Optimal Commodity Taxation with Imperfect Competition

by

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Optimal Commodity Taxation with Imperfect Competition.

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Abstract: This paper derives optimal commodity tax rules for general equilibrium models with imperfect competition. This is achieved by constructing functions for each imperfectly competitive industry describing the effect of taxation upon prices and profits, the construction is applicable to most forms of imperfect competition. Intermediate goods prices appear as important determinants of tax rates and it is shown that the implication of the Diamond-Mirrlees theorem, that intermediate goods remain untaxed, is inapplicable when the competitive assumption is relaxed.

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Section 1: Introduction.

This paper extends the theory of optimal commodity taxation to a general equilibrium model with imperfect competition. Important features of the model are non linear responses of prices to tax changes, the connections between industries via their demand and cost functions and the effects of taxation upon intermediate goods prices. The conclusion of the Diamond-Mirrlees theorem, that intermediate goods should remain untaxed, is shown to be invalid once the competitive assumption is relaxed.

Previous analyses of optimal taxation have been within the Arrow-Debreu paradigm of perfect competition, an avenue of research exemplified by Diamond and Mirrlees (1971). However many policy issues in taxation, for example the special car tax and the duties on whisky, petrol and cigarettes, are concerned with industries that are far from the competitive ideal. To address these, and other similar questions, it is necessary that previous theory be extended to allow for imperfect competition.

The first element of this extension is to understand how imperfectly competitive industries respond to changes in the tax rate upon their output. Stern (1987) and Seade (1985) have previously analysed this issue and I provide here an alternative presentation which is applicable to a variety of imperfectly competitive market forms. The important conclusion for this paper is that these responses, the "direct" effects of taxation, are non-linear.

In a general equilibrium model the market strategy of an imperfectly competitive industry is defined conditionally upon prices ruling in all markets. If any of these change, for instance due to a modification of the tax system, this will result in equilibriating responses in imperfectly competitive markets. These responses, which I will refer to as "induced" effects, form the second element of interest and I demonstrate below how they are incorporated into tax rules.

One consequence of the Diamond-Mirrlees theorem has been that intermediate goods have been relegated to the side-lines in the analysis of taxation. With perfect competition this is a valid means of proceeding. Without it, however, consideration has to be given to how the prices of intermediate goods respond to taxation, indeed, there will be both direct and induced effects.

The task of this paper is to demonstrate how these factors and, additionally, the effects of taxation upon profits are combined to provide rules that characterise optimal commodity taxes.

To bring out clearly the consequences of each the paper is divided into three central sections. Section two concentrates on combining direct and induced effects upon final prices alone. This is achieved by restricting the model to one where labour, supplied by households, is the only input into production. Throughout the paper the wage rate, w, is employed as an untaxed numeraire. For each imperfectly competitive industry I demonstrate how functions may be derived relating the price of that industry's output and the profit level of each firm in the industry to all other goods prices and the tax rate faced by that industry. These functions, and their counterparts for more general production technologies, are of critical importance throughout. An optimal tax rule is derived and analysed in two ways: the first considers balanced budget tax policies for a pair of industries, the second makes the rule comparable with the standard presentation for the competitive model, for instance Atkinson and Stiglitz (1980), by use of the Slutsky equation.

Section three relaxes the restriction on the production technology and allows any good to act as both an input and an object of final consumption, although I still assume that intermediate goods remained untaxed. The tax rules derived make apparent the importance of intermediate goods prices. Another aspect of interest becomes the ability of imperfectly competitive firms to discriminate between intermediate and final consumers.

The results of section three suggest that non-taxation of intermediate goods will not be optimal for a model with imperfect competition. This is confirmed by the analysis of section four even when all intermediate goods are produced by perfectly competitive firms. Extending the model to incorporate the optimal taxation of intermediate goods is then discussed.

Although the model employed is very general there are two situations for which it is not applicable: price-setting oligopoly with an homogenous product and free-entry oligopoly when consumers possess a preference for variety. The latter is discussed in another paper (Myles (1987a)). Finally, I have throughout assumed existence of equilibrium and stability. Given the context of the analysis this does not appear unreasonable.

Section 2: Labour as sole input.

This section derives the optimal tax rule when labour is the sole input into production and its price, the wage rate, acts as numeraire. This restrictive assumption upon technology is made in order to avoid complications concerning intermediate goods prices. It is relaxed in section three. I also assume that perfectly competitive industries produce with constant returns to scale, in conjunction with the first assumption this fixes the pre-tax prices of their output.

The central feature of the analysis is the construction of functions governing the price of each good produced under conditions of imperfect competition and the profits of each firm producing it which have as their arguments the tax rate faced by the industry and all other prices. In essence these functions capture the comparative statics behaviour of these industries and permit the analysis of taxation to be placed within a consistent general equilibrium framework.

2.1: Construction.

Assuming that there are N goods I partition these into two subsets K and J with cardinality K and J respectively, where $K \cup J = N$ and $K \cap J = \phi$. The set K denotes goods produced under constant returns to scale by perfectly competitive firms and J those produced under conditions of imperfect competition.

For a typical good $k \in K$ the post-tax price q_{Kk} is

$$q_{Kk} = p_{Kk} + t_{Kk}$$

p_{Kk} is the pre tax price and t_{Kk} the tax rate. Under the assumptions of constant returns to scale and numeraire wage rate p_{Kk} is fixed. That $\partial q_{Kk}/\partial t_{Kk} = 1$ and $\partial q_{Kk}/\partial t_{Kl}$, $l \neq k$, $= \partial q_{Kk}/\partial t_{Jj}$, all $j \in J$, $= 0$ will remain implicit throughout the following analysis.

In contrast, for a good $j \in J$ I shall assume it is produced by an industry of n_j firms, $n_j \geq 1$. The industry faces a demand function

$$X^{Jj} = X^{Jj}(q_{K1},\ldots,q_{Kk},\ldots,q_{KK},q_{J1},\ldots,q_{Jj},\ldots,q_{JJ})$$

For quantity setting industries the partial inverse is

$$q_{Jj} = q_{Jj}(q_{K1},\ldots,q_{Kk},\ldots,q_{KK},q_{J1},\ldots,X^{Jj},\ldots,q_{JJ})$$

with $X^{Jj} = \sum_{i \in j} x^{Jji}$, x^{Jji} the output of the i'th firm in the industry. Each firm i has cost function

$$C^{Jji} = C^{Jji}(x^{Jji};\ w)$$

and seeks to maximise profits

$$\Pi^{Jji} = x^{Jji}.q_{Jj} - C^{Jji}(x^{Jji};\ w) - t_{Jj}.x^{Jji}$$

For each firm optimal outputs are given from this as

$$x^{Jji} = x^{Jji}(q_{K1},\ldots,q_{Kk},\ldots,q_{KK},q_{J1},\ldots,t_{Jj},\ldots,q_{JJ};\ w)$$

and aggregating over the n firms

$$X^{Jj} = \sum_{i \in j} x^{Jji} = X^{Jj}(q_{K1},\ldots,q_{Kk},\ldots,q_{KK},q_{J1},\ldots,t_{Jj},\ldots,q_{JJ};\ w)$$

This can be substituted into the partial inverse to give

$$q_{Jj} = h_j(q_{K1},\ldots,q_{Kk},\ldots,q_{KK},q_{J1},\ldots,t_{Jj},\ldots,q_{JJ};\ w)$$

h_j is central to the analysis and captures the outcome of market interaction between firms and consumers, its structure captures the strategic interdependence between firms. Derivatives of h_j are written

$$h_{jK1},\dots,h_{jKK},h_{jJ1},\dots,h_{jJj},\dots,h_{jJJ}$$

where

$$h_{jKk} = \partial q_{Jj}/\partial q_{Kk} = \partial q_{Jj}/\partial t_{Kk},$$

$$h_{jJj} = \partial q_{Jj}/\partial t_{Jj}$$

and

$$h_{jJl} = \partial q_{Jj}/\partial q_{Jl}.$$

Hence

$$\partial q_{Jj}/\partial t_{Jl} = h_{jJl}.h_{lJl}.$$

Note that a number of these derivatives may well be zero.

Similarly, substituting

$$x^{Jji} = x^{Jji}(q_{K1},\dots,q_{Kk},\dots,q_{KK},q_{J1},\dots,t_{Jj},\dots,q_{JJ};\ w)$$

and $h_j(\quad)$ into the definition of profit yields a function for each firms profit level

$$\Pi_{Jji} = \Pi_{Jji}(q_{K1},\dots,q_{KK},q_{J1},\dots,t_{Jj},\dots,q_{JJ};\ w)$$

where Π_{Jji} is the profit of the i'th firm in the j'th industry. If the industry permits free-entry the profit function above and its derivatives will be identically zero.

To treat price setting behaviour I will consider each firm in isolation, as noted in the introduction this excludes oligopoly with homogenous output. With this assumption each firm faces a demand function

$$X^{Jj} = X^{Jj}(q_{K1},\ldots,q_{Kk},\ldots,q_{KK},q_{J1},\ldots,q_{Jj},\ldots,q_{JJ})$$

and chooses q_{Jj} to maximise

$$\Pi^{Jj} = X^{Jj}.q_{Jj} - C^{Jj}(X^{Jj};\ w) - t_{Jj}.X^{Jj}$$

which will have solution

$$q_{Jj} = h_j(q_{K1},\ldots,q_{Kk},\ldots,q_{KK},q_{J1},\ldots,t_{Jj},\ldots,q_{JJ};\ w)$$

This derivation is sufficiently general to permit many forms of market structure and conduct. Any oligopolistic interaction is captured in the relation of h_j to the primitives of demand and cost functions.

Substituting $h_j(\quad)$ for q_{Jj} in the definition of profit gives a function

$$\Pi^{Jj} = \Pi^{Jj}(q_{K1},\ldots,q_{KK},q_{J1},\ldots,t_{Jj},\ldots,q_{JJ};\ w)$$

2.2: Examples.

To illustrate the possible forms of these functions I now present two examples: fixed number Cournot and free entry Cournot. The direct effects of taxation have received previous treatments in the literature, primarily by Seade (1985) and Stern (1987), to provide some variation I will present these results in terms of the direct demand function rather than the indirect demand function employed by Seade. This can be justified in two ways. Firstly, a direct demand function estimated from an empirical exercise may not possess an analytical inverse, this is commonplace when demand takes the form of an integral over some distribution. Secondly, this approach provides an alternative statement of the sufficient conditions for overshifting and of tax-induced profit increases, it also yields a considerable simplification of the final equations for free-entry oligopoly. For these examples the induced effects of taxation are also presented.

Example 1: Fixed Number Cournot.

To induce a symmetrical outcome it is assumed that firms have identical costs given by

$$C^{Jji} = C(x_i), \text{ with } C(0) \geqslant 0,\ C' > 0.$$

The necessary condition for a maximum of profits, with Cournot conjectures, is

$$(q_{Jj} - t_{Jj} - C') + x_i \partial q_{Jj}/\partial X^{Jj} = 0$$

and the second order condition (SOC) is

$$2\partial q_{Jj}/\partial X^{Jj} + x_i \partial^2 q_{Jj}/\partial X^{Jj2} - C'' < 0 \qquad (1)$$

(1) can also be written as

$$(\partial q_{Jj}/\partial X^{Jj})^2 (2\partial X^{Jj}/\partial q_{Jj} + (q_{Jj} - t_{Jj} - C')\partial^2 X^{Jj}/\partial q_{Jj}{}^2 - C''(\partial X^{Jj}/\partial q_{Jj})^2) < 0 \qquad (2)$$

an expression that will prove useful below.

From these follows

$$\frac{\partial h_j}{\partial q_{Li}} = -\frac{(n^{-1}(1 - C''\partial X^{Jj}/\partial q_{Jj})\partial X^{Jj}/\partial q_{Li} + (q_{Jj} - t_{Jj} - C')\partial^2 X^{Jj}/\partial q_{JJ}\partial q_{Li})}{(n+1)n^{-1}\partial X^{Jj}/\partial q_{Jj} + (q_{Jj} - t_{Jj} - C')\partial^2 X^{Jj}/\partial q_{Jj}{}^2 - n^{-1}C''(\partial X^{Jj}/\partial q_{Jj})^2} \qquad (3)$$

$L = K, J \quad Ll \neq Jj$

Setting n = 1 gives the induced response when industry Jj is a monopoly.

At this level of generality it is the denominator of (3) that is especially interesting. Recall that (2) is the second-order-condition for profit maximisation and use this to write the denominator as

$$(\partial X^{Jj}/\partial q_{Jj})^2(\text{SOC} \quad (2 \ (n+1)n^{1})\partial X^{Jj}/\partial q_{Jj} - (n-1)n^{-1}C'') \qquad -(4)$$

Since $\partial X^{Jj}/\partial q_{Jj} < 0$ the second term in the brackets will be strictly negative for all $n > 1$. Therefore, if in the limit as $n \to \infty$

$$\text{SOC} \quad \partial X^{Jj}/\partial q_{Jj} \quad C'' > 0$$

dq_{Jj}/dq_{Li} will change sign at some value of n, say n^*. A necessary condition for this to occur is that the demand function is strictly convex. As n approaches n^* from above (below), (3) will tend to either $+\infty$ if the numerator is positive (negative) or $-\infty$ if negative (positive). However, Seade (1980b) has shown that negativity of (4) is a sufficient, though not necessary, condition for stability of Cournot oligopoly. Rather than impose this as a further restriction on the model I note the possibility that if (4) were to become positive the equilibrium may be unstable. Note that this is a result of the Cournot conjectures as can readily be seen by changing the model to one with collusive conjectures ($\partial x_j/\partial x_i \quad 1$) in which case the denominator of (3) will once again be the second order condition for profit maximisation. Consequently if the demand function is strictly convex the number of firms in the industry may be an important variable, provided the equilibrium is stable. For practical applications this suggests that industries with "few" firms may require different treatment to those with "many".

Turning to the numerator of (3) it can readily be seen that this may be of either sign. It will always be positive when both $\partial XJj\partial q_{Li}$ and $\partial^2 X^{Jj}/\partial q_{Jj}\partial q_{Li}$ are positive. Note that the sign of $\partial X^{Jj}/\partial q_{Li}$ may be taken as a rough measure of the relationship between X^{Jj} and X^{Li}, in what follows I will call the goods substitutes if $\partial X^{Jj}/\partial q_{Li}$ is positive, complements otherwise. However, it is negativity of the numerator that+ appears the interesting case and there are several combinations of factors that will give this. Negativity will follow from the goods being complements and the demand function having a positive second cross derivative, however it is still possible with $\partial X^{Jj}/\partial q_{Li} > 0$ if $\partial^2 X^{Jj}/\partial q_{Jj}\partial q_{Li}$ is sufficiently less than zero.

The induced effect upon profits can be calculated as

$$\frac{d\Pi_{Jji}}{dq_{Li}} = \frac{a}{(n+1)n^{-1}\partial X^{Jj}/\partial q_{Jj}+(q_{Jj}-t_{Jj}-C')\partial^2 X^{Jj}/\partial q_{Jj}{}^2-n^{-1}C''(\partial X^{Jj}/\partial q_{Jj})^2}$$

where

$$\begin{aligned} a = -(q_{Jj}-t_{Jj}-C')(2n^{-1}\partial X^{Jj}/\partial q_{Jj}.\partial X^{Jj}/\partial q_{Li} + (n-1)n^{-1}(q_{Jj}-t_{Jj}-C') \\ \partial X^{Jj}/\partial q_{Jj}.\partial^2 X^{Jj}/\partial q_{Jj}\partial q_{Li} + n^{-1}(q_{Jj}-t_{Jj}-C')\partial X^{Jj}/\partial q_{Li}\partial^2 X^{Jj}/\partial q_{Jj}{}^2 \\ - n^{-1}C''\partial X^{Jj}/\partial q_{Li}(\partial X^{Jj}/\partial q_{Jj})^2)\end{aligned}$$

Seade (1985) calculated the direct effects of taxation for oligopoly. Written in terms of the direct demand function to make them compatible with the analysis above these are

$$\frac{dh_j}{dt_{Jj}} = -\frac{\partial X^{Jj}/\partial q_{Jj}}{(n+1)n^{-1}\partial X^{Jj}/\partial q_{Jj}+(q_{Jj}-t_{Jj}-C')\partial^2 X^{Jj}/\partial q_{Jj}{}^2-n^{-1}C''(\partial X^{Jj}/\partial q_{Jj})^2} \quad (5)$$

and

$$\frac{d\Pi_{Jj}}{dt_{Jj}} = \frac{b}{\left[(n+1)n^{-1}\partial X^{Jj}/\partial q_{Jj} + (q_{Jj}-t_{Jj}-C')\partial^2 X^{Jj}/\partial q_{Jj}^2 - n^{-1}C''(\partial X^{Jj}/\partial q_{Jj})^2\right]^2} \quad (6)$$

where

$$b = (q_{Jj}-t_{Jj}-C').\partial X^{Jj}/\partial q_{Jj}(2n^{-1}\partial X^{Jj}/\partial q_{Jj} + (q_{Jj}-t_{Jj}-C').\partial^2 X^{Jj}/\partial q_{Jj}^2 - n^{-1}C''(\partial X^{Jj}/\partial q_{Jj})^2)$$

Equation (5) may be used to derive a necessary condition for over-shifting. If $\partial h_j/\partial t_{Jj}$ is to be greater than one, assuming constant marginal costs,

$$0 < \partial X^{Jj}/\partial q_{Jj} + n(q_{Jj}-t_{Jj}-C')\partial^2 X^{Jj}/\partial q_{Jj}^2$$

or, after substitution from the first-order condition

$$(X^{Jj}.\partial^2 X^{Jj}/\partial q_{Jj}^2)/(\partial X^{Jj}/\partial q_{Jj})^2 > 1$$

which can be written

$$(1/(\partial X^{Jj}/\partial q_{Jj})).E' > 1$$

with E' the elasticity of the gradient of the direct demand function. Note that $\partial^2 X^{Jj}/\partial q_{Jj}^2 > 0$ is a necessary condition for over-shifting.

Applying the same analysis to (6) profits will rise as a result of a tax increase when

$$(1/(\partial X^{Jj}/\partial q_{Jj})).E' > 2.$$

Example 2: Oligopoly with Free Entry.

It is assumed that the effect of entry is to reduce equilibrium profits to zero, or by suitable redefinition of the cost function, to some lower bound. I also make the strong assumption that the industry is sufficiently large to permit n to be treated as a continuous variable. Both the direct and induced effects are calculated in the following derivation. The derivations are contained in Myles (1987b).

Firstly, for the induced effects,

$$\frac{dh_j}{dq_{Li}} = \frac{-(q_{Jj}-C'-t_{Jj})\partial^2 X^{Jj}/\partial q_{Li}\partial q_{Jj}}{2\partial X^{Jj}/\partial q_{Jj}+(q_{Jj}-C'-t_{Jj})\partial^2 X^{Jj}/\partial q_{Jj}{}^2-C''(\partial X^{Jj}/\partial q_{Jj})^2} \qquad -(7)$$

The denominator of (7) is the second-order condition for each firm's maximisation problem and is assumed negative so that the direction of dh_j/dq_{Li} is the same as that of $\partial^2 X^{Jj}/\partial q_{Li}\partial q_{Jj}$.

Although not directly relevant for the optimal tax problem the effect of the price change upon the size of the industry is

$$\frac{dn}{dq_{Li}} = \frac{1}{x}\frac{\partial X^{Jj}}{\partial q_{Li}} - \frac{(n-1)\partial^2 X^{Jj}/\partial q_{Li}\partial q_{Jj}}{2\partial X^{Jj}/\partial q_{Jj}+(q_{Jj}-t_{Jj}-C')\partial^2 X^{Jj}/\partial q_{Jj}{}^2-C''(\partial X^{Jj}/\partial q_{Jj})^2} \qquad -(8)$$

If the goods are substitutes, in the sense defined above, the first term in (8) is positive so that a positive second cross derivative is sufficient to guarantee entry. Consequently, for substitutes, if the price rises entry occurs. However it is not possible to have price increases and exit occurring simultaneously. With complements the first term is negative so that it is possible for exit to accompany price increases.

In a similar manner the direct effects are found to be

$$\frac{dh_j}{dt_{Jj}} = -\frac{2\partial X^{Jj}/\partial q_{Jj} - C''(\partial X^{Jj}/\partial q_{Jj})^2}{2\partial X^{Jj}/\partial q_{Jj}+(q_{Jj}-C'-t_{Jj})\partial^2 X^{Jj}/\partial q_{Jj}{}^2-C''(\partial X^{Jj}/\partial q_{Jj})^2} \qquad -(9)$$

and

$$\frac{dn}{dt_{Jj}} = -\frac{1}{x}\frac{\partial X^{Jj}}{\partial q_{Jj}} - \frac{(n-1)\partial^2 X^{Jj}/\partial q_{Jj}{}^2}{\partial X^{Jj}/\partial q_{Jj}+(q_{Jj}-C'-t_{Jj})\partial^2 X^{Jj}/\partial q_{Jj}{}^2-C''(\partial X^{Jj}/\partial q_{Jj})^2} \quad (10)$$

From (9) may be derived a condition that delineates those cases for which over shifting will occur. Re-arrangement reveals that when marginal cost is constant

$$dh_j/dt_{Jj} > 1 \text{ when } \partial^2 X^{Jj}/\partial q_{Jj}{}^2 > 0.$$

Consequently, with linear demand forward-shifting will be 100%, with convex demand there will be under shifting.

Similarly (10) may be rearranged to provide a necessary condition for entry to occur. Setting (10) greater than zero and multiplying across by the denominator of the second term, entry will occur when

$$(2/x)(\partial X^{Jj}/\partial q_{Jj})^2 + ((q_{Jj}-t_{Jj}-c)/x)\partial^2 X^{Jj}/\partial q_{Jj}{}^2 . \partial X^{Jj}/\partial q_{Jj} - (n-1)\partial^2 X^{Jj}/\partial q_{Jj}{}^2 < 0$$

Using the fact that profit maximisation implies

$$(q_{Jj}-t_{Jj}-c)/(x\partial q_{Jj}/\partial X^{Jj}) = -1$$

this expression may be written as

$$(1/(\partial X^{Jj}/\partial q_{Jj})).E' > 2.$$

These examples have demonstrated certain interesting properties of the h_{Jj} and H_{Jji} functions for two particular models of imperfect competition. I now return to the derivation of optimal tax rules in a general model for which there are no prior restrictions regarding the conduct or structure of the imperfectly competitive industries.

2.3: Tax Rules.

The components of the model are:

1) Social Welfare: for this model, and throughout the paper, I will employ the utilitarian sum of indirect utilities. This welfare function is chosen to provide a common baseline from which alternative results may be assessed. Thus

$$S.W. = \sum_{h\in H} V^h(q_{K1},\ldots,q_{KK},q_{J1},\ldots,q_{JJ},w,\Pi^h)$$

where H is the set of households, h a typical member and Π^h his profit income. Note that $\Pi^h = \Sigma_{j\in J}\Sigma_{i\in j}\theta^h{}_{Jji}\Pi_{Jji}$ and $\Sigma_{h\in H}\theta^h{}_{Jji} = 1$.

2)Government Revenue Constraint: the government is attempting to raise revenue to purchase a quantity of labour with value R. Hence

$$\sum_{k\in K} t_{Kk}X_{Kk} + \sum_{j\in J} t_{Jj}X_{Jj} = R$$

Where X_{Kk}, X_{Jj} are the aggregate demands.

3) A set of J functions $h_j(\quad)$ and $\#\Sigma_{j\in J}n_j$ functions $\Pi^{Jji}(\quad)$.

The argument lying behind this formulation is essentially equivalent to that of Diamond and Mirrlees (1971). The market will ensure that excess demand for goods is zero, similarly firms' and consumers' budget constraints will be met. Restricting the government to meet its budget constraint then ensures that the labour market is in equilibrium.

Combining these the optimal tax problem becomes:

Choose $t_{K1},\ldots,t_{KK},t_{J1},\ldots,t_{JJ}$ to maximise

$$L = \sum_{h\in H} V^h(q_{K1},\ldots,q_{KK},q_{J1},\ldots,q_{JJ},w,\Pi^h)$$

$$+ \lambda\left[\sum_{k\in K} t_{Kk}X_{Kk} + \sum_{j\in J} t_{Jj}X_{Jj} - R\right]$$

The first order conditions for this problem may be written

$$\frac{\partial L}{\partial t_{K1}} = \sum_{h\in H}\frac{\partial V^h}{\partial q_{K1}} + \sum_{h\in H}\sum_{j\in J}\frac{\partial V^h}{\partial q_{Jj}}.h_{jK1} + \sum_{h\in H}\sum_{j\in J}\frac{\partial V^h}{\partial \Pi^h}.\sum_{i\in j}\theta^h_{Jji}.\frac{\partial \Pi^{Jji}}{\partial q_{K1}}.h_{jK1}$$

$$+ \sum_{h\in H}\sum_{j\in J}\frac{\partial V^h}{\partial \Pi^h}.\sum_{i\in j}\theta^h_{Jji}.\sum_{s\in J,s\neq j}\frac{\partial \Pi^{Jji}}{\partial q_{Js}}.h_{sK1} + \lambda\Bigg[X_{K1}$$

$$+ \sum_{k\in K} t_{Kk}.\frac{\partial X_{Kk}}{\partial q_{k1}} + \sum_{k\in K}\sum_{j\in J} t_{Kk}.\frac{\partial X_{Kk}}{\partial q_{Jj}}.h_{jK1}$$

$$+ \sum_{j\in J} t_{Jj}.\frac{\partial X_{Jj}}{\partial q_{K1}} + \sum_{j\in J}\sum_{s\in J} t_{Jj}.\frac{\partial X_{Jj}}{\partial q_{Jj}}.h_{sK1}\Bigg] \qquad (11)$$

which describes the choice of tax rate for a typical good $1\in K$, and

$$\frac{\partial L}{\partial t_{J1}} = \sum_{h\in H}\sum_{j\in J}\frac{\partial V^h}{\partial q_{Jj}}.h_{jJ1}.h_{1J1}$$

$$+ \sum_{h\in H}\sum_{j\in J}\frac{\partial V^h}{\partial \Pi^h}\sum_{s\in J,s\neq 1}\sum_{i\in j}\theta^h_{Jji}.\frac{\partial \Pi^{Jji}}{\partial q_{Js}}.h_{sJ1}h_{1J1}$$

$$+ \sum_{h\in H}\frac{\partial V^h}{\partial \Pi^h}\sum_{i\in 1}\theta^h_{J1i}.\frac{\partial \Pi^{J1i}}{\partial t_{J1}} + \lambda\Bigg[X_{J1} + \sum_{k\in K}\sum_{j\in J} t_{Kk}.\frac{\partial X_{Kk}}{\partial q_{Jj}}.h_{jJ1}.h_{1J1}$$

$$+ \sum_{j\in J}\sum_{s\in J} t_{Jj}\frac{\partial X_{Jj}}{\partial q_{Js}}.h_{sJl}.h_{lJl} \Bigg] \qquad -(12)$$

for a good $l \in J$.

These equations give the precise information required to calculate the optimal tax rates. This amounts to a cardinalised indirect utility function for each consumer and a cost function for each firm. With these the expressions may be evaluated in two steps. Firstly, the terms h_{jKl} and h_{jJl} can be replaced by expressions of the form drived in the examples above or such as is appropriate for each industry, the derivatives of profit by the companion terms and, secondly, the derivatives of aggregate demands may be replaced with derivatives of indirect utility using Shephard's lemma.

Two possible procedures suggest themselves for collecting the utility information required for implementation of a rule such as this. One would be to parsimoniously parametrise a common stochastic utility function and, as is common practise in the modal choice literature, estimate the distribution of unobserved parameters from market data. The summations above would be replaced with integrals over the relevant distributions. Alternatively, one could specify the utility function and obtain data on some, or all, parameters by surveys means whilst estimating remaining parameters from market data.

As equations (11) and (12) are rather sterile as they stand I now pursue two alternative ways of forcing some interpretation out of them. Firstly, I will consider the case of only two goods with one produced by a competitive industry and, secondly, I will set these equations in a form comparable with existing formulae by making use of the Slutsky equation.

With only two goods the necessary conditions are

$$\partial L/\partial t_{K1} = \Sigma_h \partial v^h/\partial q_{K1} + \Sigma_h \partial v^h/\partial q_{J1}.h_{1K1} + \Sigma_h \partial v^h/\partial \Pi^h.\theta^h{}_{J1}\partial \Pi^{J1}/\partial q_{K1}$$
$$+ \lambda(X_{K1} + t_{K1}.\partial X_{K1}/\partial q_{K1} + t_{K1}.\partial X_{K1}/\partial q_{J1}.h_{1K1}$$
$$+ t_{J1}.\partial X_{J1}/\partial q_{K1} + t_{J1}.\partial X_{J1}/\partial q_{J1}.h_{1K1})$$

and

$$\partial L/\partial t_{J1} = \Sigma_h \partial v^h/\partial q_{J1}.h_{1J1} + \Sigma_h \partial v^h/\partial \Pi^h.\theta^h{}_{J1}\partial \Pi^{J1}/\partial t_{J1}$$
$$+ \lambda(X_{J1} + t_{K1}.\partial X_{K1}/\partial q_{J1}.h_{1J1} + t_{J1}.\partial X_{J1}/\partial q_{J1}.h_{1J1})$$

If it is assumed that the government are aiming for a balanced budget and that profits accrue to an actor outside the model then

$$t_{J1} = (X_{J1}(\Sigma_h \partial v^h/\partial q_{K1} + \Sigma_h \partial v^h/\partial q_{J1}.h_{1K1}) - X_{K1}\Sigma_h \partial v^h/\partial q_{J1}.h_{1J1})a^{-1} \qquad (13)$$

where

$$a = h_{1J1}(\Sigma_h \partial v^h/\partial q_{J1}(\partial X_{J1}/\partial q_{K1} - X_{J1}/X_{K1}.\partial X_{K1}/\partial q_{K1})$$
$$- \Sigma_h \partial v^h/\partial q_{K1}(\partial X_{J1}/\partial q_{J1} - X_{J1}/X_{K1}.\partial X_{K1}/\partial q_{J1}))$$

If the goods are substitutes ($\partial X_{K1}/\partial q_{J1}$ and $\partial X_{J1}/\partial q_{K1} > 0$) then $a > 0$ and these equations provide the simple rule

$$t_{J1} < 0 \text{ if } X_{J1}(\Sigma_h \partial v^h/\partial q_{K1} + \Sigma_h \partial v^h/\partial q_{J1}.h_{1K1}) - X_{K1}\Sigma_h \partial v^h/\partial q_{J1}.h_{1J1} > 0$$

and

$$t_{J1} > 0 \text{ if } X_{J1}(\Sigma_h \partial v^h/\partial q_{K1} + \Sigma_h \partial v^h/\partial q_{J1}.h_{1K1}) - X_{K1}\Sigma_h \partial v^h/\partial q_{J1}.h_{1J1} < 0$$

The first of these is the interesting case because it implies the subsidisation of imperfectly competitive firms. The conditions necessary for its occurrence can be stated loosely as "large h_{1J1}, negative h_{1Kl}" so that a high degree of over-shifting or a negative induced effect will lead to subsidisation. The reasoning behind this result is straightforward. If taxes are overshifted the same will apply to any subsidy payment and, although the subsidy must be met by a tax on competitive firms, the final result of the policy may be a beneficial reduction of the general price level. This argument is reinforced if the imperfectly competitive industry also reduces its price in response to the tax on the competitive industry.

Returning to the equations for the general case the Slutsky equation can be used to write these as

$$\sum_{k\in K} t_{Kk} \sum_{h\in H} S^h{}_{kl} + \sum_{k\in K} t_{Kk} \sum_{j\in J} \sum_{h\in H} S^h{}_{kj} h_{jKl} + \sum_{j\in J} t_{Jj} \sum_{h\in H} S^h{}_{jl}$$

$$\sum_{j\in J} t_{Jj} \sum_{s\in J} \sum_{h\in H} S^h{}_{js} h_{sKl} = -\Big[X_{Kl} - \frac{1}{\lambda}\Big[\sum_{h\in H} \alpha^h X^h{}_{Kl}$$

$$+ \sum_{h\in H} \sum_{j\in J} \alpha^h X^h{}_{Jj} h_{jKl} - \sum_{h\in H} \sum_{j\in J} \alpha^h \sum_{i\in j} \theta^h{}_{Jji} \cdot \frac{\partial \Pi^{Jji}}{\partial q_{Kl}} . h_{jKl}$$

$$\sum_{h\in H} \sum_{j\in J} \alpha^h \sum_{i\in j} \theta^h{}_{Jji} \cdot \sum_{s\neq j} \frac{\partial \Pi^{Jji}}{\partial q_{Js}} . h_{sKl} \Big] - \sum_{k\in K} t_{Kk} \sum_{h\in H} X^h{}_{Kl} \cdot \frac{\partial X^h}{\partial \Pi} h^{Kk}$$

$$\sum_{k\in K} t_{Kk} \sum_{j\in J} \sum_{h\in H} X^h{}_{Jj} \cdot \frac{\partial X^h}{\partial \Pi} h^{Kk} . h_{jKl} - \sum_{j\in J} t_{Jj} \sum_{h\in H} X^h{}_{Kl} \cdot \frac{\partial X^h}{\partial \Pi} h^{Jj}$$

$$\sum_{j\in J} t_{Jj} \sum_{s\in J} \sum_{h\in H} X^h{}_{Js} \cdot \frac{\partial X^h}{\partial \Pi} h^{Jj} . h_{sKl} \Big] \quad \text{for all } l \in K \tag{14}$$

and

$$\sum_{k\in K} t_{Kk}\sum_{j\in J}\sum_{h\in H} s^h_{kj} h_{jJl} h_{lJl} + \sum_{j\in J} t_{Jj}\sum_{s\in J}\sum_{h\in H} s^h_{js} h_{sJl} h_{lJl}$$

$$\Big[\quad X_{Jl} \qquad \frac{1}{\lambda}\Big[\sum_{h\in H}\sum_{j\in J}\alpha^h x^h_{Jj} h_{jJl}\cdot h_{lJl}$$

$$\sum_{h\in H}\sum_{j\in J}\alpha^h\sum_{i\in j}\theta^h_{Jji}\cdot\sum_{s\neq l}\frac{\partial\Pi_{Jji}}{\partial q_{Js}}\cdot h_{sJl}\cdot h_{lJl} - \sum_{h\in H}\alpha^h\sum_{i\in l}\theta^h_{Jli}\cdot\frac{\partial\Pi_{Jli}}{\partial t_{Jl}}\Big]$$

$$\sum_{k\in K} t_{Kk}\sum_{j\in J}\sum_{h\in H} x^h_{Jj}\cdot\frac{\partial x^h}{\partial\Pi^h}{}_{Kk}\cdot h_{jJl}\cdot h_{lJl} -$$

$$\sum_{j\in J} t_{Jj}\sum_{s\in J}\sum_{h\in H} x^h_{Js}\cdot\frac{\partial x^h}{\partial\Pi^h}{}_{Jj}\cdot h_{sJl}\cdot h_{lJl}\Big] \quad \text{for all } l\in J \qquad (15)$$

It is instructive to compare these equations with those obtained in the standard model. If it is accepted that the product of a tax rate and the substitution term is an approximation to the demand change resulting from the imposition of the tax system then the left-hand-side of (14) represents the change in demand for good l as a direct result of the tax system (the first and third terms) plus the changes in demand for all goods in J as a result of price changes induced by the l'th tax rate. For each good in K this total change in demand must be equal to a sum composed of: demand for good l, the changes in demand for all goods due to variations in profit income weighted by the demand for good l and the good's own tax rate, change in demand for goods in J due to profit variations weighted by own demand, the tax rates of all goods and the induced price changes and, finally, a welfare term which incorporates the welfare effect of profit changes.

For goods in J the equation follows much the same pattern. The left-hand-side measures demand changes for all goods in J due to the direct effect of taxation for good 1 and the induced effects for the other goods. The right-hand-side again is a sum of own demand, changes in demand for goods in J as a result of profit changes and a corresponding welfare term.

It is evident that the complexity of these expressions prevents the precise interpretation that is possible for the competitive model. Their value lies in their applicability, particularly for situations that involve the assessment of the optimality of adjustment of a small number of tax rates taking the remaining tax system as given. An example of such an application is discussed in Myles (1987b).

Section 3: Other Inputs.

The model of section two was severely restricted by the assumption that labour was the sole input, I now seek to remove this assumption and to allow for more general production technologies. In doing this it will become apparent the tax formulae derived indicate that the tax treatment of intermediate goods is an interesting question. For the competitive model this is covered by the Diamond-Mirrlees theorem which at present has no parallel for models with imperfect competition. In this section I shall follow conventional theory, and practice, in assuming that intermediate goods remain untaxed. The failure of the Diamond-Mirrlees for imperfect competition will be demonstrated in section four and the taxation of intermediate goods will also be considered.

As for section two I will construct functions describing firms' responses to changes in the tax system. When intermediate goods are present the ability of imperfectly competitive firms to discriminate between their customers becomes an important determinant of the form of these equations. If discrimination by imperfectly competitive firms between purchases made for final consumption and those for use in production elsewhere is possible such firms are required to choose a pair of output or price strategies. Without discrimination they set only a single output or price. As discrimination is the most interesting case, in an analytical sense, I shall tackle this first.

When intermediate goods are present changes in the tax system will affect their prices creating another avenue through which tax policy will feed back into the economic system. In this case the effects work via firms' cost functions, changes in the price of the inputs they employ resulting in modifications to their own pricing policy. As shown below these cost effects considerably complicate optimal tax formulae.

To facilitate understanding of the general model I will first present one of the simplest examples of the class of models I wish to discuss. The model assumes that there are two price-setting monopolists and that each monopolist uses as inputs both labour and the good produced by the other monopolist, the treatment adopted is a simple one and ignores game-theoretic considerations in order to illustrate the features most relevant for taxation. I also assume that each monopolist is able to discriminate between final consumers and purchases made for use as inputs. The comparative statics describing the effects of price changes will be derived and then incorporated into optimal tax rules.

3.1: Two Monopolists.

To distinguish between final and intermediate demands I shall write quantities of final goods as X_i^C and of intermediate goods X_i^F, the subscript referring to the firm that has produced the output. Similarly, the price for intermediate output will be f_i. Assuming that each firm uses as inputs labour and the other firm's output, the problems facing firms 1 and 2 are:

$$\text{Firm 1: } \max_{q1,f1} \ \Pi^1 = (q_1 - t_1)X_1^C + f_1X_1^F - C^1$$

$$\text{Firm 2: } \max_{q2,f2} \ \Pi^2 = (q_2 - t_2)X_2^C + f_2X_2^F - C^2$$

The close interdependence between the two firms implies that any equlibrium must be a joint functional equilibrium. However complex this equilibrium, the resulting demand functions faced by the firms must, if the equilibrium is to be consistent, take the following forms:

$$X_1^C = X_1^C(q_1, q_2, w), \quad X_2^C = X_2^C(q_1, q_2, w)$$

$$X_1^F = X_1^F(t_2, f_1, q_1, w), \quad X_2^F = X_2^F(t_1, f_2, q_2, w)$$

and the cost functions are defined as

$$C^1 = C^1(f_2, w;\ X_1^C + X_1^F), \quad C^2 = C^2(f_1, w;\ X_2^C + X_2^F)$$

The appearence of tax rates and final-goods prices in the second pair of functions results from demand X_i^F being a derived demand from firm j's maximisation and hence is functionally dependent on all variables that are parametric to firm j but enter directly into j's profit function.

Concentrating upon firm 1 the necessary conditions for maximising profit are

$$X_1^C + (q_1 - t_1)\partial X_1^C/\partial q_1 + f_1 \partial X_1^F/\partial q_1 = C_0^1 \partial X_1^C/\partial q_1 + C_0^1 \partial X_1^F/\partial q_1 \quad (16)$$

and

$$X_1^F + f_1 \partial X_1^F/\partial f_1 = C_0^1 \partial X_1^F/\partial f_1 \quad (17)$$

where C_0^1 represents the derivative of the cost function with respect to its third argument. Equivalent equations may be derived for firm 2. q_1 and f_1 are determined jointly and any changes in t_1, t_2, f_2 and q_2 will affect both f_1 and q_1.

To determine the effect upon firm 1's pricing policy of changes in 2's behaviour and of changes in tax rates consider differential changes in t_1, t_2, q_2 and f_2 in (16) and (17) for which the resultant changes in f_1 and q_1 are the solution to the following pair of equations:

$$\left[2.\frac{\partial X_1{}^C}{\partial q_1} + (q_1-t_1).\frac{\partial^2 X_1{}^C}{\partial q_1{}^2} + f_1.\frac{\partial^2 X_1{}^F}{\partial q_1{}^2}\right]dq_1 + \left[\frac{\partial X_1{}^C}{\partial q_2} + (q_1-t_1).\frac{\partial^2 X_1{}^C}{\partial q_1 \partial q_2}\right]dq_2 +$$

$$\left[\frac{\partial X_1{}^F}{\partial q_1} + f_1\frac{\partial^2 X_1{}^F}{\partial q_1 \partial f_1}\right]df_1 + \left[f_1\frac{\partial^2 X_1{}^F}{\partial q_1 \partial t_2}\right]dt_2 = \left[C_{01}{}^1.\frac{\partial X_1{}^C}{\partial q_1} + C_{01}{}^1.\frac{\partial X_1{}^F}{\partial q_1}\right]df_2 +$$

$$\left[C_{00}{}^1.\left[\frac{\partial X_1{}^C}{\partial q_1}\right]^2 + C_0{}^1.\frac{\partial^2 X_1{}^C}{\partial q_1{}^2} + 2C_{00}{}^1\frac{\partial X_1{}^C}{\partial q_1}.\frac{\partial X_1{}^F}{\partial q_1} + C_0{}^1.\frac{\partial^2 X_1{}^F}{\partial q_1{}^2} + C_{00}{}^1.\left[\frac{\partial X_1{}^F}{\partial q_1}\right]^2\right]dq_1$$

$$+\left[C_{00}{}^1.\frac{\partial X_1{}^C}{\partial q_2}.\frac{\partial X_1{}^F}{\partial q_1} + C_{00}{}^1.\frac{\partial X_1{}^C}{\partial q_2}.\frac{\partial X_1{}^C}{\partial q_1} + C_0{}^1.\frac{\partial^2 X_1{}^C}{\partial q_1 \partial q_2}\right]dq_2 +$$

$$\left[C_{00}{}^1.\frac{\partial X_1{}^C}{\partial q_1}.\frac{\partial X_1{}^F}{\partial f_1} + C_{00}{}^1.\frac{\partial X_1{}^F}{\partial q_1}.\frac{\partial X_1{}^F}{\partial f_1} + C_0{}^1.\frac{\partial^2 X_1{}^C}{\partial q_1 \partial f_1}\right]df_1 +$$

$$\left[C_{00}{}^1.\frac{\partial X_1{}^C}{\partial q_1}.\frac{\partial X_1{}^F}{\partial t_2} + C_{00}{}^1.\frac{\partial X_1{}^F}{\partial q_1}.\frac{\partial X_1{}^F}{\partial t_2} + C_0{}^1.\frac{\partial^2 X_1{}^C}{\partial q_1 \partial t_2}\right]dt_2 + \left[\frac{\partial X_1{}^C}{\partial q_1}\right]dt_1$$

which is the total derivative of (16), and

$$\left[2.\frac{\partial X_1{}^F}{\partial f_1} + f_1.\frac{\partial^2 X_1{}^F}{\partial f_1{}^2}\right]df_1 + \left[\frac{\partial X_1{}^F}{\partial q_1} + f_1.\frac{\partial^2 X_1{}^F}{\partial f_1 \partial q_1}\right]dq_1 +$$

$$\left[\frac{\partial X_1{}^F}{\partial t_2} + f_1.\frac{\partial^2 X_1{}^F}{\partial f_1 \partial t_2}\right]dt_2 = \left[C_{00}{}^1.\frac{\partial X_1{}^C}{\partial q_1}\frac{\partial X_1{}^F}{\partial f_1} + C_{00}{}^1.\frac{\partial X_1{}^F}{\partial q_1}\frac{\partial X_1{}^F}{\partial f_1} + C_0{}^1.\frac{\partial^2 X_1{}^F}{\partial q_1 \partial f_1}\right]dq_1$$

$$+\left[C_{00}{}^{1}\cdot\frac{\partial X_1{}^{F}}{\partial f_1}\frac{\partial X_1{}^{C}}{\partial q_2}\right]dq_2 + \left[C_{00}{}^{1}\frac{\partial X_1{}^{F}}{\partial f_1}\frac{\partial X_1{}^{F}}{\partial f_1} + C_0{}^{1}\cdot\frac{\partial^2 X_1{}^{F}}{\partial f_1{}^{2}}\right]df_1 +$$

$$\left[C_{01}{}^{1}\cdot\frac{\partial X_1{}^{F}}{\partial f_1}\right]df_2 + \left[C_{00}{}^{1}\frac{\partial X_1{}^{F}}{\partial f_1}\frac{\partial X_1{}^{F}}{\partial t2} + C_0{}^{1}\cdot\frac{\partial^2 X_1{}^{F}}{\partial f_1\partial t_2}\right]dt_2$$

from (17). These can be solved to yield the derivatives: dq_1/dt_1, df_1/dt_1, dq_1/dq_2, df_1/dq_2, dq_1/df_2, df_1/df_2 and dq_1/dt_2, df_1/dt_2.

The final two terms of interest are the effect of t_1 and t_2 upon profits. Recalling the expression for profit

$$\Pi^1 = (q_1 - t_1)X_1{}^{C} + f_1X_1{}^{F} - C^1$$

this can be written as a function of t_1 and t_2 as follows: q_1 may be expressed as a function $q_1 = q_1[t_1,t_2,q_2,f_2]$ and f_1 as $f_1 = f_1[t_1,t_2,q_2,f_2]$. In turn, for the comparative statics of firm 1 q_2 and f_2 become $q_2 = q_2[t_1,t_2,q_1,f_1]$ and $f_2 = f_2[t_1,t_2,q_1,f_1]$. Expressing the functional dependence profits become

$$\Pi^1 = (q_1 - t_1)X_1{}^{C}[q_1,q_2] + f_1X_1{}^{F}[t_2,f_1,q_1] - C^1[f_2,X_1{}^{C}[q_1,q_2]+X_1{}^{F}[t_2,f_1,q_1]]$$

Thus the derivatives of profit are

$$d\Pi^1/dt_1 = X_1{}^{C}(a_1 - 1) + (q_1 - t_1 - C_0{}^{1})(a_1\partial X_1{}^{C}/\partial q_1 + a_2\partial X_1{}^{C}/\partial q_2) + a_3X_1{}^{F} - C_1{}^{1}a_4 + (f_1 - C_0{}^{1})(a_1\partial X_1{}^{F}/\partial q_1 + a_3\partial X_1{}^{F}/\partial f_1)$$

and

$$d\Pi^1/dt_2 = X_1^C(b_1-1) + (q_1-t_1-C_0^1)(b_1\partial X_1^C/\partial q_1 + b_2\partial X_1^C/\partial q_2) + b_3X_1^F$$
$$- C_1^1b_4 + (f_1- C_0^1)(b_1\partial X_1^F/\partial q_1 + b_3\partial X_1^F/\partial f_1)$$

with $a_1 = [\partial q_1/\partial t_1 + \partial q_1/\partial q_2.\partial q_2/\partial t_1 + \partial q_1/\partial f_2.\partial f_2/\partial t_1]$

$a_2 = [\partial q_2/\partial t_1 + \partial q_2/\partial q_1.\partial q_1/\partial t_1 + \partial q_2/\partial f_1.\partial f_1/\partial t_1]$

$a_3 = [\partial f_1/\partial t_1 + \partial f_1/\partial q_2.\partial q_2/\partial t_1 + \partial f_1/\partial f_2.\partial f_2/\partial t_1]$

$a_4 = [\partial f_2/\partial t_1 + \partial f_2/\partial q_1.\partial q_1/\partial t_1 + \partial f_2/\partial f_1.\partial f_1/\partial t_1]$

and $b_1 = [\partial q_1/\partial t_2 + \partial q_1/\partial q_2.\partial q_2/\partial t_2 + \partial q_1/\partial f_2.\partial f_2/\partial t_2]$ for example.

I am now in a position to derive optimal tax formulae for this model. Assuming a utilitarian social welfare function and share parameters θ_i^h, $i = 1, 2$ describing the distribution of profits across households the equations describing the optimal tax rules are:

$$\partial L/\partial t_1 = \Sigma_h\partial V^h/\partial q_1.[\partial q_1/\partial t_1 + \partial q_1/\partial q_2.\partial q_2/\partial t_1 + \partial q_1/\partial f_2.\partial f_2/\partial t_1]$$
$$+ \Sigma_h\partial V^h/\partial q_2.[\partial q_2/\partial t_1 + \partial q_2/\partial q_1.\partial q_1/\partial t_1 + \partial q_2/\partial f_1.\partial f_1/\partial t_1]$$
$$+ \Sigma_h\partial V^h/\partial\Pi^h.\theta^h{}_1\partial\Pi^1/\partial t_1 + \Sigma_h\partial V^h/\partial\Pi^h.\theta^h{}_2\partial\Pi^2/\partial t_1$$
$$+ \lambda(X^C{}_1 + [t_1.\partial X^C{}_1/\partial q_1+t_2\partial X^C{}_2/\partial q_1](\partial q_1/\partial t_1 + \partial q_1/\partial q_2.\partial q_2/\partial t_1$$
$$+ \partial q_1/\partial f_2.\partial f_2/\partial t_1) + [t_1.\partial X^C{}_1/\partial q_2+t_2\partial X^C{}_1/\partial q_2](\partial q_2/\partial t_1$$
$$+ \partial q_2/\partial q_1.\partial q_1/\partial t_1 + \partial q_2/\partial f_1.\partial f_1/\partial t_1))$$
$$= 0$$

with an equivalent expression for t_2. To complete the analysis these equations would be evaluated using the comparative statics expressions derived above.

This tax rule is distinguished from those presented in the previous chapter by the inclusion of terms relating to changes in f_1 and f_2, the intermediate goods prices, and the effect of tax t_i on price q_j, $j \neq i$. These take effect via the adjustment of q_2 to the change in production costs and the dependence of intermediate demands upon tax rates.

There are two implications arising from the existence of these intermediate good price effects. Firstly, simple arguments such as that following (13) will need to be extended and, secondly, they indicate that the presumption of zero taxation for intermediate goods requires close scrutiny. This issue is taken up in section four.

3.2: Generalisation.

Having illustrated the structure of the tax rules in section 3.1 I now extend the model of chapter two to incorporate intermediate goods. The analysis will concentrate upon optimal tax formulae rather than the nature of comparative statics, consequently instead of presenting the algebraic details of oligopoly with discrimination I shall provide a general sketch of the procedure.

As before the each oligopoly will be composed of n quantity setting firms, each firm $i \in j$ choosing an output x_i^{Cj} for final consumption and x_i^{Fj} to satisfy intermediate demand.

The inverse demand functions facing the industry are

$$q_j = q_j(\Sigma_{i\in j} x_i^{Cj}, q_{Js, s\in J, s\neq j}, q_{Kk, k\in K})$$

$$f_j = f_j(\Sigma_{i\in j} x_i^{Fj}, \Sigma_{i\in j} x_i^{Cj}, q_{Js, s\in J, s\neq j}, q_{Kk, k\in K}, t_{Js, s\in J, s\neq j}, t_{Kk, k\in K}, f_{Js, s\in J, s\neq j}, f_{Kk, k\in K}, w)$$

The inclusion of tax rates and final goods prices in f_j is justified by the argument of the previous section.

With R_j the set of inputs used in production of j and C^j the cost function common to all firms producing x^j, profit may be written

$$\Pi^{Jji} = x_i^{Cj}.[q_j(\Sigma_{i\epsilon j}x_i^{Cj},q_{Js,s\epsilon J,s\neq j},q_{Kk,k\epsilon K}) - t_j] - C^j(f_{rj,rj\epsilon Rj},w;x_i^{Cj}+x_i^{Fj}) + x_i^{Fj}.f_j(\Sigma_{i\epsilon j}x_i^{Fj},\Sigma_{i\epsilon j}x_i^{Cj},q_{Js,s\epsilon J,s\neq j},q_{Kk,k\epsilon K},t_{Js,s\epsilon J,s\neq j},t_{Kk,k\epsilon K},f_{Js,s\epsilon J,s\neq j},f_{Kk,k\epsilon K},w)$$

Maximisation of profits will lead to outputs

$$x_i^{Cj} = x_i^{Cj}(t_{Js,s\epsilon J},t_{Kk,k\epsilon K},q_{Js,s\epsilon J,s\neq j},q_{Kk,k\epsilon K},f_{Js,s\epsilon J,s\neq j},f_{Kk,k\epsilon K},w)$$

$$x_i^{Fj} = x_i^{Fj}(t_{Js,s\epsilon J},t_{Kk,k\epsilon K},q_{Js,s\epsilon J,s\neq j},q_{Kk,k\epsilon K},f_{Js,s\epsilon J,s\neq j},f_{Kk,k\epsilon K},w)$$

Substituting these equilibrium quantities into the partial inverse demand function gives

$$q_j = q_j(nx_i^{Cj}, q_{Js,s\epsilon J,s\neq j}, q_{Kk,k\epsilon K})$$

and

$$f_j = f_j(nx_i^{Fj}, nx_i^{Cj},q_{Js,s\epsilon J,s\neq j},q_{Kk,k\epsilon K},t_{Js,s\epsilon J,s\neq j},t_{Kk,k\epsilon K},f_{Js,s\epsilon J,s\neq j},f_{Kk,k\epsilon K},w)$$

These functions capture the comparative statics behaviour of this industry in a manner analogous to the $h_j(\quad)$ of section 2. To incorporate them into an equilibrium structure I will denote them, for a typical industry l, as

$$q_{Jl} = g_{Jl}(q_{Kk,k\epsilon K}, q_{Jj,j\epsilon J,j\neq l}, t_{Kk,k\epsilon K}, t_{Jj,j\epsilon J}, f_{Jj,j\epsilon J,j\neq l}, f_{Kk,k\epsilon K})$$

and

$$f_{Jl} = e_{Jl}(q_{Kk,k\in K},\ q_{Jj,j\in J,j\neq l},\ t_{Kk,k\in K},\ t_{Jj,j\in J},\ f_{Jj,j\in J,j\neq l},\ f_{Kk,k\in K})$$

Employing the entire set of these it is possible to substitute into the profit function, in the manner of section 3.1, to derive a further set of expressions

$$\Pi^{Jji} = \Pi^{Jji}(t_{Kk,k\in K},\ t_{Jj,j\in J})$$

which characterise the effect of tax changes upon profits. A corresponding treatment of price setting may be developed in an analogous manner.

For the competitive firms

$$q_{Kl} = g_{Kl}(t_{Kl},\ f_{Jj,j\in J},\ f_{Kk,k\in K,k\neq l})$$

where f_{Kk} is given definitionally by $q_{Kk} - t_{Kk}$. With this extended notation $\partial q_{Jl}/\partial q_{Kl} = g_{JlqKl}$ for example. Note that some of the derivatives, particularly with respect to intermediate prices, may well be zero.

The maximisation problem remains as before:

$$\max_{tK1,\dots,tKK,tJ1,\dots,tJJ} L = \sum_{h\in H} V^h(q_{K1},\dots,q_{KK},q_{J1},\dots,q_{JJ},w,\Pi^h)$$

$$+ \lambda\left[\sum_{k\in K} t_{Kk}X_{Kk}^C + \sum_{j\in J} t_{Jj}X_{Jj}^C - R\right]$$

The first order conditions for a typical good $l \in K$ may be written

$$\frac{\partial L}{\partial t_{Kl}} = \sum_{h\in H}\sum_{j\in J}\frac{\partial V^h}{\partial q_{Jj}}\left[g_{JjqKl} + \sum_{s\in J, s\neq j} g_{Jjqjs}g_{jstkl} + \sum_{s\in J, s\neq j} g_{Jjfjs}e_{jstkl}\right]$$

$$+ \sum_{h\in H} \frac{\partial v^h}{\partial q_{Kl}} + \sum_{h\in H}\sum_{j\in J} \frac{\partial v^h}{\partial \Pi^h} \cdot \sum_{i\in j} \theta^h{}_{Jji} \cdot \frac{\partial \Pi^{Jji}}{\partial t_{Kl}}$$

$$+\lambda\Big[\; X_{Kl}{}^C + \sum_{k\in K} t_{Kk} \cdot \frac{\partial X_{Kk}{}^C}{\partial q_{kl}} + \sum_{j\in J} t_{Jj} \cdot \frac{\partial X_{Jj}{}^C}{\partial q_{Kl}}$$

$$+ \sum_{k\in K}\sum_{j\in J} t_{Kk} \cdot \frac{\partial X_{Kk}{}^C}{\partial q_{Jj}} \cdot \Big[g_{JjqKl} + \sum_{s\in J', s\neq j} g_{Jjqjs} g_{jstkl} + \sum_{s\in J', s\neq j} g_{Jjfjs} e_{jstkl} \Big]$$

$$+ \sum_{j\in J}\sum_{s\in J} t_{Jj} \cdot \frac{\partial X_{Jj}{}^C}{\partial q_{Jj}} \cdot \Big[g_{JjqKl} + \sum_{s\in J', s\neq j} g_{Jjqjs} g_{jstkl} + \sum_{s\in J', s\neq j} g_{Jjfjs} e_{jstkl} \Big]\Big]$$

-(18)

and for a typical good $l \in J$

$$\frac{\partial L}{\partial t_{Jl}} = \sum_{h\in H}\sum_{j\in J} \frac{\partial v^h}{\partial q_{Jj}} \Big[g_{JjtJl} + \sum_{s\in J, s\neq j} g_{JjqJs} \cdot g_{JstJl} + \sum_{s\in J, s\neq j} g_{JjfJs} \cdot e_{JstJl}\Big]$$

$$+ \sum_{h\in H}\sum_{k\in K} \frac{\partial v^h}{\partial q_{Kk}} \sum_{j\in J} g_{KkfJj} \cdot e_{JjtJl} + \sum_{h\in H}\sum_{j\in J} \frac{\partial v^h}{\partial \Pi^h} \sum_{i\in l} \theta^h{}_{Jji} \cdot \frac{\partial \Pi^{Jji}}{\partial t_{Jl}}$$

$$+ \lambda\Big[\; X_{Jl}{}^C + \sum_{k\in K}\sum_{s\in K} t_{Kk} \cdot \frac{\partial X_{Kk}{}^C}{\partial q_{Ks}} \cdot \sum_{j\in J} g_{KsfJj} \cdot e_{JjtJl}$$

$$+ \sum_{j\in J}\sum_{k\in K} t_{Jj} \cdot \frac{\partial X_{Jj}{}^C}{\partial q_{Kk}} \cdot \sum_{s\in J} g_{KkfJs} \cdot e_{JstJl}$$

$$+ \sum_{k\in K} t_{Kk} \cdot \sum_{j\in J} \frac{\partial X_{Kk}{}^C}{\partial q_{Jj}} \Big[g_{JjtJl} + \sum_{s\in J, s\neq j} g_{JjqJs} \cdot g_{JstJl} + \sum_{s\in J, s\neq j} g_{JjfJs} \cdot e_{JstJl}\Big]$$

$$+\sum_{j\in J} t_{Jj}\cdot\sum_{j\in J}\frac{\partial X_{Jj}^{C}}{\partial q_{Kk}}\left[g_{JjtJl}+\sum_{s\in J,s\neq j} g_{JjqJs}\cdot g_{JstJl}+\sum_{s\in J,s\neq j} g_{JjfJs}\cdot e_{JstJl}\right]\Bigg]$$

-(19)

The interpretation of the components of (18) are, taking each term in turn, as follows:

1-3)The effect upon welfare of:

the induced price changes of goods in J

the increase in q_{Kl}

changes in profits due to tax t_{Kl}

5)Own demand

6-7)Demand effects of increase in q_{Kl} on:

goods in K

goods in J.

8)Demand effects due to induced price changes for:

goods in K

goods in J.

Similarly, for (19)

1-3)The effect upon welfare of:

the induced price changes of goods in J

price changes of goods in K due to change in f_{Jj}'s

changes in profits caused by t_{Jl}

4)Own demand

5-6)Demand effects of increase in q_{Kl} upon:

goods in K

goods in J

7-8)Demand effects of changes in q_{Jj}'s upon:

goods in K

goods in J

The complexity of these equations is self-evident and their interpretation is somewhat difficult. However the major lesson to be learnt is clear: within the context of the discrimination model adopted here the behaviour of the prices of intermediate goods produced by imperfectly competitive firms is as important as that of final goods when assessing optimal taxes. As with the standard model the prices of intermediate goods produced by perfectly competitive industries do not appear in the equations. If welfare-improving tax changes were the object of interest the same points would apply. These factors indicate the relevance of a study of intermediate good taxation. I do not propose to extend these equations further but will now analyse tax rules when no discrimination is possible.

3. 4: Non-Discrimination.

If firms are unable to practice discrimination each industry will face a demand function X_j^T where

$$X_j^T = X_j^C(q_j, q, w) + X_j^F(q_j-t_j, q-t, t_j, t, w)$$

where q and t are now vectors of other prices and taxes. The total demand is

$$X_j^T = X_j^T(q_j, q, q_j-t_j, q-t, t_j, t, w)$$

which can be partially inverted to

$$q_j = q_j(X_j^T, q, t_j, q-t, t, w).$$

and used to derive comparative states.

The tax rules with this model may be derived from (18) and (19) by replacing the functions $g_{J1}(\ \cdot\ )$ and $e_{J1}(\ \cdot\ )$ with a single function, derived from the comparative statics sketched above, representing a single price q_{J1}'. With this new system for each good produced by an imperfectly competitive industry changes in the tax upon that good will still have effects through both the demand side and via the cost functions of firms that employ the good as an input. Consequently, although the details of the analysis differ from those for the discrimination case the tax rules will retain the same form.

Section 4: The Taxation of Intermediate Goods.

The analysis to this point has followed conventional practice in assuming that intermediate goods remain untaxed. However it is clear from the form of the tax rules derived above that this is unlikely to be optimal in the presence of imperfect competition, the non-linearity of responses to taxation indicating that there may exist the possibility of trading-off increases in final goods prices against greater reductions in the prices of intermediate goods. In the light of these arguments I now turn to the taxation of intermediate goods.

To bring out most clearly the factors at work I will first analyse welfare-improving and optimal taxes for two simple models each with one monopoly and a single competitive industry. In the first the monopolist's entire output is sold to the perfectly competitive industry whose output constitutes the model's final good. The roles are reversed in the second model. These models will demonstrate that the taxation of intermediate goods produced by both imperfectly and perfectly competitive industries will be optimal in a broad range of circumstances. Attention will be focused upon efficiency arguments by assuming the existence of a single consumer who consumes the final good, receives the monopoly's profits and supplies labour.

4.1: Monopoly Production of Intermediate Good.

Each firm in the competitive industry is assumed to have a fixed-coefficient production function, units of measurement are normalised so that each unit of output, y, requires one unit of labour and one unit of the monopolist's output, x. Writing the after-tax price of x as ξ_x, with the tax levied upon it i_x, the after-tax price of y is

$$q_y = w + \xi_x$$

so that

$$\partial q_y/\partial \xi_x = 1$$

The monopoly produces x employing labour alone and sells only to the competitive industry, with total cost given by

$$C(w; x) + xi_x$$

with price ξ_x chosen to maximise profits, where

$$\Pi = \xi_x . x - C(w; x) - xi_x$$

and x = y.

Writing the consumer's indirect utility function as

$$V = v(q_y, w, \Pi)$$

welfare improving tax changes are the solution to;

Find di_x, dt_y s.t. dV > 0, dR = 0.

Starting from a position with $i_x = t_y = 0$

$$dV = [\partial V/\partial q_y + \partial V/\partial \Pi . \partial \Pi/\partial q_y]dt_y + [\partial V/\partial q_y . \partial q_y/\partial \xi_x . \partial \xi_x/\partial i_x]di_x$$

$$+ [\partial V/\partial \Pi . \partial \Pi/\partial i_x + \partial V/\partial \Pi . \partial \Pi/\partial q_y . \partial q_y/\partial \xi_x . \partial \xi_x/\partial i_x]di_x$$

and

$$dR = 0 = xdi_x + ydt_y$$

but as x = y, $di_x = -dt_y$. Substituting into dV

$$dV = [\partial V/\partial \Pi.\partial \Pi/\partial i_x + \partial V/\partial \Pi.\partial \Pi/\partial q_y.\partial q_y/\partial \xi_x.\partial \xi_x/\partial i_x + \partial V/\partial q_y.\partial q_y/\partial \xi_x.\partial \xi_x/\partial i_x - \partial V/\partial q_y - \partial V/\partial \Pi.\partial \Pi/\partial q_y]di_x$$

Now $\partial q_y/\partial \xi_x = 1$ hence

$$dV = [\partial V/\partial \Pi.\partial \Pi/\partial i_x + \partial V/\partial \Pi.\partial \Pi/\partial q_y.\partial \xi_x/\partial i_x + \partial V/\partial q_y.\partial \xi_x/\partial i_x - \partial V/\partial q_y - \partial V/\partial \Pi.\partial \Pi/\partial q_y]di_x$$

or

$$dV = [\partial V/\partial \Pi.(\partial \Pi/\partial i_x + \partial \Pi/\partial q_y(\partial \xi_x/\partial i_x - 1)) + \partial V/\partial q_y.(\partial \xi_x/\partial i_x - 1)]di_x \quad -(20)$$

It is equation (20) that demonstrates why imperfect competition makes the taxation of intermediate goods desirable and underlines the differences between this model and that of perfect competition. If x were produced by a perfectly competitive industry the expression in brackets would be zero; the profit terms would not appear and $\partial \xi_x/\partial i_x$ would be identically 1. Hence no changes di_x could be welfare-improving for perfect competition. In contrast the expression cannot be signed unambiguously for imperfect competition, what can be stated is that it will only be zero for an exceptional combination of factors.

If overshifting occurs, $\partial \xi_x/\partial i_x > 1$, a sufficient condition for the subsidisation of the intermediate good is that both profit derivatives, $\partial \Pi/\partial i_x$ and $\partial \Pi/\partial q_y$, are negative, with overshifting taxation requires at least one of these to be positive. The intuitive argument lying behind this result is that overshifting implies that any subsidy will be magnified in the reduction in the price of the intermediate good, this reduction offsetting the increase in tax levied upon the final good leading finally to a lower final price. This mechanism will increase welfare unless the reduction in monopoly profits is sufficient to offset the lower price. Equation (20) also demonstrates how undershifting will point towards the taxation of the intermediate good.

The conclusions to be drawn from this model are that intermediate goods should not be exempt from taxation and, for this particular model, that welfare improving tax changes would in general, given the prevalence of overshifting in the simulations of Myles (1987b), move towards the subsidisation of intermediate goods produced by imperfectly competitive industries. However it is by no means clear whether the latter conclusion will extend to more general models, although the intuitive explanation indicates that it might, but the former certainly will.

The above discussion of welfare improving tax changes has demonstrated that the Diamond-Mirrlees theorem will not extend to models of imperfect competition and has also illustrated the major factors that will influence the rates of tax placed upon intermediate goods. I now extend the analysis to optimal taxation considering first the tax rules for the simple model above.

Assuming, as above, that the government budget should be balanced the optimal tax problem may be written

$$\max_{ix,ty} v(q_y, w, H) \text{ s.t. } 0 = i_x x + t_y y$$

The necessary conditions are

$$\partial V/\partial q_y + \partial V/\partial \Pi . \partial H/\partial q_y + \lambda(i_x \partial y/\partial q_y + y + t_y \partial y/\partial q_y) = 0$$

and

$$\partial V/\partial q_y . \partial \xi_x/\partial i_x + \partial V/\partial \Pi . \partial H/\partial i_x + \partial V/\partial \Pi . \partial H/\partial q_y . \partial \xi_x/\partial i_x$$

$$- \lambda(y + i_x \partial y/\partial q_y . \partial \xi_x/\partial i_x + t_y \partial y/\partial q_y . \partial \xi_x/\partial i_x) = 0$$

where I have used $\partial q_y/\partial \xi_x = 1$ and $\partial x/\partial q_y = \partial y/\partial q_y$. Eliminating λ between these equations and using the budget constraint to simplify the expression the optimal tax scheme is characterised by the solution to the equation

$$[\partial V/\partial q_y . \partial \xi_x/\partial i_x + \partial V/\partial \Pi . \partial \Pi/\partial i_x + \partial V/\partial \Pi . \partial \Pi/\partial q_y . \partial \xi_x/\partial i_x] = [\partial V/\partial q_y] \quad -(21)$$

For perfect competition (21) will be satisfied definitionally, so that any pair of tax rates with $t_y = -i_x$ will be a solution. This reflects the structure of the model: any budget balance tax scheme will result in an identical final good price. With imperfect competition this will not be the case, except for special circumstances, and the previous analysis has already discussed the factors that determine the direction of the outcome.

<u>4.2: Competitive Factor Production.</u>

It has now been established that intermediate goods produced by imperfectly competitive firms should fall within the scope of the tax system, by reversing the roles of the two industries I shall now establish that the same is true for intermediate goods produced by competitive industries.

The reversed model now has intermediate goods produced by a competitive industry and I will assume constant coefficient technology so that

$$\xi_y = w + i_y$$

Similarly, the monopoly produces with costs given by

$$C(\xi_y, w; x) + xt_x$$

and, finally, the single consumer has indirect utility function

$$V = v(q_x, w, \Pi)$$

For variations dt_x and di_y the change in welfare is

$$dV = [\partial V/\partial q_x . \partial q_x/\partial t_x + \partial V/\partial \Pi . \partial \Pi/\partial t_x]dt_x$$

$$+ [\partial V/\partial q_x . \partial q_x/\partial g_y . \partial \xi_y/\partial i_y + \partial V/\partial \Pi . \partial \Pi/\partial \xi_y . \partial \xi_y/\partial i_y$$

$$+ \partial V/\partial \Pi . \partial \Pi/\partial q_x . \partial q_x/\partial \xi_y . \partial \xi_y/\partial i_y]di_y$$

and from the budget constraint

$$dt_x = -(y/x)di_y$$

Using this and that $\partial \xi_y/\partial i_y = 1$

$$dV = [-(y/x)(\partial V/\partial q_x . \partial q_x/\partial t_x + \partial V/\partial \Pi . \partial \Pi/\partial t_x) + \partial V/\partial q_x . \partial q_x/\partial \xi_y$$

$$+ \partial V/\partial \Pi . \partial \Pi/\partial \xi_y + \partial V/\partial \Pi . \partial \Pi/\partial q_x . \partial q_x/\partial \xi_y]di_y$$

To simplify the argument assume that $y = x$, hence

$$dV = [\partial V/\partial q_x(\partial q_x/\partial \xi_y - \partial q_x/\partial t_x)$$

$$+ \partial V/\partial \Pi(\partial \Pi/\partial \xi_y + \partial \Pi/\partial q_x . \partial q_x/\partial \xi_y - \partial \Pi/\partial t_x)]di_y \quad (22)$$

Equation (22) characterises the direction that the welfare improving policy should take with regard to the tax treatment of an intermediate good produced by a perfectly competitive industry. It is most important to note that the term in square brackets will only be zero for an unusual combination of circumstances, and it is only in these circumstances that there will be no welfare improving changes. Hence

(22) is sufficient to dispel any notion that perfectly-competitively produced intermediate goods should remain exempt from taxation.

The features of the model that are behind this conclusion are once more the non-linear responses to taxation, this is clear from (22) by setting profit effects equal to zero and setting tax effects linear to give

$$dV = -\,\partial V/\partial q_x . di_y$$

and

$$dV = \partial V/\partial q_x . dt_x$$

so that no balanced budget changes will increase welfare. As would be expected values of $\partial q_x/\partial \xi_y > 1$ work in favour of subsidising y, in these cases the subsidy will be overreflected in its effect upon final price.

4.3: Tax Rules.

Following these developments it is now possible to discuss a complete system of optimal taxes for the model of section three. The analysis above has demonstrated that all intermediate goods, regardless of the structure of the industry producing them, should be brought into the tax system. If the model has N goods the tax system will be described by 2N first-order conditions plus a budget constraint. There will also be four typical first-order conditions: two for the choice of taxes on final goods and two for those on intermediate goods.

The maximisation problem becomes:

Choose $t_{K1},\ldots,t_{KK},t_{J1},\ldots,t_{JJ},i_{K1},\ldots,i_{KK},i_{J1},\ldots,i_{JJ}$ to maximise

$$L = \sum_{h \in H} V^h(q_{K1}, \ldots, q_{KK}, q_{J1}, \ldots, q_{JJ}, w, \Pi^h)$$
$$+ \lambda \Big[\sum_{k \in K} t_{Kk} X^C_{Kk} + \sum_{j \in J} t_{Jj} X^C_{Jj}$$
$$+ \sum_{k \in K} i_{Kk} X^F_{Kk} + \sum_{j \in J} i_{Jj} X^F_{Jj} - R \Big]$$

It is possible to derive and present four typical first order conditions for this maximisation, however their length and complexity is such that a discussion of their form is preferable to a presentation.

Starting with the model of section three if discrimination is assumed possible the first step towards incorporating taxes on intermediate goods is to replace f_{J1} and f_{Kk} with ξ_{J1} and ξ_{Kk} respectively and extending the functions $g_{J1}(\cdot)$, $e_{J1}(\cdot)$ and $\Pi_{Jji}(\cdot)$ to incorporate the intermediate good tax i_{J1}. Prices of goods produced by competitive firms should also include the tax i_{K1}. Having made these changes the analysis will proceed to generate four equations in the manner of (16) and (17) above, each taking into account changes in intermediate prices in addition to final prices.

Section 5: Summary and Conclusions.

This paper has been devoted to extending optimal tax formulae to imperfectly competitive models. This extension has required several existing strands of the literature to be brought together and some further extensions to be made. In defining optimal tax rules imperfect competition implies that firms will condition their pricing policies upon the tax rates that they face directly and upon the prices of other goods via the demand decisions of consumers and other firms. Consequently when a tax rate is adjusted its repercussions for other prices must be traced through the economic system. Under the alternative sets of conditions specified above these are captured by the adjustment equations derived.

Similarly the direct effect of tax changes can only be evaluated by working through the individual firms' maximisation problems; even with a fixed cost structure the helpful linearity of perfect competition is lost. Furthermore, once it is possible for firms to earn positive profits account must also be taken of how these are affected by taxes and of the recipients' responses to variations in their profit income.

In section two I presented as examples the derivations of the direct and induced effects for oligopoly without entry. Some of these results have already been noted in the literature, most notably in Seade (1985), but their presentation here is somewhat original. To complement the results for fixed-number oligopoly corresponding derivations were performed when entry was unrestricted.

Having derived the relevant results optimal tax rules were formulated taking into account the complexities of imperfect competition. Essentially the equations presented here are straightforward extensions of existing formulae for competitive models and indeed collapse to these when induced effects are set to zero and direct effects are assumed linear.

As the general equations are rather uninformative two approaches were tried to obtain some insight into their structure. Restricting the number of goods, in addition to labour, to two and assuming profits accrued to an actor outside the model it was possible to find conditions for which a balanced budget policy would lead to subsidisation of the good produced by the imperfectly competitive firms. The second approach involved the use of the Slutsky equation to render the equations comparable to those usually presented for the competitive model. From these it could be seen that, rather than considering only the reduction in demand for each good as a result of the tax system, account had also to be taken of the induced demand effects upon other goods.

Section three extended the analysis to more general production technologies but retained the assumption of zero taxation for intermediate goods. However, even with this restriction, the effects that taxation has upon intermediate goods prices become an integral feature of optimal tax rules with imperfect competition.

If zero taxation of intermediate goods is assumed the equations show it is still correct that the prices of intermediate goods produced by perfectly competitive industries may be excluded from consideration. However those from imperfectly competitive industries cannot. The reason for this distinction is clear from the comparative statics exercises and is also closely connected with the 100% shifting of competitive firms. In essence, with all other prices given a competitive firm's pre-tax price is also its intermediate goods price. This remains unaffected by taxation, to the first-order at least. However, with imperfect competition this neat distinction breaks down. Even when no discrimination is possible starting from a no-tax position and considering the imposition of a tax, only in exceptional circumstances will the post-tax price differ from the pre-tax rate by the value of the tax. In almost all circumstances the post-tax price less tax will not be equal to the no-tax price.

When discrimination is possible the mechanism at work is considerably more transparent and any claim that intermediate goods prices will not be affected by taxation even more tenuous.

All of these results point towards the conclusion that tax should be levied upon all forms of output and, except for exceptional combinations of circumstances, this was demonstrated in section four. The basis of this demonstration was again the non-linear responses of imperfectly competitive firms and the fact that tax policy could exploit these to obtain reductions in the price level for final goods which would be welfare-improving provided they were not offset by reductions in profit income.

In conclusion, the Diamond-Mirrlees theorem cannot be extended to allow imperfect competition even if intermediate goods are produced only by competitive industries. This implies that the tax treatment of intermediate goods should be considered an integral part of optimal tax theory.

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