Price Adjustment within a Framework of Symmetric Oligopoly
An Analysis of Pricing in 380 U.S. Manufactoring Industries
1958-71

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Abstract

This paper is concerned with the generation and testing of predictions on price adjustment from a model of symmetric oligopoly. Two types of industry demand regimes are considered, linear, and iso-elastic. It is shown that these can be distinguished by a simple test, and linear demand is given strong support. It is then shown that the industry conjectural variation may be treated as a function whose properties can be established by non-linear estimation of the price adjustment equation.

The purpose of this paper is to show that the predictions of symmetric oligopoly theory on price adjustment under a regime of linear industry (inverse) demand schedules accord well with the actual pricing behaviour of 380 industries in U.S. manufacturing over the period 1958 through 1971. Two parts of the literature are pulled together: empirical studies of inter-industry price determination, which have examined the impact of concentration on pricing without providing any theoretical underpinning, and standard neoclassical oligopoly theory. The approach outlined below allows for a simple test to distinguish between two possible demand regimes, linear, versus iso-elastic, industry demand schedules. A further point of interest is that estimates of the industry conjectural variation can be derived for each industry.

I. Theoretical Framework

In this section we follow the method and notation of Seade [1980]. With Y the output of the industry, profits for each of the n identical firms in the industry are

$$\Pi^f = y^f p(Y) - c(y)^f$$

and profit maximisation requires

$$\frac{d\Pi}{dy^f} = p + y^f p' \frac{dY}{dy^f} - c^{f'}$$

If we now assume that $\frac{dY}{dy}f$, the conjectural variation, is the same for all firms in the industry then we can re-arrange the first-order condition, writing

$$p + \lambda y p' - c' = 0$$

for all firms, where $\lambda = dY/dy^f$. There are n firms, hence n equations of this form, and since $ny = Y$, we have

(1) $$np + \lambda Y p' = nc'.$$

Although the precise cost function does not present any theoretical problems, there is an empirical issue which is that while theory works with marginal cost, data is based upon unit variable cost. It would certainly be a great convenience if these two were equal. We therefore assume a linear total cost function, $c^f = ay^f + b^f$, so that $a = UVC = MC$, and

(2) $$np + \lambda Y p' = nc' = na.$$

To solve for equilibrium price we must specify the demand relationship. We shall consider two types of (inverse) industry demand schedules, linear, given by $p = g_1 - hY$, and iso-elastic, given by

$$p = g_2 Y^{-\beta}, \quad (0<\beta<1).$$

Equation (2) may now be manipulated to generate expressions for equilibrium industry output Y^*, and equilibrium price p^* in terms of the parameters of the model, a, g,[1/] n, h, β and λ . In the linear demand case these are respectively, (see Table 1 below) $n(g_1-a)/h(n+\lambda)$ and $(g_1\lambda+na)/(n+\lambda)$. Under iso-elastic demand, they are respectively, $\{na/g_2(n-\lambda\beta)\}^{-1/\beta}$ and $na/(n-\lambda\beta)$. Under either demand regime the expression for p* has the customary properties; for example as n increases, price falls, and in the limit price tends towards marginal cost. It is an attractive feature of this theory that any number of firms can be handled within the same simple framework, so that the theory is not restricted to those industries conventionally viewed as oligopolies; the only restriction is that perfectly competitive industries cannot exist, since with the cost function we have assumed, marginal cost pricing would result in losses.

The assumed properties of λ are set out in Seade (op. cit., p. 481). Within an industry, all firms share the same view about the value of λ . In general, λ will lie between 1 and n, the former representing Cournot response, and the latter full collusion, in which case Y* and p* take their monopoly values. A natural presumption is that $\lambda = \lambda(n)$. We consider this point in some detail below. We may therefore write $p = p(a,g,n,\lambda,h \text{ or } \beta)$. We assume that a, g and n may change independently of each other over time; λ will change with n, $d\log\lambda/d\log n$ being denoted by ε ; h and β are assumed constant. Differentiating p with respect to a, g and n and expressing in terms of proportionate change we have

(3) $\Delta p = e_a\Delta a + e_g\Delta g + e_n\Delta n,$[2/]

where Δi indicates a proportionate change in variable i, and e_i is the elasticity of price with respect to variable i. But the elasticities e_a, e_g, and e_n can be derived from p^* in each demand regime, and these are shown in rows 3 through 5 of Table 1. Viewing (3) as a potential regression equation, we are building up a series of predictions about the coefficients of the regression. From rows 3 and 4 in Table 1 we see that while e_a and e_g are constants in iso-elastic demand, under a linear demand regime they are variables responding to the number of firms in the industry as shown in rows 6 and 7. The third elasticity e_n does not vary systematically with n under either demand regime. In the evaluation of the first derivatives (see Appendix), shown in rows 6, 7 and 8, λ is treated as $\lambda(n)$.

TABLE 1 - Expressions for certain variables under two demand regimes

	Variable	Linear Demand	Iso-elastic demand
1.	Y^*	$n(g_1-a)/h(n+\lambda)$	$\{na/g_2(n-\beta\lambda)\}^{-1/\beta}$
2.	p^*	$(g_1+na)/(n+\lambda)$	$na/(n-\beta\lambda)$
3.	e_a	$na/(g_1\lambda+na)$	1
4.	e_g	$g_1\lambda/(g_1\lambda+na)$	0
5.	e_n	$-n\lambda(g_1-a)(1-\varepsilon)/(g_1\lambda+na)(n+\lambda)$	$-\beta\lambda(1-\varepsilon)/(n-\beta\lambda)$
6.	de_a/dn	>0	0
7.	de_g/dn	<0	0
8.	de_n/dn	indeterminate	indeterminate

It is interesting that to establish the qualitative results of rows 6 and 7 $\lambda'(n)$ is not constrained to be negative. But what is necessary is that $d\log\lambda/d\log n$ must be less than unity. There is a strong element of common sense here. If $d\log\lambda/d\log n$ equalled unity then λ and n would rise in equal proportions and n/λ, which Seade calls the number of effective firms, would remain constant as n increased. It is clearly desirable that n/λ increases with n, so that the number of effective firms will increase, though not necessarily in proportion with n. Accordingly we impose the constraint that $d\log\lambda/d\log n < 1$. It is also interesting to note that when n is large e_a and e_g under linear demand tend towards the values shown under iso-elasticity, that is, unity and zero respectively. But as n falls, e_a becomes less than, and e_g greater than their iso-elastic counterparts. Thus the question of discriminating between the two demand regimes turns on the issue whether the coefficients in (3) on Δa and Δg are functions of n. In sum, (3) can be written as

(4) $\Delta p = \{n/(k\lambda+n)\}\Delta a + \{k\lambda/(k\lambda+n)\}\Delta g$
$-\{n\lambda(k-1)(1-\varepsilon)/(k\lambda+n)(n+\lambda)\}\Delta n$

under linear demand, where $k = g_1/a$, or

(5) $\Delta p = \Delta a - \{\beta\lambda(1-\varepsilon)/(n-\beta\lambda)\}\Delta n$ under iso-elastic demand.

k is hereafter assumed not to vary across industries.

Clearly (5) is equivalent to (4) with two restrictions introduced simultaneously, namely, that the coefficient on Δa is unity, and on Δg, zero. It is therefore a simple matter to test for the validity of these restrictions. But in addition (4) can be estimated directly by non-linear techniques, in the process of which the nature of λ can be explored.

II. Empirical Specification

The most interesting results are derived by estimating (4) by non-linear methods; these are discussed below. For the moment we focus on linear estimates, for two reasons. Firstly, we can conduct a simple test to establish whether the restrictions implied in (5) are valid. The null hypothesis is that the restrictions are true, that is, the coefficient on Δa is unity and on Δg zero. Incorporating these restrictions, we have the regression equation

(5a) $(\Delta p - \Delta a)_i = \alpha_0 + \alpha_1 \Delta n_i + u_i$, and the alternative hypothesis,

(5b) $\Delta p_i = \beta_0 + \beta_1 \Delta a_i + \beta_2 \Delta g_i + \beta_3 \Delta n_i + u_i$

The standard comparison of the two RSS can be performed. These results are reported in Table 2. But a second reason for linear regression is that if we write as an approximation of (4)

(6) $$\Delta p_i = \alpha_0 + \alpha_1 \Delta a_i + \alpha_2 N_i \Delta a_i + \alpha_3 \Delta g_i + \alpha_4 N_i \Delta g_i + a_5 \Delta n_i + \alpha_6 N_i \Delta n_i + u_i$$

where the prefixes in N_i represent multiplicative terms, N_i referring to the number of firms in the i^{th} industry, then we have a formulation rather like that used by Wilder, Williams and Singh (W-W-S) (1977) and by De Rosa and Goldstein (D-G) (1982), except that they used the concentration ratio, rather than N, in their interaction terms (and did not have Δn on the right hand side). For purposes of comparison a regression was run using the 4-firm concentration ratio as adjusted by Shepherd (1970), thus,

$$(7) \quad \Delta p_i = \beta_0 + \beta_1 \Delta a_i + \beta_2 CR_i \Delta a_i + \beta_3 \Delta g_i + \beta_4 CR_i \Delta g_i + \beta_5 \Delta n_i + \beta_6 CR_i \Delta n_i + u_i \; .$$

It turns out that a better fit is gained in (6) if $\log N_i$ rather than N_i is used in the interaction terms. It further transpires that when the term $N_i \Delta n_i$ is introduced, the coefficients α_5 and α_6 become insignificant. Accordingly the term $N_i \Delta n_i$ is omitted, as is $CR_i \Delta n_i$ in (7). The predictions on the coefficients in (6) can be made with the help of Table 1.

When $N_i = 1$, and $\log N_i = 0$, $N_i \Delta a_i$ and $N_i \Delta g_i$ disappear; hence, α_1 and α_3 may be interpreted as the elasticity of price with respect to marginal cost and demand, respectively, under pure monopoly, in a linear demand regime. As the number of firms increases, (see row 6 in Table 1) we expect this elasticity to rise; hence we expect α_2 to be positive. From row 7 we expect α_4 to be negative; the price response to demand shifts is less the more competitive the industry. We also note that $\alpha_1 = 1 - \alpha_3$. We expect α_5 to be negative.

The constant term α_0 presents a bit of a problem. The theory set out above has no role for a constant term, and yet all the published work in this area finds a positive and significant intercept. The tenor of the treatment of the constant term in the literature is that it reflects 'inflationary' pressure on prices, over and above, seemingly, any effect through demand or cost shifts. For example, W-W-S write 'the intercept term is the empirical manifestation of inflationary inertia, reflecting the expectations of price and wage setters'. A simpler explanation suggests itself which is more in the spirit of marginalist analysis.

Over a period of several years we would expect prices to reflect changes in fixed costs. Nordhaus (1972), for example, has expressed surprise that a cost of capital variable has seldom been included in price equations. In principle then we should like to include in our estimating equation a variable which measures the proportionate annual increase in unit fixed cost. However such a variable does not appear to be readily measurable. If this variable is omitted, then under certain well-known conditions[3/] the effect of the omission will show up as a positive constant. We suggest that this is the meaning of the constant term in the regression results. The results of regressing (6) and (7) are shown below in Table 2. However (4) can be estimated directly by non-linear techniques, and this is discussed in detail below.

III. Previous estimates of inter-industry price equations

Studies of inter-industry pricing behaviour published over the last fifteen years have typically tried to answer the question, whether more concentrated industries helped or hindered the inflationary process. To that end, proportionate price change was regressed against variables measuring cost and demand shifts, plus interactive terms based upon these variables and the degree of concentration.

The coefficients of the interactive terms were then given special attention. Although several studies of long-run price dynamics at the industry level have been published the picture of the factors which influence price movements is still confusing. Ripley and Segal (R-S) (1973) concluded that concentrated industries passed on a smaller proportion of unit labour cost changes than did non-concentrated industries. This finding gained support from Lustgarten (1975), and was further verified in Wilder, Williams, and Singh (W-S-S) (1977) who, focussing on unit variable cost, rather than unit labour cost, concluded that 'concentrated industries pass along a smaller proportion of unit variable cost changes into price changes than do less concentrated industries'. However, DeRosa and Goldstein (D-G) (1982) threw a spanner into the works by showing that the elasticity of price with respect to unit variable cost did not vary with the degree of concentration; moreover, they found that price was sensitive to shifts in demand (not found by W-W-S) and also that this sensitivity was greater in concentrated industries (again not found by W-W-S). Lustgarten (1975) had found that price was demand sensitive, but that the sensitivity was independent of the degree of concentration. Clearly all combinations are possible ! Another recurring question has been whether price is more sensitive to cost increases than to cost decreases. R-S found no evidence of such an effect; W-W-S found that for industries with rising UVC the intercept term in the price equation was significantly lower, while D-G found this effect and also that the slope coefficient for such industries was significantly lower.

Now it is interesting that the researchers mentioned above have worked on different time-periods, or have asked slightly different questions, one from another. For example, R-S examined a cross-section of 395 industries for the period 1958 to 1969, in which the dependent variable is defined as the average of the proportionate changes in price from 1958 to 1969, for each industry. Thus we might describe this study as a 'longish-run' analysis. Lustgarten studied the period 1958 to 1970, using time-series and pooled data procedures. Thus he focussed in the main on year-to-year changes. For W-W-S, the period was 1958 to 1972. Their interest was cross-section analysis, but for each year separately, so that their dependent variable was the proportionate price change for a given year. D-G adopted a similar procedure, but for a shorter time-period, 1972-76. It seems that the results are sensitive to the time-scale under consideration. And yet this is no surprise since there may well be lags in adjusting to disturbances like a shift in a marginal cost schedule, or in a demand schedule. The shorter the period under consideration the more likely we are to be misled by the tactics of price adjustment, and the more likely it is that shift parameters are crowding in on one another, forcing on the industry a series of disturbances which will take time to resolve. Hence, it is likely that for simple models at least, the true relationships will emerge only from a study of longer-run price behaviour.

But that is 'true' in this context ? D-G comment that 'the underlying theoretical rationale for such effects of concentration on pricing behaviour are not, in our view, well established'. W-W-S attempt a theoretical framework which follows Eckstein and Fromm (1968) in that it sets out marginalist responses to changes in unit variable cost. Nordhaus (1970) has also set out a (long-run) marginalist framework. But these various attempts suffer from the drawback that they are rooted in the _firm_, whereas the empirical work has the _industry_ as its unit of observation. Now it is true that in the case of a pure monopoly the firm and the industry are one and the same thing; but there are no pure monopolies at the 4-digit level in manufacturing. Thus rather than focussing on theoretical predictions about _firm's_ behaviour, we must seek to establish how the _industry_ price adjusts to changes in its determinants industry-wide, hence the theoretical framework in Section I above.

IV. Data

The data set used in this study will be familiar to many researchers. Based in the Annual Survey of Manufactures, it contains information on variable costs, price, and output for a maximum of 412 4-digit manufacturing industries. The price data originated in the Division of Research and Statistics of the Federal Reserve Board. Previous empirical studies in this area have shown, arguably, that estimates of coefficients in the price equation are highly sensitive to the period under consideration, and accordingly, in order to reveal any underlying relationships this study focussed on a longer period and one of relative calm, 1958-71, excluding the disruptions of the early 1970's , such as price controls, oil shocks, etc. However, rather than year-to-year changes, this study defines its estimating variables as averages of these year-to-year changes, which is the definition used by Ripley and Segal. In terms of equation (4) above,

$$\Delta p = \frac{1}{13} \sum_{t=1958}^{1970} (P_{t+1} - P_t)/P_t$$

Unit variable cost (UVC) is the sum of payroll plus raw material costs, divided by an output index: thus

$$\Delta a = \Delta UVC = \frac{1}{13} \sum_{t=1958}^{1970} (UVC_{t+1} - UVC_t)/UVC_t \ .$$

Δg is of course not readily observed. W-W-S and D-G used as a demand proxy the proportionate change in the ratio of inventories to sales (IS), the notion being that an industry experiencing growth in demand will probably be observed to have a falling IS. If prices are sensitive to demand pressure, then price increases should be associated with a falling IS ratio. This approach has much to commend it for short-run work when industries are presumably adjusting to disequilibria, but seems less acceptable in longer run studies. Nonetheless, given the absence of good demand proxies, we use this measure, changing the sign to assist interpretation. Thus

$$\Delta g = -\Delta IS = -\frac{1}{13} \sum_{t=1958}^{1970} (IS_{t+1} - IS_t)/IS_t \ .$$

Finally, we have from the 1958 and 1967 Census of Manufactures N_{58} and N_{67}, the number of companies in each industry. Thus

$$\Delta n = \frac{1}{9} \ (N_{67} - N_{58})/N_{58} \ .$$

The results using N_{58} for the interactions in equation (6) and in equation (8) were slightly better than any other year. A variable N_{65}, the midpoint of N_{63} and N_{67} was calculated and used in the estimates. It proved slightly inferior in equations (6) and (8) to N_{58}. All reported results incorporate N_{58}.[4/]

V. Results of linear regressions

The results shown in Table 2, Column (1) indicate a solid measure of support for the relationships described in equation (6), which are based upon a linear demand regime. All the explanatory variables are significant at the 5% level (or less) with the signs as predicted. The elasticity of price with respect to UVC for a pure monopolist is .691,[5/] and for the industry with the largest value of N, SIC 2421 with N = 15371, this elasticity is 1.02. The pure monopolist translates demand shifts into price changes to a greater extent than other

TABLE 2 : Results of linear regressions of price movements in 380 manufacturing industries, 1958-71

Variables	(1)	(2)	(3)	(4)
Intercept	.005	.005	.005	.003
	(13.7)	(13.8)	(14.3)	(11.7)
ΔUVC	.691	.979	.869	
	(13.7)	(26.1)	(47.9)	
NΔUVC	.034			
	(3.9)			
ΔIS	.147	-.056	.047	
	(2.8)	(1.6)	(3.1)	
NΔIS	-.019			
	(2.8)			
ΔN	-.018	-.018	-.023	-.023
	(2.8)	(2.9)	(3.5)	(3.3)
CRΔUVC		-.002		
		(3.3)		
CRΔIS		.002		
		(3.2)		
R^2	.87	.87	.86	.03
RSS	.00924	.00914	.00968	.01129
No. of Observations	380	380	380	380

Notes: (i) in column (4) the dependent variable is ΔP-ΔUVC
(ii) absolute t values are in parentheses.
(iii) $\log N_{58}$ is used in the interaction terms in column (1).

industries. On these figures, industries with N > 1900 show no price response at all to shifts in demand. The impact of changes in the number of firms is as predicted, though quantitatively small. These results are supported by the results in column (2), which pertain to equation (7). The same general message is produced when the concentration ratio, rather than N_i is used in the interaction terms. These results confirm the finding of W-W-S, that more concentrated industries pass on less of their cost shifts, but also show, which W-W-S did not, that more concentrated industries translate more of their demand shifts into price changes.[6/]

Columns (3) and (4) show the information needed to test the validity of the joint restrictions on the coefficients given in equation (5a). The null hypothesis, that the restrictions are valid, is overwhelmingly rejected, with F(2,376) calculated to be 31.2, against a value for F at the 99% level of 6.80. Linear demand schedules are given strong support.

VI. Estimating the conjectural variation

Several authors (Gollop and Roberts, 1979, Iwata, 1974, Rogers, 1983) have attempted to estimate the conjectural variation at the level of the firm. Here we are interested in the industry level, assuming that in each industry all firms have identical conjectures. It will be noted that the conjectural variation, λ , appears as a parameter to be estimated in (4). Our procedure was to substitute alternative functional forms for λ, beginning with λ constant. We selected the function which produced the lowest residual sum of squares, and then compared this version of (4) with the approximation in (6) using a test suggested by Davidson and Mackinnon (1981) for comparing alternative models.

What sort of functional form might we expect λ to follow? A common assumption, for example in Seade, is that $\lambda'(n)$ is negative. But this is unlikely to be generally correct. Note that if $\lambda_i = dY/dy_i$, we may write

$\lambda_i = 1 + (n-1)\theta_i$, where $\theta_i = dy_j/dy_i$ for $j \neq i$, and is identical across firms. We assume $0<\theta<1$, and $d\theta/dn<0$.
θ may be described as the pure degree of collusion, equalling one for full collusion and zero for Cournot-response. It is easy to show that if $\lambda'(n)$ is to be negative then the elasticity of θ with respect to n, $\varepsilon_{\theta n}$, must be less than $n/(1-n),(n>1)$. Of course there is no reason why $\varepsilon_{\theta n}$ should lie in this range, and so $\lambda'(n)$ positive cannot be ruled out. However it turns out that λ is best described not by a monotonic function at all, but by one which falls over a short range of low values of n, and then increases with n. A function of the form

$$\lambda = \xi_0 \mid n-\xi_1 \mid ^{\xi_2}$$

was substituted for λ, where ξ_0, ξ_1, and ξ_2 are parameters to be estimated. The point about this function is that for those industries in which $n < \xi_1$, $\lambda'(n)$ is negative; the estimated value of ξ_1 represents a turning point beyond which $\lambda'(n)$ is positive.[7/] A further amendment arises from the fact that, as seen in (4), the coefficients on Δa and Δg sum to unity. Since we do not have Δg as a variable, but only its proxy, ΔIS, which may bear a rather uncertain relationship to Δg, it seems too severe to build in the above restriction in the non-linear estimation. Accordingly a new parameter, PROXY, was introduced multiplicatively with ΔIS, so that the coefficients on ΔUVC and ΔIS.PROXY sum to unity. The usual test is applied to PROXY. The final version of the relationship to be estimated is therefore

$$(8) \quad \Delta p = \gamma_0 + \Delta UVC\,\{n/(k\lambda+n)\} + \Delta IS.PROXY\,\{k\lambda/(k\lambda+n)\} - \Delta n\{\lambda n(k-1)(1-\varepsilon)/(k\lambda+n)(n+\lambda)\}$$

with $\lambda = \xi_0 | n-\xi_1 |^{\xi_2}$.

The results are shown in Table 3.

TABLE 3 : Results of non-linear estimates of parameters in equation (8)

Parameter	Estimate
γ_0	.005 (14.5)
PROXY	.283 (2.5)
k	1.056 (15.5)
ξ_0	1.651 (3.2)
ξ_1	-9.10 (39.4)
ξ_2	.537 (3.5)
RSS	.008968
No.of Observations	380

Note: absolute t statistics are in parentheses.

The results shown in Table 3 indicate that all the parameters of the model are statistically significant.[8] The conjectural variation is of the form

$$\lambda = 1.651 |n-9.10|^{.537}$$

This suggests that for the four industries with n less than 9.1, λ falls as n increases, indicating a relatively rapid fall in the pure collusion coefficient Θ. For $n \geq 10$, λ increases with n.[9] The values of λ for the first 8 industries ranked in ascending order of n are shown in Table 4.

TABLE 4: Values of λ for certain industries

SIC	n	λ
3332	5	3.52
3334	6	3.03
2895	8	1.74
2824	9	0.48
2823	10	1.56
3331	11	2.33
3333	11	2.33
2111	12	2.92

At the opposite end of the scale, SIC 2421, with 15371 companies, has an estimated value of 292.3 for λ and an implied value of Θ of 0.02. Apart from SIC 2824 the values of λ for all the industries in the sample lie above unity. Although this formulation produces a substantial improvement in the fit, it does bring with it certain problems. One should note that the elasticity of λ with respect to n is $.537n/(n-9.1)$, which for $10 < n < 20$ has a value greater than unity. Over this range, (15 industries, in fact), θ actually increases with n. It is clear that further investigation into the precise functional relationship between λ and n is merited. A further problem arises with the estimated value of k. At 1.06 it is clearly too small; k, or g/a, is equal in a linear demand regime to $1+1/|e|$ where $|e|$ is the absolute price elasticity of demand at the point where the (industry) marginal cost schedule intersects the demand schedule. A value of k nearer to 2 might have been expected. On the whole, however, the results indicate plausible values for the λ function.

Finally, a test for choosing from among alternative models was performed, the C test as described in Davidson and MacKinnon (1981). The maintained hypothesis, H_0, is the non-linear model described by equation (8). The alternative hypothesis H_1 is the linear approximation in equation (6). With $H_0: \Delta p = f(n,Z)$ and $H_1: \Delta p = g(n,Z)$ we regress

$$\Delta p_i - \hat{f}_i = \alpha_1(\hat{g}_i - \hat{f}_i) + u_i,$$ which produces $\hat{\alpha}_1 = 0.19$,

and a t-statistic of 0.82, which suggests that H_0 is "true"; reversing the procedure, and regressing

$$\Delta p_i - \hat{g}_i = \alpha_2 (\hat{f}_i - \hat{g}_i) + u_i$$ produces $\hat{\alpha}_2 = 0.81$ and a t-statistic of 3.50. The nonlinear specification is therefore preferred.

VII. Conclusions

It has been shown that a model of symmetric oligopoly can be applied with a degree of success to pricing behaviour in a wide range of industries with vastly differing numbers of constituent firms, and which are certainly not oligopolies in the conventional sense. Predictions on price formation can be generated for linear and iso-elastic demand regimes and a simple test shows that linear demand schedules are strongly preferred. Non-linear estimates allow alternative functional forms for the conjectural variation to be compared. The best formulation is one in which the conjectural variation falls with n over a few low values of n, and then rises with n thereafter. This model is shown to be preferred to a linear approximation. Several questions remain to be explored. Among the most interesting would be the effects of assuming that firms are asymmetric, and the implications of treating k as an endogenous variable.

APPENDIX

(i) $$e_a = na/(g_1\lambda + na)$$

where $\lambda = \lambda(n)$, and $d\log\lambda/d\log n = \varepsilon$

$$\frac{de_a}{dn} = \frac{\partial e_a}{\partial n} + \frac{\partial e_a}{\partial \lambda}\frac{d\lambda}{dn}$$

$$= \frac{(g_1\lambda + na)a - na^2 - g_1 na\lambda'}{(g_1\lambda + na)^2}$$

$$= \frac{g_1\lambda a - g_1 na\lambda'}{(g_1\lambda + na)^2}$$

$$= \frac{g_1\lambda a\ (1 - \varepsilon)}{(g_1\lambda + na)^2}$$

which is positive if $\varepsilon < 1$

(ii) $$e_g = g_1\lambda/(g_1\lambda + na)$$

$$\frac{de_g}{dn} = \frac{\partial e_g}{\partial n} + \frac{\partial e_g}{\partial \lambda}\frac{d\lambda}{dn}$$

$$= \frac{-ag_1\lambda + \left[(g_1\lambda + na)g_1 - g_1^2\lambda\right]\lambda'}{(g_1\lambda + na)^2}$$

$$= \frac{-ag_1\lambda + nag_1\lambda'}{(g_1\lambda + na)^2}$$

$$= \frac{-ag_1\lambda\ (1 - \varepsilon)}{(g_1\lambda + na)^2}$$

<u>which is negative if $\varepsilon < 1$</u>

(iii) $$e_n = \frac{-n\lambda(g_1 - a)(1 - \varepsilon)}{(g_1\lambda + na)(n - \lambda)}$$

$$= \frac{-n(g_1\lambda - a)(\lambda - n\lambda')}{(g_1\lambda + na)(n + \lambda)}$$

$$\frac{de_n}{dn} = \frac{\partial e_n}{dn} + \frac{\partial e_n}{\partial\lambda}\frac{d\lambda}{dn} + \frac{\partial e_n}{\partial\lambda'}\frac{d\lambda'}{dn}$$

Writing D for $(g_1\lambda + na)(n + \lambda)$, we have

$$\frac{\partial e_n}{\partial n} = \frac{D(g_1 - a)(2n\lambda' - \lambda') + n(g_1 - a)(\lambda - n\lambda')\left[a(n + \lambda) + (g_1\lambda + na)\right]}{D^2}$$

$$\frac{\partial e_n}{\partial\lambda}\frac{d\lambda}{dn} = \frac{-D(g_1 - a)n + n(g_1 - a)(\lambda - n\lambda')\left[g_1(n + \lambda) + (g_1\lambda + na)\right]\lambda'}{D^2}$$

$$\frac{\partial e_n}{\partial\lambda'}\frac{d\lambda'}{dn} = \frac{n^2(g_1 - a)}{D}\lambda''$$

This can be rearranged to give

$$\frac{de_n}{dn} = \frac{(g_1 - a)(\lambda - n\lambda')(n^2a - g_1\lambda^2 + 2g_1\lambda\lambda'n + n^2a\lambda' + g_1n^2\lambda')}{D^2} + \frac{n^2(g_1 - a)\lambda''}{D}$$

<u>which cannot be signed in general</u>

(iv) $$e_n = \frac{-\beta\lambda(1 - \varepsilon)}{(n - \beta\lambda)}$$

$$= \frac{-\beta(\lambda - n\lambda')}{(n - \beta\lambda)}$$

$$\frac{de_n}{dn} = \frac{\partial e_n}{\partial n} + \frac{\partial e_n}{\partial \lambda}\frac{d\lambda}{dn} + \frac{\partial e_n}{\partial \lambda'}\frac{d\lambda'}{dn}$$

$$\frac{\partial e_n}{\partial n} = \frac{(n - \beta\lambda)\beta\lambda' + \beta(\lambda - n\lambda')}{(n - \beta\lambda)^2}$$

$$\frac{\partial e_n}{\partial \lambda}\frac{d\lambda}{dn} = \frac{\left[-\beta(n - \beta\lambda) - \beta^2(\lambda - n\lambda')\right]\lambda'}{(n - \beta\lambda)^2}$$

$$\frac{\partial e_n}{\partial \lambda'}\frac{d\lambda'}{dn} = \frac{\beta n\lambda''}{n - \beta\lambda}$$

This can be rearranged into

$$\frac{de_n}{dn} = \frac{\beta\lambda(1 - \varepsilon)(1 - \beta\lambda')}{(n - \beta\lambda)^2} + \frac{\beta n\lambda''}{(n - \beta\lambda)},$$

<u>which cannot be signed in general.</u>

Footnotes

1/ That is, g_1 or g_2, depending upon which demand regime we are in.

2/ Δg indicates under a linear demand regime a slope-preserving shift in the demand schedule; under an iso-elastic demand regime it indicates an elasticity-preserving shift.

3/ These are that the coefficient on the omitted variable is positive, and that the correlations between the omitted variable and the explanatory variables are small.

4/ The main change was that with N_{65} the t statistic on the coefficient of $N_i \Delta IS_i$ fell to 1.6, compared with 2.8 in Table 2.

5/ Note that 0.691 is too high for the monopolist's e_a. For the case when $n=1=\lambda$, $e_a=a/(g_1+a)$ which, since g_1 must exceed a, has an upper bound of 0.5.

6/ Another issue has been the question of an asymmetrical response to cost increases versus cost decreases. We could find no such asymmetry; slope and intercept dummies proved insignificant, a result which proponents of marginalism should welcome.

7/ Instead of estimating λ, it is of course possible (and indeed attractive) to estimate Θ, substituting $1 + (n-1)\Theta$ for λ. This was tried, but produced somewhat inferior results to those reported. The turning point in λ gives it a slight edge, in terms of RSS.

8/ The value of e_a for the monopolist is now, with $n=1=\lambda$, 0.49.

9/ This highlights a problem in terminology. Seade refers to λ as the degree of collusion. But the degree of collusion so defined increases with n for most industries. It seems more appropriate to refer to Θ as the (pure) degree of collusion where $\Theta = (\lambda-1)/(n-1)$.

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