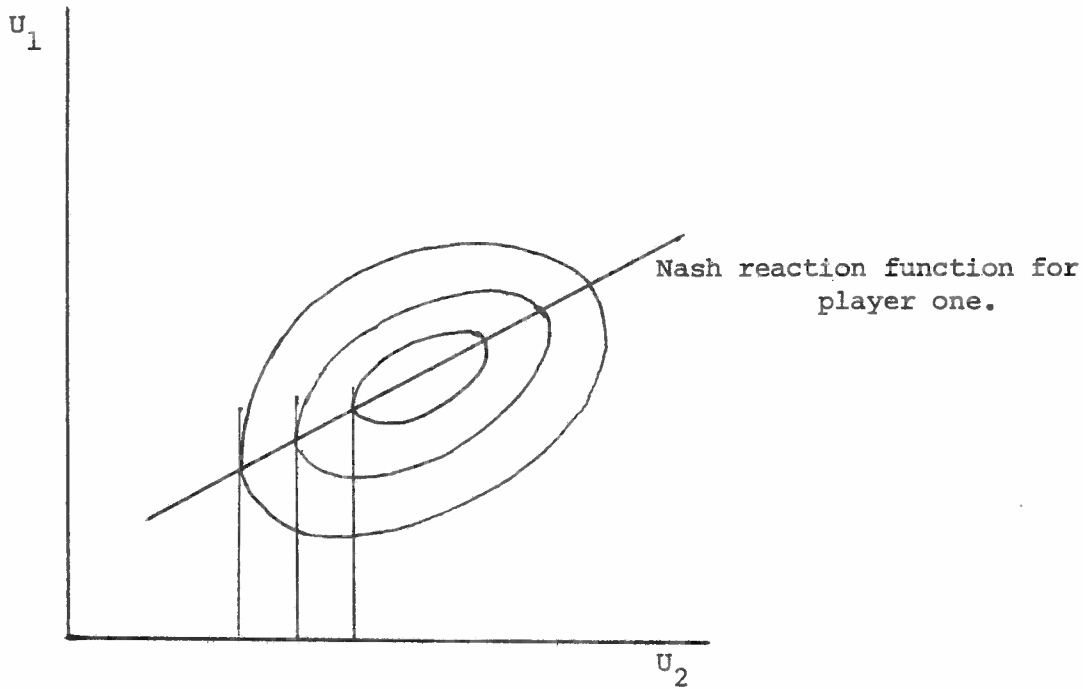
$$\begin{aligned} x &= \sum_i -B_i R_i^{-1} B_i^T Q_i (x - x_i^a) \\ &= [I + \sum_i B_i R_i^{-1} B_i^T Q_i B_i]^{-1} [\sum_i B_i R_i^{-1} B_i^T Q_i x_i^a] \end{aligned} \quad (13)$$

An intuitive understanding of the Nash solution can be gained by considering a diagrammatic representation of the two player case. Substitution of the state equation into the loss function yields an indirect loss function in terms of the controls of the two players.

$$\begin{aligned} \frac{1}{2} [& u_1^T (B_1^T Q_1 B_1 + R_1) u_1 + u_2^T B_2^T Q_1 B_2 u_2 + 2u_1^T B_1^T Q_1 B_2 u_2 \\ & + x_1^{aT} Q_1 x_1^a - 2u_1^T B_1^T Q_1 x_1^a - 2u_2^T B_2^T Q_1 x_1^a] \end{aligned} \quad (14)$$

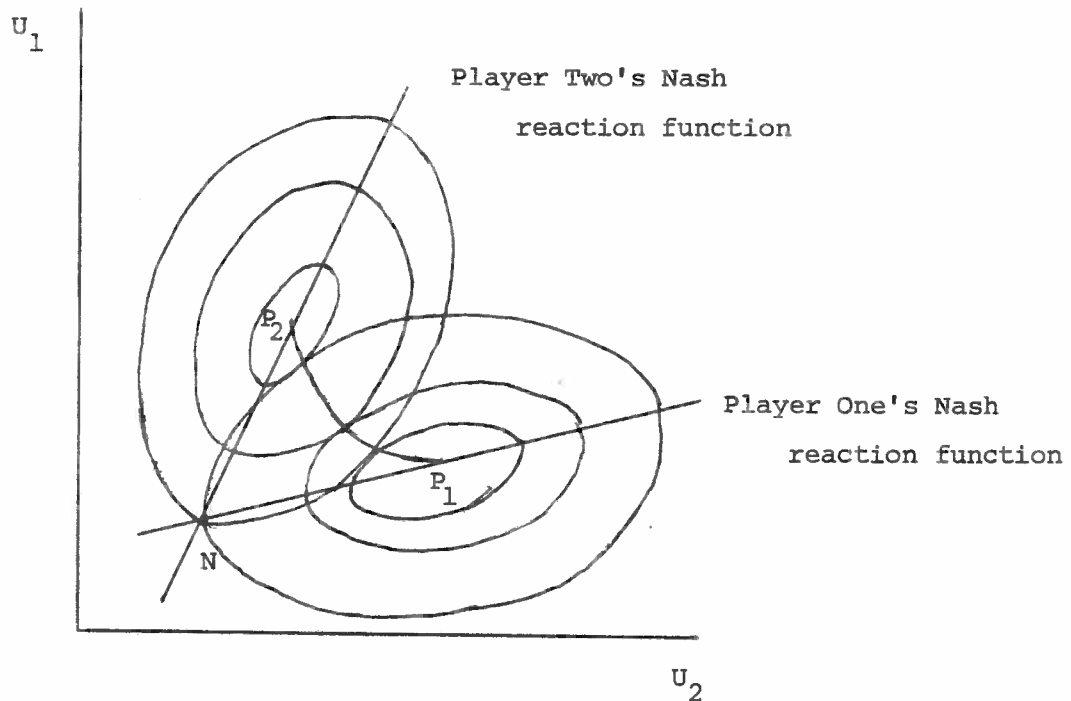
The iso-utility contours of the i 'th player can therefore be drawn as a set of concentric ellipses in u_1, u_2 space. This is illustrated in figure one.

Figure One.



The Nash reaction function for player one is then obtained by fixing the control variable of player two and finding the iso-utility contour closest to the central or 'bliss' point which is consistent with this. The intersection of the two player's Nash reaction functions is an equilibrium in the sense that at this point both players hypotheses about each other's behaviour are proved correct. This is illustrated in figure two.

Figure Two



N is the point of Nash equilibrium in figure two. In addition we have drawn the set of Pareto efficient points (labelled P_1, P_2).

The Nash equilibrium will not be Pareto efficient as long as players set differing or inconsistent targets on the states. It may well be that the Q_1 matrices are not of full rank. For example in the interdependent economies example the state vector may consist of the output levels of the two countries and the balance of payments position. Country 1 may not care explicitly about Country 2's output i.e. the element in the Q_1 matrix corresponding to it can be set equal to zero. However there is an indirect effect via the Balance of Payments which will be reflected in the indirect loss function. Thus it is not that Country 1 sets a different target on Country 2's output, it is simply that Country 2's targeted output level is inconsistent with Country 1's targeted Balance of Payments.

Stackelberg leader games

In this section we consider a two player game in which one player takes the other's control as independent of his actions i.e. exhibits Nash behaviour while the other correctly anticipates the way in which his fellow player responds to his decisions. We shall refer to the first player as a Nash follower and the second player as a Stackelberg leader. The follower's control problem is exactly the same as in the Nash case and his decision rule can go through as :

$$u_2 = [I + R_2^{-1} B_2^T Q_2 B_2]^{-1} [-R_2^{-1} B_2^T Q_2 B_1 u_1 + R_2^{-1} B_2^T Q_2 x_2^a] \quad (15)$$

The leaders' control problem becomes one of minimising (16)

$$\frac{1}{2} \left[(x - x_1^a)^T Q_1 (x - x_1^a) + u_1^T R_1 u_1 \right] \quad (16)$$

subject to

$$x = B_1 u_1 + B_2 u_2 \quad (17)$$

and

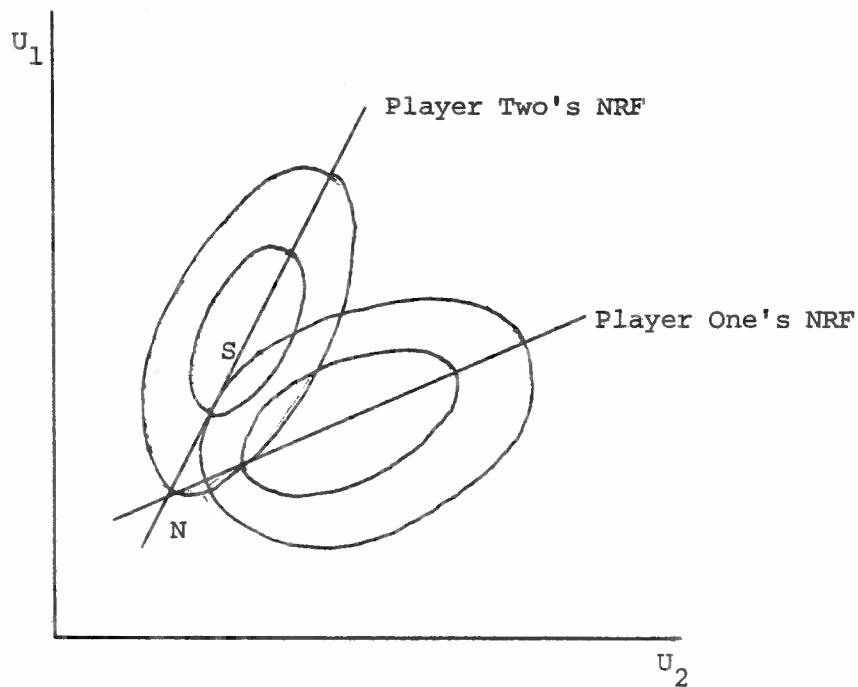
$$u_2 = G_2 x_2^a - G_2 B_1 u_1 \quad \text{where } G_2 \text{ is defined as in (15)} \quad (18)$$

Setting up the Lagrangean and taking the first order conditions with respect to the controls we get:

$$u_1 = -R_1^{-1} [B_1^T - (B_1 G_2 B_1)^T] p_1 \quad (19)$$

which is a function of the follower's Nash reaction function. The analysis could easily be extended to deal with a number of Nash followers or alternatively a hierarchy of leaders. Staying with the simple case for now we can again get an intuitive understanding of the nature of the solution by looking at the players' indifference map in u_1, u_2 space. This is illustrated in figure three.

Figure Three



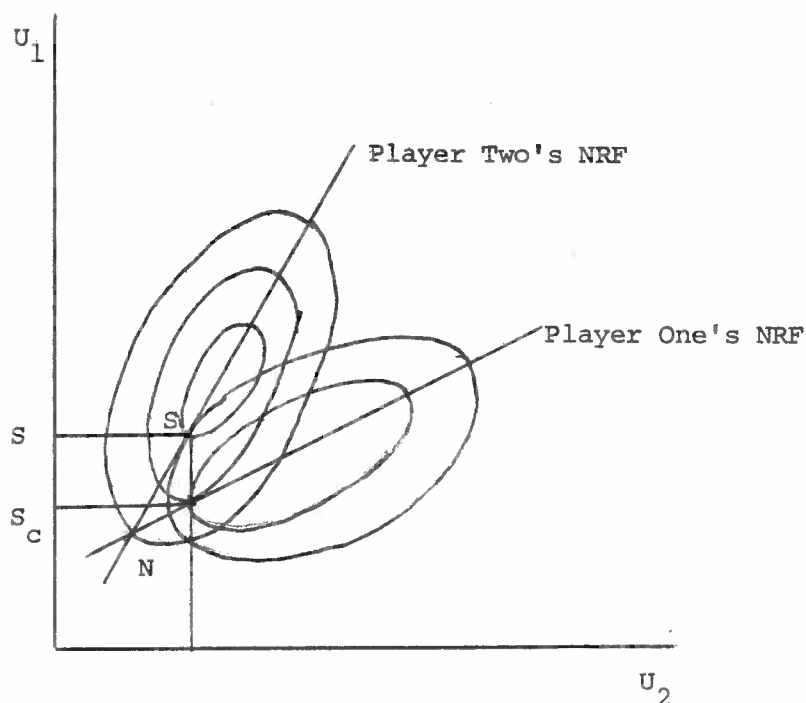
S , the point where player two's Nash reaction function is tangential to one of player one's iso-utility contours, is the Stackelberg leadership solution. It should be noted that this may well produce superior welfare for both leader and follower which is the case illustrated in the diagram, however there is no guarantee of this. It is still unlikely that the Stackelberg solution will produce a Pareto efficient outcome.

A problem associated with the Stackelberg leader solution is that of cheating i.e. the phenomenon by which the leader can announce a

strategy, cause the follower to adopt a strategy on his Nash reaction function consistent with it, and then adopt a different strategy. This is discussed for both static and dynamic games by Hamalainen (1981). There are two types of cheating of interest for our purposes, two-stage cheating and perfect cheating.

The two-stage cheating strategy is to announce the Stackelberg strategy then to choose a point on the Nash reaction function when the follower has implemented his control. This is illustrated in figure four.

Figure Four

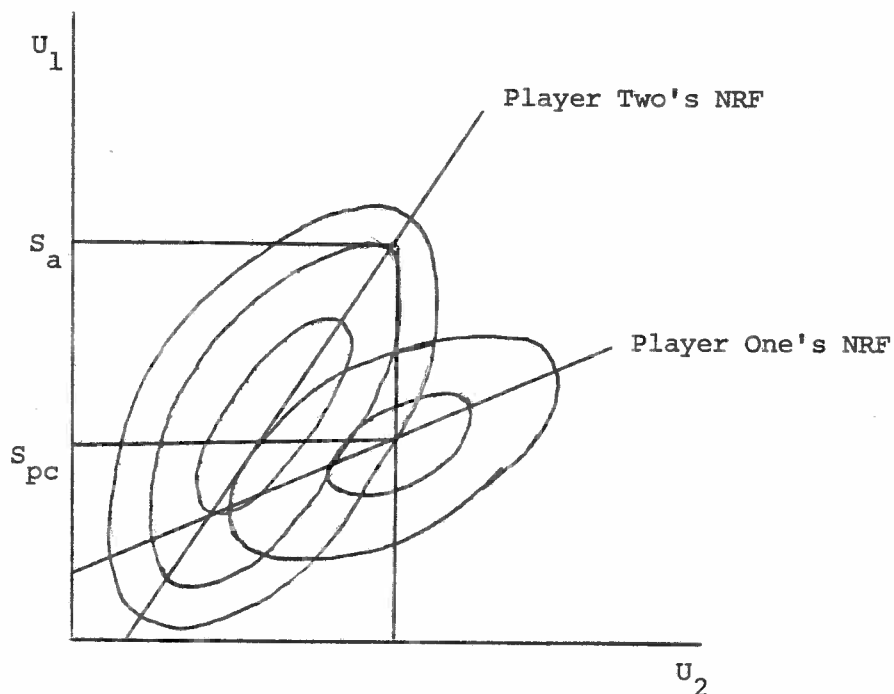


S_c shows the two-stage cheating solution. It is unambiguously inferior to the genuine Stackelberg solution for the follower but may still be superior to the Nash solution.

Perfect cheating implies that the leader announces a control which causes the follower to adopt a strategy consistent with the leader

attaining his bliss point. This is illustrated in figure five.

Figure Five



S_a is the leader's announced control while S_{pc} is the control he actually employs.

It should be noted that the quadratic objective function (and therefore linear reaction function) is sufficient to ensure the existence of the perfect cheating strategy. More general objective functions may lead to non-existence problems.

It should also be noted that the perfect cheating solution is Pareto efficient, lying on one of the end-points of the Pareto efficient locus. The cost of Pareto efficiency though is the imposition of a unit weight on the leader's loss function.

Pareto Efficient Solutions.

We can compute Pareto efficient solutions by assuming a single controller with a loss function which is a weighted combination of the individual players' loss functions as in (20).

$$\frac{1}{2} \sum w_i [(x-x_i^a)^T Q_i (x-x_i^a) + u_i' R_i u_i] \quad (20)$$

This is minimised subject to the state vector.

$$x = \sum B_i u_i \quad (21)$$

First order conditions from the Lagrangean yield:

$$u_i = -w_i^{-1} R_i^{-1} B_i^T p \quad (22)$$

In order to get the decision rules in terms of the states rather than the shadow prices we differentiate the Lagrangean with respect to the states to get:

$$p = \sum w_i Q_i (x-x_i^a) \quad (23)$$

which substituted into (22) gives us:

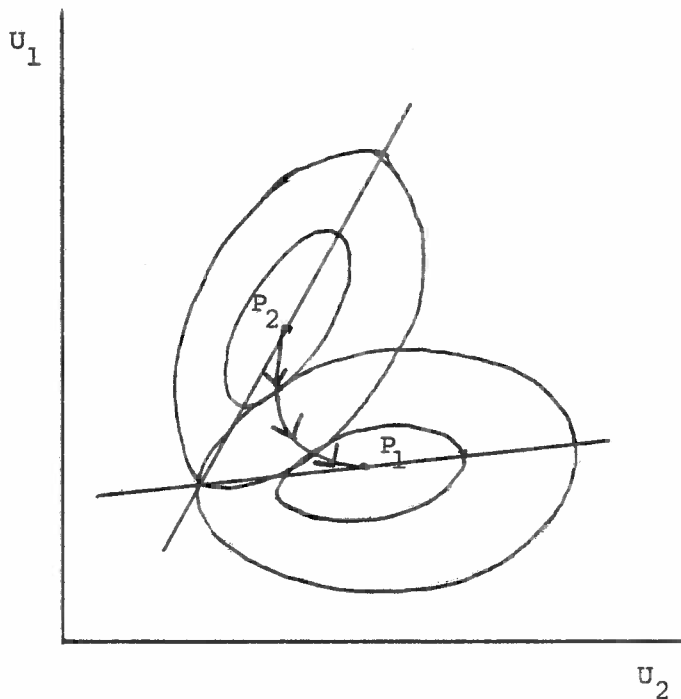
$$u_i = -w_i^{-1} R_i^{-1} B_i^T \sum w_i Q_i (x-x_i^a) \quad (24)$$

which can in turn be substituted into the state equation to yield:

$$x = \left[I + \sum B_i w_i^{-1} R_i^{-1} B_i^T Q_i \right]^{-1} \left[\sum B_i w_i^{-1} R_i^{-1} B_i^T Q_i x_i^a \right] \quad (25)$$

the eventual outcome therefore depends on the targets of the individual players and the weights the controller or arbitrator assigns to them. What the controller effectively does is determine which point on the Pareto efficient locus is chosen by assigning weights to the players. This is illustrated in figure six:

Figure Six



The arrows indicate an increasing weight assigned to player one's loss function.

Policy optimisation in the open economy

The small open economy. (Fiscal policy, fixed exchange rates.)

The first case we consider is a very simple economy characterised

solely by an IS curve, where the level of exports are taken to be independent of its actions. The structure of the economy is therefore given by:

$$y = cy + g + x - my \quad (26)$$

The loss function of the policy authorities is taken to be:

$$w = \frac{1}{2}(y - y^a)^2 + \frac{\beta}{2}(S - S_a)^2 + \frac{\theta}{2}g^2, \quad S = x - my \quad (27)$$

from (6) we get the general relationship between the controls and the targets for a single controller optimisation problem:

$$u = [I + R^{-1}B^TQB]^{-1} [R^{-1}B^TQx^a] \quad (28)$$

which for this particular case can be written:

$$g = \frac{1}{\Delta} \left[\frac{-1}{\theta(s+m)} ya - \frac{\beta m}{\theta(s+m)} Sa \right] \quad (29)$$

where:

$$\Delta = 1 + \frac{1}{\theta(s+m)} + \frac{\beta m^2}{\theta(s+m)}$$

The properties of this decision rule make intuitive sense, the higher the output target the greater is government spending and the higher the Balance of Payments surplus target the lower the level of government spending. Note that if we set a Balance of Payments target of zero the decision rule appears to depend only on the target level of

output but Balance of Payments effects still enter via the parameter Δ . If there is a positive Balance of Payments surplus target and the weight on the Balance of Payments increases then there is an unambiguous negative effect on the level of government spending. If however there was a target Balance of Payments deficit then the effect on government spending can be either positive or negative according to:

$$\frac{\partial l/\Delta\theta(s+m)}{\partial\beta} y^a \gtrless \frac{\partial\beta m/\Delta\theta(s+m)}{\partial\beta} s^a \quad (30)$$

Fiscal policy with two interacting economies and a fixed exchange rate

Nash solutions (symmetric countries)

Each economy is assumed to be fully represented by a fixed price IS curve and one country's imports are the other country's exports. Targets are put on output and the Balance of Payments and the welfare function is assumed to be of the form:

$$W = \frac{1}{2}(y_i - y_i^a)^2 + \frac{\beta}{2}(s_i - s_i^a)^2 + \frac{\theta}{2}g^2 \quad (31)$$

The structure of the world economy is given by:

$$\begin{bmatrix} y_1 \\ y_2 \\ s_1 \end{bmatrix} = \begin{bmatrix} \alpha_1 & \alpha_2 \\ \alpha_2 & \alpha_1 \\ m(\alpha_2 - \alpha_1) & -m(\alpha_2 - \alpha_1) \end{bmatrix} \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} \quad (32)$$

where:

$$\alpha_1 = \frac{s+m}{\Delta} \quad \Delta = s(s+2m) \quad S = \text{the marginal propensity to save}$$

$$\alpha_2 = \frac{m}{\Delta} \quad M = \text{the marginal propensity to import}$$

This represents the constraint set subject to which the welfare or loss function (31) is minimised. Making the Nash assumption of zero conjectural variations enables us to use the general formula for the Nash reaction function (11) to write down country one's decision rule in terms of its output and Balance of Payments targets and country two's level of government spending:

$$g_1 = \frac{-(\alpha_1 \alpha_2 - \beta m^2 (\alpha_2 - \alpha_1)^2)}{\theta + \alpha_1^2 + \beta m^2 (\alpha_2 - \alpha_1)^2} g_2 + \frac{\alpha_1}{\theta + \alpha_1^2 + \beta m^2 (\alpha_2 - \alpha_1)^2} y_1^a - \frac{\beta m (\alpha_2 - \alpha_1)}{\theta + \alpha_1^2 + \beta m^2 (\alpha_2 - \alpha_1)^2} s_1^a \quad (33)$$

The slope of the Nash reaction function can be either positive or negative according to:

$$\beta \geq \frac{\alpha_1 \alpha_2}{m^2 (\alpha_2 - \alpha_1)^2} = \frac{s+m}{ms^2} \quad (34)$$

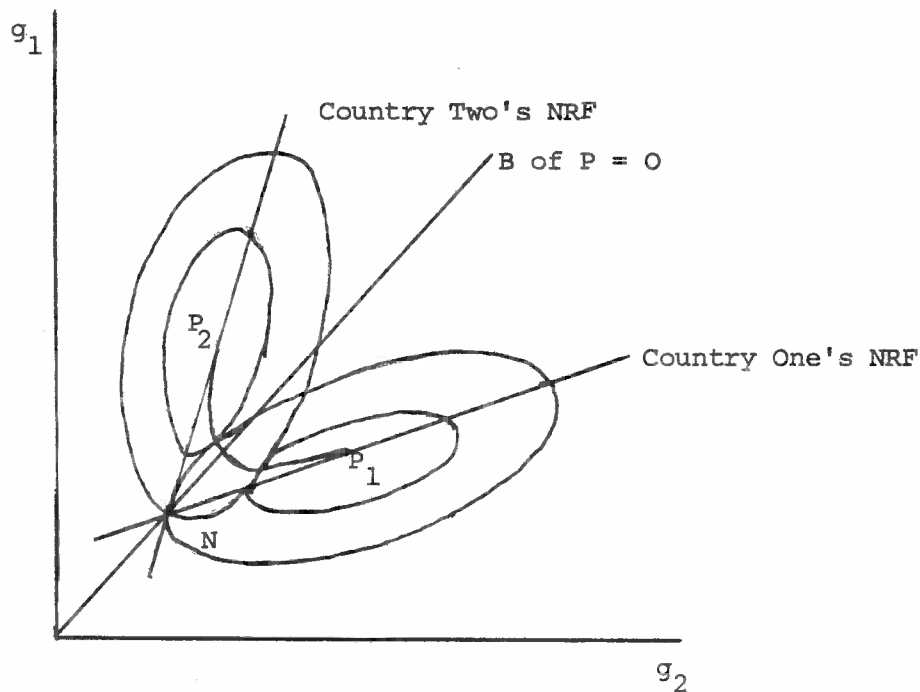
therefore we can state the following results. The Nash reaction function is more likely to be upward sloping:

- (1) The greater the weight put on the Balance of Payments target.
- (2) The more open the economy is, as measured by the marginal propensity to import.

(3) The larger is the marginal propensity to save.

Consider the case where both countries set a Balance of Payments surplus target. Clearly this amounts to a zero-sum game since one country's surplus is the other's deficit. The Nash solution is Pareto inefficient with a deflationary bias. This can easily be seen by reference to figure seven:

Figure Seven

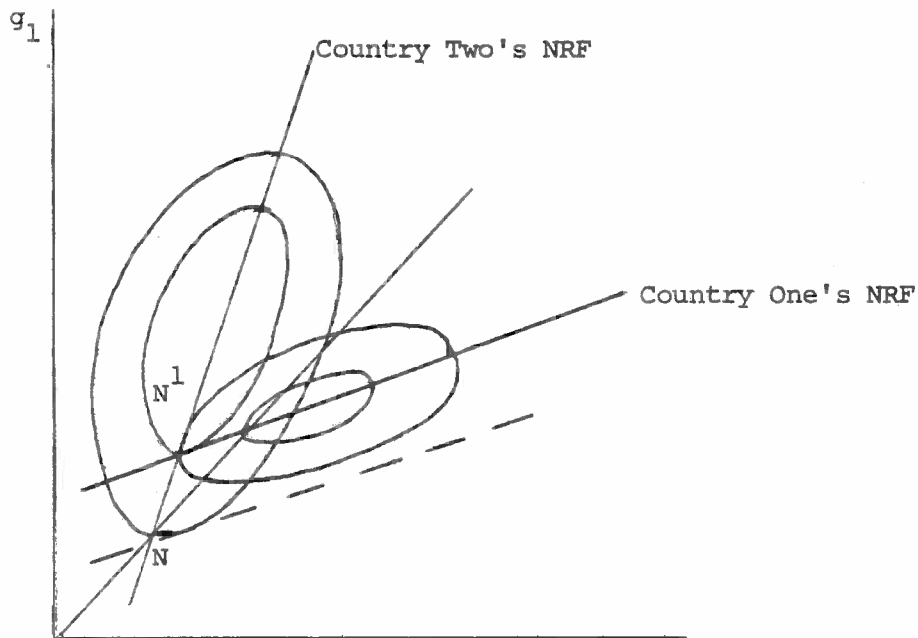


The 45° line indicates the combinations of g_1 and g_2 consistent with Balance of Payments equilibrium in both countries. P_1 and P_2 are the ideal or 'bliss' points of the two countries, the locus of tangencies of the iso-utility contours lying between P_1 and P_2 is the set of Pareto efficient points. Since N the Nash solution lies below the Pareto efficient point on the 45° line we can conclude that there is a deflationary bias. The reason for the Nash solution lying on the 45° line is the symmetry of the problem and hence the result that:

$$\left(\frac{\partial g_1}{\partial g_2} \right)_{\text{Country two}} = 1 / \left(\frac{\partial g_1}{\partial g_2} \right)_{\text{Country one}} \quad (35)$$

we also require symmetric targets for the Nash solution to lie on the 45° line. Allowing the countries to have different targets produces some interesting comparative static results. Figure eight illustrates the case where country one has either a larger output or smaller Balance of Payments target than country two.

Figure Eight



The effect is expansionary when compared with the symmetric targets case. In addition we can see that country one's Balance of Payments moves into deficit. Zero Balance of Payments targets by both countries would produce a Pareto efficient Nash equilibrium. The rather unrealistic case of both countries having a Balance of Payments deficit target would produce a Nash equilibrium with an inflationary bias from the Pareto efficient locus.

In addition to varying the targets, varying the relative weight on the targets alters the equilibrium by changing both the slope and the intercept of the Nash reaction function. Other things being equal an increase in the slope and a fall in the intercept of the Nash reaction function will occur when the weight on the Balance of Payments increases. The effect on the level of aggregate world demand is therefore uncertain.

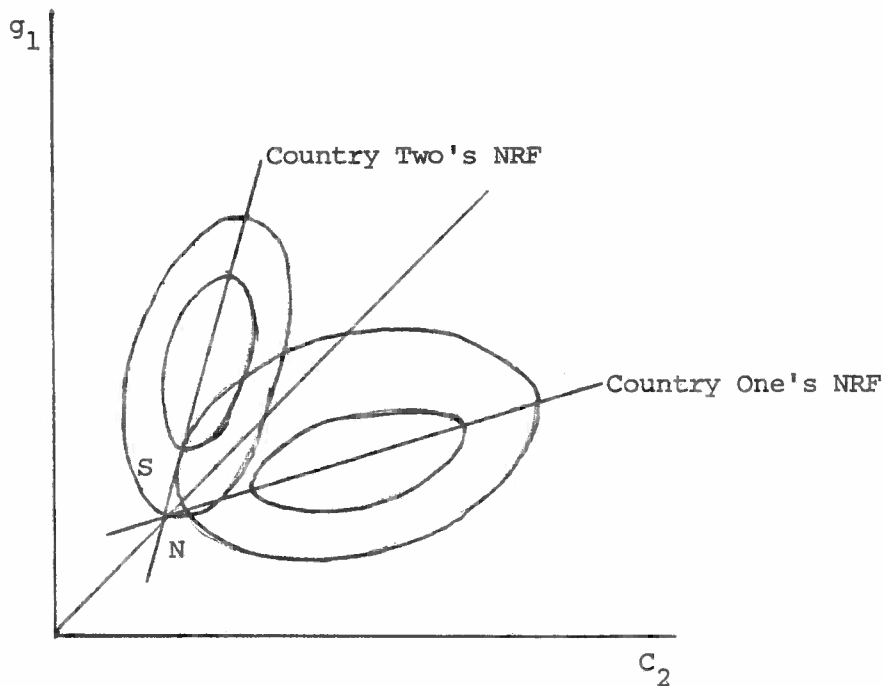
In the two country case it would surely become obvious that they were engaged in a zero-sum game. However more realistic situations could be considered. One argument is that the two countries could be thought of as part of the world economy with the rest of the world determining the deficit that they have to share. A plausible scenario is to view the pricing behaviour of the primary producers determining the aggregate deficit but each individual country within the west trying to minimise its share of this.

Stackelberg leader solutions

When we move to Stackelberg leader models we consider cases where one country can choose his position on the other Nash reaction function. What gives it this power may vary according to circumstances. In many cases it is the institutional framework of the world economy which has allocated roles to countries, for example the US gained a central role in world economic decision making through the importance of the dollar in the Bretton Woods system. Alternatively the effectiveness with which countries can set their policy instruments may govern the allocation of roles. For instance one 'country' could be thought of as the EEC which has sufficient coordination in economic policies to respond to announced US policy decisions but not to act jointly to influence the US decision making process.

In the case of Nash games the sign of the slope of the Nash reaction functions was not particularly important and all the results carried through whether it was positive or negative. In the case of Stackelberg leader games there are some quite important qualitative differences when the sign changes. Retaining the assumption of symmetry consider first the case where both Nash reaction functions slope upwards. Country one is now assigned the role of leader and chooses a point on country two's reaction function tangential to one of his iso-utility curves. As illustrated in figure nine this involves an increase in government spending by both countries and a Balance of Payments deficit for country one.

Figure Nine

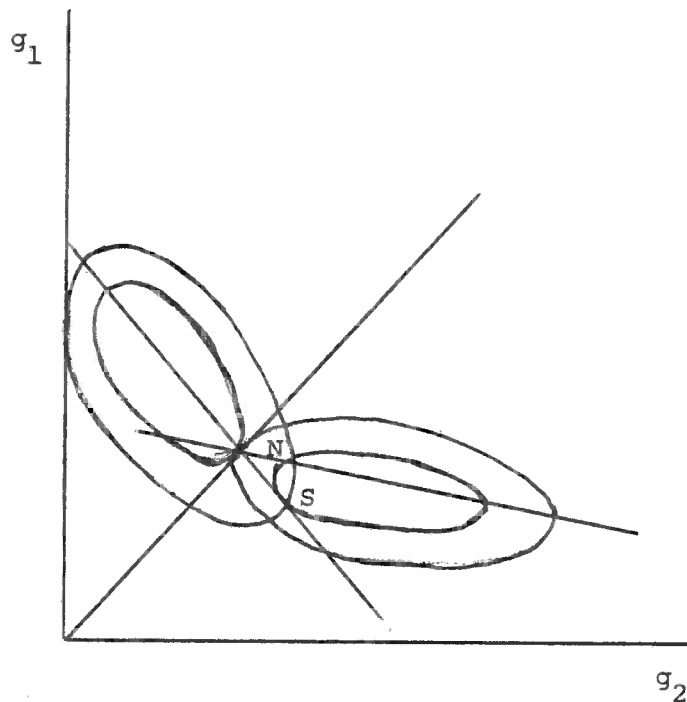


Note also that the Stackelberg solution results in a Pareto superior outcome.

In the case where the Nash reaction functions are downward sloping the Stackelberg solution involves a cut in government spending by the

leader and an increase by the follower relative to their Nash levels.
This is illustrated in figure ten.

Figure Ten



The Stackelberg equilibrium has a Balance of Payments surplus for the leader and shows a decrease in welfare for the follower relative to the Nash solution. Obviously allocating the role of leader to one country is not a sufficient condition for an expansion of world output within this model.

Another problem which arises within the Stackelberg context is that of cheating. With upward sloping Nash reaction functions this would involve the leader announcing that he is going to expand to a greater extent than he actually intends doing. In the case where they are downward sloping it would involve the leader announcing that he is going to be more contractionary than he intends.

Monetary policy with two interacting economies and a fixed exchange rate

Nash games

In the small open economy with fixed exchange rates monetary policy is ineffective because the Balance of Payments deficit caused when income expands leads to an erosion of the domestic monetary base. When we move into a two country framework it regains some effectiveness since each country can have a significant effect on the world money stock. This is the case analysed by Hamada (1974). The control variables are the domestic credit bases of the two countries with the actual values of their money stocks being partially determined by the Balance of Payments. Each country has a loss function of the form:

$$W_i = \frac{1}{2}(Y_i - Y_i^a)^2 + \frac{\beta}{2}(S_i - S_i^a)^2 + \frac{1}{2}C_i^2 \quad (36)$$

with S_i being defined as reserve changes or :

$$S_i = (Ky_i - \lambda r - C_i) \quad (37)$$

The structure of the economy can be written as :

$$\begin{bmatrix} Y_1 \\ Y_2 \\ S_1 \end{bmatrix} = \begin{bmatrix} \alpha_1 & -\alpha_2 & \alpha_3 & \alpha_3 \\ -\alpha_2 & \alpha_1 & \alpha_3 & \alpha_3 \\ \frac{1}{2}(\alpha_1 + \alpha_2) & -\frac{1}{2}(\alpha_1 + \alpha_2) & -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} g_1 \\ g_2 \\ C_1 \\ C_2 \end{bmatrix} \quad (38)$$

$$\alpha_1 = \frac{2\lambda(s+m) + Kb}{\Delta} \quad \Delta = 2(\lambda s + Kb)(s+2m)$$

$$\alpha_2 = \frac{Kb - 2\lambda m}{\Delta} \quad (39)$$

$$\alpha_3 = \frac{b(s+2m)}{\Delta}$$

λ = coefficient of the interest rate in the demand for money function

K = coefficient on income in the demand for money function

s = marginal propensity to save

m = marginal propensity to import

b = coefficient of the interest rate in the IS curve
i.e. the responsiveness of investment to the interest rate

for details of the structure of this model and the fiscal policy model see Mundell (1968).

Using the general formula for the Nash reaction function (11) we can write down country one's reaction function in terms of its targets on output and the Balance of Payments and country two's level of domestic credit. Government spending is assumed to be fixed in both countries for ease of exposition and to permit diagrammatic analysis. However there is nothing in principle to stop the countries using both policy instruments simultaneously. Country one's reaction function is :

$$c_1 = \frac{-\alpha_3^2 + \beta/4}{\theta + \alpha_3^2 + \beta/4} c_2 + \frac{\alpha_3}{\theta + \alpha_3^2 + \beta/4} y_1^a - \frac{\beta/2}{\theta + \alpha_3^2 + \beta/4} s_1^a \quad (40)$$

The condition for a positive slope is:

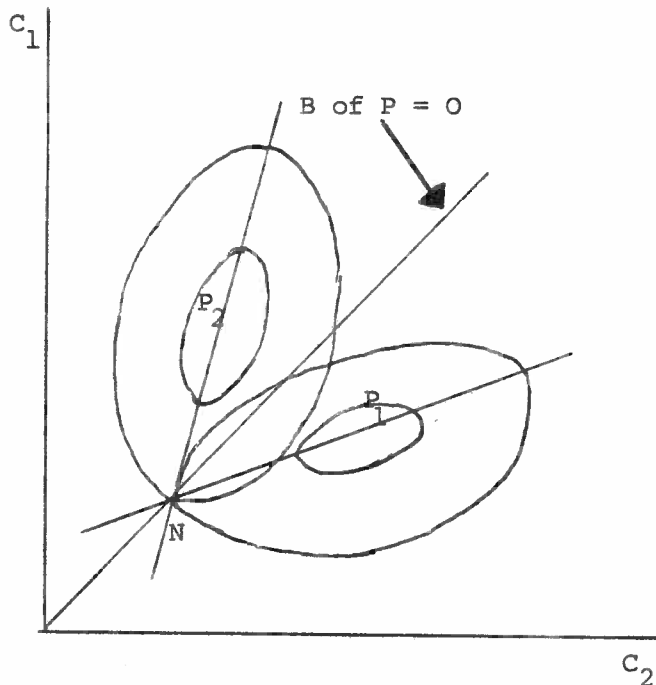
$$\beta > 4\alpha_3^2 \quad \text{or} \quad \beta > \left(\frac{b}{\lambda(s+Kb)} \right)^2 \quad (41)$$

Therefore there is more likely to be a positive slope:

- (1) The greater the weight on the Balance of Payments.
- (2) The less responsive investment is to the rate of interest.
- (3) The more responsive the demand for money is to the rate of interest.
- (4) The larger the marginal propensity to save.
- (5) The more responsive the demand for money is to the level of income.

The slope of the Nash reaction functions is always less than one in absolute value, therefore the Nash solution exhibits a deflationary bias from the Pareto efficient locus, c.f. Figure eleven.

Figure Eleven

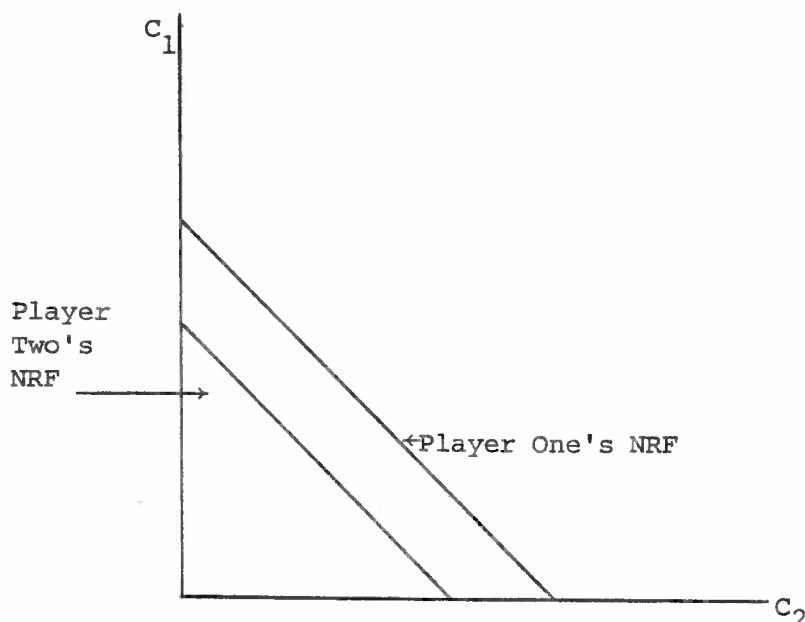


As in the previous example if, for some reason, the countries were to set a Balance of Payments deficit target then the Nash solution would exhibit an

inflationary bias from the Pareto efficient locus. Comparative static exercises again show that a larger output target or smaller Balance of Payments surplus target for country one will lead to output expanding in that country but the Balance of Payments moving into deficit.

A special case which arises is where there is zero weight put on the Balance of Payments and costless adjustment of the domestic credit base (this can be found by letting θ in equation (40) tend to zero. The slope of the Nash reaction functions are -1 in this case. If the output targets differ then the country with the larger target supplies the whole of the world money base, c.f. Figure twelve:

Figure Twelve



Although this is an interesting special case its practical importance should not be over-stated.

Stackelberg leader games.

The results for the fiscal policy example go straight through here and can simply be restated as :

For upward sloping reaction functions:

- (1) The effect is expansionary for both countries and there is an improvement in welfare for both.
- (2) The leader has the temptation to cheat by announcing a strategy more expansionary than he intends.

For downward sloping reaction functions:

- (1) The leader cuts his domestic credit level and the follower expands his. The leader's welfare improves and that of the follower deteriorates.
- (2) The leader has the temptation to cheat by announcing a strategy more contractionary than he intends.

It is likely that if the game is repeated then the leader will find it less profitable to cheat due to the loss of credibility this will entail.

Conclusions

This paper has been concerned with the methodology of analysing policy optimisation with several controllers rather than specific policy problems. Nevertheless some strong themes have emerged which prove applicable to more complicated models. For example the sub-optimality of decentralised decision making carries through to most models. The need for dynamic models can be seen by the inability of our framework to handle problems of price inflation. An intertemporal framework capable of handling such problems is given by Miller and Salmon (1983). Fixed exchange rate models have a limited applicability to current policy problems but are not entirely irrelevant when countries are unwilling to allow unconstrained depreciation to correct Balance of Payments deficits for fear of inflationary pressure.

The problem of cheating in Stackelberg leader models has been introduced but again we feel that an intertemporal framework might produce some more interesting results here. This is particularly the case when the leaders credibility becomes eroded as his realised strategy differs from his announced strategy.

Appendix AParabolic loss functions

The discussion in the main section is exclusively in terms of quadratic loss functions. It has been suggested that these may not always provide a good approximation to the true loss function of the policy authorities. In particular the assumption that positive and negative deviations from the target Balance of Payments are treated symmetrically is open to question. As an alternative therefore we have experimented with loss functions which are linear in the Balance of Payments and quadratic in deviations of output from a target level. The results are set out below.

Fiscal policy with fixed exchange rates

The loss function of the policy authorities is:

$$W = \frac{1}{2}(Y_1 - Y_1^*)^2 - \beta_1 S_1 + \frac{\theta}{2} g_1^2 \quad (A1)$$

and the reduced-form equations for the world economy taken from the main text are:

$$\begin{bmatrix} Y_1 \\ Y_2 \\ S_1 \end{bmatrix} = \begin{bmatrix} \alpha_1 & \alpha_2 \\ \alpha_2 & \alpha_1 \\ m(\alpha_2 - \alpha_1) & -m(\alpha_2 - \alpha_1) \end{bmatrix} \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} \quad (A2)$$

We do not have a general equation for the Nash reaction function in this case but it is relatively easy to solve from first principles.

Differentiating A1 with respect to g_1 yields:

$$\frac{\partial W}{\partial g_1} = (Y_1 - Y_1^*) \frac{\partial Y_1}{\partial g_1} - \beta \frac{\partial S_1}{\partial g_1} + \theta g_1 = 0 \quad (A3)$$

which yields the Nash reaction function:

$$g_1 = \frac{\alpha_1}{\alpha_1^2 - \beta m(\alpha_2 - \alpha_1) + \theta} Y_1^* - \frac{\alpha_1 \alpha_2}{\alpha_1^2 - \beta m(\alpha_2 - \alpha_1) + \theta} g_2 \quad (A4)$$

which unlike the quadratic example has an unambiguously negative slope.

Monetary policy with fixed exchange rates

The policy authorities loss function is again assumed to be A1.

B_1 is redefined to mean the increase in foreign currency reserves. The reduced form equations for the world economy taken from the main text are:

$$\begin{bmatrix} Y_1 \\ Y_2 \\ S_1 \end{bmatrix} = \begin{bmatrix} \alpha_1 & -\alpha_2 & \alpha_3 & \alpha_3 \\ -\alpha_2 & \alpha_1 & \alpha_3 & \alpha_3 \\ \frac{1}{2}(\alpha_1 + \alpha_2) & -\frac{1}{2}(\alpha_1 + \alpha_2) & -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} g_1 \\ g_2 \\ c_1 \\ c_2 \end{bmatrix} \quad (A5)$$

The Nash reaction function is:

$$C_1 = \frac{\alpha_3 Y_1^*}{\alpha_3^2 - \beta/2 + \theta} - \frac{\beta}{2(\alpha_2^3 - \beta/2 + \theta)} C_2 \quad (A6)$$

The condition for a positive slope is:

$$\beta > 2(\alpha_3^2 + \theta) = \frac{1}{2} \left(\frac{b}{\lambda_S + Kb} \right)^2 + 2\theta \quad (A7)$$

which is more likely to be satisfied:

- (1) the greater the weight on the Balance of Payments
- (2) the less the responsiveness of investment to the rate of interest
- (3) the greater the responsiveness of the demand for money to income

- (4) the greater the responsiveness of the demand for money to the rate in interest
- (5) the greater the marginal propensity to save
- (6) the smaller the costs of using monetary policy.

Appendix B

Costless control

In the text Lagrange multipliers were used to derive the Nash reaction functions.

$$u_i = \left[I + R_i^{-1} B_i^T Q_i B_i \right]^{-1} \left[-R_i^{-1} B_i^T Q_i \left(\sum_{j \neq i} B_j Y_j \right) + R_i^{-1} B_i^T Q_i X_i^a \right] \quad (B1)$$

if there are more targets than instruments then a control problem exists even if control is costless. However R_i^{-1} is not defined. The Nash reaction function can be computed in two ways. One is by direct substitution of the constraint set into the objective function. Another is to take the limit of the Nash reaction function as R_i tends to zero. If we rewrite B1 as

$$u_i = \left[R + B_i^T Q_i B_i \right]^{-1} \left[-B_i^T Q_i \left(\sum_{j \neq i} B_j U_j \right) + B_i^T Q_i X_i^a \right] \quad (B2)$$

Then clearly $\lim_{R \rightarrow 0} (B2)$ exists. It is also easily shown that this

gives the same answer as direct substitution.

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