RISK AVERSION, INEQUALITY AVERSION AND OPTIMAL CHOICE OF DISTRIBUTIONS

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Synopsis

This paper considers explicitly costly choice between mean-preserving distributions of a random variable. First, we extend a theorem of Diamond-Rothschild-Stiglitz to our environment. We then apply the result to risk and inequality analysis. W.r.t. the former, we generalise Ehrlich and Becker's seminal analysis of self-protection. W.r.t. the latter, we establish a sufficient condition for lump-sum-tax-financed and proportional tax-financed expenditure upon reducing inequality in pre-tax income or abilities to increase with society's absolute inequality aversion. This requires everyone's relative inequality aversion to lie within the interval [1,2]. We draw upon empirical evidence to show: Norway may satisfy this requirement; the U.S. may not. Additionally, we examine the impact of variations in national income upon proportional tax-financed inequality reduction.

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RISK AVERSION, INEQUALITY AVERSION, AND OPTIMAL CHOICE OF DISTRIBUTIONS

In a large number of important decision contexts involving risk (or concepts formally akin to risk, such as economic inequality), economic agents take actions which directly influence the probability distributions which confront them. For example, in Ehrlich and Becker's (1972 [7]) seminal analysis of self-protection, an individual affects the risk of pecuniary loss by spending on safety precautions.[(1)]

Again, attempts to reduce inequality by promoting "equality of opportunity" - via, say, tax-financed educational expenditure and most forms of social engineering - can be usefully characterized as attempts to change the distribution of abilities to make it "less unequal" in some sense. E.g., some might interpret the U.K. policy of comprehensive education in precisely these terms. While having a statistically insignificant impact upon mean ability (as measured, perhaps, by mean G.C.E. qualifications), the policy can be argued to have resulted in regression to the mean from both tails of the distribution. Indexing different ability levels by e, under this policy we have, in effect, educational expenditure converting someone who might have been an e_1-person under a different regime to an e_2-person. To the extent that real incomes or utilities are directly related to abilities, a policy with such an outcome can also result in a less unequal distribution of utilities.

An interesting question which arises in such environments is: what, if any, is the relationship between an individual's risk aversion (resp., society's inequality aversion) and the riskiness (resp., inequality) of the distribution of income or chosen abilities? Diamond and Stiglitz (D-S, 1971 [6]) dealt with this question in their influential paper when they considered, *inter alia*, risk aversion and career choice. However, a careful reading of their work reveals that they did not consider explicitly *costly* choice between distributions in a manner which allowed for interaction between the cost of choice and the decision-maker's utility function in each state.

This is potentially an important omission because, e.g., choosing a career which is less risky either atemporally or intertemporally frequently involves some sacrifice of expected income as well as direct expenditure upon schooling/training costs, books, journals, tools and the like. At the societal level, choice of less unequal distributions of income often involves not only the costs of attempted social engineering via the educational system, but also the resource costs of administering the social security safety net.

Authors subsequent to D-S, notably Dean Hiebert (1983, [5]), Honda (1983 [11]), and Paroush (1981 [16]) have extended D-S's work to consider environments in which choice between distributions is costly. Moreover, this was done for problems explicitly involving more than one decision variable. However, these authors confined attention to one of two cases. (i) Where the mean-preserving change in the distribution of the relevant randon variable (r.v.) occurred only via a multiplicative transformation of the original r.v., hence its variance, leaving intact the *shape* of the distribution (Honda and Paroush); (ii) Where there were effectively only two possible states of the world (Dean Hiebert).(2)

In this paper we will consider optimal costly choice of distributions in a more general context than Dean Hiebert, Honda or Paroush by requiring only that mean-preserving changes in the riskiness of the r.v. satisfy the famous Diamond-Rothschild-Stiglitz (D-R-S) integral conditions (D-S, 1974, [6]). However, this is done only for a single decision variable problem.

Section I formulates the model and derives the main result linking choice of riskiness (resp., inequality) in the randon component of wealth to the decision-maker's risk (resp., inequality) aversion. The cost of choice in this section is interpreted as either a state-invariant expenditure upon self-protection or a lump-sum tax to finance egalitarianism. We elucidate briefly the sources of differences from D-S's corresponding theorems. II obtains some corollaries for a special utility function and for special relationships between wealth and the r.v. III extends the analysis to consider proportional tax financed egalitarianism. This enables us to analyze explicitly, albeit in a highly stylized model, the optimal relationships between national income and the rate of taxation and total expenditure upon egalitarianism. In IV we examine two recent studies which, respectively, derive estimates for the index of relative inequality aversion and the index of relative risk aversion. This enables us to bring some empirical evidence to bear upon our theoretical findings. V concludes. An Appendix contains the proofs of the main results.

I. THE MODEL AND MAIN RESULT

We will first introduce and interpret the notation appropriate to a context of risky choice and subsequently extend it to inequality analysis.

Following D-S, consider a family of utility functions $U[\omega,\rho]$, at least thrice-differentiable and once-differentiable in the first and second argument, respectively. Here, ρ is an ordinal measure parameterising the degree of Arrow-Pratt (Arrow, 1970, [2]; Pratt, 1964, [17]) absolute risk aversion exhibited by $U(\cdot)$ at ω and satisfies D-S's Theorem 3. The decision-maker's wealth at arbitrary x, e and c, $\omega \equiv \omega(x,e,c,) > 0$, is a twice continuously differentiable function of each of: e, the realisation of the r.v. (or state of the world), $\omega_e \neq 0$; c, the expenditure on choice of distribution of e; x, a parameter or carrier variable which, in some extensions, could be treated as endogenous.(3) $U(\cdot)$ exhibits strict risk aversion: $U_{\omega\omega} < 0$.

We assume $e \in [\underline{e},\bar{e}] \equiv \mathcal{E}$ and $e \sim F[e,\gamma(c)]$. $\mathcal{E}$ is non-empty. γ parameterises different mean-preserving distributions for e. F is at least thrice continuously differentiable in its arguments. An increase in γ represents an increase in the riskiness of F in the sense of D-S and thus satisfies (for differential changes) the D-R-S conditions (cf. [6], eqns. (1) and (3)),

$$\int_{\underline{e}}^{\bar{e}} F_{\gamma}(e,\gamma)\, de = 0 \tag{1}$$

and

$$T(s,\gamma) \equiv \int_{\underline{e}}^{s} F_{\gamma}(e,\gamma)\, de \geq 0, \quad \forall s \in [\underline{e},\bar{e}] \tag{2}$$

(1) is the mean-preserving condition; (2) is the second degree stochastic dominance condition for *all* risk averters to prefer the "less risky" distribution.

As effecting mean-preserving decreases in risk for the random component of wealth is assumed costly, it is natural to presume

$$\gamma'(c) < 0, \quad \forall c \in C \tag{3}$$

C being a compact, non-empty interval. The presumption of diminishing returns to risk reduction ($\gamma'' \geq 0$) is neither completely easy to defend, nor, happily, strictly necessary for our analysis. Equally indefensible, except for theoretical convenience, is our assumption that the technology of risk reduction (resp., inequality reduction) is common to all individuals (all societies). However, this assumption is indispensable for some *cet. par.* comparative static deductions.

We are interested in the impact of changes in ρ upon c. At arbitrary ρ the decision maker solves

$$\underset{c}{\text{Max.}}\ EU = \int_{\underline{e}}^{\bar{e}} U[\omega(x,e,c),\rho]\ F_e(e,\gamma(c))de \qquad (4)$$[4]

We will assume the conditions for the existence of a regular interior maximiser, $c^*(\rho)$, are satisfied.[5] We deal only with interior solutions.

The first-order necessary condition (FONC) for the above optimisation is

$$\int_{\underline{e}}^{\bar{e}} U_\omega[\omega,\rho]\omega_c F_e(e,\gamma(c^*))de + \gamma'(c^*) \int_{\underline{e}}^{\bar{e}} U[\omega,\rho]F_{e\gamma}(e,\gamma(c^*))de = 0. \qquad (5)$$

This should be compared with D-S's (1974, [6], eqn. (16), p. 345) condition for optimal choice of distribution.

The essential difference is in the presence of the first term of (5). This reflects the interaction between the cost of the choice of distribution and each state's utility function.[6]

Using routine comparative statics (CS) techniques, it is possible to demonstrate the following when (1) and (2) are satisfied.

<u>Theorem 1</u>: If $U_{\omega\rho}$, $U_{\omega\omega\rho}$, ω_c, ω_{ee} are sign-preserving on $\Sigma \times C$ and if

$$\text{(A)(i)} \quad U_{\omega\rho} \leqq 0, \qquad \text{(ii)} \quad U_{\omega\omega\rho} \leqq 0; \tag{6}$$

$$\text{(i)} \quad \omega_c \leqq 0, \qquad \text{(ii)} \quad \omega_{ee} \geqq 0 \tag{7}$$

with at least one strict inequality in each of (6) and (7), then $c^{*\prime}(\rho) > 0$, i.e., the decision-maker will choose a less risky distribution for the random component of wealth; if

$$\text{(B)(i)} \quad U_{\omega\rho} \geqq 0, \qquad \text{(ii)} \quad U_{\omega\omega\rho} \geqq 0; \tag{8}$$

and

$$\text{(i)} \quad \omega_c \leqq 0, \qquad \text{(ii)} \quad \omega_{ee} \geqq 0 \tag{9}$$

with at least one strict inequality in each of (8) and (9), then $c^{*\prime}(\rho) < 0$

<u>Proof</u>: See the Appendix.

We must stress that (6) and (7) or (8) and (9) are only *sufficient* conditions. Other combinations of signs for $U_{\omega\rho}$, $U_{\omega\omega\rho}$, ω_c and ω_{ee} which yield either $c^{*\prime}(\rho) > 0$ or $c^{*\prime}(\rho) < 0$ are not precluded.

These sufficient conditions might seem rather stringent when compared with D-S's which, typically, only required the sign of an analogue of $U_{\omega\omega\rho}$ together with that (1) and (2) (or a more stringent analogue of (2)) should hold. However, we will demonstrate presently that our requirements are relatively innocuous in some plausible environments. Of course, at the technical level, the additional sufficiency requirements present in Theorem 1 arise from the influence of the additional term in our FONC, (5).

Note that our mean-preserving changes in the distribution of e relate only to a partial determinant of wealth. Thus, the mean of ω can increase or decrease in c. (For special $\omega(\cdot)$'s examined below, the mean declines in c unambiguously.) Hence, even in the case where $c^{*\prime}(\rho) > 0$, thus $\gamma^{*\prime}(\rho) \equiv \gamma'(c^*(\rho))\, c^{*\prime}(\rho) < 0$, we would need to be careful in interpreting the outcome as indicating that, *cet. par.*, an increase in risk aversion motivates choice of a less risky distribution of final (net) wealth.[7]

D-S were well aware of this problem in environments with final wealth only some function of the r.v. This motivated their consideration of mean utility preserving changes in risk. Equality of mean utility is a natural competitive equilibrium condition under uncertainty and a referent in certain other related contexts (e.g., cf. Kanbur, 1979 [12]). However, while costly choice between mean-preserving distributions of an r.v. which leave mean net wealth monotonically related to the level of risk chosen seems perfectly intuitive, costly choice between mean utility preserving distributions leaving mean utility constant bar the cost of the choice presents separability problems. Simultaneously, within the context of equality analysis, we certainly do not wish to consider costly choice preserving mean net utility because expected social welfare would then be invariant w.r.t. choice of distribution. Therefore, despite the equivalence of mean utility preserving changes in risk and mean preserving changes in risk for an r.v. (D-S, 1974, [6], Theorem 3), we will not extend our analysis to consider costly mean utility preserving risk changes here.

The key to Theorem 1, particularly the counter-intuitive possibility that $c^{*\prime}(\rho) < 0$ (and the main source of the difference from D-S), is the trade-off between the disutility of increased risk (or inequality) in a component of income and the disutility of having to expend resources to mitigate the relevant increase. These resources would otherwise certainly be available for consumption.

The intuition underlying Theorem 1 perhaps can be aided further by noting corollaries valid when $\omega(x,e,c)$ takes plausible special forms. Consider cases where

$$\omega(x,e,c) \equiv \omega^1(x,e) - c \tag{10}$$

$$\omega(x,e,c) \equiv \omega^2(x) + ke - c,\ k > 0, \text{ a constant} \tag{11}$$

$$\omega(x,e,c) \equiv \omega^3(x)e - c \tag{12}$$

Then

<u>Corollary 1</u>: If (10) holds and $\omega^1_{ee}(x,e) \geq 0$ on $\mathcal{E} \times C$, then $c^{*\prime}(\rho) > 0$ if $U_{\omega\rho} \leq 0$ and $U_{\omega\omega\rho} \leq 0$ on $\mathcal{E} \times C$ with at least one strict inequality; $c^{*\prime}(\rho) < 0$ if $U_{\omega\rho} \geq 0$ and $U_{\omega\omega\rho} \geq 0$ on $\mathcal{E} \times C$ with at least one strict inequality.

<u>Proof</u>: By Theorem 1 and by inspection, noting that $\omega_c < 0$ unambiguously.

<u>Corollary 2</u>: If (11) or (12) holds (i.e. ω is linear in e on $\mathcal{E} \times C$), then $c^{*\prime}(\rho) > 0$ if $U_{\omega\rho} \leq 0$ and $U_{\omega\omega\rho} \leq 0$ on $\mathcal{E} \times C$ with at least one inequality strict; $c^{*\prime}(\rho) < 0$ if $U_{\omega\rho} \geq 0$ and $U_{\omega\omega\rho} \geq 0$ on $\mathcal{E} \times C$ with at least one inequality strict.

Proof: As for Corollary 1, noting that $\omega_{ee} = 0$ additionally now.

In the context of our model, cases (10)-(12) represent, *inter alia*, generalisations to a continuum of states of Ehrlich and Becker's two states example of an environment where an individual's self-protective expenditure (c) can affect the probability distribution of losses confronted. It should be clear that, in a many-states environment, some restriction of the admissable variations in the distributions of loss, hence of gross wealth, is required for tractability. (11) and (12) together with (1) and (2) provide a restriction to distributions preserving mean gross wealth prior to self-protective expenditure.

Inequality Analysis

Above we alluded to the familial relationship between inequality and risk analysis (a relationship exploited by Atkinson, 1970 [3], among others). This relationship allows a natural reinterpretation of our model and results so far.

Under this reinterpretation, our decision-maker is a population of normalized size unity. Its constituents are indexed by $e \in E$ with distribution $F[e,\gamma(c)]$. ($\int_{\underline{e}}^{\bar{e}} F_e(e,\gamma(c))de = 1$.) An e-person has net ("post-tax") income $\omega(x,e,c)$. $\omega(\cdot)$ can be considered the "earnings function" relating ability or "schooling", e, to net income at arbitrary c (a government instrument, such as a lump-sum tax) and x (a parameter, such as common working life) when individuals behave optimally. The distribution of ability is subject to social choice (e.g., via educational expenditure and other aspects of redistributive social policy) and we confine attention to choice restricted between distributions preserving mean ability in the population. Also, to keep things simple in this paper, we assume there are no disincentive effects to taxation.

The society has the utilitarian social welfare function (SWF) given by (4). In the spirit of Atkinson (p. 251), ρ is now interpreted as an index of absolute inequality aversion.

Assuming $\omega(\cdot)$ takes one of the forms (11) or (12), our problem is then to see how optimal choice of inequality in gross income - i.e., prior to the cost of egalitarianism - varies with society's absolute inequality aversion.

If the society exhibits constant absolute inequality aversion, then identical absolute decreases in net income for all individuals increases inequality if inequality is measured by Atkinson's index.(8) Thus, greater lump-sum-financed expenditure designed to achieve a less unequal distribution of the pre-tax income while preserving mean pre-tax income if society's absolute inequality aversion increased actually increases inequality, neglecting the impact of the redistribution, according to Atkinson's index. Corollary 2 gives sufficient conditions for this outcome. However, if inequality is measured by Kolm's (1976, [14], p. 419)"leftist" index,(9) inequality is unaffected by lump-sum taxation of all individuals. Therefore, we could neglect the influence of variations in lump-sum taxation (l.s.t.) upon overall inequality. Corollary 2 then gives sufficient conditions for an increase in society's absolute inequality aversion to result in either an increase or a decrease in redistributive expenditures, hence in inequality.

The above comments apply to egalitarianism financed by l.s.t., an instrument generally regarded as inegalitarian, *per se*, except in the context of Kolm's leftist index. Because of this, a given sum raised by l.s.t. can be welfare-improved upon by a revenue-neutral shift to other forms of tax for the purposes of reducing inequality. To that extent, our sufficient conditions for an increase in society's absolute inequality aversion to result in a reduction in inequality are "over-sufficient".(10) This motivates our consideration of proportional tax financed expenditure in Section III.

II. AN EXAMPLE

We will confine attention to utility functions exhibiting constant absolute risk or inequality aversion (RA or IA, resp.). These have the general form

$$U[\omega(x,e,c),\rho] \equiv A-B\cdot\exp\{-\rho\omega(x,e,c)\}, \quad A,B > 0 \qquad (13)$$

Thus (suppressing functional arguments),

$$\begin{aligned} U_\rho &= \omega B\cdot\exp(-\rho\omega) && \text{(i)} \\ U_{\omega\rho} &= (1-\rho\omega)B\cdot\exp(-\rho\omega) && \text{(ii)} \qquad (14) \\ U_{\omega\omega\rho} &= -\rho(2-\rho\omega)B\cdot\exp(-\rho\omega) && \text{(iii)} \end{aligned}$$

Hence,

$$U_{\omega\rho} \left\{\begin{smallmatrix}\geq\\=\\<\end{smallmatrix}\right\} 0 \text{ as } \rho\omega \left\{\begin{smallmatrix}\leq\\=\\>\end{smallmatrix}\right\} 1 \text{ (i)}; \; U_{\omega\omega\rho} \left\{\begin{smallmatrix}\geq\\=\\<\end{smallmatrix}\right\} 0 \text{ as } \rho\omega \left\{\begin{smallmatrix}\geq\\=\\<\end{smallmatrix}\right\} 2 \text{ (ii)} \qquad (15)$$

But, depending on the context, $\rho\omega$ is either simply the Arrow-Pratt index of relative risk aversion (RRA) (Arrow, 1970, [2]; Pratt, 1964, [17]), or the analogous index of relative inequality aversion (Atkinson, 1970, [3], p. 251). Thus, utilizing Corollary 2, we have the following.

Corollary 3: If individuals (societies) possess constant but different RA (IA) and possess otherwise identical preferences, and identical mean incomes, then the individuals' self-protective expenditure (societies' l.s.t. - financed inequality reducing expenditure) is increasing in RA(IA) - i.e., $c^{*\prime}(\rho) > 0$ - if RRA(RIA) $\in [1,2]$, $\forall\ e \in [\underline{e},\bar{e}]$.

Proof: By Corollary 2 and (15).

Despite its simplicity, Corollary 3 seems of some importance for inequality analysis. We know that constant IA implies increasing RIA (analogously with RA and RRA - e.g., cf. D-S (p. 352, eqns. (34)). That RIA should have the latter feature is argued by Atkinson (e.g., p. 251), among others. Similarly, Arrow (1970), [2], essay 3, and elsewhere) argues strongly for increasing RRA. He demonstrates that this must be true over some range of wealth for bounded utility functions. Indeed, Arrow demonstrates that RRA tends to a limit > 1 for "large" wealth but also must be < 1 for "small" wealth. Thus, by exact analogy, we would expect that RIA $\in [1,2]$ for e-persons with incomes "middling to high" in some sense.

Given this, Corollary 3 suggests the following. If we compared societies with any particular common and suitably "low" mean income and common level of inequality and asked how much resources each would be prepared to utilise in order to reduce inequality, it is quite possible that the societies would be prepared to sacrifice less resources the greater was their absolute inequality aversion. Certainly, $c^{*\prime}(\rho)$ is likely to be of ambiguous sign, *a priori*, for such societies.(11)

Similarly, if we repeated the exercise for societies with any given suitably "high" common mean income and common level of inequality, a similar outcome would be likely.

However, were the identical exercise repeated for societies with a suitably "middling to high" common mean income, and such that RIA $\in [1,2]$, $\forall e$, for individuals in these societies, then the societies would be prepared to devote greater resources to reducing inequality the greater was their absolute inequality aversion. In fact, leading on from this, Corollary 3 also suggests the following, counter-intuitively. If mean income is suitably "middling to high" but "not too unequally distributed" already in a given society (thus RIA $\in [1,2]$, $\forall e$, or for "most" e), the more likely is greater expenditure upon achieving even more equality as absolute inequality aversion increases.

The observations so far seem to capture the notion that, at a given level of inequality and inequality aversion, everyone in a society might be "too poor" to be inclined to devote resources to reducing inequality, or everyone might be already "too rich" to care.[13a]

If, as is typically true with RA cf. RRA, IA is negligibly small cf. RIA, more can be said. $\rho(2-\rho\omega)$ will then be negligibly small in absolute value cf. $(1-\rho\omega)$. In that event, from (14), the term in $U_{\omega\rho}$ will dominate that in $U_{\omega\omega\rho}$ for each e-person. Then, from (15) and Corollary 2, *if* $\gamma'(c)$ *is suitably small also,*[12] it will be *approximately* true that $c^{*\prime}(\rho)\{\gtrless\}0$ as RIA$\{\gtrless\}1$. This reinstates our intuitive notions about inequality: optimal mean gross income preserving expenditure upon reducing inequality will be increasing (decreasing) in absolute inequality aversion if relative inequality aversion is relatively high (low).[13]

$$(x+e)(1-t) \equiv e_1(1-t)$$

and gross national income is now

$$\mu_1 \equiv x + E(e) \equiv E(e_1)$$

Differences in x across societies could be due to, say, differences in natural resource endowments.

Perturbing x and using routine CS techniques, we have

Theorem 2: $t^{*\prime}(\mu)$ is ambiguous. RIA ≥ 1 and $U_{111} \leq 0$, $\forall e$, are sufficient, but unnecessary, for $d[\mu_1 t^*(\mu_1)]/d\mu_1 > 0$. If $U[\cdot]$ exhibits decreasing IA, then $d[\mu_1 t^*(\mu_1)]/d\mu_1$ is ambiguous. ($U_{\omega\omega\omega} \equiv U_{111}$, and so on, for short.)

Proof: See the Appendix.

There are two obvious sources of the ambiguities revealed by Theorem 2. First, with an increase in μ_1, it is possible for $\mu_1 t^*$ to increase even if t_1^* decreases. Thus, even if the society exhibits increasing absolute inequality aversion and is prepared to pay an increasing amount to effect a given reduction in inequality as its wealth increases, this might still be consistent with a reduction in the tax rate. Second, as is well known, the amount the society will pay to effect a given reduction in inequality will decrease as its wealth increases if it exhibits decreasing absolute inequality aversion.

These observations also provide the requisite insight into the role of U_{111}. As is also well known, $U_{111} > 0$ is necessary but insufficient for decreasing IA; $U_{111} < 0$ is sufficient but unnecessary for increasing IA.

The role of RIA is equally straightforward. For our purposes, RIA measures the value of a reduction in equality effected by an equi-proportionate variation in everyone's income with the distribution of individuals unchanged. But this is precisely what we have with proportional taxation prior to consideration of the impact of changes in the shape of the distribution of individuals.

Before closing the theoretical analysis, it seems desirable to emphasize the significance of the assumed absence of disincentive effects for our model. To do this, it is only necessary to consider the outcome were the government able to levy a linear income tax, with lump-sum component a, in our environment. Its resources (R) for financing egalitarianism would then be

$$R = a + t\underline{\mu}.$$

It is trivial to show the following, which we state without proof.

Theorem 3: With two instruments, a lump-sum tax (subsidy), and a proportional tax, t, the government attains the first-best and sets $t^* = 1$, $a^* = -\underline{\mu}$.

The key to this result is simply the observation that, in our model, while egalitarianism which involves changing some e_1 persons to e_2 persons consumes resources, there are no opportunity costs to operating the tax system itself in the absence of disincentives. Therefore, the government can attain the first-best wholly by direct redistribution. Hence $R^* = 0$.

IV. EMPIRICS

Given its importance, it remains for us to examine, albeit briefly, empirical evidence on the magnitude of RIA. Two main pieces of evidence are available.

First, Christiansen and Jansen (C-J, 1978, [4])[14] derive estimates of RIA implicit in the Norwegian system of indirect taxation from cross-section data for 1975. They make the assumption that the indirect taxes and the associated induced household consumption patterns observed were chosen optimally in order to maximize an individualistic utilitarian SWF. They then ask what utilitarian SWF rationalizes the data.[15] Additionally, they assume that RIA is constant (implying decreasing IA).[16] Without us going into details of their estimation here, this produced estimates of RIA ranging from 0.867 to 1.706 under different assumptions (C-J, p. 232).

Second, while there appears to be no direct estimates of RIA for the U.S., we can draw upon empirical results for the conceptually similar RRA. As Arrow and others (e.g., Allingham, 1972, [1], p. 165) have observed, risk aversion is one explanation of the inequality aversion (and hence, derived demand for social security) typically embodied in the SWF. In their empirical work using cross-sectional data on household asset holdings in the U.S., Friend and Blume conclude that, *inter alia*,

"regardless of their wealth level, [RRAs] for households are on average well in excess of one and probably in excess of 2" (Friend and Blume, 1972, [10], p. 900).[17]

Despite their troublesome assumption of constant RIA, Christiansen and Jansen's results suggest that Norway might be in the class of societies for which a *cet. par.* increase in IA would result in increased expenditure upon reducing inequality. This accords with casual observation about the Scandinavian countries. These are noted for their relatively high levels of social expenditure motivated by egalitarianism.

For the U.S., if we accept Friend and Blume's conclusions about the size of RRA as approximately indicative of that of RIA, a different conclusion follows. It seems possible then that the U.S. might be among a class of societies for which, *cet. par.*, $c^{*\prime}(\rho)$ is ambiguous at best. It might even be that a *cet. par.* increase in IA now *reduced* the optimal amount to be spent on inequality reduction.(18)

Suppose, however, the argument suggesting the terms in $U_{\omega\omega\rho}$ could be neglected in signing $c^{*\prime}(\rho)$ were valid. Friend and Blume's and Christiansen and Jansen's results then suggest that both the U.S. and Norway are in a class of countries for which, *cet. par.*, increased IA is associated with increased inequality-reducing expenditure.

Again, using Friend and Blume's results on RRA, analogous comments about comparable individuals' self-protective responses to an increase in RA can be made.

V. CONCLUSIONS

We have studied a very simple model of costly choice of distributions applicable to either risk or inequality analysis.

W.r.t. the former, *inter alia*, we extended Ehrlich and Becker's seminal analysis of self-protection. In view of their observation that "the incentive to self-protect, unlike the incentive to insure, is not so dependent on attitudes towards risk..." (E-B, 1972, [7], pp. 639-640), it seemed natural to ask: Under what circumstances would increased risk aversion be associated with increased self-protection, appropriately defined, *cet. par.*? This paper has provided an answer. We also strengthen their observation by showing, in a more general multi-state setting, that the relationship between RA and self-protection need not be unambiguous.

W.r.t. the latter, we have integrated inequality and tax analysis when, as is reasonable empirically, it is assumed that reducing inequality consumes resources. With due allowance for the many simplifications and caveats expressed, our main result here can be stated as follows: If policies to effect mean gross income preserving or mean ability preserving changes in inequality are financed by l.s.t., or a proportional tax, there will not be, *cet. par.*, an unambiguous relationship between society's absolute inequality aversion and the extent of inequality in gross incomes or abilities unless all members' indices of relative risk aversion (RIA) fall within a critical range. We then appealed to some empirical evidence to show that Norway's RIA is likely to be within that critical range while the U.S.A.'s was not.

Analysis of how our results on inequality would be modified when taxation's disincentive effects are incorporated in the model remains on the agenda for future research. The presence of disincentive effects alongside the resource costs of egalitarianism as modelled in this paper will, in principle, enable analysis of the trade-off between reducing inequality by progressivity in the tax system and by other means.

APPENDIX Proof of Theorem 1.

Totally differentiating the FONC (5) and rearranging yields

$$dc^*/d\rho = -D^{-1}\left[\int_{\underline{e}}^{\bar{e}} \{U_{\omega\rho}\omega_c F_e + \gamma' U_\rho F_{e\gamma}\}de\right] \tag{A.1}$$

$$D \equiv \left[\int_{\underline{e}}^{\bar{e}} \{(U_{\omega\omega}(\omega_c)^2 + U_\omega \omega_{cc})F_e + 2\gamma' U_\omega \omega_c F_{e\gamma} + \gamma'' UF_{e\gamma} + (\gamma')^2 UF_{e\gamma\gamma}\}de\right] \tag{A.2}$$

$D < 0$ by the assumption that the second-order sufficient condition for the maximization is satisfied. Thus the sign of $dc^*/d\rho$ is that of the [-] factor in (A.1). Integrating the second term of this by parts and using (1) repeatedly,

$$\gamma' \int_{\underline{e}}^{\bar{e}} U_\rho F_{e\gamma} de = \gamma' \int_{\underline{e}}^{\bar{e}} T(e,\gamma)\left[U_{\omega\omega\rho}(\omega_e)^2 + U_{\omega\rho}\omega_{ee}\right]de$$

Thus,

$$\text{sign } dc^*/d\rho = \text{sign }\left[\int_{\underline{e}}^{\bar{e}} \{U_{\omega\rho}\omega_c F_e + \gamma' T(e,\gamma)[U_{\omega\omega\rho}(\omega_e)^2 + U_{\omega\rho}\omega_{ee}]\}de\right] \tag{A.3}$$

Noting that $T(e,\gamma) \geq 0$ (by (2)) and using (6)-(9), the result follows. Q.E.D.

Note, for the purposes of Corollaries 2 and 3, that when $\omega(\cdot)$ is linear in e, the third term of (A.3) vanishes.

Proof of Theorem 2.

The FONC for the proportional tax problem is

$$-\int_{\underline{e}}^{\bar{e}} U_1[e_1(1-t^*),\rho]e_1F_e[e,\gamma(\mu_1t^*)]de + \int_{\underline{e}}^{\bar{e}} U[e_1(1-t^*),\rho]F_{e\gamma}[e,\gamma(\mu_1t^*)]\gamma'\mu_1 de = 0$$

$$e_1 = x + e,\ e \in E. \tag{A.4}$$

Perturbing x is equivalent to perturbing μ_1 without affecting E(e). Doing this, totally differentiating (A.4) and rearranging yields

$$dt^*/d\mu_1 = [D_1]^{-1}\Big[\int_{\underline{e}}^{\bar{e}} \Big\{\gamma't^*(U_1e_1)F_{e\gamma}-(\gamma')^2\mu_1t^*UF_{e\gamma\gamma}-(\gamma''\mu_1t^* + \gamma')UF_{e\gamma}$$

$$+ (U_1 + (1-t^*)e_1U_{11})F_e - \gamma'\mu_1(1-t^*)U_1F_{e\gamma}\Big\}de\Big] \tag{A.5}$$

$$D_1 \equiv \Big[\int_{\underline{e}}^{\bar{e}} \Big\{-2\gamma'\mu_1(e_1U_1)F_{e\gamma} + e_1^2U_{11}F_e + (\gamma'\mu_1)^2UF_{e\gamma\gamma}+\gamma''\mu_1^2UF_{e\gamma}\Big\}de\Big]$$

$$< 0 \tag{A.6}$$

by the second-order condition.

Ambiguity of $t^{*\prime}(\mu_1)$ should be clear by inspection.

Next, note

$$d[\mu_1t^*(\mu_1)]/d\mu_1 \ \{\gtreqless\}\ 0 \text{ as } \mu_1t^{*\prime}(\mu_1) + t^*(\mu_1)\ \{\gtreqless\}\ 0$$

$$\Longleftrightarrow \text{ as } \mu_1t^{*\prime}(\mu_1)/t^*(\mu_1)\ \{\gtreqless\}\ -1.$$

Using (A.5) and (A.6), this becomes as

$$\mu_1\Big[\int_{\underline{e}}^{\bar{e}} \Big\{\gamma't^*(U_1e_1)F_{e\gamma}-(\gamma')^2\mu_1t^*UF_{e\gamma\gamma}-(\gamma''\mu_1t^*+\gamma')UF_{e\gamma}+(U_1+(1-t^*)e_1U_{11})F_e$$

$$- \gamma'\mu_1(1-t^*)U_1F_e\Big\}de\Big]\ \{\lesseqgtr\}\ -t^*D_1$$

$$\Longleftrightarrow \text{ as } \int_{\underline{e}}^{\bar{e}} \Big\{-\gamma'\mu_1t^*(e_1U_1)F_{e\gamma}-\mu_1\gamma'UF_{e\gamma}+\mu_1(U_1+(1-t^*)e_1U_{11})F_e-\gamma'\mu_1^2(1-t^*)U_1F_e$$

$$+ t^*e_1^2\, U_{11}F_e\Big\}de\ \{\lesseqgtr\}\ 0 \tag{A.7}$$

We can consider the elements of the RHS (A.7) in turn:

(1) $-\gamma'\mu_1 t^* \int_{\underline{e}}^{\bar{e}} (e_1 U_1)F_{e\gamma}de = -\gamma'\mu_1 t^*\left[x\int_{\underline{e}}^{\bar{e}} U_1 F_{e\gamma}de + \int_{\underline{e}}^{\bar{e}} eU_1 F_{e\gamma}de\right]$

$= -(\gamma'\mu_1 t^*)\left[x(1-t)^2 \int_{\underline{e}}^{\bar{e}} T(e,\gamma)U_{111}de + \int_{\underline{e}}^{\bar{e}} T(e,\gamma)\left\{2U_{11}(1-t^*)+e(1-t)^2 U_{111}\right\}de\right]$

< 0 if $U_{111} \leq 0$, $\forall e$, with at least one strict inequality. (A.8)

(2) $\int_{\underline{e}}^{\bar{e}} \left\{U_1 + e_1(1-t^*)U_{11}\right\}F_e < 0$ if RIA ≥ 1, $\forall e$, with at least one strict equality. (A.9)

(3) $-\mu_1\gamma' \int_{\underline{e}}^{\bar{e}} UF_{e\gamma}de = -\mu_1\gamma' \frac{\partial}{\partial\gamma}\left[\int_{\underline{e}}^{\bar{e}} UF_e de\right] < 0,$ (A.10)

because increases in γ represent mean-preserving increases in inequality, and assuming the regularity which permits interchanging the order of integration and differentiation. (More simply, by the FONC (A.4).)

(4) $-\gamma'\mu_1^2(1-t^*) \int_{\underline{e}}^{\bar{e}} U_1 F_{e\gamma}de = -\gamma'\mu_1^2(1-t^*) \int_{\underline{e}}^{\bar{e}} T(e,\gamma)U_{111}de < 0$

if $U_{111} \leq 0$, $\forall e$, with at least one inequality strict. (A.11)

(5) $t^* e_1^2 \int_{\underline{e}}^{\bar{e}} U_{11}F_e de < 0$ by concavity. (A.12)

By (A.7) - (A.12), $d[\mu_1 t^*(\mu_1)]/d\mu_1 > 0$ if RIA ≥ 1, $\forall e$, and $U_{111} \leq 0, \forall e$. Noting that $U_{111} > 0$ is necessary for decreasing IA, if $U[\cdot]$ exhibits decreasing IA, then $d[\mu_1 t^*(\mu_1)]/d\mu_1$ is of ambiguous sign. Q.E.D.

FOOTNOTES

1. Others who apply Ehrlich and Becker's self-protection notion include Dean Hiebert (1983, [5]), Fraser (1983a, [8]; 1983b, [9]), Laffont (1980), [15]), and Paroush (1981, [16]).
2. See Fraser (1983a, [8]) and Karni (forthcoming, [13]) for applications of the two-states model to the comparative statics of risk aversion with state-dependent preferences.
3. Particular interpretations of $\omega(\cdot)$ will be given in due course.
4. Note that we could formulate an equivalent problem yielding identical results with

$$\underset{\gamma}{\text{Max.}}EU = \int_{\underline{e}}^{\bar{e}} U[\omega(c(\gamma),x,e),\rho]F_e(e,\gamma)de.$$

5. Laffont (1980, [15], Essay 3, esp. p. 67, fn. 5 and Appendix, pp. 82-86) presents a general treatment of the requirements for quasi-concavity of EU in a similar environment.
6. Overall, (5) is merely a suitable generalization to a continuum of states of Ehrlich and Becker's (1972, [7], p. 639, eq. (28)) and Laffont's (1980, [15], p.79) conditions for optimal self-protection in the absence of market insurance.
7. A brief heuristic argument for just such an interpretation, however, is the following. It is clear that by increasing c the mean net wealth is decreased, but the more risk averse individual still prefers this to the previous situation. Thus risk must have decreased in some sense.
8. In our notation, Atkinson's (1970, 3, p.250) index, I, is given by

$$I = 1 - [\omega_{EDE}/E(\omega)]$$

where $E(\cdot)$ is the expectation operator and ω_{EDE}, the *equally distributed equivalent* income, is defined by

$$U[\omega_{EDE},\rho] \int_{\underline{e}}^{\bar{e}} F_e[e,\gamma(c)]de \; (= U[\omega_{EDE},\rho])$$
$$= \int_{\underline{e}}^{\bar{e}} U[\omega(\cdot),\rho]F_e[e,\gamma(c)]de \qquad \text{(F.1)}$$

Verification of our statement is on his page 252.

9. Kolm defined his "leftist" index, I_ℓ, for a finite population. In a continuum case the corresponding definition is

$$I_{\ell} = (1/\alpha)\log \int_{\underline{e}}^{\bar{e}} [\exp\{\alpha\cdot(E(\omega)-\omega(e))\}[F_e(e,\gamma)de, \qquad (F.2)$$

α being a non-negative parameter.

Among other things, both Atkinson and Kolm's indices enable comparisons between distributions with different means.

10. It is well-known that while l.s.t. is non-distortionary, it is usually administratively infeasible because of the difficulty in taxing leisure.

11. Here, the ambiguity of $c^{*\prime}(\rho)$ is more likely the greater was inequality within the societies to start with. This is because a low mean income society with high inequality will have many poor individuals with low incomes, hence RIA, thus with $U_{\omega\rho} > 0$, and only a few with relatively high incomes, hence high RIA, thus with $U_{\omega\rho} < 0$. By the same token, there would be many with $U_{\omega\omega\rho} < 0$ and few with $U_{\omega\omega\rho} > 0$. A similar story can be told for societies with a suitably high mean income and high inequality. I.e., the greater is the dispersion of income within given societies, so that some individuals possess RIA > 2 and others RIA < 1, the greater is the ambiguity of $c^{*\prime}(\rho)$, *cet. par.*

12. This would be true if, *cet. par.*, the reduction in inequality associated with a given increment of expenditure is "small" (perhaps because, *if* $\gamma'' > 0$, considerable expenditure has been incurred already).

13. Then, e.g., we would not expect to see an increase in IA associated with an increase is egalitarianism in countries with low mean incomes and high inequality. In these, a few individuals would have high incomes, hence high RIA (> 1, perhaps), but the majority would have low incomes, thus low RIA (< 1), which would dominate. The intuition for this is that, at the (predominantly) low RIA, the resource costs of any given reduction in inequality would dominate the calculations in a given society cf. the putative benefits from the reduction. Analogous comments are applicable to the self-protective behaviour of an individual with low mean income with wide dispersion.

14. I am indebted to Jesus Seade for referring me to Christiansen and Jansen's work.

15. For their application, this SWF defines the Norwegian government's implicit preferences over income distributions. "The implicit preferences in economic policy are defined as the preferences which make the actual policy optimal, given these preferences" (C-J, [4], p. 218). This is a form of the well-known "inverse optimum" problem.

16. Hence, they assume that the same relative income change for all individuals implies distributive neutrality. This corresponds to what Kolm (1976, [14]) calls a "rightist" notion of inequality. This also corresponds to the Atkinson index defined in fn. 8 above.

17. They also concluded that, as a first approximation, the assumption of constant RRA for households is a fairly accurate description of the market place (p. 901). However, they acknowledged that the latter follows from their treatment of investment in housing. "Other plausible treatments would imply either moderately increasing or moderately decreasing RRA" (p. 901).

18. A possible qualification on this rather conservative conclusion, apart from that we are inferring RIA from RRA, is that Friend and Blume excluded from the sample households with net worth less than $1,000 (Friend and Blume, 1975, [10], pp. 906-7). If RRA (RIA) is increasing in wealth, (in econometric jargon) there will be a "censoring" bias to their estimate.

13a. As Nick Stern has emphasized to me, it is possible to defend exactly the opposite interpretation of the behaviour of poor societies. Viz: It is precisely because they care so *much* about inequality that they are likely to reduce the regressive lump sum tax as their inequality aversion increases. However, we show in Corollary 4 below that our findings are substantially unmodified even when we consider the less regressive case of a proportional tax.

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