TAXATION AND PARAMETER UNCERTAINTY
SOME EXAMPLES

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ABSTRACT

Well-known models of optimal taxation are considered. Particular functional forms are selected to represent consumer preferences. Uncertainty about the parameters of these functions is introduced. Propositions are stated describing the consequences for optimal taxation.

I. INTRODUCTION

By accident or design economists have almost invariably separated questions of parameter estimation and policy optimisation. As a result, studies of optimal taxation, such as Ramsey (1927), Mirrlees (1971), Stern (1976) and Seade (1977) and the literature on the formulation of empirical models, such as Hendry and Richard (1982 and 1983), have developed quite independently. The former generally treat demand system parmeters as if they were certain, or could be known with certainty given information on consumers' characteristics; and the latter discuss criteria for model selection, and the properties of estimators, with little or no reference to their intended used.

Rather than attempting a general unification of parameter estimation and policy optimisation, this paper concentrates on two connections between them. Given that econometric models provide uncertain estimates, the first question is how to adjust optimal tax rules for uncertainty. A case of particular interest is that of 'certainty equivalence' (Simon, 1956, Theil, 1957) when the policy choice is independent of parameter uncertainty, and this result is shown to apply to particular tax problems. For example in a Ramsey tax model with a linear expenditure system (see model A.4) optimal taxes are not affected by uncertainty about any one of the marginal propensities.

The second question is how parameter estimates may be revised in the knowledge that they will be used in a particular policy optimisation problem. The case to be considered is that where the policy rule regards all parameters as if they were known with certainty. Information about uncertainty, and the social welfare function, may then be incorporated in a 'certainty equivalent parameter estimate'. In practice, this might reveal whether, in the context of a specific policy choice, a particular parameter estimate should be regarded as an upper or lower bound.

Section 2 describes general methods for adjusting optimal taxes for uncertainty. These methods were applied to five examples; the results are discussed in section 3. Section 4 concludes.

2. METHOD

For tractability it is assumed that only one parameter is uncertain. This is necessary in order to be able to infer from an increase in the variance (or spread) of that parameter that the riskiness of the environment has unambiguously increased. Clearly, it would be desirable to consider cases of multi-dimensional uncertainty arising from many parameters, however it would then become difficult to say whether a simultaneous increase in some variances, and reduction in others led to an increase or decrease in overall riskiness. Atkinson and Bourginon (1985) have suggested criteria (stochastic dominance) for making such a justment. However here attention is restricted to the simple case of uncertainty about one parameter only.

The numbers of instruments available is also an important consideration. The single-instrument case is generally more tractable,however, weaker results also apply in the case of many instruments.

2.1 A single instrument

In its most general form the criteria for the choice of tax rate may be described by the social welfare function, W,

$$W = W(t,\underset{\sim}{\theta}) \qquad (1)$$

where t is the tax instrument and $\underset{\sim}{\theta}$ is a vector of parameters describing the influence of the tax rate on social welfare. It is assumed, for the time being, that other instruments are fixed, or are determined by rules independent of t, and hence may be regarded as parameters in the present context.

The basis of the methodology adopted here is comparison between a situation of certainty where the welfare function (1) applies, with a situation of uncertainty, where decisions are made on the basis of expected welfare.

$$EW = \int_{-\infty}^{+\infty} W(t,\underset{\sim}{\theta}) f(\theta_j) d\theta_j \qquad (2)$$

where $f(\theta_j)$ is the probability distribution of parameter j.

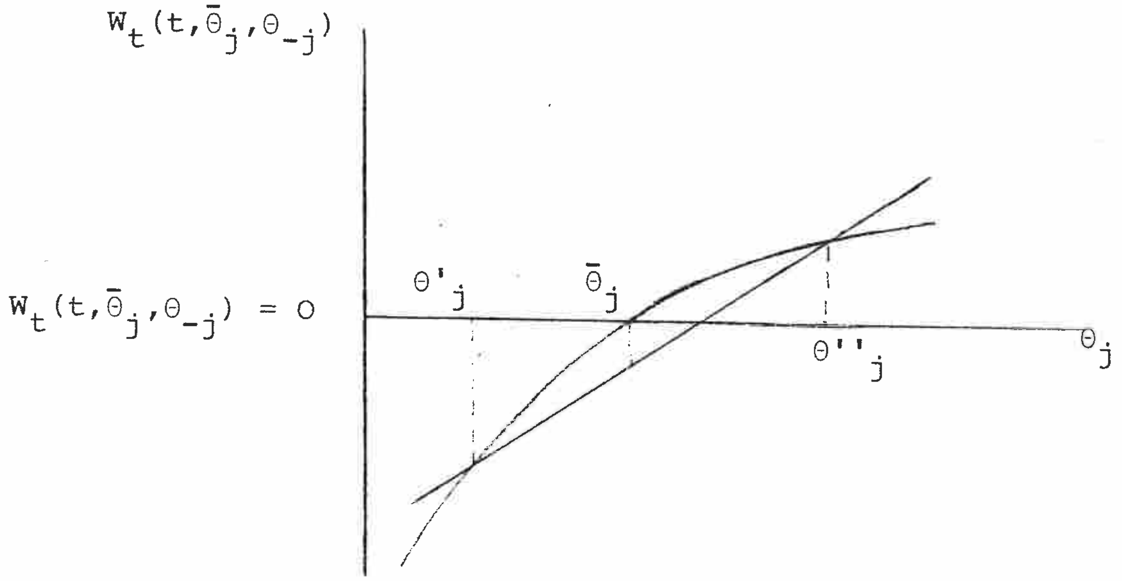
Diamond and Stiglitz (1974) provide a theorem relating the sign of the difference between the maximum values of the welfare and expected welfare functions to the concavity or convexity of the first order condition. An approximation due to Malinvaud (1969) for the magnitude of this difference is also exploited. Points other than the maxima of the functions are examined by considering the tax rates which yield a fixed amount of revenue.

Consider the problem under certainty. The necessary and sufficient conditions for a welfare maximum are

$$W_t(t,\bar{\theta}_j,\underset{\sim}{\theta}_{-j}) = 0 \; ; \quad W_{tt}(t,\bar{\theta}_j,\theta_{-j}) < 0$$

Given the parameters $\underset{\sim}{\theta}_{-j}$ and $\bar{\theta}_j$, t is the optimal tax rate. By holding t at its optimal value and allowing θ_j to take values other than $\bar{\theta}_j$ it is possible to plot the first order condition; Figure 1 also assumes that it is concave in θ_j.

Figure 1 : Increase in uncertainty about θ_j : the concavity of the first order condition



Now, if θ_j takes values of θ_j' and θ_j'' each with probability $\frac{1}{2}$ (there is a mean preserving spread around $\bar{\theta}_j$), the expected value of W_t falls below zero. From the second order condition, W_t increases as t falls, therefore a reduction in t is required to restore optimality. Thus, an increase in uncertainty about θ_j requires an increase (decrease) in t if W_t is concave (convex) in θ_j. This argument is formalised in Theorem 1.

Theorem 1 (Diamond and Stiglitz, 1974)

The optimal tax rate increases, is constant, or decreases with uncertainty about parameter j according to whether the derivative W_t is convex, linear, or concave in Θ_j, i.e.

$$W_{\Theta j\Theta jt} \gtreqless \Rightarrow \frac{\partial t^*}{\partial \mathrm{var}\Theta j} \gtreqless 0$$

Note, this is true for more general measures of risk than variance - mean preserving spread for example. Here attention is restricted to means and variances; this approach may be regarded as a first approximation to more general classes of distributions (Samuelson, 1970). Direct application of this theorem to optimal tax problems reveals when tax recommendations on the basis of certain parameters may be regarded as upper or lower bounds, i.e. it indicates the desired direction of adjustment. The value of the optimal tax rate under uncertainty may be estimated by taking a second order Taylor expansion of the welfare function, around the mean $\bar{\Theta}_j$ of the parameter.

$$EW[t, f(\Theta_j), \Theta_{-j}] \simeq W(t, \bar{\Theta}_j; \Theta_{-j}) + \frac{1}{2} W_{\Theta j\Theta j} \mathrm{Var}\Theta_j$$

Differentiate with respect to t, and set both sides equal to zero

$$0 = EW_t \simeq W_t + \frac{1}{2} W_{\Theta j\Theta jt} \mathrm{Var}\Theta_j = 0$$

The solution to the left-hand side is t^* and to the right is t^{**}.

This may be summarised as follows:

<u>Approximation 1</u> (Malinvaud, 1969)

<u>The solution to the problem under uncertainty may be approximated by adjusting the first order conditions for the problem under certainty in the following way:</u>

$$W_t(t^{**},\bar{\Theta}_j,\Theta_{-j}) + \frac{1}{2} W_{\Theta j\Theta jt}\text{Var}\Theta_j = \underset{\sim}{0}$$

<u>Then the solution to this adjusted FOC is approximately equal to that of the problem under certainty</u>

$$t^{**} \simeq t^*$$

Analogous results apply to the certainty equivalent problem. This requires the introduction of a bias into the estimate of Θ_j, such that the resulting certainty equivalent parameter estimate Θ_j^* may be used in policy optimisation as if it were known with certainty. Thus the choice of Θ_j^* would ensure that t^* satisfies

$$W_t(t^*,\Theta_j^*) = 0$$

Theorem 2 shows the conditions under which a parameter estimate should be regarded as an upper or lower bound in the context of a particular optimisation problem.

Theorem 2

The certainty equivalent parameter estimate is above (below) the expectation of the parameter when the ratio of the second and first derivatives of the objective function is positive (negative).

$$\frac{W_{\Theta j\Theta t}}{W_{\Theta jt}} \gtreqless 0 \Rightarrow \Theta_j^* - \bar{\Theta}_j \gtreqless 0$$

Proof:

Defining r as the degree of uncertainty, the first order condition for welfare maximisation is

$$EW_t(t,\underset{\sim}{\Theta},r) = 0$$

Holding $\underset{\sim}{\Theta}$ constant and differentiating yields

$$\frac{dt}{dr} = -\frac{EW_{tr}}{EW_{tt}} \qquad (3)$$

By definition, the certainty equivalent parameter estimate is required to satisfy

$$W_t(t,\Theta) = 0$$

which implies

$$\frac{dt}{d\Theta_j} = -\frac{W_{t\Theta j}}{W_{tt}} \qquad (4)$$

combining (3) and (4)

$$\frac{d\theta_j}{dr} = \frac{EW_{tr}}{EW_{tt}} \quad \frac{W_{tt}}{W_{t\theta j}}$$

Since the second derivatives with respect to t are negative for maxima, the sign of $d\theta_j/dr$ is the same as $EW_{tr}/W_{t\theta j}$. The proof of Theorem 1 (Diamond and Stiglitz, 1974) showed by integration that the sign of EW_{tr} is the same as that of $W_{\theta j\theta jt}$. Therefore

$$\frac{d\theta_j}{dr} \gtreqless 0 \iff \frac{W_{\theta j\theta jt}}{W_{\theta jt}} \gtreqless 0$$

and hence

$$\theta_j^* - \bar{\theta}_j \gtreqless 0 \iff \frac{W_{\theta j\theta jt}}{W_{\theta jt}} \gtreqless 0$$

□

The magnitude of this difference may be estimated using a technique similar to approximation 1.

In the certainty equivalent problem θ_j^* is defined such that if it were used in policy optimisation as if there was no uncertainty, t^* would be the policy choice:

$$W_t(t^*;\theta_j^*,\theta_{-j}) = 0$$

Expanding around $\bar{\Theta}_j$ and equating to zero

$$W_t \simeq W_t(t^*,\bar{\Theta}_j,\Theta_{-j}) + W_{t\Theta j}(\Theta_j^*-\bar{\Theta}_j) = 0 \tag{5}$$

From Approximation 1,

$$W_t(t^*,\bar{\Theta}_j,\Theta_{-j}) \simeq W_t(t^{**},\bar{\Theta}_j,\Theta_{-j}) = -\frac{1}{2} W_{\Theta j\Theta jt}\mathrm{Var}\Theta_j \tag{6}$$

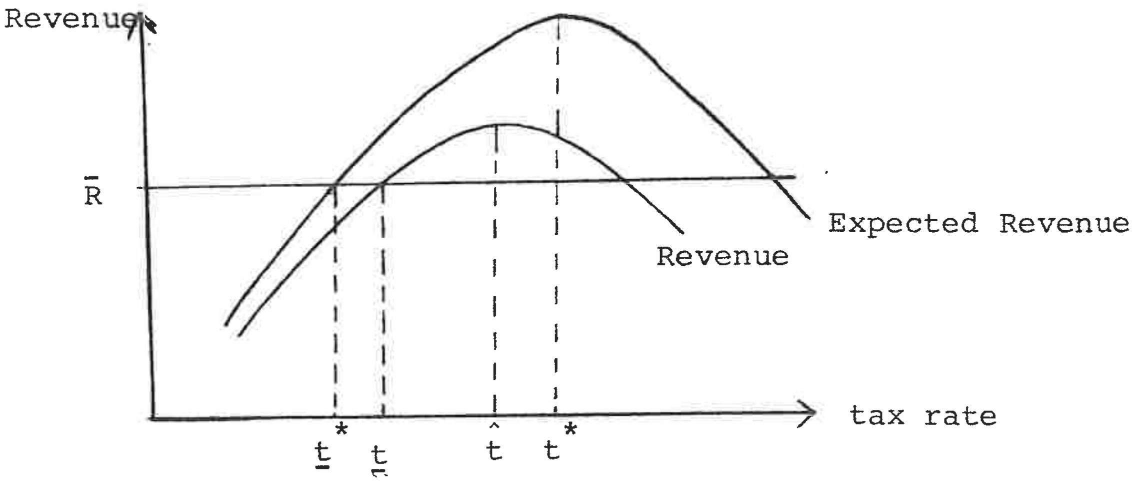
Substituting (5) into (6) yields

<u>Approximation 2</u>

$$\Theta_j^* - \bar{\Theta}_j \simeq \frac{1}{2}\frac{W_{\Theta j\Theta jt}}{W_{\Theta jt}}\mathrm{Var}\Theta_j$$

These theorems and approximations all relate to the maxima of the appropriate welfare functions. A wide variety of tax issues require the examination of other properties of the functions. Consider, for example, the case where the government is required to raise a given amount of revenue. This is illustrated in Figure 2 where the revenue under certainty, and expected revenue are plotted for different values of t. The functions are assumed to be concave with unique global maxima at $\hat{t}$ and t^* respectively.

FIGURE 2 : Revenue and expected revenue functions



In the absence of other constraints it would seem natural to choose the lower of the two tax rates which satisfy the revenue requirement. Therefore, if the expected revenue curve lies everywhere above the revenue curve, uncertainty means a lower tax rate. If, in addition expected revenue may be approximated by the Taylor expansion

$$ER(t,\underset{\sim}{\theta}) \simeq R(T,\bar{\underset{\sim}{\theta}}_j, _{-j}) + \frac{1}{2} R_{\theta j\theta j}\mathrm{Var}\theta_j$$

then the vertical distance between the revenue and expected revenue curves is given by $\frac{1}{2} R_{\theta j\theta j}\mathrm{Var}\theta_j$. Hence

Approximation 3

The lowest tax rate $\underline{t}$ which on average raises $\bar{R}$ decreases (increases) with uncertainty if $R_{\theta j\theta j}$ is positive (negative).

$$R_{\theta j\theta j} \gtreqless 0 \Rightarrow \frac{d\underline{t}}{d\mathrm{Var}\theta_j} \gtreqless 0$$

2.2 Many Instruments

By defining $\underset{\sim}{t}$ as a vector of tax instruments, the Diamond Stiglitz theorem may be generalised as follows. Expected welfare depends on the uncertain parameter Θ_j whose distribution $F(\Theta_j, r)$ has a spread, or riskiness, described by the scalar r.

$$EW = \int_0^1 W(\Theta, \underset{\sim}{t}) F_{\Theta j}(\Theta_j, r) d\Theta_j$$

The first order conditions

$$EW_{ti} = 0 \quad \text{for all } i \tag{7}$$

implicity define a relationship between the vector of taxes, and the riskiness of Θ_j. Defining EW_t as the vector of derivatives of EW, and totally differentiating yields

$$\frac{\partial}{\partial r} (EW_t) dr + \frac{\partial}{\partial \underset{\sim}{t}} (EW_t) d\underset{\sim}{t} = 0 \tag{8}$$

For a unique global maximum, the matrix of second derivatives of EW with respect to $\underset{\sim}{t}$ must be negative definite; therefore (8) may be rearranged

$$\frac{d\underset{\sim}{t}}{dr} = - (EW_{tt})^{-1} \frac{\partial}{\partial r} (EW_t) \tag{9}$$

It is a straightforward generalisation of Diamond and Stiglitz to show that every element in $\frac{\partial}{\partial r}(EW_t)$ has the same sign as the corresponding element in $W_{t\theta j\theta j}$ and is zero if $W_{t\theta j\theta j} = 0$. However, this does not necessarily have implications for the sign of $\frac{d\underset{\sim}{t}}{dr}$, since there may be over-riding cross effects from the matrix of second derivatives. But, the following theorem is obvious from (9).

Theorem 3

If each element of the vector of derivatives of $EW(\theta,t)$ is linear in θ_j then the vector of optimal instruments is not affected by risk.

$$(W_{t\theta j\theta j} = \underset{\sim}{0}) \Rightarrow \left[\frac{\partial}{\partial r}(EW_t) = \underset{\sim}{0}\right] \Rightarrow \left(\frac{d\underset{\sim}{t}}{dr} = \underset{\sim}{0}\right)$$

Given uncertainty and many instruments, the government's revenue constraint requires careful interpretation. It is impossible to know in advance exactly how much revenue will result from the application of a particular set of tax instruments, therefore the revenue constraint which would be applied under certainty must be modified. There are two alternatives:

(i) Ex ante revenue constraint. The government is required to plan to raise a given amount of revenue R. This may be written

$$\underset{\sim}{t} . \underset{\sim}{\bar{x}} = R$$

where $\underset{\sim}{t}$ is the vector of tax rates and $\underset{\sim}{\bar{x}}$ is the vector of expected demand for each commodity. Because the world is risky and errors are made in forecasting consumers' responses, it is unlikely that the planned revenue target will be precisely fulfilled; however, under this interpretation the constraint states only that the target would be met if the forecasts turned out to be exactly true.

ii) <u>Ex Post revenue constraint.</u> Under this interpretation exactly the right amount of revenue is always recovered because one instrument t_n is determined ex post. This is written

$$t_n = \frac{R - \sum_{i=1}^{n-1} t_i x_i}{x_n}$$

where the x's on the right-hand side are the actual demands, not forecasts. Usually indirect tax models are applied to consumer demands, and it is difficult in that context to imagine the tax rate on the n^{th} commodity being determined after the demand for all goods include the n^{th} have been revealed. However, this kind of constraint is a plausible model in other areas of government policy-making. For example, the scheme of prices at which Area Boards buy electricity from the Central Electricity Generating Board, has elements which are determined ex post (see Electricity Council, 1981). The Area Boards formulate expectations about the prices which they are likely to face, and behave accordingly; then at the end of the year the Central Electricity Generating Board makes adjustments to the announced price schedule to ensure that revenue targets are fulfilled.

The distinction between ex ante and ex post revenue constraints is crucial in determining the influence of uncertainty on optimal taxes. The reason is as follows: the Diamond Stiglitz theorem indicates the importance of the curvature of the first order condition with respect to the uncertain parameter. The ex ante revenue constraint depends on nothing but the expected value of this parameter and contributes no curvature to the program. However, the ex post revenue constraint depends on the entire probability distribution of the unknown parameters and this makes a difference to the convexity of the first order conditions.

The order of differentiation is a convenient way of capturing these contrasting effects. The ex ante revenue constraint amounts to first differentiating the welfare function twice with respect to the uncertain parameter, and then using the constraint.

$$\frac{\partial}{\partial t_i} (W_{\theta j \theta j}) = \frac{\partial}{\partial t_i} W_{\theta j \theta j} + \frac{\partial}{\partial t_n} W_{\theta j \theta j} \frac{dt_n}{dt_i}$$

Notice that the term dt_n/dt_i, when the revenue constraint enters, is not differentiated with respect to θ. But if the order of differentiation is reversed

$$\frac{\partial^2}{\partial\theta_j \partial\theta_j} (W_{ti}) = \frac{\partial^2}{\partial\theta_j \partial\theta_j} \left(\frac{\partial w}{\partial t_i} + \frac{\partial w}{\partial t_n} \frac{dt_n}{dt_i}\right)$$

curvature of dt_n/dt_i is incorporated and hence the revenue constraint has been imposed ex post.

3. DISCUSSION OF RESULTS

Cursory inspection of the theorems and approximations in the previous section reveals that the effect of introducing uncertainty in optimal tax models will depend crucially on the source of uncertainty and the properties of the objective function. It would be quite surprising if broad generalisations emerged; it would be more reasonable to expect that results which apply to one tax model do not apply to others. Therefore this section reports the examination of five models. They are divided into two categories; income, and commodity taxation. The results are summarised in Table 1. Clearly, these examples are all quite special cases of the general problem of taxation under uncertainty, but despite the restrictiveness of their assumptions, they may be of some interest per se. Each example uses a model, and a functional form for preferences which is well known in the literature, thus Table 1 may be regarded as an attempt to throw light on previously neglected aspects of well-established models.

3.1 Linear Income Tax Models

The first three examples are based on the linear income tax model where the government is allowed only one instrument. The tractability of this model is the consequence of quite restrictive assumptions. The first, following Rawls (1971), is that zero weight is allocated to all individuals except the least well off, thus social welfare is identical to the utility of that individual. The second requires that all individuals face the same tax rates and that tax revenue is redistributed equally across the

TABLE 1 : Summary of Results

Model	Instrument	Demand Specification	Result under certainty	Result under Uncertainty	Certainty equivalent parameter estimates
A.1	linear income tax	$U = x^{\alpha}(T-\ell)^{(1-\alpha)}$	$t = \frac{1-\sqrt{(1-\alpha)}}{\alpha}$	$\frac{\partial t^*}{\partial var\alpha} > 0$	$\alpha^* > \hat{\alpha}$
A.2	linear income tax	$\ell = \alpha w(1-t) + \beta m + \gamma$	N/A	$\frac{\partial t^*}{\partial var\alpha} = \frac{\partial t^*}{\partial var\gamma} = 0$ $sign\left\{\frac{\partial t^*}{\partial var\beta}\right\} = sign\left\{\alpha\sigma(1-t) + \gamma\right\}$	$\beta^* > \bar{\beta}$ if $(\alpha, \gamma > 0,\ \beta < 0)$
A.3	linear income tax	$\ell = a+b(1-t)w+c(1-t)^2w^2 + dI+ed^2+fIw(1-t)+gm$	$t = 0.491$	$t = 0.522$	-
A.4	indirect taxes t_1, t_2	$U = \sum_{i=0}^{2} \beta_i \log(x_i-\gamma_i)$	N/A	$\frac{\partial t_i^*}{\partial var\beta_j} = 0$ for all i,j ex ante rev constraint $\frac{\partial t_1^*}{\partial var\gamma_1} < 0; \frac{\partial t_1^*}{\partial var\gamma_2} > 0$ ex post rev constraint $\frac{\partial t_1^*}{\partial var\gamma_j} < 0$ for all j	ex ante rev constraint $\gamma_i^* < \bar{\gamma}_i$ i = 1,2 ex post rev constraint $\gamma_i^* > \bar{\gamma}_i$ for all i
A.5	indirect taxes t_1, t_2, transfers m	$U = \sum_{i=0}^{2} \beta_i \log(x_i-\gamma_i)$	Uniformity	$\frac{\partial t_i^*}{\partial var\beta_j} = 0$ for all i,j $(var\gamma_j \neq 0) \Rightarrow (t_1^* \neq t_2^*)$ for all j	-

whole population. This assumptions, combined with identical preferences, ensures that no policy change can change the ranking of individuals; (all changes are horizontally equitable, King, 1983) hence the welfare function equals the same individual's utility function before and after the change in income taxation.

The Rawlsian criterion requires an explanation of why some individuals are better-off than others. Stern (1976) provides a variety of models of individual differences, two of which are used here. In examples A2 and A3, differences in skills are represented by a distribution of wages; individuals have identical preferences but the less skillful face lower wages. An alternative model, used in example A1, also assumes identical preferences, but wages are constant and individuals differ according to their time endowments.

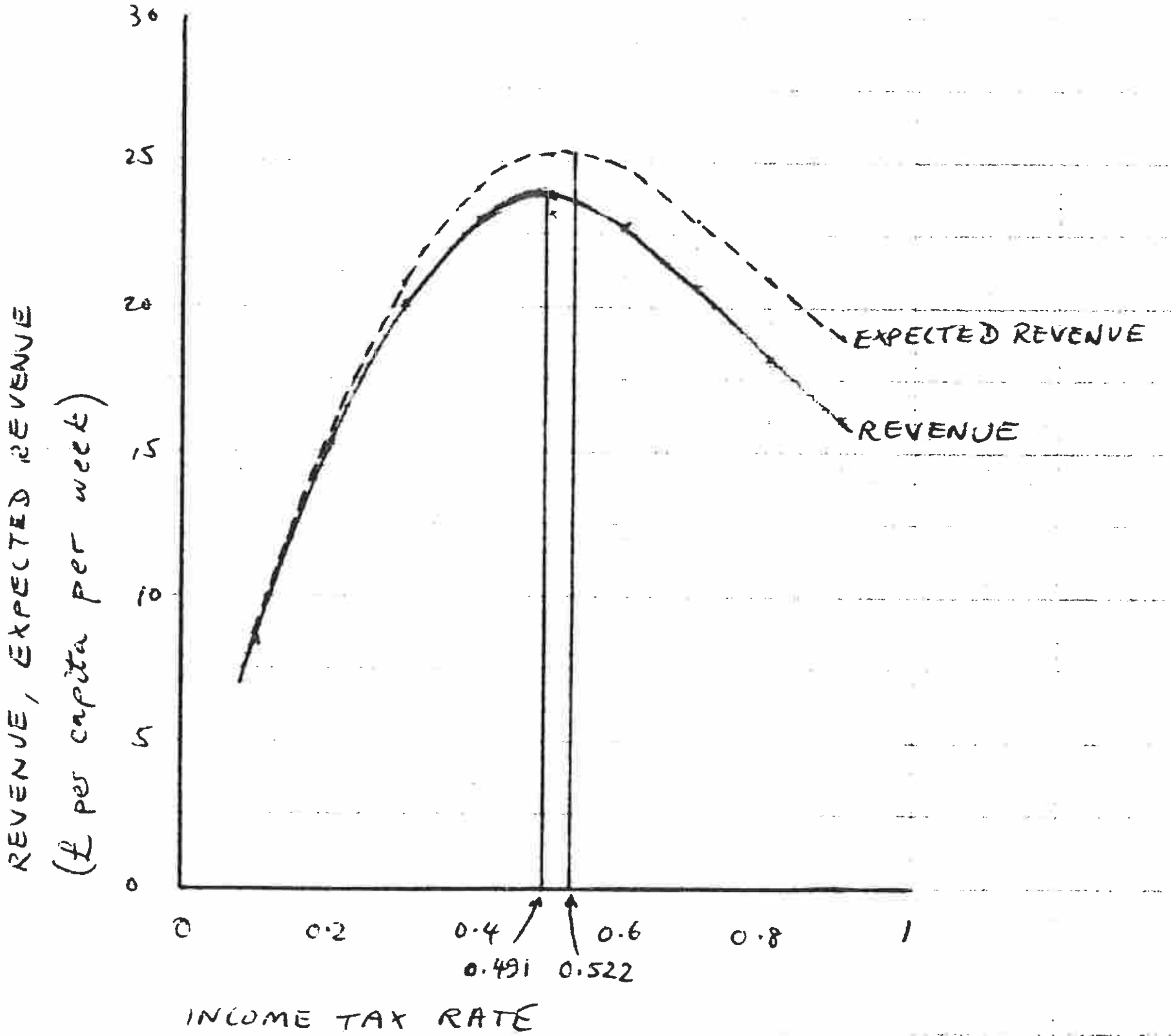
The combination of linear income taxation with equal redistribution, and a Rawlsian welfare function greatly simplifies the government objectives. The utility of the least well-off increases with transfers, therefore the government maximises tax revenue. The tractability of this model has made it the subject of considerable literature, Mirrlees (1971), Atkinson (1976) and Phelps (1973) have examined it as a particular example of more general models, and have used it for optimal tax calculations. Sheshinski (1972) restricts attention to linear income-tax models and describes propositions which apply to more general welfare functions. Feldstein (1973) concentrates on the disincentive effects of income taxation on labour supply. Broome (1975) uses the most restrictive assumption of Cobb Douglas preferences which form the basis of example A1.

Under uncertainty, the results may be summarised as follows: firstly (model A1) with Cobb Douglas preferences, uncertainty about the share parameter increases the optimal income tax rate, and the magnitude of the adjustment is likely to be large (in the order of 25% to 30%). The solution to the certainty equivalent problem indicates that the standard parameter estimate may be regarded as a lower bound. In this case the parameter estimate may be increased by about 80% and regarded as known with certainty. Secondly, if the labour supply function is linear in non-labour income (models A2 and A3) only uncertainty about the coefficient on non-labour income matters, the magnitude and direction of uncertainty adjustment depending on parameter values. Model A3 uses the econometric estimates of Brown et al, and finds that the incorporation of uncertainty increases the revenue maximising tax rate by 6.3%. The influence of uncertainty on the Laffa curve is illustrated in Figure 3.

3.2 Indirect Tax Models

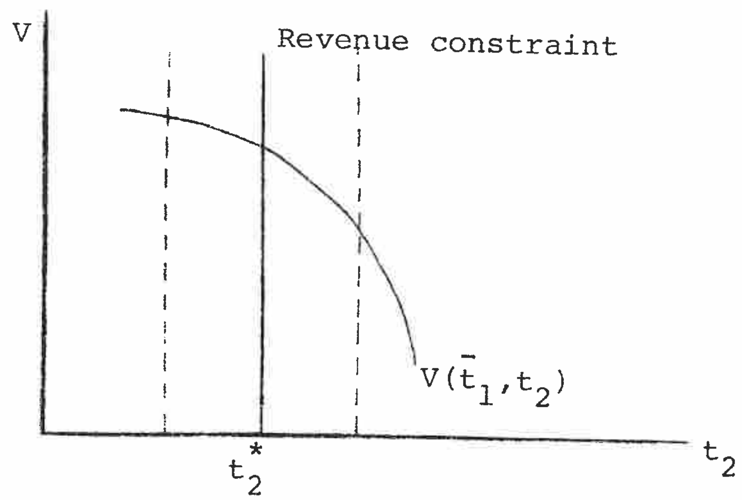
Two indirect tax models are examined. The first, A4, concentrates on efficiency issues, and assumes that there is only one consumer. Preferences are represented by the linear expenditure system. Even in this simple framework it is impossible to derive analytic expressions for the optimal tax rates in terms of the demand parameters. However, the effect of uncertainty may be precisely characterised. Uncertainty about the marginal propensities does not affect

FIGURE 3. Tax Simulations Based on Quadratic Labour Supply Function, Brown, Levin and Ulph (1976)



optimal tax rates, but uncertainty about the subsistence levels does. If the model is simplied to two commodities and labour (Corlett Hague, 1953), results may be derived about the qualitative effects of uncertainty about the λ's. With an ex post revenue constraint, the suprising result emerges that uncertainty about any of these parameters reduces the optimal tax on good one. To see why this happens, consider first the choice which would be made under certainty: t_1 would be increased up to the point where the marginal welfare losses resulting from a higher t_1 were exactly offset by the marginal welfare gains from the reductions in t_2 permitted by the revenue constraint. Holding t_1 fixed, it is possible to plot the outcomes for the objective function of different values of t_2. This function will be donward sloping - lower t_2 means higher welfare - and concave.

FIGURE .4 : Indirect utility holding t_1 constant



Under certainty, given the fixed level of t_1, only one value of t_2 satisfies the revenue constraint, so it may be illustrated by a vertical line in Figure .4 When uncertainty is introduced, the value of t_2 is determined ex post by the unknown parameter. This is depicted by a spread around t_2^*. Clearly, because of the concavity of $V(\bar{t}_1,t_2)$ the welfare losses which come about if t_2 turns out to be high are greater than the gains if it turns out to be low. Therefore a risk averse decision maker will want to reduce the chance that t_2 may be higher than expected, and this is achieved by reducing t_1. This is true whatever the source of uncertainty, and contrasts with the results under the ex ante revenue constraint.

If the revenue constraint applies ex ante, t_1 is increased by uncertainty about γ_2 and reduced by uncertainty about γ_1. The interpretation of this result also relies on the concavity of the welfare function. When the revenue constraint applies ex ante it is unlikely to be fulfilled precisely when demands are revealed. Then concavity implies that a revenue surplus will entail a greater loss of welfare than the gain which would be associated with a deficit. To hedge against this risk, the deployment of tax instruments is shifted towards goods whose demands are more certain. Thus if γ_1 is uncertain the demand for x_1 is relatively risky so taxation, relative to a situation of perfect knowledge is shifted away from good one, increasing the burden on good two.

Model A5 examines uniform commodity taxes, where the model has many consumers and thus incorporates considerations of equity. The linear expenditure system has the property of quasi-homotheticity, and given the possibility of identical lump sum grants to all consumers, this is known to imply the optimality of uniform taxes (see Sadka, 1977, Deaton, 1981, Atkinson and Stiglitz, 1981, p.433). Model A5 uses these assumptions and shows that in this context uniformity is not affected by the introduction of uncertainty about any one of the marginal propensities. However, when one of the subsistence levels is uncertain it is desirable to have different indirect tax rates across goods.

A more convincing argument in favour of uniform commodity taxation has been suggested by Deaton and Stern (1985). In contrast with the models examined in this paper, they express the optimal tax rule in terms of covariances rather than parameter estimates, so rather different methods are required to adjust for uncertainty. Deaton and Stern's argument proceeds as follows: a powerful and frequently used tool for redistribution of income is lump-sum grants based on the demographic characteristics of the recipient. When such a system of 'demogrants' is combined with the empirical assertion that Engle curves are linear, uniform commodity taxation emerges. However it can be shown that demographic characteristics are observed with some error, and that this error is related to society's distributional predelictions, then the uniformity result no longer holds, and it becomes desirable to use the system of indirect taxation to compensate for errors in the administration of demogrants. Stern (1982) discusses errors in the administration of a system of lump-sum grants to compare its merits with optimal non-linear income taxation. The argument here indicates the consequence of such errors for the structure of indirect taxation.

4. CONCLUDING REMARKS

In principle it would be possible to avoid any distinction between parameter estimation and policy optimisation. Rather than using statistical criteria to derive estimates from the data, and then selecting optimal taxes according to a welfare function, the whole procedure may be condensed into one operation. Policies may be chosen to maximise welfare given observed data, making policy rules and parameter estimates redundant. This paper makes only a small step towards such a unified approach however it does draw attention to two pieces of information which may be neglected as a result of the separation of estimation and optimisation. Firstly, parameter uncertainty may be included in optimal policy rules; and secondly, the 'best estimate' depends on the kinds of policy choice for which it is intended.

The techniques which have been put through their paces in the examples may be directly applicable to certain categories of policy decisions. In an institutional setting where policies are subject to a large number of constraints it is possible that only one degree of freedom remains. In electricity pricing, for example, although there are a multitude of different rates for different kinds of supply, their relationship to each other is severely constrained. For practical purposes, the standard domestic supply price may be taken as the basis from which other rates are derived. In addition, if there is predominantly one source of uncertainty, such as the price elasticity of electricity demand, the policy problem has the required characteristics and the techniques in Section 2 may be applied.

Generalisation of the optimal tax examples is difficult. The results are sensitive to the precise specification of the model. If for example, the linear labour supply function included a term in squared non-labour income, the clear-cut results would evaporate and uncertainty about any of the parameters would affect the optimal policy choice. Similarly the results are not robust to monotonic transformations of the utility function. If it were squared, the labour supply function would include complicated cross-product terms, and once again, uncertainty about any of the parameters would matter. However, the examples have illustrated the following general propositions. Firstly, adjustment for uncertainty may have a significant effect on optimal taxation, but the magnitude of the adjustments is likely to be small compared with the approximation based on perfect certainty. Secondly, the magnitude of the adjustment is likely to depend on the government's policy constraints: to raise an amount of revenue close to the maximum; to maximise revenue; or to raise a small amount of revenue.

References:

Abbott, M. and O.Ashenfelter (1976) Labour Supply, Commodity Demand, and the Allocation of Time, Review of Economic Studies, Vol.43, pp.389-411.

Atkinson, A.B. and F.Bourginon (1984) "Income Distribution and Differences in Needs", Taxation Incentives and the Distribution of Income Discussion Paper No.64, April.

Atkinson, A.B. H.N.Stern, Gomulka (1980) On the Switch from Direct to Indirect Taxation, J.Pub.E., 14, 195-224.

Atkinson, A.B. and J.E.Stiglitz (1972) The Structure of Indirect Taxation and Economic Efficiency, Journal of Public Economics, 1.

——————————— (1980) Lectures on Public Economics, McGraw Hill.

Broome, J. (1975) An Important Theorem on Income Tax, RES 42, 649-652.

Brown, C.V., E.Levin and D.T.Ulph (1976) Estimates of Labour Hours Supplied by Married Male Workers in Great Britain, Scottish Journal of Political Economy, Vol.23, No.3, November, 261-277.

Corlett, W.J., D.C.Hague (1953) Complementarity and the Excess Burden of Taxation, Review of Economic Studies, pp.21-30.

Deaton, A. (1979) Optimally Uniform Commodity Taxes, Economics Letters, 2, 357-361.

Deaton, A., and J.Muellbauer (1980) Economics and Consumer Behaviour, Cambridge University Press.

Deaton, A. (1981) Optimal Taxes and the Structure of Preferences, Econometrica, Vol.49, No.5, September, p.1245.

Deaton, A. and Stern, N.H. (1985) Optimally Uniform Commodity Taxes, Taste Differences and Lump-Sum Grants, Discussion Paper, June

Deaton, A.S. (forthcoming) "Econometric Issues for Tax Design in Developing Countries" in Newbery Stern (eds.) The Theory of Taxation for Developing Countries, World Bank, Oxford University Press.

Diamond, P.A. and J.E.Stiglitz (1974) Increases in Risk and in Risk Aversion, Journal of Economic Theory 8, 337-360.

Dixit, A. (1975) Welfare Effects of Tax and Price Changes, J.Pub.E. 4, 103-123.

Edgeworth, F.Y. (1897) The Pure Theory of Taxation, EJ 7, 46-70.

Electricity Council (1981) Review of the Structure of the Bulk Supply Tariff, Electricity Council.

Feldstein, M. (1973) On the Optimal Progressivity of the Income Tax, J.Pub.E. 2, 357-376.

Hausman, J.A. (1980) The Effect of Wages, Taxes, and Fixed Costs on Women's Labour Force Participation, J.Pub.E. Vol.14, 161-194.

Hendry, D.F. and J.F.Richard (1982) On the Formulation of Empirical Models in Dynamic Econometrics, Journal of Econometrics, Vol.20, October, pp.9-33.

——— (1983) The Econometric Analysis of Economic Time Series, International Statistical Review, Vol.51, pp.1-38.

King, M.A. (1983) An Index of Inequality : With Applications to Horizontal Equity and Social Mobility, Econometrica, Vol.51, No.1 January, pp.99-115.

Luch, C., A.A.Powell and R.A.Williams, (1977), Patterns on Household Deman and Saving, Oxford University Press, World Bank.

Malinvaud, E. (1969) First Order Certainty Equivalence, Econometrica, Vol.37, No.4, October, pp.706-718.

Mirrlees, J.A. (1971) An Explanation in the Theory of Optimum Income Taxation, Review of Economic Studies 38, 175-208.

Neuman J. von and O.Morgenstern (1947) Theory of Games and Economic Behaviour, Princeton : Princeton University Press.

Newbery, D.M.G. and N.H.Stern (eds.) (forthcoming) The Theory of Taxation for Developing Countries, World Bank, Oxford University Press.

Newbery, D.M.G. and J.E.Stiglitz (1981) The Theory of Commodity Price Price Stabilization, Clarendon Press, Oxford.

Phelps, E.S. (1973) The Taxation of Wage Income for Economic Justice, Quarterly Journal of Economics, Vol.87, 331-354.

Rawls, J. (1971) A Theory of Justice, Harvard University Press, Cambridge, Massachusetts.

Ramsey, F.P. (1927) A Contribution to the Theory of Taxation, Economic Journal, 37, 47-61.

Sadka, E. (1977) A Theorem on Uniform Taxation, Journal of Public Economics, vol.7, pp.387-391.

Samuelson (1970) The Fundamental Approximation Theorem of Portfolio Analysis in Terms of Means, Variances and Higher Moments, RES, vol.37, pp.537-542.

Sandmo, A. (1981) Income Tax Evasion, Labour Supply, and the Equity-Efficiency Tradeoff, Journal of Public Economics 16, 265-288.

Seade, J. (1977) On the Shape of Optimal Tax Schedules, Journal of Public Economics, vol.7, pp.203-35, No.2, April.

Sheshinski, E. (1972) The Optimal Linear Income Tax, RES, 39, 297-302.

Shoemaker (1982) The Expected Utility Model : Its Variants, Purposes, Evidence and Limitations, Journal of Economic Literature, XX No.2, pp.529-63, June.

Simon, H.A. (1956) Dynamic Programming Under Uncertainty with a Quadratic Criterion Function, Econometrica, Vol.24.

Stern, N.H. (1976) On the Specification of Models of Optimum Income Taxation, J.Pub.E. 6, 123-162.

——— (1982) 'Optimal Taxation with Errors in Administration', J.Pub.E. vol.17, No.2, March, pp.181-211.

——— (1984a) On the Specification of Labour Supply Functions, Taxation Incentives and the Distribution of Income, Discussion Paper 50, May.

——— (1984b) Optimal Taxation and Tax Policy, IMF Staff Papers, Vol.31, No.2, June, pp.339-378.

——— (forthcoming) "An Introduction to the Theory of Optimal Commodity and Income Taxation", in Newbery, Stern (eds.) The Theory of Taxation for Developing Countries, World Bank, Oxford University Press.

Theil, H. (1957) A Note on Certainty Equivalence in Dynamic Planning, Econometrica, Vol.25.