Ownership Concentration and the Theory of the Firm: a Simple-Game-Theoretic Approach Applied to US Corporations in the 1930's

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This paper is circulated for discussion purposes only and its contents should be considered preliminary.

1. INTRODUCTION

Capital market constraints on the firm are traditionally described as working through two mutually reinforcing mechanisms. First, a direct limitation on management discretion operates through accountability to shareholders. Larger shareholders are assumed continuously to monitor company performance particularly in its effect on profitability and equity values. In the event of a departure from profit maximisation they will organise to use their voting power to force changes in company policy or, in the limit, to replace the existing top-level management with one more acceptable to them. Behind this institutional threat lies the second constraint, the possibility of an increase in share concentration leading to a takeover should the share price fall low enough or the threat prove ineffective (for example if concentration is too low).

Most writers have tended to emphasise the takeover mechanism, regarding the capacity of dispersed shareholders to organise to constrain management otherwise as being weak. However, Aoki (1983) has recently pointed to evidence of a trend towards increasing concentration with the rise of institutional shareholders particularly pension funds in Europe, Japan and the United States. He concludes that "... the tendency toward a managerial revolution in the sense of wider dispersal of shareholders' control is now definitely reversed". An implication of this is that large shareholders find themselves locked in and must deal directly with management.

They cannot easily sell their holdings without adversely affecting the share price. He also suggests that the takeover mechanism is losing its role in disciplining management which fails to maximise share values.[1/]

Share concentration has an important role in managerial theories but its treatment has usually been essentially ad hoc. Most writers have pointed to low concentration in many companies and an apparent lack of organisation by shareholders as evidence of a loose constraint.[2/] This is contrasted with evidence of a tight constraint in other companies which are majority-controlled or where there is a sizeable minority holding. Moreover, little attention has been given to the possibility of shareholders forming coalitions to exert control.[3/]

Progress in dealing systematically with this question has been made by Cubbin and Leech (1983) who developed methods of measuring control on the basis of a probabilistic model of shareholder voting. This approach has subsequently been applied by Leech (1984) as part of a critique of the methodology and results of Berle and Means.

The present paper sets out to do four things. First, it applies the theory of simple games to the measurement of share concentration and shareholder control. Second, it presents a static theory of the firm with an explicit shareholder constraint. Third, it develops a dynamic theory of the firm in which share concentration is determined rationally within a model of owner control. Fourth, the method of measuring shareholder control is applied to the TNEC data set for the largest 200 non-financial corporations in the United States in 1937, allowing direct comparisons to be made with the findings of Berle and Means.

2. OWNERSHIP CONCENTRATION AND MINORITY CONTROL

That a consensus has never been reached on the prevalence of management control is a reflection in part of a lack of an adequate theoretical framework for modelling the relation between control and share concentration. This has meant that the findings of surveys aimed at assessing the extent of the managerial revolution have tended to appear arbitrary and inconclusive. In part it is also due to differences in the type and quality of data used.

We distinguish two types of approach among the relatively small number of surveys which have been conducted for the US and UK. On the one hand, the approach of Berle and Means was based exclusively on measurement of share concentration with little attempt at direct observation of control. Management control was inferred from an absence of an arbitrary level of share concentration. Although the quality of the share ownership data used by Berle and Means was very poor, it was the sole basis of their study. The same method was later used by Larner(1970) with good ownership data and these results have been widely cited as confirmation of the trend towards a managerial revolution postulated by Berle and Means. The other group of studies have emphasised the need, not only for accurate statistics of ownership concentration, but also to widen the scope to include direct observation of indicators of the type of control. These studies have been closer to a case-study approach with information collected on the identities of directors and relationships between them and leading shareholders as evidence of control based on ownership. In this group are the surveys by TNEC (1940) and later by Burch (1972) for the US and Florence

(1961) for the UK. Despite their differences of approach and results they have not had the same impact as those in the first group. More recently Nyman and Silberston (1978) have used a similar type of data and argued for a case-study approach.

The standard approach to determining the relationship between the shareholding distribution and control has been to infer owner control from the existence of a large concentration of ownership whether a single individual, a family, a group of families, a corporation or a financial institution. The precise level of concentration assumed to give control has varied between five percent and twenty percent. Studies in the second group have also allowed the existence of controlling coalitions which are not observed in the shareholding data by looking at the concentration of groups of holdings, for instance the largest ten or twenty.

The results are obviously very sensitive to assumptions about the precise figure chosen. Once we allow that control can exist without a majority holding, then there is a need for a model of the relation of power to shareholding. All the studies mentioned have made assumptions about this, if only implicitly. Florence (1961), however, discusses this question and anticipates the game-theoretic approach adopted in the present paper: "the bulk of the shareholders with middle-sized as well as small holdings are usually of such a quantity and quality that they cannot and do not form a coherent party. Most of them do not vote and, if and when they do, their votes are as likely to go one way as another - to be scattered indifferently. In this situation the 'resolute' person or a small coherent 'resolute' group of persons determined on a certain policy or certain key appointment such as that of directors, could win, even with a concentration of a minority of voting shares as low as, say 10%".

Allowing control to be held by a coalition of leading shareholders rather than a single shareholder raises two distinct questions. First, is the shareholding distribution sufficiently concentrated to give the coalition enough voting power to have control? Second, does the coalition have the capacity to form itself into a 'resolute' group capable of organising to take and exercise control? The first question is analysed using the theory of simple games in the next three sections. The second question is analysed by consideration of the costs of organisation and control. These two factors are then put together to form the basis of a theoretical perspective on the firm. In this theory firms are classified into management-controlled and minority-controlled. But we can equally well employ the dichotomy in Williamson's (1964) managerial discretion model. The firm is assumed to maximise a managerial utility function subject to an inequality constraint on the level of profits. Qualitative differences between firms arise depending on whether this constraint is satisfied as an equality or a strict inequality.

3. SIMPLE GAMES AND MULTILINEAR EXTENSIONS

We consider a shareholder voting game with characteristic function V whose domain is the 2^N subsets of the set of players $N = \{1,2,...,N\}$. (The symbol N is here used interchangeably for the set of players and the number of players.) The function V has the properties:

$V(\phi) = 0, \quad V(N) = 1, \quad V(S) = 1$ if S is a winning coalition

$V(S) = 0,$ if S is a losing coalition

$V(S \cup \{i\}) \geq V(S) + V(\{i\})$ if $i \notin S$.

Values of such games have been defined in terms of the Shapley value (the Shapley-Shubik power index) and the Banzhaf power index. Both approaches have been applied to shareholder games by the present author (Leech (1985)). Although the two indices have important differences in their mathematical properties and exhibit widely differing asymptotic behaviour, they appear to agree closely in empirical application to shareholder-voting games.

The Shapley-Shubik index is a N-vector of values whose i^{th} element is given by the well-known formula:

$$(1) \qquad \phi_i = \sum_{\substack{S \subset N \\ i \notin S}} \frac{s!N-s-1!}{N!} \left[V(S \cup \{i\}) - V(S)\right], \quad i = 1,...,N$$

where s is the number of members of coalition S. Its interpretation is as a measure of the likelihood that player i will be able to change a coalition from losing to winning. The Banzhaf index has a similar interpretation but assumes a different system of coalition formation.[4/] See Leech (1985) for further details.

Direct computation of these indices is not possible except where the number of players is very small. In empirical applications where N is large good approximations can be obtained using Owen's method of multilinear extensions (Owen (1972)).

The domain of the characteristic function V is the set of corners of the unit N-cube which define all possible voting outcomes. The multilinear extension is a function whose domain is the whole cube which corresponds to V at the corners. Specifically, the multilinear extension is the function f on I^N where $I = [0,1]$, defined by

$$(2) \qquad f(x_1,\ldots,x_N) = \sum_{S \subseteq N} \left\{ \prod_{i \in S} x_i \prod_{i \notin S} (1-x_i) \right\} V(S)$$

for $0 \leq x_i \leq 1$, $i = 1,\ldots,N$. At a particular voting outcome S we have $x_i = 1$ if $i \varepsilon S$ and $x_i = 0$ otherwise and therefore $f = V(S)$.

The multilinear extension can be interpreted probabilistically. If player i has probability x_i of joining coalition S then the term in braces is the probability that S will form, assuming independence. The multilinear extension can therefore be thought of as the expected value of a random coalition. If one player, say j, is assumed to belong to every coalition, i.e. $x_j = 1$, then f can be interpreted as the probability that the majority coalition will include player j.

Empirical calculations of the power indices are based on different probabilistic interpretations of the multilinear extension with specific assumptions about the probabilities x_i. Let the i^{th} partial derivative of (2) be denoted by $f_i(x)$ where $x = (x_1, \ldots, x_N)$. The multilinear extension (2) can be written:

$$(3) \qquad f(x) = x_i \sum_{\substack{S \subset N \\ i \notin S}} \prod_{j \in S} x_j \prod_{\substack{j \notin S \\ j \neq i}} (1-x_j)\, V(S \cup \{i\})$$

$$+ (1-x_i) \sum_{\substack{S \subset N \\ i \in S}} \prod_{\substack{j \in S \\ j \neq i}} \prod_{j \in S} x_j\, (1-x_j) V(S) .$$

Differentiating (3) gives:

$$(4) \qquad f_i(x) = \sum_{\substack{S \subset N \\ i \notin S}} \prod_{j \in S} x_j \prod_{\substack{j \notin S \\ j \neq i}} (1-x_j) \left[V(S \cup \{i\}) - V(S)\right] .$$

The Shapley value is obtained from (4) on setting $x_j = t$ for all j and inte grating t out. Thus, (4) becomes:

$$(5) \qquad \int_0^1 f_i(t,t,\ldots,t)dt = \sum_{\substack{S \subset N \\ i \notin S}} \int_0^1 t^s(1-t)^{N-s-1} dt \left[v(S \cup \{i\}) - v(S)\right].$$

The integral in (5) is well known and we can write

$$\int_0^1 t^s(1-t)^{N-s-1}dt = s!(N-s-1)!/N!.$$

Substituting into (5) gives the Shapley value, (1),

$$\phi_i = \int_0^1 f_i(t,t,\ldots,t)dt .$$

Expression (4) has a probabilistic interpretation as the expected incremental value of a random subset with the addition of player i. For simple games it can be interpreted as the probability that the addition of player i to a random subset changes it from losing to winning. The Shapley-Shubik index is obtained by assuming a common probability and integrating it out.

The Banzhaf index can also be obtained from (4) on setting $x_j = 1/2$ for all $j \neq i$. This gives the non-normalised form of the index as the probability that the addition of player i to a random losing subset will make it winning assuming all subsets are equiprobable.

The theory of multi-linear extensions is set out in Owen (1972 and 1975). Applications of the power indices to shareholder-voting games using real-world data are described in

Leech (1985).

The theory developed in the remainder of the present paper can be developed in terms of either of these power indices and is completely general. Our prime concern, however, is with the measurement of the voting power of coalitions of leading shareholders and analysing how power changes as the size of the leading coalition changes. Moreover, since we are concerned with the question of control rather than the overall distribution of power, it is unnecessary to calculate the power index for each shareholder. Our approach, especially in the empirical section, uses a simplified measure of power which has the interpretation as the proportion of voting outcomes in which the leading coalition is in the majority.[5/] This is described in the next section.

4. CONCENTRATION AND THE POWER OF COALITIONS

We initially take a simplified view in which the number of holdings is fixed. No distinction is made between the right to disposal of voting rights and beneficial ownership (which would be relevant, for example, in the case of a trust or pension fund). Attention is restricted to examining the relationship between control over the rights to shareholdings (including the right to buy and sell) and power within the firm.

Symbolically, there are N holdings represented by the ordered sequence $p_1, p_2, \ldots, p_N$ where $p_i \geq p_{i+1}$ for all i and $\Sigma p_i = 1$. Concentration is measured by the function $P(n) = \sum_{i=1}^{n} p_i$ which relates the proportion of votes held by a coalition of n leading shareholders to n. This concentration curve is represented graphically in Figure 1 for both an equally distributed case and a general case. For graphical convenience it is drawn as a continuous curve.

Each leading coalition [6/] defines a game [7/] with $N-n+1$ players, assuming members of the coalition vote as a single bloc, with weights $P(n), p_{n+1}, \ldots, p_N$. The power index for the coalition, $\alpha(n)$, can be obtained for each n. Formally, using a probabilistic interpretation of the multilinear extension, we assume subsets of players to be formed at random all of which include the leading coalition. If each subset (voting outcome) is regarded as equiprobable then this is equivalent to setting $x_i = 1/2$ for all $i \neq 1$ in the function f. Thus $\alpha(n) = f(1, 1/2, 1/2, \ldots, 1/2) = \Pr\left[P(n) + \sum_{\substack{i \in S \\ i > n}} p_i > 1/2\right]$.

It is apparent that the power index has the property that $1/2 \leq \alpha(n) \leq 1$.

For large N this probability can be found easily by assuming player i contributes a number of votes q_i according to a probability distribution whereby $q_i = p_i$ and $q_i = 0$ with equal probability. The votes of player i have mean $p_i/2$ and variance $p_i^2/4$. Denote the votes of subset S by $q(S)$. Then $q(S) = P(n) + \sum_{i=n+1}^{N} q_i$ is a random variable with mean $P(n) + \frac{1}{2} \sum_{i=n+1}^{N} p_i = P(n) + \frac{1}{2}(1-p(n)) = \frac{1}{2} P(n) + \frac{1}{2}$. Its variance is $\frac{1}{4} \sigma(n)^2$ where $\sigma(n)^2 = \sum_{i=n+1}^{N} p_i^2$. Moreover, it is normally distributed. Letting Φ denote the normal distribution function, $\Phi(a) = Pr[z < a]$ where z is a standard normal random variable, then

$$(6) \qquad \alpha(n) = Pr\left[q(S) > \frac{1}{2}\right] = 1 - \Phi\left(\frac{\frac{1}{2} - \frac{1}{2}P(n) - \frac{1}{2}}{\frac{1}{2}\sigma(n)}\right)$$

$$= \Phi(P(n)/\sigma(n)).$$

The power index $\alpha(n)$ is formally calculated as the probability that a leading coalition of size n can win a majority if all voting outcomes are equally probable.[8/] Another interpretation is as the proportion of voting outcomes which produce a majority for the coalition. On this latter interpretation probability calculus is required only for computational purposes to provide an algorithm for empirical application. It is not necessary to assume random behaviour. The game theory analysis is not concerned with behaviour but with the measurement of power in a fundamental, strategic sense.[9]

The index $\alpha(n)$ has close affinities with the Banzhaf power index which measures the likelihood of a "swing" for each player in the sense of that player being the marginal member of a minimally winning subset. The Banzhaf index is defined for each player and can be normalised to provide a distribution of power among the players. Since the present analysis focuses on the power of leading coalitions as a function of coalition size, normalisation is unnecessary. There is a simple relation between $\alpha(n)$ and the non-normalised Banzhaf index (the "swing probability"), $\beta(n)$, say. It is shown in Leech (1985) that $\beta(n) = 2\alpha(n) - 1$. There is no advantage to using $\beta(n)$ in the present application, however, since we define control below in terms of an extreme value of the power index close to unity and for such values $\beta(n) \approx \alpha(n)$.

5. POWER AND CONTROL

Power curves corresponding to the two concentration curves are shown in Figure 1. Consider first the equal distribution AB. Since $p_i = 1/N$ for all i, all shareholdings individually have equal voting power. Voting outcomes in which the majority includes a coalition of size 1 are one half the total and therefore $\alpha(1) = 1/2$. Obviously a coalition of size $N/2$ is a majority coalition, $P(N/2) = 1/2$ and $\alpha(N/2) = 1$. The power curve in this case is therefore DE. Assuming an increase in concentration with constant N gives the curve BC. The size of the majority leading coalition is reduced to n' where $P(n') = 1/2$, and $\alpha(n') = 1$. Also $\alpha(1) > 1/2$ and the power curve is now FG. Concentration has reduced the size of a majority leading coalition and increased the power of each leading coalition.

The power function is the basis of the definition of control. Allowing the possibility of minority control means that a controlling coalition can be defined with a power index of less than unity (working or factual control). Letting this value of the power index be $\alpha^* < 1$, the size of a controlling coalition must satisfy $\alpha^* \leq \alpha(n)$. In Figure 1 the two minimal controlling coalitions are of sizes $\bar{n}$ and n^* respectively.

Figure 2 shows the case where concentration increases and N falls as the largest shareholder buys out others. The initial distribution is represented by AB with N_0 holdings.

~~The concentration curve CD represents the distribution~~ which results after the largest shareholding has increased by the addition of a group of large holdings.

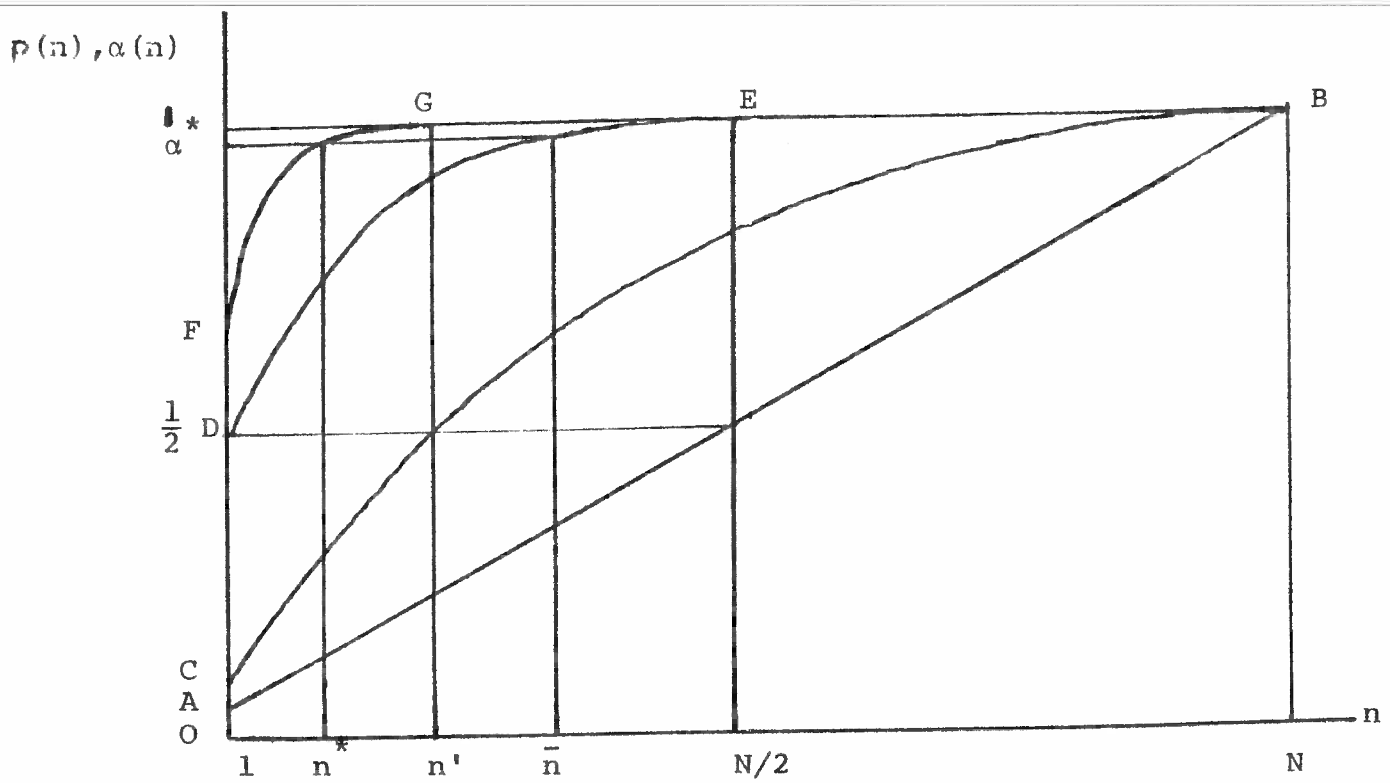

FIGURE 1 : The concentration curve and the power curve

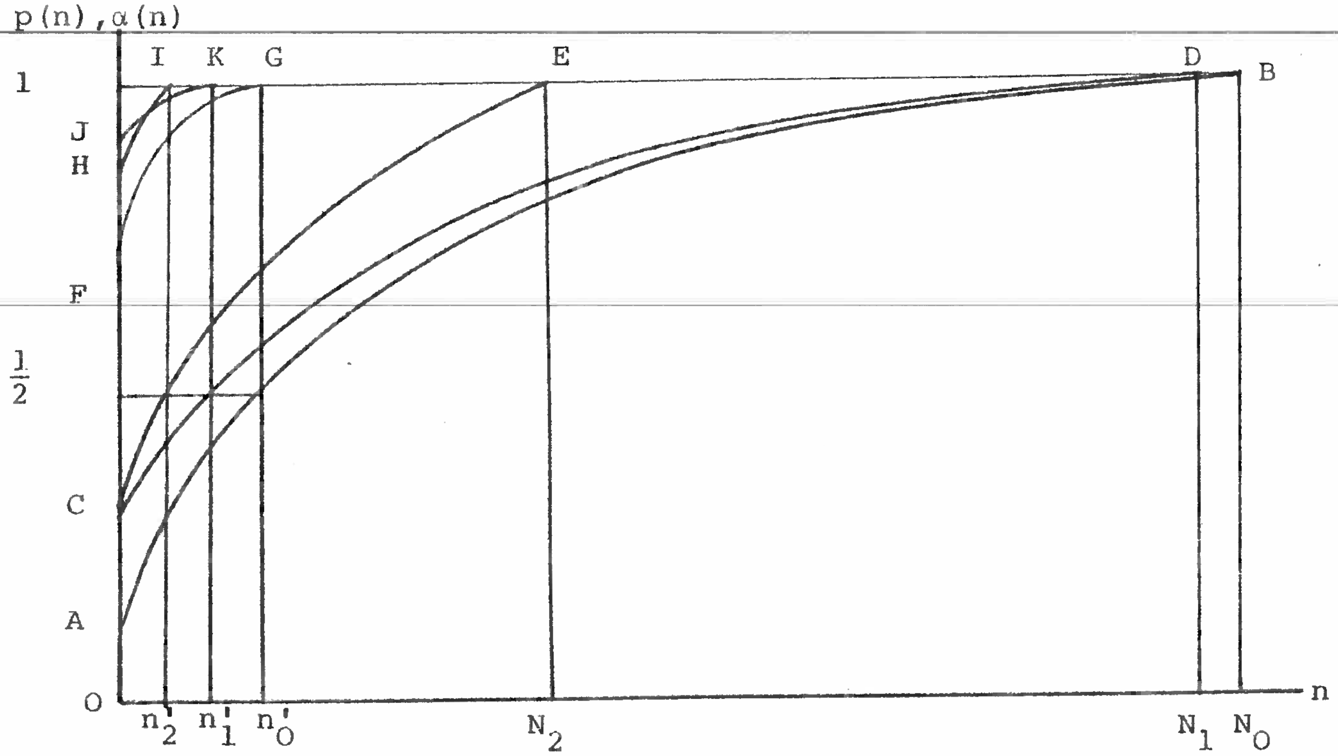

FIGURE 2 : Changes in concentration with changing N

The number of holdings has reduced to N_1 and the concentration curve shifts to the left. The corresponding power curve moves left from FG to JK. The opposite case where the largest holding increases by the same amount by acquisition only from the lower end of the size distribution is represented by the curve CE. In this case the number of holdings falls to N_2 and the concentration curve shifts vertically. The associated power curve is HI.

Comparing the two post-redistribution power curves, JK and HI, two features are apparent. First, the greater enhancement in the power of the single largest holding occurs in the former case (amalgamation of large holdings). Second, the power curve HI is steeper than JK, which means that it is impossible to rank the two redistributions in terms of their effect on the power of leading coalitions in general.

6. COSTS OF ORGANISATION AND CONTROL

The previous three sections have developed a definition of owner control based on the theory of simple games. A controlling coalition always exists, its size depending on the concentration of the shreholding distribution and the value of the power index assumed to correspond to control. This analysis therefore has no economic implications without consideration of the feasibility or costs of forming such coalitions. Such costs also depend on concentration of ownership. 10/

Associated with each leading coalition is a level of cost incurred in its formation and organisation into a cohesive group and which would be incurred if it had control. The cost function is written $c(n) = f + v(n)$ and is assumed to have fixed and variable components.

Fixed elements comprise costs incurred by controlling shareholders in the overall direction of the firm together with the fixed component of the costs of organising and maintaining a controlling coalition. These include

(i) Information costs of monitoring and evaluating strategic policies with regard to investment, diversification, research and development, marketing, financial structure, etc.

(ii) Costs incurred in the supervision of management to enforce accountability to shareholders.

(iii) The fixed component of the organisation of a controlling coalition. These include costs of shareholder relations, shareholders' meetings and proxy machinery.

These costs are likely to depend on the type of shareholders which constitute the leading coalition. Fixed costs of control are lower where the leading shareholders are companies or financial institutions than where they are individuals.

The variable costs reflect the feasibility of the formation of leading coalitions and are an increasing function of coalition size. These costs depend on the capacity of leading shareholders to collude and depend on sociological as well as organisational factors. They are probably unquantifiable in practice even with good quality shareholding data or in a case study. However, qualitative statements about them can be made. They will be considerably reduced by personal contact which allows the perception and pursuit of common interests. Thus they will be lower for a coalition which contains members of the same family or associated families than for one consisting of unrelated individuals. They will be lower for a coalition of similar financial institutions. These costs may be reduced by interlocking directorships which facilitate personal contact and the identification of community of interests. Costs of organisation include the cost of communication and of disseminating information about the firm among members of the coalition. It is assumed that variable costs of organisation increase steeply beyond a certain coalition size as communication and the perception of common interests becomes more difficult with the inclusion of more members.

7. STATIC THEORY OF THE FIRM

It is assumed, in the tradition of the managerial theories, that managers and shareholders have fundamentally different and conflicting interests in the firm. Shareholders' utility depends exclusively on the market value of the firm while managers are willing to trade-off profits in the pursuit of other goals: staff expenses, sales or growth. The benefit accruing to shareholders from control under these assumptions is the difference between the maximum attainable value of the firm and whatever its value would be under unrestricted management control, gross of an allowance for the costs of organisation and control. This reduction in equity value resulting from managerial discretion is denoted by D and the benefit function for a leading coalition of size n is $B(n) = DP(n)$. The cost of organisation and control incurred by members of that coalition is $C(n)P(n)$. (It is assumed that these costs, being incurred only when the leading coalition has control, are charged to the firm.)

Figure 3 illustrates the case of a minority-controlled firm. Given the concentration curve and the control criterion α^*, a controlling coalition is of size n^*. The cost of organisation and control for such a coalition would be $c(n^*)$ and the coalition's share in such costs $c(n^*)P(n^*)$ which is represented by BC. This coalition would form and take control in the case where the unrestricted pursuit of managerial objectives would otherwise reduce share values by D_1 giving rise to the benefit function $B_1(n)$. The net benefit to the controlling coalition is therefore AB. Shareholder control will result in a reduction in the discretionary element to D_2. This model is equivalent to a Williamson model of managerial discretion with a binding constraint on profits.

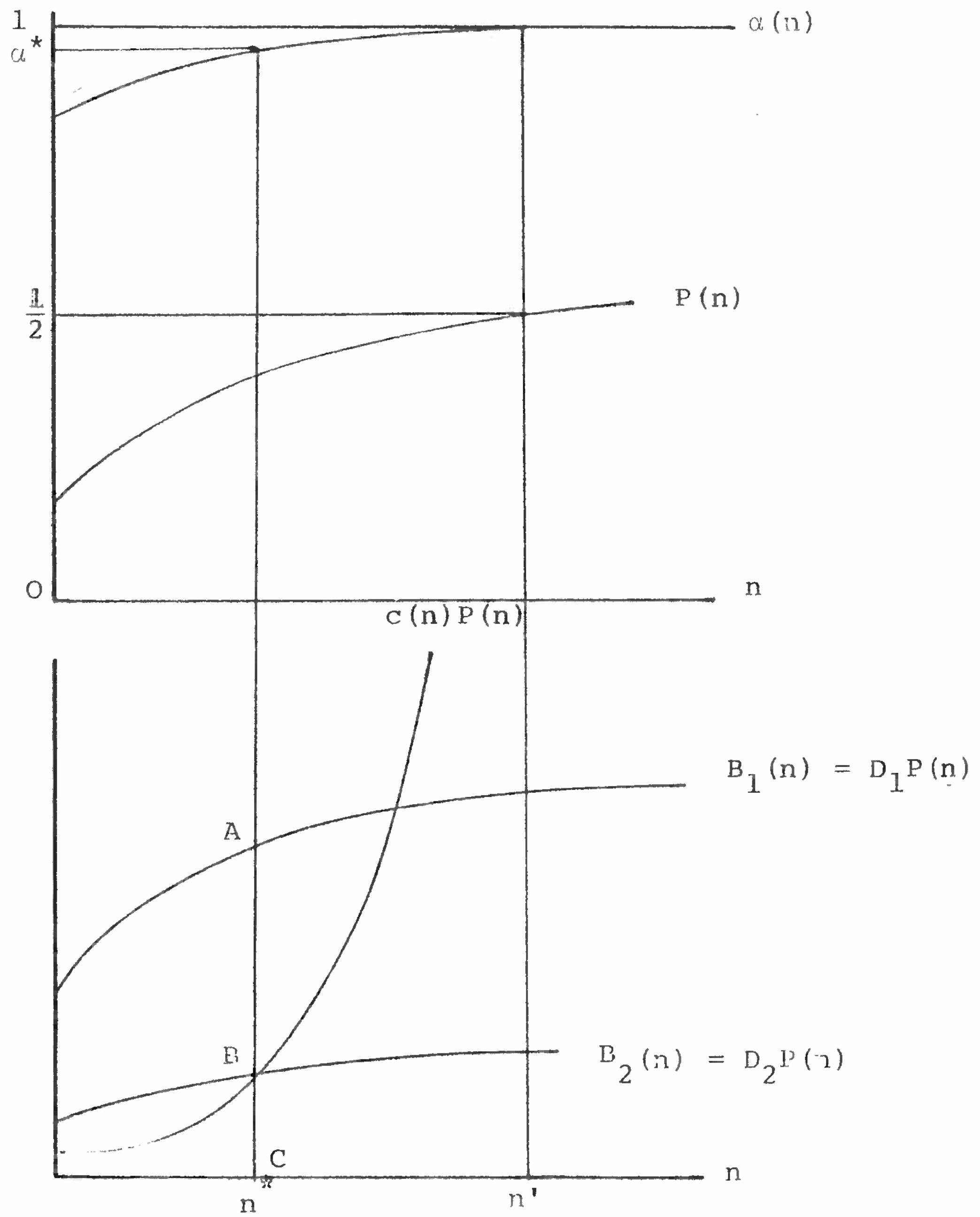

FIGURE 3 : Minority Control

Figure 4 shows the case of management control or alternatively a firm with a non-binding profit constraint. Either share concentration is lower or the costs of organisation and control are higher. The benefit function is $B_1(n)$ and for a controlling coalition, the costs exceed the benefits. The shareholder constraint is inoperative as long as the discretionary element remains less than D_3. The constraint on management is $D \leq C(n^*)$.

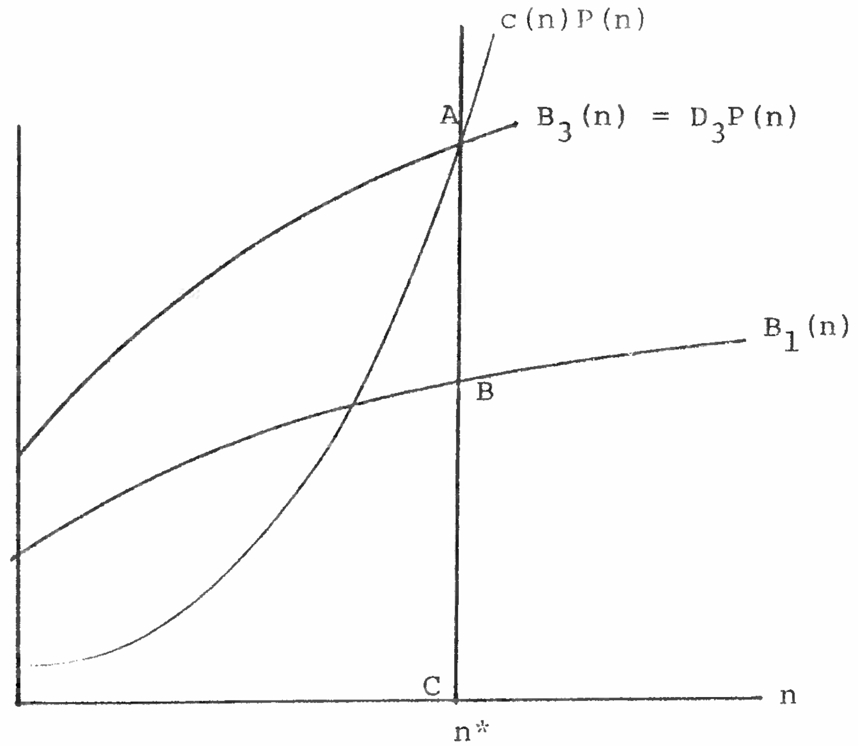

FIGURE 4 : Management Control

8. REMARKS ON TAKEOVER

The managerial theories assume the existence of a "security constraint" on profitability reflecting the threat of takeover if the share price falls below some level. Little attention has been given to constraints from shareholders of the type suggested in this paper. In the absence of a single large shareholding, the takeover threat is the only capital market response to non-value-maximising behaviour usually described. However, allowing the endogenous formation of shareholder coalitions, as in the previous section, enables takeovers to be viewed within the same perspective as ownership concentration.[11/]

The essential, defining feature of the managerial theories is their emphasis on an assumed divergence of interests between managers and owners. Relative to this, differences in utility functions among shareholders are regarded as unimportant. However, an implication of the assumption of shareholder unanimity for the theory of takeovers as a constraint on management is that it is not necessary for a takeover raider to acquire a fifty percent holding. It is sufficient for the raider to acquire a large enough holding to permit the organisation of a controlling coalition. The takeover can be viewed as a particular means by which ownership concentration, which has fallen too low to permit owner control, can be restored through outside intervention. This view, which is a logical implication of the conditions assumed by managerial theories, is in contrast with the takeover as a means by which one company acquires another which is highly profitable. In the latter case there is conflict of interests between existing shareholders and the takeover raider and the latter must acquire a majority holding for control.

Figure 5 represents a takeover within this model. The initial concentration curve is represented by $P_0(n)$ with associated power curve $\alpha_0(n)$ and controlling coalition of size n_0^*. In the lower diagram the costs and benefits of shareholder control are in aggregate rather than accruing to leading coalitions as before. The firm is initially management-controlled since $c(n_0^*) > D$ and we envisage a takeover by an outsider acquiring shareholdings on the open market, increasing the size of the largest holding from $P_0(1)$ to $P_1(1)$.[12/] The two opposite cases discussed in section 5 above are considered : acquisition of predominantly large holdings giving the concentration and power curves P_1 and α_1; and acquisition of predominantly small holdings giving the curves P_2 and α_2. The diagram is drawn with the intersection of the two power curves above the level of the control criterion α^* and therefore $n_2^* > n_1^*$. The increase in concentration is enough to make $c_0(n_2^*) = D$ and therefore give shareholder control by acquisition of small holdings. Whether shareholder control emerges in the other case depends on the effect on the cost function, since amalgamating large holdings has the effect of increasing the variable cost element. In Figure 5 it would not give control since $c_1(n_1^*) > D$. However, if the initial cost function c_0 were less steep this would make c_1 less steep and we would have $c_1(n_1^*) < D$. It is therefore not possible to make general qualitative statements about the relative effectiveness of takeovers in the two polar cases considered.

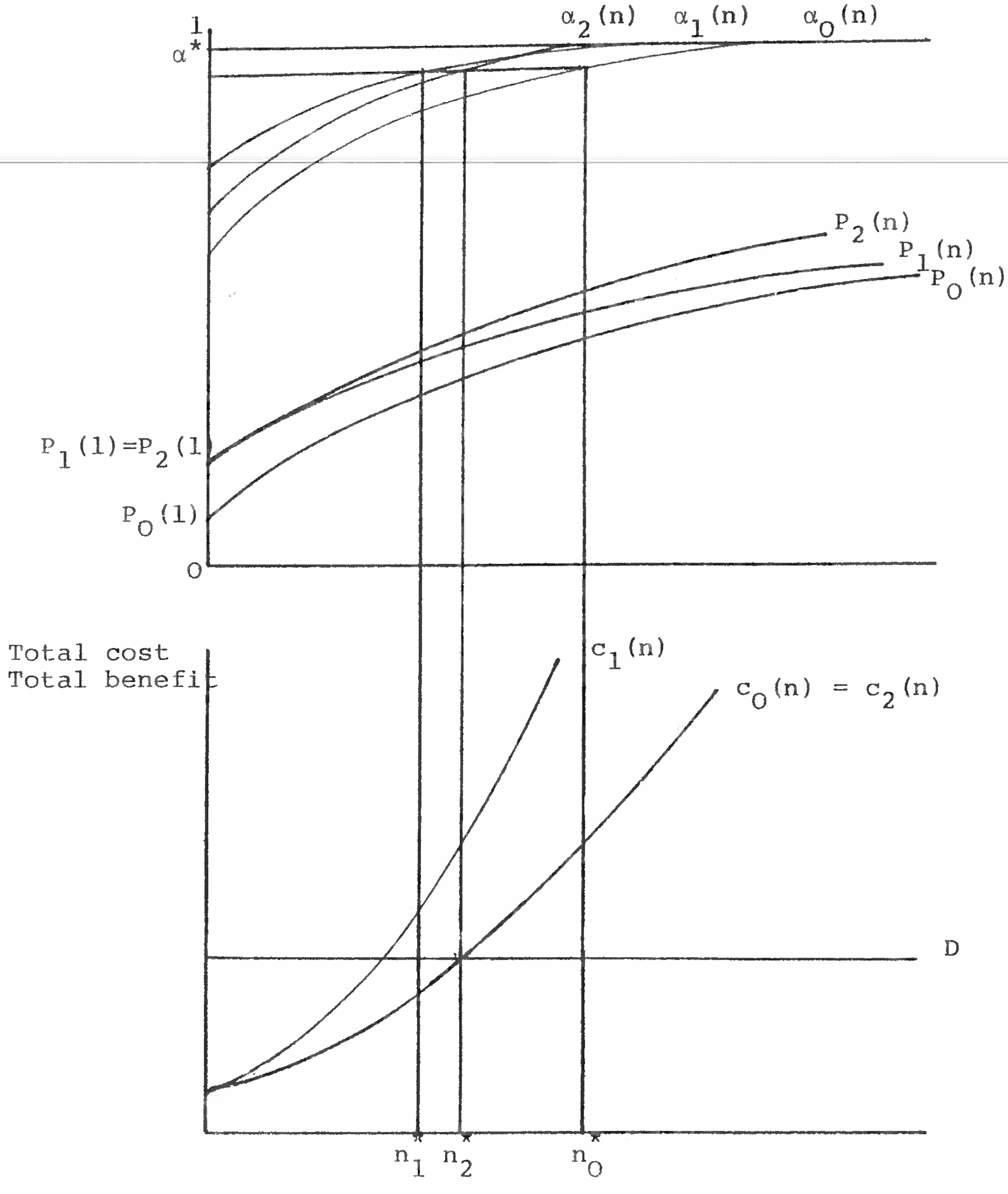

FIGURE 5 : Takeovers

9. DYNAMIC THEORY

We consider the problem faced by an owner-controlled firm with profitable investment opportunities requiring the raising of new equity finance. Existing shareholders may be unable or unwilling to supply this additional capital either because of limitations to their personal wealth or their need to maintain a sufficiently diversified portfolio. Expansion therefore entails new shareholders and a consequent loss of control, in some sense, by the initial controlling group.

The relevance of this for the theory of the firm depends on the nature of the loss of control. A reduction in share concentration can produce a loss of control to management or to a new controlling coalition of larger size. In the latter case the firm remains owner-controlled and the loss-of-control aspect of the expansion can be built into a dynamic optimisation model.

The effect of expansion on the power curve depends on the distributional changes which occur and which result from the manner in which new capital is raised. In general these changes will also affect the costs of organisation and control. In this section we define the problem as the optimal determination of the size of the controlling coalition and we assume that expansion is accompanied by a reduction in concentration.

Suppose the firm is initially controlled by a minority coalition of size n_O. The concentration curve is $P_O(n)$, $1 \leq n \leq N_O$ and the holding of the i^{th} shareholder is p_{Oi}. The value of the power index associated with control is $\alpha^* = \alpha_O(n_O)$, where $\alpha_O(n)$ is the power curve associated with P_O. The initial share capital is K_O and new shares are issued to increase the capital to $K_g = (1+g)K_O$, a growth rate of g. The new concentration curve is $P_g(n)$, $1 \leq n \leq N_g$ where $N_g - N_O$ is the increase in the number of shareholdings. Individual holdings are now denoted by p_{gi}. The controlling coalition is now of size n_g, defined by $\alpha^* = \alpha_g(n_g)$.

This does not allow us to define the problem completely, however, since n_g depends on the form of the post-expansion distribution described by P_g. Further assumptions are required about the way new share capital is raised in terms of its effect on the concentration curve. For simplicity, we assume that the effects of expansion on concentration are predictable, leading to an unambiguous reduction in concentration and that expansion has no effect on the costs of organisation and control. The second condition would not be met in general if there was a change in the size ranking of shareholders, most particularly at the higher end of the distribution. A clear case which would meet both requirements wouldbe one in which the nominal value of existing holdings remained unchanged and all new holdings were small: $p_{gi} \leq p_{gN_O}$, for all $i > N_O$ and for all g.

Under these assumptions there is a unique relationship between the size of the controlling coalition and the growth rate : $g = g(n_g)$.

Expansion opportunities are described by a profit function relating profits (gross of the costs of organisation and control) to the growth rate. This function, $\Pi(g)$ is assumed to have the general shape illustrated in Figure 6, first increasing sharply and then falling beyond some point.

FIGURE 6 :

The problem facing the controlling coalition can now be defined as that of maximising its own profits with respect to g, or equivalently n_g. That is, the objective function is

$$P_g(n_0)\left[\Pi(g(n_g)) - C(n_g)\right].$$

Diagrammatically, this problem is represented in Figure 7.

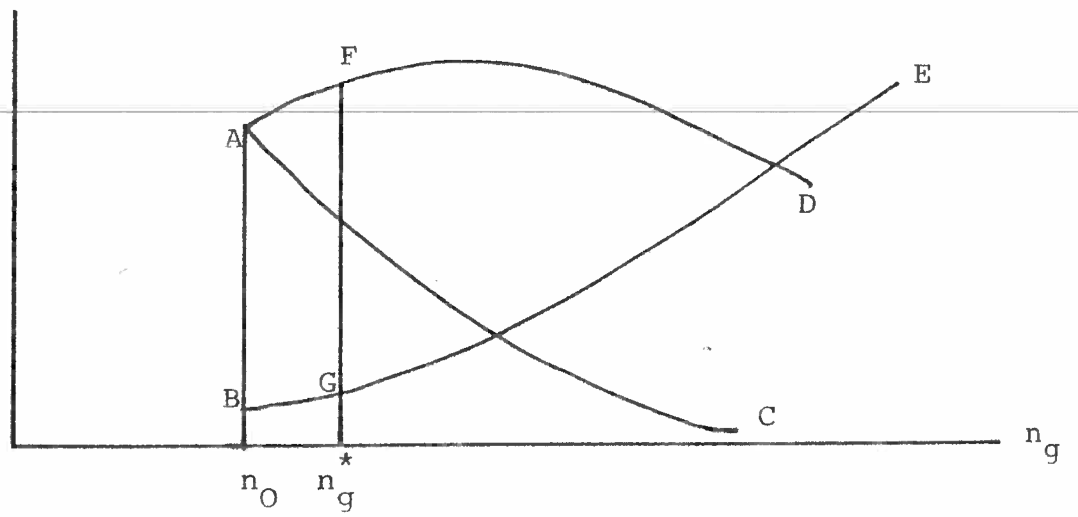

FIGURE 7 : Optimal Expansion of Controlling Coalition

The initial position is at point A with a net profit accruing to the coalition of AB. Expansion implies a loss of control and a reducing proportional holding of the initial coalition. Curve AC represents the gross profits accruing to the initial coalition allowing for the distributional changes (and loss of control) but without assuming any growth. AC is therefore the quantity $P_g(n_0)\Pi(0)$ plotted against n_g. The curve AD is the coalition's share of the profits from growth, $P_g(n_0)\Pi(g(n_g))$ and BC is $P_g(n_0)C(n_g)$.

The optimum expansion implies a controlling coalition size of n_g^* (a growth rate of $g^* = g(n_g^*)$ with net profits of

FG defined by the first order condition that the slope at F equals the slope at G. The second order condition requires the second derivative at F to be less than the second derivative at G.

This model describes the optimal expansion of a minority-controlled firm. It is also necessary to analyse the transition from majority control to minority control which occurs earlier in the life cycle of the firm. It is obvious that this requires only a minor modification in terms of an appropriate choice of the control criterion α^*. Before expansion a majority-controlled firm has $P_0(n_0) \geq 1/2$ and $\alpha_0(n_0) = 1$. After expansion, $\alpha_g(n) < 1$ for relevant values of g and n. The initial controlling coalition must therefore choose a value for α^* which defines a limit to the loss of control it is prepared to experience.

10. A SPECIAL CASE

A special case occurs under the assumptions made above: (i) that existing shareholdings are constant in nominal terms; (ii) that all new shareholdings are small: $p_{gi} \geq p_{\alpha N_0}$ for $i > N_0$. For illustrative purposes we also assume: (iii) that all new holdings are equal : $p_{gi} = p_g$ for $i > N_0$. The new shareholding distribution is

$$p_{gi} = \begin{cases} p_{0i}/(1+g) & 1 \leq i \leq N_0 \\ p_g & N_0 < i \leq N_\alpha \end{cases}$$

and $p_g(n) = p_O(n)/(1+g)$ for $n \leq N_O$. Since all expansion is financed by new shareholders, $K_g - K_O = (N_g-N_O)p_g K_g$ and therefore $(N_g-N_O)p_g = g/(1+g)$.

A controlling coalition is defined by the condition $\alpha^* = \alpha_g(n_g)$ and therefore $P_g(n_g) = z^* \sigma_g(n_g)$. We can now obtain n_g in terms of the characteristics of the pre-expansion concentration curve and the scale of expansion, g. If $n < N_O$, then

$$\sigma_g(n)^2 = \sum_{i=n+1}^{N_O} p_{gi}^2 + (N_g-N_O)p_g^2$$

$$= \frac{\sigma_O(n)^2}{(1+g)^2} + (N_g-N_O)p_g^2 \ .$$

This expression depends on the value of p_g. Bounds on $\sigma_g(n)^2$ can be obtained by considering the most and least concentrated cases. The most concentrated case has $p_g = p_{gN_O} = p_{ON_O}/(1+g)$. Therefore, $(N_g-N_O)p_g^2 = g p_{ON_O}/(1+g)^2$. The least concentrated case is the limiting case where the number of shareholders becomes infinitely large and p_g goes to zero. Therefore,

$$(N_g-N_O)p_g^2 = gp_g/(1+g) \rightarrow 0 \quad \text{as} \quad p_g \rightarrow 0.$$

Hence the bounds on $\sigma_g(n)^2$ are

$$\sigma_O(n)^2/(1+g)^2 < \sigma_g(n)^2 \leq [\sigma_O(n)^2 + gp_{ON_O}]/(1+g)^2.$$

Taking the upper bound value, the controlling coalition can be defined by

$$P_O(n_g) = z^*(\sigma_O(n_g)^2 + gp_{ON_O})^{1/2}$$

which defines a unique monotonic relation between g and n_g. Thus the problem facing the initial controlling coalition is then to maximise its own profits with respect to g (or n_g) subject to the continued existence of a controlling coalition after expansion. That is,

$$\max \frac{P_O(n_O)}{1+g} [\Pi(g) - c(n_g)]$$

subject to $P_O(n_g) = z^*(\sigma_O(n_g) + gp_{ON_O})^{1/2}$.

11 OWNERSHIP CONCENTRATION AND CONTROL IN US CORPORATIONS IN THE 1930's : THE TNEC DATA

The study by the Temporary National Economic Committee (TNEC) of the distribution of ownership in large US corporations, TNEC (1940), remains a unique data source. Although it is now out of date as a description of reality, it has historical value as a parallel study to that of Berle and Means who analyses substantially the same sample of companies in terms of control. Although the ownership information available to Berle and Means was seriously incomplete, their results are still widely quoted in the literature on the managerial revolution. A partial reassessment of their results has been made by the present author, Leech (1984), but this has been limited by the paucity of useable data provided in their original study. The TNEC study on the other hand provides high quality ownership distributions for each company. For these reasons of comparability with Berle and Means (and therefore current relevance) and availability of good data we feel it is worthwhile to perform a secondary analysis of the TNEC study. In this section we provide a survey of ownership control in the top 200 nonfinancial corporations using the game-theoretic methods developed above.

As mentioned above in section 2 the TNEC study differed from that of Berle and Means in several respects besides that of having statutory powers of data gathering. As a study of corporate control (rather than the distribution of the ownership of corporate wealth), its perspective was very different. It was primarily concerned with identifying and describing centres of ownership control rather than finding evidence of the extent of the managerial revolution. As a result their classification of control was very different, employing three categories : family control (by one family or an interest group comprising several families or a group of business associates); control by other corporations, and; companies with no centre of ownership control. This last category was not identified necessarily with management control but rather they concluded that their analysis of record holdings had failed to disclose a centre of ownership control. They did allow, however, that in many of them the chief officers may have been in a position of control through the power of the proxy voting machinery, although this was not revealed in the analysis. The ownership control group was subdivided according to the concentration of shareholdings into: "majority", "predominant minority" (a single group with 30 to 50 percent of the voting stock); "substantial minority" (between 10 and 30 percent); and "small minority" (less than 10 percent). The level of concentration assumed to be required for control therefore differed from the 20 percent figure used by Berle and Means.

The TNEC study was also different in that it looked at immdiate, rather than ultimate, control. Thus a company in which the centre of ownership control was found to be another nonfinancial company was assigned to the category of "control by another corporation" rather than the control category of the latter. Instrumentalities of control (such as trusts, estates and personal holding companies) were, however, taken account. Another important difference was that the TNEC employed other evidence that shareholding distributions alone to identify centres of control. A centre of ownership control was found if either there was a large concentration of stock in the hands of an identified group or, where the distribution was less concentrated, dominant shareholders were represented in the management and remaining shareholdings were highly dispersed.

The overall results of the TNEC study were that, out of the top 200 nonfinancial corporations, 77 were classified as under family control, 56 controlled by other corporations, 6 were classified as under joint control of family and corporate interest groups (an intermediate group), and in 61 cases no centre of control was found. Family control was relatively more prevalent in manufacturing and merchandising and relatively rare among railroads and public utilities. Control by other corporations was relatively common among utilities. Companies without an identified centre of ownership control were rare among utilities but relatively common among manufacturing companies and even more so among railroads.

The present purpose in using the TNEC data is to apply the theory of simple games to investigate the extent of ownership control in the sense of controlling coalitions defined in section 5 above. The analysis can not be said be identify actual controlling coalitions but provides a way of interpreting share concentration data in terms of the potential for ownership control.

12. THE IDENTIFICATION OF CONTROLLING COALITIONS AND THE TNEC DATA

The TNEC study provides lists for each company of the largest 20 shareholders of record identified by name and type of holder for each class of share. The information provided allows the identification of obvious interest groups and in our analysis holdings which belong to a common group have been consolidated into a single holding. In particular holdings by members of the same family (or group of associated families) have been combined together with their instrumentalities (e.g. family trusts and foundations). Also holdings of other corporations have been consolidated, for example direct corporate holdings and those of employee welfare funds, but no attempt has been made to group holdings of financial institutions. Thus the basic data for each company is a set of the top k holdings, $p_1, p_2, \ldots p_k$ where $p_i \geq p_{i+1}$. Each holding is also classified by type.

The general methodology employed is to compute the power index for each leading coalition of size $n \leq k$ and compare it with some value assumed to correspond with working control. Thus for each coalition it is necessary to compute $P(n)$, $\sigma(n)$ and hence $\alpha(n) = (P(n)/\sigma(n))$. The company is assumed to have a centre of ownership control with a controlling coalition of size n^* if $\alpha(n^*) \geq \alpha^*$.

This approach, described in general in section 5, provides us with no information about the costs of organisation and control

TABLE 1 : Analysis of Common Stock ($\alpha^* = 0.99$)

	Group						Size rank									
	Industrial		Utility		Railroad		1-50		51-100		101-150		151-200		All	
	#	%	#	%	#	%	#	%	#	%	#	%	#	%	#	%
Majority	10	8.5	23	42.6	7	24.1	9	18	15	30	12	24	4	8	40	20
Controlling Coalition:																
1	24	20.5	12	22.2	6	20.7	18	36	6	12	7	14	11	22	42	21
2-10	53	45.3	14	25.9	9	31.0	12	24	17	34	19	38	28	56	76	38
11-20	17	14.5	3	5.6	4	13.8	5	10	6	12	8	16	5	10	24	12
None found	13	11.1	2	3.7	3	10.3	6	12	6	12	4	8	2	4	18	9
TOTAL	117	100	54	100	29	100	50	100	50	100	50	100	50	100	200	100

TABLE 2: Analysis of Common Stock ($\alpha^* = 0.999$)

	Group						Size rank								All	
Type	Industrial		Utility		Railroad		1-50		51-100		101-150		151-200		All	
	#	%	#	%	#	%	#	%	#	%	#	%	#	%	#	%
Majority	10	8.5	23	42.6	7	24.1	9	18	15	30	12	24	4	8	40	20
Controlling Coalition:																
1	18	15.4	10	18.5	5	17.2	15	30	4	8	5	10	9	18	33	16.5
2-10	44	37.6	11	20.4	7	24.1	10	20	15	30	12	24	25	50	62	31
11-20	12	10.3	4	7.4	3	10.3	4	8	2	4	7	14	6	12	19	9.5
None found	33	28.2	6	11.1	7	24.1	12	24	14	28	14	28	6	12	46	23
Total	117	100	54	100	29	100	50	100	50	100	50	100	50	100	200	100

TABLE 3 : Analysis of Common Stock in Terms of Type of Leading Shareholder

	Individual	Other Corporation	Subsidiary Corporation	Insurance Company	Investment Company	Foundation	Bank	Joint	Total
Majority	13	27	-	-	-	-	-	-	40
Controlling Coalition:									
1	21	16	-	-	4	-	-	1	42
2-10	40	9	3	4	11	-	9	-	76
11-20	6	-	-	2	7	1	8	-	24
None	7	1	1	1	1	1	6	-	18
Total	87	53	4	7	23	2	23	1	200

The control criterion assumes $\alpha^* = 0.99$. Definitions of the types of shareholder are as follows: "Individual": individuals; families; groups of identified families; personal and family holding companies and foundations; "Other Corporation": other nonfinancial corporations; parent companies; "Subsidiary Corporation": subsidiary companies; own stock holdings; own employee welfare or pension funds; "InsuranceCompany": insurance companies; fraternal organisations; unions, etc; "Investment Company": investment companies; employee welfare funds of other companies; brokers and investment bankers as beneficial holders; "Foundation": foundations not included in other categories; "Bank": banks, brokers, etc., beneficiariesnot disclosed; banks as beneficial holders; "Joint": joint corporate and family control.

and will always give a controlling coalition (in the extreme case of an equal distribution, a majority coalition). However this is not a serious limitation since it is assumed that the costs of organisation and control are small for small coalition sizes and only such coalitions can be examined with the present data since $k \leq 20$ in each case. Reasons for restricting the analysis to the top 20 holdings are not stated in the TNEC report but the question has been considered by Florence (1961), who also used data in this form. He uses two arguments: (i) twenty is "a number not too large to prevent some sort of personal contact and yet large enough to control among companies or corporations a sufficient proportion of votes to give a virtual majority"; (ii) twenty is about the normal size for a committee or board. Thus it may not be too strong to assume that costs of organisation and control are quite small for all the leading coalitions which can be formed with the available data.

A second, more fundamental, problem arises from the use of truncated data. Computation of the power index requires the calculation of $\sigma(n)$ which depends on the whole distribution. The best we can do is to obtain bounds on $\sigma(n)$ on extreme assumptions about the dispersion of non-observed holdings. Assuming $p_i = p$ constant for $i > k$, the upper bound is obtained when $p = p_k$ and $N = k + (1-P(k)/p_k$ (assuming away rounding error). Thus, $\sigma(n)^2 \leq \sum_{i=n+1}^{k} p_i^2 + (1-p(k)p_k$. The lower bound is the limit as p goes to zero and, therefore, $\sigma(n)^2 > \sum_{i=n+1}^{k} p_i^2$. Thus the truncation of the distribution may not be too serious a problem provided these bounds are not too far apart. In the empirical analysis the upper bound value has been used and, although this biases the analysis heavily away from finding a controlling coalition (embodying an extreme assumption about share concentration), it does appear to provide a basis for discrimination among companies.

A further problem surrounds the choice of the power index for control, α^*. There is an element of arbitrariness in this and a value of 99 percent is preferred (that is a coalition can win 99 percent of potential votes). In order to test the sensitivity of the results to this parameter the analysis has also been made using $\alpha^* = 99.9$ percent.

The results of the analysis of common voting stock are reported in Tables 1, 2 and 3. In Table 1, the 99 percent value has been assumed for α^* and the upper bound for $\sigma(n)$ used (this has been used throughout). Among the 160 companies without majority control, a controlling coalition has been found in 142 cases. None was found for 18 companies. In 42 cases $n^* = 1$, the controlling coalition contained one shareholder and in 76 cases the coalition size was between 2 and 10. The analysis by size of company shows that both controlling coalitions with one member and no controlling coalition are more common among the largest fifty companies than any other group. In every size group over three quarters of companies are either majority-controlled or have a controlling minority coalition of size no greater than 10. The analysis in terms of type of company shows a similar incidence of no controlling coalition for industries and railroads and a high rate of majority control for utilities. The proportion of companies with a one-member controlling coalition is about the same in all groups but the proportion where the coalition size is between 2 and 10 is high among industrials. These results support the conclusions in Leech (1984), which were based on a sample of mainly railroads.

Table 2 reports the same analysis using the 99.9 percent control criterion for the power index. This shows a much higher proportion of companies for which no controlling coalition has been found (23 percent against 9 percent in Table 1). This still means, however, that 77 percent are either majority-controlled or have a controlling minority coalition.

In Table 3 the companies are analysed in terms of the type of leading shareholder, that is, the largest shareholder. The control criterion assumed is the same as in Table 1. In 80 companies which are deemed to be owner controlled the largest holding is by an individual or family and in a further 52 it is a corporation. In three companies the largest holding is by a subsidiary corporation and therefore it is not possible to infer anything about the type of control for these - to ascertain control for these companies requires analysis without reference to the largest holding which can be regarded as an instrumentality by which control is exercised and retained by the existing controlling group whether a coalition of shareholders or management. Investment companies have minority control as a coalition of one in four cases and have the largest holding in a controlling coalition in a further 18 companies. There are no instances of either majority control or single-member minority coalition control by banks although they are the largest shareholder in 17 cases where there is a controlling coalition. Banks are more frequently than any other type the largest shareholder where no controlling coalition is found.

The analysis above has been in terms of common voting stock for most companies and voting stock for those which had no common stock. This might reasonably be assumed to give some information about control. However the picture is complicated by the fact that different companies have several classes of shares with different voting rights. A more accurate analysis would be based on data which combined different types of share into a single distribution of voting rights. Unfortunately this cannot be done with the TNEC data, not just because of differential voting rights attaching to different classes of voting shares, but because the data is in the form of the top 20 holdings within each class and this makes meaningful amalgamation impossible. In order to provide some checks on the results a further analysis has been done using data on the distribution of ownership of shares of all classes, without regard to voting rights.

The data from all the share distributions reported for each company were used to assemble a distribution of the largest k' shareholdings. Let $\bar{p} = \max_j \left\{p_{k_j}\right\}$ where k_j is the number of holdings of type j in the data after appropriate consolidation and p_{k_j} is the smallest holding observed. Then a holding p_{ij} (the i^{th} holding of type j, $i = 1,\ldots,k_j$) is included in the distribution if $p_{ij} \geq \bar{p}$. Then k' is the number of holdings in this distribution after further consolidation. This gives a distribution of the ownership of the k' largest holdings of all shares.

Table 4 provides an analysis of this distribution treating all shares as voting. Comparing Table 4 with Table 3, fewer companies would now be majority-controlled and fewer of them show no controlling coalition. More companies would be minority-controlled by a coalition of one and more would have a controlling coalition containing no more than 10 members. The same pattern is found for each company type with a big reduction in the numbers of majority controlled utilities and railroads.

TABLE 4 : Analysis of All Types of Stock

	Industrial	%	Utilities	%	Railroads	%	All	%
Majority	8	6.8	14	25.9	2	6.9	24	12
Minority Controlling Coalition								
1	26	22.2	18	33.3	8	27.6	52	26
2-10	54	46.2	19	35.2	11	37.9	84	42
11-20	16	13.7	2	3.7	6	20.7	24	12
None found	13	11.1	1	1.9	2	6.9	16	8
Total	117	100	54	100	29	100	200	100

13. SUMMARY AND CONCLUSIONS

This paper has described a model of the shareholder constraint on the firm and of the relationship between ownership concentration and corporate control based on the theory of simple games. Shareholder organisation is conceived in terms of leading coalitions which are costly to form and which possess power within the context of shareholder voting. Control is defined in terms of a power index for the coalition within the shareholder-voting game. The static theory of the firm is developed on the basis of a comparison of the costs of organisation and control for a controlling coalition and the benefits gained by it. A dynamic theory of an owner-controlled firm is developed which has the property that reducing share concentration and transfer of control among shareholder coalitions are optimal. The takeover constraint is discussed and it is suggested that takeovers might appropriately be modelled within the same model. The simple-game theoretic approach is used to analyse the TNEC data set for the top 200 non-financial corporations in the US in 1937. Using this approach the ownership concentration observed would suggest that control by minority coalitions of shareholders was feasible for the great majority of these companies (whether or not such coalitions existed in actuality) and there is little evidence on the basis of share concentration alone to support a general belief in a pervasive separation of ownership and control.

Footnotes

1/ "... rather it is becoming a means for large acquisition-minded corporations to take over well-managed small and medium firms".

2/ Usually equated to management control.

3/ One exception is Yarrow (1976).

4/ The Shapley value is based on orderings of players and treats each ordering equally. The Bauzhaf approach disregards orderings and gives equal weight to each coalition.

5/ This index is described in section 4 and applied in the subsequent sections but the theory can equally well be developed in terms of one of the conventional indices. Using the Shapley-Shubik index, for example, would require only minor and obvious modifications to sections 4-10.

6/ Attention is confined to leading coalitions throughout the analysis since a coalition with a given combined holding has the fewest members and therefore the costs defined in section 6 will cet.par. be lower than for any other coalition.

7/ The asymmetric treatment of shareholders in this game - the partitioning into members of the leading coalition and the rest - does not imply differences in motivation. Shareholder utility functions are assumed to be increasing in the present value of the firm. Rather, shareholders outside the leading coalition lack information and are unorganised.

8/ $\alpha(n)$ is similar to the degree of control defined by Cubbin and Leech (1983) on the assumption of probabilistic voting with an allowance for abstentions.

9/ The Shapley-Shubik index for the leading coalition can be obtained by letting the probability distribution of q_i be as follows: $q_i = p_i$ with probability t and $q_i = 0$ with probability $1-t$. Then $q(S)$ is normally distributed with mean $E(q(S)) = P(n) + t(1-P(n)) = t + (1-t)P(n)$ and variance $var(q(S)) = t(1-t)\sigma(n)^2$. Letting $i = 1$ denote the leading coalition, the index is $\phi_1 = \int_0^1 f_1(t,t,\ldots,t)dt$. But f_1 is the probability that the addition of player 1 to a random subset (assuming this voting model) changes it from losing to winning. That is,

$$(7) \quad f_1(t,\ldots,t) = Pr\left[\frac{1}{2} < q(S) < \frac{1}{2} + P(n)\right].$$

Hence, integrating (7) with respect to t gives ϕ_1.

The non-normalised Banzhaf index can also be obtained from (7) directly by setting $t = \frac{1}{2}$.

10/ This analysis has close similarities to that of Yarrow (1976) who models shareholder intervention in terms of a cost for leading coalitions of variable size. However he assumes that every leading coalition has control by being able to count on enough support from other shareholders in the event of an intervention. Thus the only influence of ownership concentration in his model occurs through its effect on the cost function and the problem of control is not addressed.

11/ This point is also made by Yarrow (1976) who regards a takeover as a special case of shareholder intervention by a coliation of size 1.

12/ Strictly, the diagram represents simply an increase in the size of the largest holding whether an increase in the existing largest shareholding or by an outsider. In the latter case it would be necessary for consistency to assume the takeover raider has acquired the existing largest holding as well as additional holdings.

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